Quasinormal modes and stability criterion of dilatonic black holes in 1 + 1 and 4 + 1 dimensions

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We study the stability of black holes that are solutions of the dilaton gravity derived from string-theoretical models in two and five dimensions under scalar field perturbations, using the Quasinormal Modes (QNMs) approach. In order to find the QNMs corresponding to a black hole geometry, we consider perturbations described by a massive scalar field nonminimally coupled to gravity. We find that the QNMs frequencies turn out to be pure imaginary leading to purely damped modes, in the range $0 < \zeta < 1/4$ of nonminimal coupling constant (ζ), and the QNMs acquires a real part if $\zeta > 1/4$ that is in agreement with the literature of dilatonic black holes. Our result exhibits the unstable behavior of the considered geometry against scalar perturbations. We study the instability for different values of nonminimal coupling constant. We extend our results to the $4 + 1$ dimensional dilatonic black hole, where the metric is the product of a two-dimensional asymptotically flat geometry and a three-sphere with constant radius, which are completely decoupled from each other. The exact solution for the QNMs was obtained in the five-dimensional case.

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I. INTRODUCTION

Two-dimensional theories of gravity have recently attracted much attention [1–3] as simple toy models that possess many features of gravities in higher dimensions. They also have black hole solutions which play an important role in revealing various aspects of spacetime geometry and quantization of gravity, and are also related to string theory [4,5].

On the other hand, there is also a growing interest in five-dimensional dilatonic black holes in the last few years, since it is believed that these black holes can shed some light to the solution of the fundamental problem of the microscopic origin of the Bekenstein-Hawking entropy. The area-entropy relation $S_{\rm BH} = A/4$ was obtained for a class of five-dimensional extremal black holes in Type II string theory using D-brane techniques [6], while in Ref. [4] the U-duality that exists between the five-dimensional black hole and the two-dimensional charged black hole was exploited [5] to microscopically compute the entropy of the latter. For that reason, it is important to understand the dynamics of matter fields and the metric perturbations in such black hole backgrounds in order to find stable solutions. One of the key issues worth studying is so-called quasinormal modes (QNMs), known as the "ringing" of black holes, that play an essential role in the analysis of classical aspects of black holes physics.

In this work we are interested in the stability of the $1 + 1$-dilatonic black hole using the QNMs' approach. Quasinormal modes associated with perturbations of different fields were considered in different works [7], and for AdS and dS space [8–16]. A similar situation occurs in $2 + 1$ dimension [17–19], and the acoustic black holes [20–22]. Quasinormal modes of dilatonic black holes in $3 + 1$ dimensions can be seen in Refs. [23–25].

Determination of QNMs for a specific geometry implies solving the field equations for different types of perturbations (scalar, fermionic, vectorial, etc.), with suitable boundary conditions that reflect the fact that this geometry describes a black hole. Quasinormal modes for a scalar classical perturbation of black holes are defined as the solutions of the Klein-Gordon equation characterized by purely ingoing waves at the horizon, $\Phi \sim e^{-i\omega(t+r)}$, since at least a classically outgoing flux is not allowed at the horizon. In addition, one has to impose boundary conditions on the solutions in the asymptotic region (infinity), and for that it is crucial to use the asymptotic geometry of the spacetime under study. In the case of an asymptotically flat spacetime, the condition we need to impose over the wave function is to have purely outgoing waves $\Phi \sim e^{-i\omega(t-r)}$ at the infinity [8]. In general, the QNMs are given by $\omega_{\rm QNM} = \omega_R + i\omega_I$, where ω_R and ω_I are the real and imaginary parts of the frequency $\omega_{\rm QNM}$, respectively. Therefore, the study of QNMs can be implemented as one simple test for studying the stability of the system. In this sense, any imaginary frequency with the wrong sign would mean an exponentially growing mode, rather than a damping of it.

In this work we analytically compute the QNMs of $1 + 1$-dilatonic black hole, in order to test stability of the system. The organization of this article is as follows: In Sec. II we specify the $1 + 1$-dilatonic black hole. In Sec. III we determine the QNMs and we establish a criterion for the stability of the system. In Sec. IV we study the problem of QNMs for the five-dimensional dilatonic black hole. Finally, we finish with the conclusions in Sec. V.

II. 1 + 1-DILATONIC BLACK HOLE

In order to have a gravity theory with dynamical degrees of freedom in two-dimensional spacetime, we consider the gravity coupled to a dilatonic field described by the action

$$S_g = \frac{1}{2\pi}\int d^2x\sqrt{-g}e^{-2\phi}(R + 4(\nabla\phi)^2 + 4\lambda^2). \quad (1)$$

It is worthwhile noticing that the two-dimensional critical string theory [26] has been an inspiration of many articles, since it is a simple toy model possessing black hole solutions which can be a starting point to solve the problems of Hawking radiation and the information loss inside black holes [27–30].

It was also proved some time ago, that the dilatonic black hole is a solution of an exact conformal field theory, namely, the WZW model with gauge group $SL(2, R)/U(1)$. This solution can be derived by solving the two-dimensional beta function equations of the string theory, that is effectively a two-dimensional graviton-dilaton system. The equations of motion for the graviton and dilaton are given by

$$\beta^G_{\mu\nu} = R_{\mu\nu} + 2\nabla\mu\nabla\nu\phi = 0, \quad (2)$$

$$\beta^\phi = \Box\phi - 2(\nabla\phi)^2 + 2\lambda^2 = 0. \quad (3)$$

A general static metric describing a black hole in this theory can be written as

$$ds^2 = -f(r)d\tau^2 + \frac{dr^2}{f(r)}, \quad (4)$$

where $f(r) = 1 - e^{-\phi}$ and $\phi = (r - r_0)/r_0$. If we change the coordinate as $x = \frac{r-r_0}{r_0}$, then the function $f(r(x)) = f(x)$ becomes $f(x) = 1 - e^{-x}$ and the horizon of the black hole is located at $x = 0$. This solution represents a well-known string-theoretic black hole [4,5,26,31].

III. QUASINORMAL MODES

In order to study the QNMs, we consider a scalar field with nonminimal coupling to gravity, propagating in the background of the dilatonic black hole. This system is described by the action [31]

$$S[\varphi] = -\frac{1}{2}\int d^2x\sqrt{-g}((\nabla\varphi)^2 + (m^2 + \zeta R)\varphi^2), \quad (5)$$

where ζ is a parameter from the nonminimal coupling. The field equations reads

$$(\Box - \mu^2 - \zeta R)\varphi = 0, \quad (6)$$

where $\mu = r_0 m$. In terms of the coordinate x and assuming a solution in the form $\varphi = e^{-i\omega t}R(x)$, the radial Eq. (6) can be written as

$$f\partial_x^2 R(x) + e^{-x}\partial_x R(x) - \left(\frac{\omega^2}{f} - \mu^2 - \zeta e^{-x}\right)R(x) = 0. \quad (7)$$

Next, we define a new variable, $z = 1 - e^{-x}$, so that the radial equation adopts the form

$$z(1-z)\partial_z(z(1-z)\partial_z R(z)) + (\omega^2 - z\mu^2 - \zeta' z(1-z))R(z) = 0, \quad (8)$$

where $\zeta' = \zeta/r_0^2$ is a new parameter. With the change $R(z) = z^\alpha(1-z)^\beta F(z)$, the last equation reduces to the hypergeometric differential equation for the function $F(z)$, that is,

$$z(1-z)F''(z) + (c - (a + b + 1)z)F'(z) - abF(z) = 0. \quad (9)$$

Here, the coefficients a, b and c are given through the relations

$$c = 2\alpha + 1, \qquad a + b = 2(\alpha + \beta) + 1, \qquad ab = (\alpha + \beta)(\alpha + \beta + 1) - \zeta', \quad (10)$$

from where we obtain the expressions for the coefficients,

$$a = \tfrac{1}{2}(1 + 2\alpha + 2\beta - \sqrt{1 - 4\zeta'}), \quad (11)$$

$$b = \tfrac{1}{2}(1 + 2\alpha + 2\beta + \sqrt{1 - 4\zeta'}), \quad (12)$$

and for the exponents α and β,

$$\alpha = \pm i\omega, \quad (13)$$

$$\beta = \pm\sqrt{\omega^2 - \mu^2}. \quad (14)$$

Without loss of generality, above we choose the negative signs. It is well-known that the hypergeometric equation has three regular singular points, at $z = 0$, $z = 1$ and $z = \infty$, and it has two independent solutions in the neighborhood of each point [32]. The solutions of the radial equation reads as follows:

$$F(z) = C_1 F_1(a, b, c; z) + C_2 z^{1-c} F_1(a - c + 1, b - c + 1, 2 - c; z), \quad (15)$$

where $F_1(a, b, c; z)$ is the hypergeometric function and C_1, C_2 are constants. The solution for $R(z)$ is then

$$R(z) = C_1 z^{-i\omega}(1-z)^{-i\sqrt{\omega^2-\mu^2}} F_1(a, b, c; z) + C_2 z^{i\omega}(1-z)^{-i\sqrt{\omega^2-\mu^2}} F_1 \times (a - c + 1, b - c + 1, 2 - c; z). \quad (16)$$

Note that, when $c = 1$, two solutions become linearly dependent and the general solution represents a bound state. This point was discussed in Ref. [31].

In the neighborhood of the horizon ($z = 0$), the function $R(z)$ behaves as

$$R(z) = C_1 e^{-i\omega \ln z} + C_2 e^{i\omega \ln z}, \tag{17}$$

and the scalar field φ can be written in the following way:

$$\varphi \sim C_1 e^{-i\omega(t+\ln z)} + C_2 e^{-i\omega(t-\ln z)}. \tag{18}$$

From the above expression it is easy to see that the first term corresponds to an ingoing wave, while the second one represents an outgoing wave in the black hole. For computing the QNMs, we have to impose that there exist only ingoing waves at the horizon so that, in order to satisfy this condition, we set $C_2 = 0$. Then the radial solution at the horizon is given by

$$R(z) = C_1 z^{-i\omega}(1-z)^{-i\sqrt{\omega^2-\mu^2}} F_1(a, b, c; z). \tag{19}$$

In order to implement the boundary conditions at the infinity ($z = 1$), we use the linear transformation $z \to 1 - z$, and then we apply the Kummer's formula [32] for the hypergeometric function. We obtain

$$R(z) = C_1 z^{-i\omega}(1-z)^{-i\sqrt{\omega^2-\mu^2}} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} F_1(a, b, a+b-c+1; 1-z) + C_1 z^{-i\omega}(1-z)^{i\sqrt{\omega^2-\mu^2}} \times \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} F_1(c-a, c-b, c-a-b+1; 1-z). \tag{20}$$

The above solution near the infinity ($z = 1$) takes on the form

$$R(z) = C_1 (1-z)^{-i\sqrt{\omega^2-\mu^2}} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + C_1 (1-z)^{i\sqrt{\omega^2-\mu^2}} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}, \tag{21}$$

while the solution for the scalar field near the infinity behaves as

$$\varphi \sim C_1 e^{-i\sqrt{\omega^2-\mu^2}(t+\ln(1-z))} \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} + C_1 e^{-i\sqrt{\omega^2-\mu^2}(t-\ln(1-z))} \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}. \tag{22}$$

In order to compute the QNMs, we also need to impose the boundary conditions on the solution of the radial equation at infinity, meaning that only purely outgoing waves are allowed there. Therefore, the second term in the above expression should be zero, what is fulfilled only at the poles of $\Gamma(a)$ or $\Gamma(b)$. Since the gamma function $\Gamma(x)$ has the poles at $x = -n$ for $n = 0, 1, 2, \ldots$, the wave function satisfies the considered boundary condition only upon the following additional restriction:

$$a = -n, \tag{23}$$

or

$$b = -n, \tag{24}$$

where $n = 0, 1, 2, \ldots$. These conditions determine the form of the quasinormal modes, that is, from Eqs. (12) and (33), we find

$$\omega = -\frac{i}{4}\left(1 - \sqrt{1-4\zeta'} - \frac{(1+\sqrt{1-4\zeta'})\mu^2}{n+n^2+\zeta'} + n\left(2 - \frac{2\mu^2}{n+n^2+\zeta'}\right)\right). \tag{25}$$

The expression (25) for frequencies shows a possible instability of the black hole under scalar perturbations, which could imply an exponentially growing mode if the wrong sign of the pure imaginary frequency had been chosen (positive). This issue is clarified in Figs. (1 and 2. Figure 1 shows the instability arising in the fundamental mode for scalar perturbations that excite this mode, in the range $0 \leq \zeta' \leq 1/4$. Note that in this range of the nonminimal coupling parameter the quasinormal modes are purely imaginary, as in the 2 + 1-dilatonic case [33]. The plot in the figure corresponds to a mass $\mu = 1$. If we consider an arbitrary mass for the scalar field, the instability is also present, and depends on the values of μ with respect to n. In order to explicitly show this fact, we rewrite Eq. (25) in the following form:

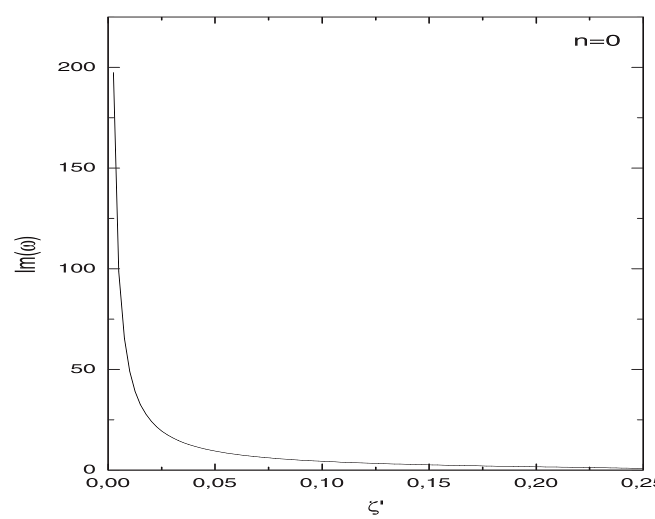

FIG. 1. The imaginary part of the QNM's frequency of the fundamental mode as a function of the nonminimal coupling parameter. This plot shows an unstable behavior of a scalar perturbation that excites the fundamental mode. We have taken $\mu = 1$.

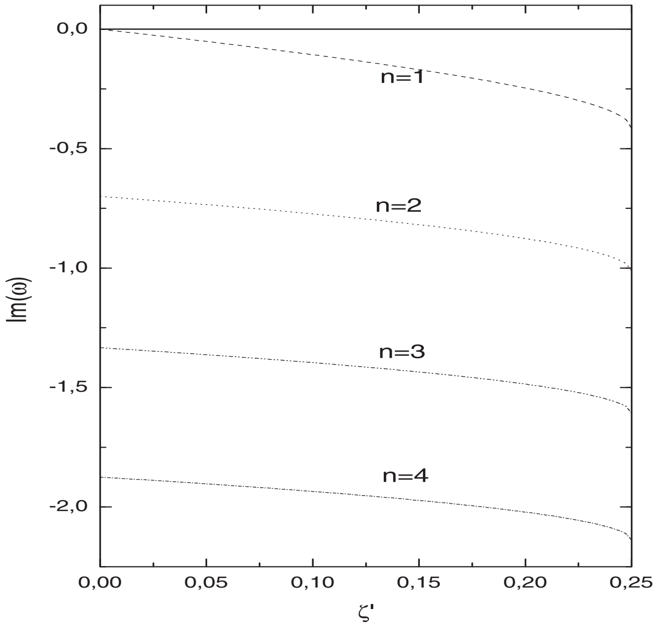

FIG. 2. The imaginary part of the QNM's frequency as a function of the nonminimal coupling parameter, for several overtones. This plot shows a stable behavior of scalar perturbations for all overtones. We have taken $\mu = 1$.

$$\omega_{2D} = -i\frac{1}{4(n+n^2+\zeta')}\Delta(n, \zeta', \mu^2), \tag{26}$$

where

$$\Delta(n, \zeta', \mu^2) \equiv (1 + 2n - \sqrt{1-4\zeta'})(n+n^2+\zeta') - (1+2n+\sqrt{1-4\zeta'})\mu^2. \tag{27}$$

Clearly, a possible instability depends on the sign of $\Delta(n, \zeta')$. First, we examine the behavior of Δ of the fundamental mode for different values of coupling constant ζ'. In the minimal coupling case ($\zeta' = 0$)

$$\Delta(0, 0, \mu^2) = -2\mu^2, \tag{28}$$

and this expression shows that the instability arises in the fundamental mode (see Fig. 1). For the conformal coupling case ($\zeta' = 1/4$)

$$\Delta(0, 1/4, \mu^2) = 1/4 - \mu^2, \tag{29}$$

and if $\mu > 1/2$ we have instability. In the range $0 < \zeta' < 1/4$, the following relation is satisfied $1 + \sqrt{1-4\zeta'} > 1$ so that $\text{sgn}(\Delta(n, \zeta'))$ is fully dependent of μ^2. For example if $\mu^2 = 1$, then $\text{sgn}(\Delta(n, \zeta')) = -1$ and we have instability, a similar behavior occurs if $\mu^2 > 1$. When $\mu^2 < 1$ we check if $\Delta(0, \zeta', \mu^2) \gtrless 0$ then $(\zeta' + \mu^2) \gtrless \mu^2$, and it is straightforward to check that $\Delta < 0$, that means instability. In the same line, for the overtones we see that the overtones $n < \mu$ guarantees the instability under scalar perturbations in the minimal case. A similar situation occurs in the conformal case, where

$$\omega = -i\frac{((1+2n)^2 - 4\mu^2)}{4+8n}, \tag{30}$$

if $n < \mu - 1/2$. This strong instability is due to the fact that n is a mode number and when we perturb the black hole, all the modes are excited, and not only those with $n > \mu - 1/2$. In summary, the two-dimensional dilatonic black hole shows an unstable behavior against scalar perturbations. This results was shown in Ref. [34] where the instability of 1 + 1 dilatonic black holes has been shown using metric perturbations. In the range of parameters $\zeta' > 1/4$, the frequency of QNMs acquires a real part,

$$\omega = -\frac{\sqrt{4\zeta'-1}}{4}\left(1 + \frac{\mu^2}{n+n^2+\zeta'}\right) - \frac{i}{4}\left(1 - \frac{\mu^2}{n+n^2+\zeta'} + n\left(2 - \frac{2\mu^2}{n+n^2+\zeta'}\right)\right). \tag{31}$$

Figure 3 shows the behavior of both the real and imaginary parts of QNMs. In this range, we observe that the black hole is stable for all QNMs for $\zeta' > 1$. When $\mu \neq 1$, for

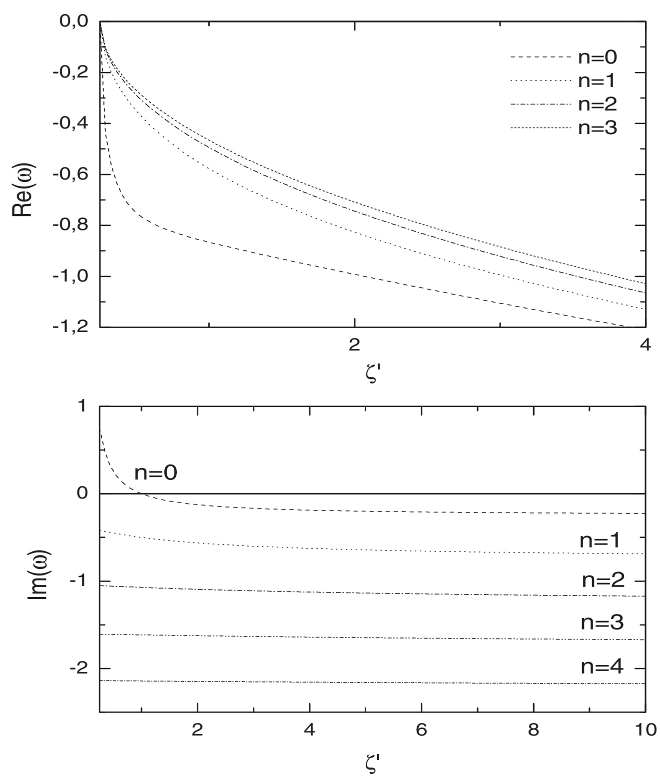

FIG. 3. The upper panel shows the real part of the QNM's frequency as a function of the nonminimal coupling parameter for several overtones, in the case of the two-dimensional black hole. Note that, for a high nonminimal parameter, the real part coalesce. The lower panel shows the imaginary part of the QNM's frequency as a function of the nonminimal coupling parameter, for several overtones. It demonstrates a stable behavior of scalar perturbations for all overtones with $\zeta' > 1$. We have taken $\mu = 1$.

the fundamental mode the instability arises for $\zeta' < \mu^2$, and for the overtones ($n \neq 0$) the instability is arise for $n(n+1) < \mu^2 - \zeta'$.

Finally, note that the real part of the QNMs, in the limit of highly damped modes (i.e., QNMs with a large imaginary part), tends to a constant, that is in agreement with Refs. [35,36]. This result satisfies the Hod's conjecture [37].

IV. DILATONIC BLACK HOLE IN FIVE DIMENSIONS

There is a growing interest in five-dimensional dilatonic black holes in last years, since it is believed that these black holes could shed some light on the fundamental problem of the microscopic origin of the Bekenstein-Hawking entropy. The area-entropy relation $S_{\rm BH} = A/4$ was obtained for a class of five-dimensional extremal black holes in Type II string theory, using D-brane techniques [6]. Also, in Ref. [4], the U-duality that exists between the five-dimensional black hole and the two-dimensional charged black hole [5] was used to microscopically compute the entropy of the latter.

The metric of the five-dimensional dilatonic black hole can be written as [4]

$$ds^2 = -\left(1 - \frac{r_0^2}{r^2}\right)\left(1 + \frac{r_0^2 \sinh^2\alpha}{r^2}\right)^{-2} dt^2 + \left(\frac{r^2}{r_0^2} - 1\right)^{-1} dr^2 + r_0^2 d{\Omega_3}^2. \tag{32}$$

This metric is the product of the two completely decoupled parts, namely an asymptotically flat two-dimensional geometry which describes a two-dimensional charged dilatonic black hole and a three-sphere with constant radius. This statement can be directly shown if we apply in the (t, r) sector the transformation defined by

$$e^{(2/r_0)x} = 2\left(\tfrac{r^2}{r_0^2} + \sinh^2\alpha\right)(m^2 - q^2)^{1/2}, \tag{33}$$

where m and q are related to the mass and charge of the dilatonic black hole [5]. Then Eq. (32) reads as follows:

$$ds^2 = -N^2 dt^2 + N^{-2} dx^2 + r_0^2 d{\Omega_3}^2, \tag{34}$$

with

$$N^2 = 1 - 2me^{-Qx} + q^2 e^{-2Qx}. \tag{35}$$

Now we consider the uncharged dilatonic black hole metric, with $q = 0$,

$$ds^2 = -(1 - 2me^{-Qx})dt^2 + \frac{dx^2}{1 - 2me^{-Qx}}, \tag{36}$$

as the two-dimensional sector of five-dimensional dilatonic black hole that we are interested in computing for the QNMs. To complete this issue we need to solve the equation of motion associated to the action

$$S[\varphi] = -\frac{1}{2}\int d^5x\sqrt{-g}((\nabla\varphi)^2 + (m^2 + \zeta R)\varphi^2), \tag{37}$$

where ζ is a parameter from nonminimal coupling. The field equation reads as follows:

$$(\Box - \mu^2 - \zeta' R + \nabla^2_{(S^3)})\varphi = 0, \tag{38}$$

where $\mu = r_0 m$, $\zeta' = \frac{\zeta}{r_0^2}$ and $\nabla^2_{(S^3)}$ is the Laplace-Beltrami operator in the S^3 sphere. We adopt the following ansatz,

$$\varphi \sim \Phi(t, x) Y(\chi, \theta, \phi), \tag{39}$$

where Y is a normalizable harmonic function on S^3, i.e., it satisfies the equation $\nabla^2_{(S^3)} Y = \alpha Y$, that in terms of the coordinates in S^3 can be written as

$$ccs^2\chi\left(\frac{\partial}{\partial\chi}\left(\sin^2\chi\frac{\partial Y}{\partial\chi}\right) + ccs^2\theta\left(\frac{\partial}{\partial\theta}\left(\sin^2\theta\frac{\partial Y}{\partial\theta}\right)\right) + ccs\theta\frac{\partial^2 Y}{\partial\phi^2}\right) = \alpha Y^{(nlm)}, \tag{40}$$

and its solutions are given by

$$Y^{(nlm)}(\chi, \theta, \phi) = \left(\frac{2^{2l+1}(n+1)(n-l)!l!^2}{\pi(n+l+1)!}\right) \times \sin^l\chi C_{n-l}^{(l+1)}(\cos\chi) Y^{(lm)}(\theta, \phi). \tag{41}$$

Here, $C_{n-l}^{(l+1)}(\cos\chi)$ are the Gegenbauer polynomials [32,38], $Y^{(lm)}(\theta, \phi)$ are the S^2 scalar harmonics, and the coefficient is chosen to normalize the harmonics. The eigenvalues are

$$\alpha = -n(n+2), \qquad |m| \leq l \leq n = 0, 1, 2, \ldots. \tag{42}$$

Therefore, in this Ansatz, we can write Eq. (38) in the following form:

$$(\Box - \mu^2 - \zeta' R + n(n+2))\Phi(t, x) = 0, \tag{43}$$

that is identical to Eq. (6) where the term $n(n+2)$ is an additive constant. If we repeat the analysis made in the previous section, we find that the frequencies of the QNMs are given by

$$\omega_{5D} = -\frac{i}{4}\left(1 - \sqrt{1 - 4\zeta'} - \frac{(1 + \sqrt{1 - 4\zeta'})\mu^2 - n(n+2)}{n' + n'^2 + \zeta'} + n'\left(2 - \frac{2\mu^2 - 2n(n+2)}{n' + n'^2 + \zeta'}\right)\right), \tag{44}$$

with n (angular number) and n' (modes number) are integer numbers. Analogously to the $1+1$ case, we rewrite the Eq. (44) as follows:

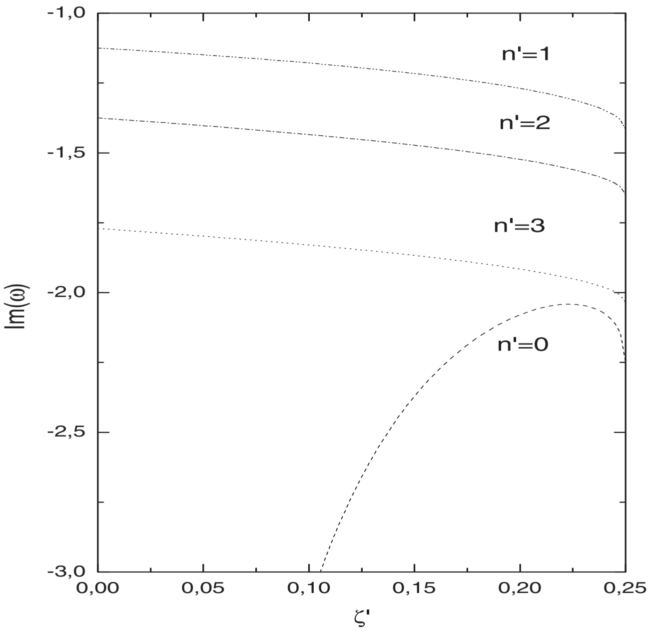

FIG. 4. The imaginary part of the QNM's frequency as a function of the nonminimal coupling parameter is illustrated for several overtones. This plot shows the stable behavior of scalar perturbations for all overtones of the five-dimensional dilatonic black hole.

$$\omega_{5D} = \omega_{2D} - i\frac{1}{4(n' + n'^2 + \zeta')}(1 + 2n')n(n + 2). \tag{45}$$

The last expression shows a similar behavior to the two-dimensional black hole in the range $0 \leq \zeta' \leq 1/4$, when $n = 0$ (s-waves). If $n \neq 0$, the situation is completely different due to the inclusion of the transverse part that ensures the stability of the five-dimensional black hole over all QNMs. This result is shown in Fig. 4 for $n = 1$ and $\mu = 1$.

In the range $\zeta' > 1/4$, a behavior similar to the one of two-dimensional case is obtained; that is, the QNMs acquire the same real and imaginary parts, and the inclusion of the transverse term ensures the stability in this case as well. Note that, in the limit of high damping, the real part tends to the same constant as in the two-dimensional case.

V. FINAL REMARKS

In this paper we computed the exact values of the quasinormal modes of dilatonic black holes in 1 + 1 and 4 + 1 dimensions and we have shown that the QNMs are purely imaginary (this kind of QNM was also reported in Refs. [22,33,39–42]) in the range $0 \leq \zeta' \leq 1/4$ for the nonminimal coupling parameter. For values of this parameter in the range $\zeta' > 1/4$, we found that the QNMs acquire real parts in both two- and five-dimensional cases, and in the limit of higher damping they tend to have the same constant. This result is in agreement with the Hod's conjecture [37] and it also matches with the results obtained in Ref. [35] using the WKB approximation, and in Ref. [36] where the monodromy approach was adopted. Because the considered kind of black hole does not exhibit a real part of the frequencies of QNMs (analogous to the electromagnetic perturbations for a Schwarzschild black hole, where the real part disappears in the limit of higher damping modes) in the range $0 < \zeta' < 1/4$, and as the asymptotic values of the Re(QNMs) must reproduce the black hole entropy, Hod's proposal could imply vanished entropy. A possible verification of the Hod's proposal depends on the values of the nonminimal coupling parameter. In the context of dilatonic black hole this fact does not apply in the range $0 < \zeta' < 1/4$ because the real part of QNMs vanished. This point is still an open problem and we expect to discus it elsewhere. This shows that the Hod's conjecture is not clearly established at present, and is only fully applicable for a single horizon black hole obtained in pure Einstein gravity theory.

Additionally, we found that the fundamental modes of QNMs are unstable for all values of nonminimal coupling parameter in the range $0 < \zeta' < 1/4$. This implies the instability of dilatonic black holes in 1 + 1 dimensions against a scalar perturbation. For the range $\zeta' > 1/4$ a similar behavior occurs, and we observe that the black hole is stable for all QNMs for $\zeta' > 1$. In the case of $\mu \neq 1$ for the fundamental modes the instability arises for $\zeta' < \mu^2$, and for the overtones ($n \neq 0$) the instability arises for $n(n + 1) < \mu^2 - \zeta'$.

Finally, we would like to emphasize that this result can also be applied to compute the QNMs in the five-dimensional case [4,5], where the metric is the product of a two-dimensional asymptotically flat geometry and a three-sphere with constant radius, which are completely decoupled from each other.

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