Radiation can never again dominate matter in a vacuum dominated universe

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We demonstrate that in a vacuum-energy-dominated expansion phase, surprisingly neither the decay of matter nor matter-antimatter annihilation into relativistic particles can ever cause radiation to once again dominate over matter in the future history of the Universe.

The study of decaying particles in a cosmological context has a long history [1–10]. Although there are many variations on this theme, there is one constant result: if the Universe is initially dominated by nonrelativistic particles, and this matter undergoes a standard exponential decay into relativistic particles, then the Universe rapidly transitions from a matter-dominated phase into a radiation-dominated phase when the age of the Universe is roughly equal to the particle lifetime.

In this paper we show, rather suprisingly, that this conclusion is no longer valid at the present time, if the dark energy reflects a constant vacuum-energy density in the Universe. Once the Universe begins to enter a phase of exponential expansion, no process leading to the disappearance of matter can ever cause radiation to once again dominate over the remnant matter density, no matter how small.

Consider a nonrelativistic component with density ρ_M, decaying into a relativistic component ρ_R, and take the lifetime of this decay to be τ, with a standard exponential decay law. (For simplicity, we will assume that the decay products are "sterile", i.e., that they do not interact significantly with anything else, but our conclusions do not depend on this assumption). Then the equations governing the matter and radiation evolution are

$$\frac{d\rho_M}{dt} = -3H\rho_M - \rho_M/\tau, \tag{1}$$

$$\frac{d\rho_R}{dt} = -4H\rho_R + \rho_M/\tau, \tag{2}$$

where H is the time-dependent Hubble parameter:

$$H = \left(\frac{8\pi G\rho}{3}\right)^{1/2}, \tag{3}$$

with ρ being the total energy density. (We assume a flat universe throughout). If the decaying nonrelativistic component dominates the expansion, it is easy to show that the energy density in the nonrelativistic component is rapidly converted into relativistic energy density when $t \approx \tau$ (see,

e.g., Ref. [7]); indeed, this assumption has become part of the standard cosmological lore.

But now consider what happens for a vacuum-dominated expansion, such as the Universe is experiencing at present. We take the ratio of radiation density to matter density to be given by r

$$r \equiv \rho_R/\rho_M. \tag{4}$$

Then Eqs. (1) and (2) can be combined to yield an equation for r

$$\frac{dr}{dt} = \frac{1}{\tau} + \left(\frac{1}{\tau} - H\right)r. \tag{5}$$

In the matter-dominated era, H decreases with time, and as long as $H < 1/\tau$, we see that $r \rightarrow \infty$, as expected. However, as the Universe evolves from a matter-dominated state to a vacuum-energy-dominated state, the value of H asymptotically approaches a constant value, H_Λ, given by

$$H_\Lambda = \left(\frac{8\pi G\rho_\Lambda}{3}\right)^{1/2}, \tag{6}$$

where ρ_Λ is the (constant) vacuum-energy density. Define a "time of no return", $t_\Lambda \equiv 1/H_\Lambda$. Substituting $H = H_\Lambda \equiv 1/t_\Lambda$ into Eq. (5), we can solve this equation analytically to yield

$$\left(1 - \frac{\tau}{t_\Lambda}\right)r = \exp\left(\left[\frac{1}{\tau} - \frac{1}{t_\Lambda}\right]t\right) - 1, \tag{7}$$

where r is normalized to be zero at $t = 0$, i.e. the initial radiation density is negligible in the matter-dominated era.

In terms of the present-day Hubble parameter H_0, and the fraction of the critical density in vacuum energy, Ω_Λ, we have simply

$$t_\Lambda = H_0^{-1}\Omega_\Lambda^{-1/2} = 9.8 \times 10^9 \text{ yr } h^{-1}\Omega_\Lambda^{-1/2}, \tag{8}$$

where h is the value of H_0 in units of $100 \text{ km sec}^{-1} \text{ Mpc}^{-1}$.

The asymptotic value of r is sensitive to the relative values of τ and t_Λ. If $\tau < t_\Lambda$, then as $t \rightarrow \infty$, Eq. (7) gives $r \rightarrow \infty$, which is the same as the conventional result for

decays during the matter-dominated phase, since at such times the vacuum energy will indeed be cosmologically irrelevant. But if $\tau > t_\Lambda$, the right-hand side of Eq. (7) goes to -1 as $t \to \infty$, and r asymptotically approaches a constant, given by

$$r(t \to \infty) = \frac{t_\Lambda}{\tau - t_\Lambda}. \tag{9}$$

Thus, the ratio of radiation to matter approaches a constant; the matter never disappears relative to the radiation (although of course both densities go to zero as $t \to \infty$). Further, as long as $\tau > 2t_\Lambda$, we have $r < 1$ asymptotically; in this case the decay-produced radiation never dominates the decaying matter! Finally, there is an intermediate case, $\tau = t_\Lambda$, for which the solution given in Eq. (7) does not apply. In the case, the solution of the evolution equation is simply $r = t/\tau$. In this case, r becomes arbitrarily large as $t \to \infty$, but the increase is linear in time, rather than exponential as is the case for shorter particle lifetimes. (See also [11] which examines a different model for energy exchange).

   While this result is perhaps nonintuitive, there is a straightforward physical explanation. Radiation redshifts as one extra power of the scale factor relative to matter. For a vacuum-energy-dominated universe the scale factor is itself an exponential of the time. Thus, in the absence of decay the energy density of matter in the Universe will increase relative to radiation by an exponential function of time. If the matter density itself decreases exponentially, the two factors cancel, leading to a constant final ratio of matter density to radiation density.

   Our analytic solutions give the correct asymptotic behavior, but Eq. (5) must be integrated numerically to study the physically realistic case of a universe containing both decaying dark matter and vacuum energy. We have performed this numerical integration for several illustrative values of τ/t_Λ, (note that ρ_M/ρ_Λ at any time is a fixed function of t/t_Λ); the results are displayed in Fig. 1. This figure clearly shows the asymptotic behavior discussed above.

   Now consider our present-day universe. Taking reasonable values of $h = 0.7$ and $\Omega_\Lambda = 0.7$ in Eq. (8), we obtain a value of $t_\Lambda = 1.7 \times 10^{10}$ yr. Current constraints on the decay of the dark matter yield $\tau > t_\Lambda$. For example, Ichiki *et al.* [12], using the WMAP data, derive a 95% confidence limit of $\tau > 5.2 \times 10^{10}$ yr, which corresponds to $\tau/t_\Lambda > 3$. Thus, we conclude that we have already entered the era at which it is no longer possible for relativistic energy density from decaying dark matter to ever dominate the dark matter density itself.

   There are other mechanisms to convert the dark matter into relativistic energy density. For example dark matter within the dense cores of halos can annihilate into relativistic particles (see, e.g., Refs. [13,14]). As Krauss and Starkman have recently demonstrated [15], such processes
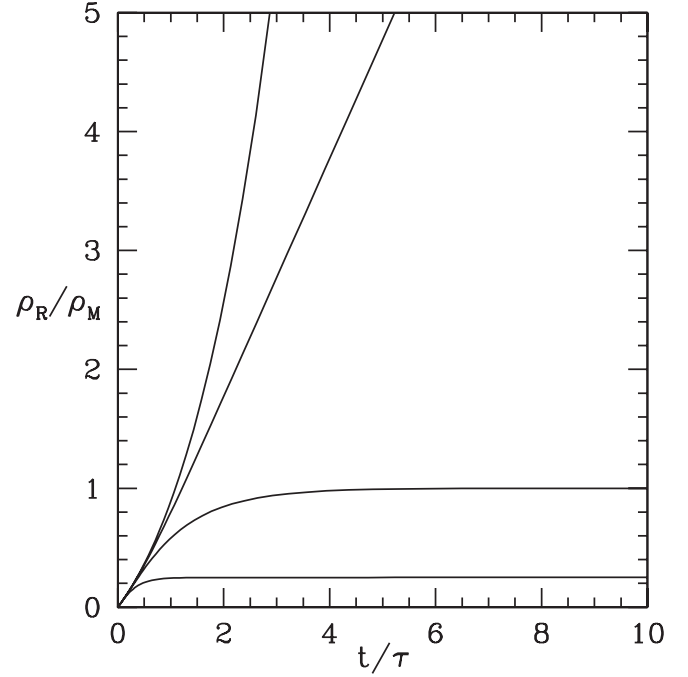

FIG. 1.   The ratio of decay-produced relativistic energy density, ρ_R, to decaying nonrelativistic dark matter, ρ_M, as a function of the time measured in units of the decaying particle lifetime τ. From top to bottom, the curves correspond to $\tau/t_\Lambda = 0.5, 1, 2, 5$.

will have dramatic consequences for the future of large scale structure.

   In general, as described in [15], at late times the annihilation rate for dark matter in halos with density n, given by $\Gamma_{ANN} = n\langle\sigma v\rangle$, will fall as t^{-1}. Nevertheless, even if it were to remain constant over time, this annihilation rate would still not be fast enough produce a radiation density that would overwhelm that in matter.

   In the case of falling annihilation rates one can in fact produce a stronger bound. If $\dot{M} = M/t$, which will be the case for bound systems whose density changes only due to annihilations as the Universe ages [16], then $M(t) = M_0/t$. In this case, as long as the expansion of the Universe is such that $R \approx t^\alpha$ where $\alpha \geq 1$ then radiation will redshift faster than the mass decreases, and the radiation density will never overwhelm the matter density. This will occur if the equation of state parameter $w = p/\rho < -1/3$ for the dominant energy in the Universe. However, for systems bound by gravity, where adiabatic expansion will accompany annihilations, supplanting annihilation in reducing core densities, then $\dot{M} = M/4t$ [15]. In this case, $M(t) = M_0/t^{1/4}$. For this case, no expansion involving radiation or matter is sufficiently slow to allow the radiation produced by annihilations to overwhelm the matter density.

   In the unphysical but conservative case where annihilation occurs at a constant rate, dark matter annihilation would then map directly onto the decaying particle problem we have already solved. With a constant annihilation

rate, each galaxy halo would act as a very massive decaying "particle", with lifetime $\tau = 1/\Gamma_{ANN}$, and our previous analysis would then hold. Studies of galaxy halo profiles today (i.e. [13,17]) imply that the characteristic annihilation time, τ, in galaxies must greatly exceed the present age of the Universe, and hence also greatly exceed t_Λ today. Moreover for annihilation into standard particles and for dark matter whose remnant abundance today was determined by freeze-out at early times, the inverse annihilation rate will in fact be many orders of magnitude larger than the current age today. In either case, our previous argument holds: the ratio of ρ_R to ρ_M will remain small regardless of the particle annihilations.

Our consideration of dark matter annihilation is particularly relevant if ordinary matter is in fact unstable, due to proton decay. In this case, our arguments tell us that even if dark matter annihilates away, remnant baryons will still dominate compared to radiation. Moreover, even once baryons start to decay significantly in the far, far future [18] they will still dominate over the energy density of their relativistic decay products until these decay products themselves become nonrelativistic, assuming they are massive. Thus, matter, even ordinary matter, will always continue to dominate over radiation for all times.

We further point out in this regard that our analysis clearly applies in the case of two decaying matter species with different timescales, as for example might occur for decaying dark matter and decaying baryons. Since the latter are likely to have a far longer lifetime, our analysis is trivially extended by considering two species with different exponential timescales for decay. Consider two such species with lifetimes τ_1 and τ_2, with $\tau_1 \ll \tau_2$. Once $t > \tau_1$, while the ratio of energy densities in the first decaying particle and the radiation produced by it will become constant, nevertheless the total energy density in both will quickly become insignificant relative to the energy density in the second, more stable, nonrelativistic species. Then when the second nonrelativistic species decays, our original analysis applies.

In the extremely far future, individual galaxies, or clusters of galaxies will become essentially isolated "island universes" [19]. Nevertheless, on sufficiently large (perhaps even superhorizon) scales these will behave like a homogenous gas of particles amidst a background vacuum-energy sea from the perspective of this analysis, and if the matter within them decays or annihilates, as discussed above, they can be treated as decaying particles and our analysis continues to hold.

To be sure, it is true that in the far future of a vacuum-dominated universe, neither matter nor radiation will significantly affect the dynamics of the Universe, and therefore which dominates over the other is, from a practical perspective, not dynamically significant. Nevertheless, many scenarios in early universe cosmology are based on decaying systems which cause the Universe to shift from matter to radiation domination. One's intuition suggests this will always be the case, and thus this result is interesting because it demonstrates that conventional wisdom about the past Universe cannot be applied to the future Universe, if it remains vacuum or dark energy dominated. In this regard, our result may also be relevant for determining the initial conditions of a post-vacuum-dominated universe, if the dark energy is not vacuum energy, but instead evolves.

This latter comment, of course, reinforces the fact that our results depend crucially on the assumption that the dark energy is, indeed, a vacuum energy (cosmological constant) with ρ_Λ = constant, and that it, itself, does not decrease with time, in which case the Hubble parameter will once again decrease with time. Thus, in spite of the many negative facets of a future with a cosmological constant [20], only persistent vacuum energy remarkably preserves this surprising eternal dominance of matter over radiation.

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