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Energy conditions and cosmic acceleration

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In general relativity, the energy conditions are invoked to restrict general energy-momentum tensors $T_{\mu\nu}$ in different frameworks, and to derive general results that hold in a variety of general contexts on physical grounds. We show that in the standard Friedmann-Lemaître-Robertson-Walker (FLRW) approach, where the equation of state of the cosmological fluid is unknown, the energy conditions provide model-independent bounds on the behavior of the distance modulus of cosmic sources as a function of the redshift for any spatial curvature. We use the most recent type Ia supernovae (SNe Ia) observations, which include the new *Hubble Space Telescope* SNe Ia events, to carry out a model-independent analysis of the energy conditions violation in the context of the standard cosmology. We show that both the null (NEC), weak (WEC), and dominant (DEC) conditions, which are associated with the existence of the so-called *phantom* fields, seem to have been violated only recently ($z \lesssim 0.2$), whereas the condition for attractive gravity, i.e., the strong energy condition (SEC) was first violated billions of years ago, at $z \gtrsim 1$.

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I. INTRODUCTION

Within the framework of the standard Friedmann-Lemaître-Robertson-Walker (FLRW) cosmology, the Universe is modeled by a space-time manifold endowed with a spatially homogeneous and isotropic metric

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right], \quad (1)$$

where the spatial curvature $k = 0$, 1, or -1, $a(t)$ is the scale factor, and we have set the speed of light $c = 1$. The metric (1) only expresses the principle of spatial homogeneity and isotropy along with the existence of a cosmic time t. However, to study the dynamics of the Universe a third assumption in this approach to cosmological modelling is necessary, namely, that the large scale structure of the Universe is essentially determined by gravitational interactions, and hence can be described by a metrical theory of gravitation such as general relativity (GR), which we assume in this work.

These very general assumptions constrain the cosmological fluid to be a perfect-type fluid

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - p g_{\mu\nu}, \quad (2)$$

where u_μ is the fluid four-velocity, with total density ρ and pressure p given, respectively, by

$$\rho = \frac{3}{8\pi G}\left[\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right], \quad (3)$$

$$p = -\frac{1}{8\pi G}\left[2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right], \quad (4)$$

where G is the Newton constant, and dots indicate derivative with respect to the time t.

A further constraint on this standard cosmological picture, without invoking any particular equation of state, arises from the so-called *energy conditions* [1–3] that limit the energy-momentum tensor $T_{\mu\nu}$ on physical grounds. These conditions can be stated in a coordinate-invariant way, in terms of $T_{\mu\nu}$ and vector fields of fixed character (timelike, null, and spacelike). In the FLRW framework, however, only the energy-momentum of a perfect fluid (2) should be considered, so that the most common energy conditions (see, e.g., [1–4]) reduce to

$$\begin{aligned}
&\textbf{NEC} \Rightarrow \rho + p \geq 0,\\
&\textbf{WEC} \Rightarrow \rho \geq 0 \quad \text{and} \quad \rho + \mathrm{p} \geq 0,\\
&\textbf{SEC} \Rightarrow \rho + 3\mathrm{p} \geq 0 \quad \text{and} \quad \rho + \mathrm{p} \geq 0,\\
&\textbf{DEC} \Rightarrow \rho \geq 0 \quad \text{and} \quad -\rho \leq \mathrm{p} \leq \rho,
\end{aligned}$$

where NEC, WEC, SEC, and DEC correspond, respectively, to the null, weak, strong, and dominant energy conditions. From Eqs. (3) and (4), one easily obtains that these energy conditions can be translated into the following set of dynamical constraints relating the scale factor $a(t)$ and its derivatives for any spatial curvature k:

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$$\mathbf{NEC} \Rightarrow -\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \geq 0, \tag{5}$$

$$\mathbf{WEC} \Rightarrow \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} \geq 0, \tag{6}$$

$$\mathbf{SEC} \Rightarrow \frac{\ddot{a}}{a} \leq 0, \tag{7}$$

$$\mathbf{DEC} \Rightarrow -2\left[\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right] \leq \frac{\ddot{a}}{a} \leq \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}, \tag{8}$$

where clearly the NEC restriction [Eq. (5)] is also part of the WEC constraints. From the theoretical viewpoint, these energy conditions have been used in different contexts to derive general results that hold for a variety of situations [5]. For example, the Hawking-Penrose singularity theorems invoke the WEC and SEC [1], whereas the proof of the second law of black hole thermodynamics requires the NEC [2].

In order to shed some light on the energy conditions interrelations from the observational side, it is important to confront the constraints arising from Eqs. (5)–(8) with the current observational data. In this regard, recently some of us [6] used the fact that the classical energy conditions can be translated into differential constraints involving only the scale factor and its derivatives, to place model-independent bounds on the distance modulus $\mu(z)$ of cosmic sources in a *flat* ($k = 0$) FLRW universe. When compared with the type Ia Supernovae (SNe Ia) data, as compiled by Riess *et al.* [7], and Astier *et al.* [8], it was shown that all the energy conditions seem to have been violated in a recent past of the cosmic evolution ($z \sim 1$), when the Universe is expected to be dominated by *normal* matter fields.

The aim of this paper is twofold. First, we extend the results of Ref. [6] by deriving model-independent bounds on $\mu(z)$ for any spatial curvature k, including the flat one ($k = 0$) as a special limiting case. Second, we confront our general bounds with the most recent SNe Ia observations, as provided recently by Riess *et al.* [9], which include the new Hubble Space Telescope (HST) SNe Ia events. This new data sample indicates that both NEC and DEC were violated only recently ($z \lesssim 0.2$), whereas the condition for attractive gravity (SEC) was first violated billions of years ago, at $z \gtrsim 1$.

II. GENERAL CONSTRAINTS ON DISTANCES FROM ENERGY CONDITIONS

The predicted distance modulus for an object at redshift z is defined as

$$\mu(z) \equiv m(z) - M = 5\log_{10} d_L(z) + 25, \tag{9}$$

where m and M are, respectively, the apparent and absolute magnitudes, and d_L, given by

$$d_L(z) = a_0(1+z)r(a), \tag{10}$$

stands for the luminosity distance in units of megaparsecs (throughout this paper the subscript 0 denotes present-day quantities). From Eq. (1), it is straightforward to show that the radial distance $r(a)$ can be written as

$$r(a) = \frac{H_0^{-1}}{a_0\sqrt{|\Omega_k|}} S_k[\sqrt{|\Omega_k|} I(a)], \tag{11}$$

where $\Omega_k = -k/(a_0H_0)^2$ is the usual definition of the curvature parameter, $I(a)$ is given by

$$I(a) = a_0H_0 \int_a^{a_0} \frac{da}{\dot{a}a}, \tag{12}$$

and the function $S_k(x)$ takes one of the following forms:

$$S_k(x) \equiv \begin{cases} \sin(x) & \text{if } \Omega_k < 0, \\ x & \text{if } \Omega_k = 0, \\ \sinh(x) & \text{if } \Omega_k > 0. \end{cases} \tag{13}$$

A. NEC/WEC

In order to derive bounds on the predicted distance modulus $\mu(z)$ from NEC/WEC we note that the first integral of Eq. (5) provides

$$\dot{a} \geq a_0H_0\sqrt{\Omega_k + (1-\Omega_k)(a/a_0)^2}, \tag{14}$$

for any value of $a < a_0$. By using the above inequality we integrate (12) to obtain the following upper bound on the radial distance:

$$r(z) \leq \frac{H_0^{-1}}{a_0\sqrt{|\Omega_k|}} S_k\left\{S_k^{-1}\left[\sqrt{\left|\frac{\Omega_k}{\Omega_k - 1}\right|}(1+z)\right] - S_k^{-1}\sqrt{\left|\frac{\Omega_k}{\Omega_k - 1}\right|}\right\}, \tag{15}$$

where $a_0/a = 1 + z$, and S_k^{-1} is the inverse function of S_k. Concerning the derivation of the above expression, two important aspects should be emphasized at this point. First, that it uses the constraint $\Omega_k < 1$ that arises from the WEC, as given by Eq. (6). Second, since the argument of the function $\sin^{-1}(x)$ is limited to $-1 \leq x \leq 1$, this restricts our analysis of a spatially closed geometry ($\Omega_k < 0$) to redshifts lying in the interval $z \leq \sqrt{(\Omega_k - 1)/\Omega_k} - 1$. Note, however, that given the current estimates of the curvature parameter from Wilkinson Microwave Anisotropy Probe (WMAP) and other experiments, i.e., $\Omega_k = -0.014 \pm 0.017$ [10], the above interval leads to $z \leq 10$, which covers the entire range of current SNe Ia observations ($z \lesssim 2$).

Now, by combining Eqs. (9), (10), and (15), we obtain the following upper bound from the NEC/WEC:

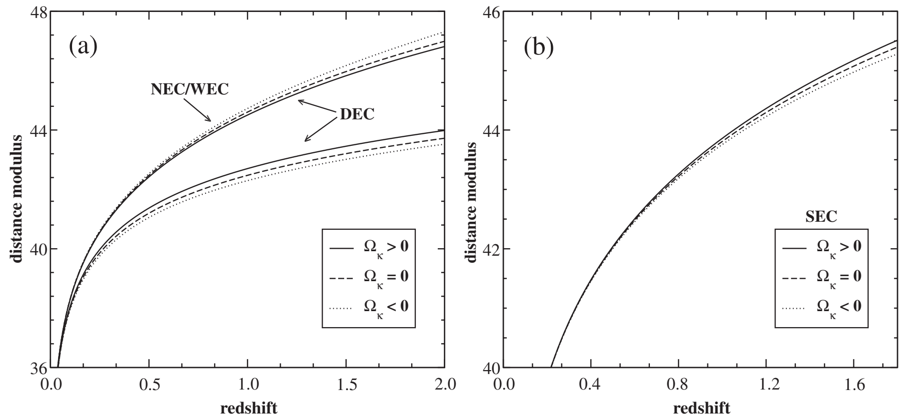

FIG. 1. Model-independent bounds on the distance modulus $\mu(z)$ as a function of the redshift for different signs of the curvature parameter Ω_k. Panel (a): Bounds from the NEC/WEC are shown in the top set of curves, while the upper and lower bounds from the DEC correspond, respectively, to the top and bottom sets of curves. Panel (b): Upper bounds from the SEC for all signs of the spatial curvature.

$$\mu(z) \leq 5\log_{10}\left[\frac{H_0^{-1}}{\sqrt{|\Omega_k|}}(1+z)S_k\left\{S_k^{-1}\left[\sqrt{\left|\frac{\Omega_k}{\Omega_k-1}\right|}(1+z)\right] - S_k^{-1}\sqrt{\left|\frac{\Omega_k}{\Omega_k-1}\right|}\right\}\right] + 25. \quad (16)$$

Clearly, if the NEC/WEC are obeyed then $\mu(z)$ must take values such that Eq. (16) holds. The three curves in the top of Fig. 1(a) show the NEC/WEC bounds on $\mu(z)$ as a function of the redshift for different signs of the curvature parameter Ω_k. To plot the curves we have used the central value of the HST key project estimate for the Hubble parameter, i.e., $H_0 = 72\ \mathrm{kms^{-1}\,Mpc^{-1}}$ [11] (that we assume throughout this paper). It is worth emphasizing that, as discussed in Ref. [6], the predicted distance modulus depends very weakly on the value adopted for the Hubble parameter.

B. SEC

Similarly to the NEC, a first integration of Eq. (7) implies $\dot{a} \geq a_0 H_0\ \forall\ a < a_0$ which, along with Eqs. (11) and (12), gives the following upper bound on the radial comoving distance:

$$r(z) \leq \frac{H_0^{-1}}{a_0\sqrt{|\Omega_k|}} S_k\{\sqrt{|\Omega_k|}\ln(1+z)\}. \quad (17)$$

Note that, differently from the previous case (NEC/WEC), the above constraint holds for any value of the curvature parameter Ω_k. From Eqs. (9), (10), and (17), the SEC bound on $\mu(z)$ reads

$$\mu(z) \leq 5\log_{10}\left[\frac{(1+z)}{H_0\sqrt{|\Omega_k|}} S_k\{\sqrt{|\Omega_k|}\ln(1+z)\}\right] + 25. \quad (18)$$

Figure 1(b) shows the SEC-$\mu(z)$ prediction as a function of the redshift. From top to bottom the curves correspond, respectively, to positive, null, and negative values of the curvature parameter Ω_k.

C. DEC

DEC provides both upper and lower bounds on the rate of expansion. In order to find the lower bound from DEC we integrate the inequality on the left-hand side of Eq. (8) to obtain $\dot{a} \leq a_0 H_0 \sqrt{\Omega_k + (1-\Omega_k)(a_0/a)^4}$. By combining this equation with Eqs. (11) and (12) we find

$$r(z) \geq \frac{H_0^{-1}}{a_0\sqrt{|\Omega_k|}} S_k\left\{-\frac{1}{2}S_k^{-1}\left[\sqrt{\left|\frac{\Omega_k}{\Omega_k-1}\right|}(1+z)^{-2}\right] + \frac{1}{2}S_k^{-1}\sqrt{\left|\frac{\Omega_k}{\Omega_k-1}\right|}\right\}, \quad (19)$$

which holds for values of $\Omega_k < 1$. Again, the above inequality, along with Eqs. (9) and (10), gives rise to the following lower bound on $\mu(z)$ from the DEC:

$$\mu(z) \geq 5\log_{10}\left[\frac{H_0^{-1}}{\sqrt{|\Omega_k|}}(1+z)S_k\left\{-\frac{1}{2}S_k^{-1}\left[\sqrt{\left|\frac{\Omega_k}{\Omega_k-1}\right|} \times (1+z)^{-2}\right] + \frac{1}{2}S_k^{-1}\sqrt{\left|\frac{\Omega_k}{\Omega_k-1}\right|}\right\}\right] + 25. \quad (20)$$

As expected from Eq. (8), the DEC upper bound coincides with the NEC constraint on $\mu(z)$, which is given by Eq. (16). Figure 1(a) illustrates this point, and also makes clear that the DEC-fulfillment gives rise to both a lower and an upper bound on the distance modulus $\mu(z)$. It is also worth mentioning that, although we have restricted our analysis to the distance modulus, the inequalities (15), (17), and (19) are general bounds that can be used in any cosmological test involving the radial comoving distance.

III. RESULTS AND DISCUSSION

In Figs. 2 and 3 we confront the energy conditions predictions for $\mu(z)$ with current SNe Ia observations. The data points appearing in the panels correspond to the new *gold* sample of 182 events distributed over the redshift interval $0.01 \lesssim z \lesssim 1.7$, as compiled by Riess *et al.* in Ref. [9], which include the new, recently discovered 21 SNe Ia by the HST. In order to perform our subsequent analyses, from now on we adopt $\Omega_k = -0.014$, which corresponds to the central value of the estimates provided by current WMAP experiments [10]. For the sake of comparison, the best-fit ΛCDM model for the new *gold* sample (corresponding to $\Omega_m \simeq 0.48$ and $\Omega_\Lambda \simeq 0.95$) is shown in all panels of Figs. 2 and 3.

Figures 2(a)–2(c) show the upper and lower-bound curves $\mu(z)$ for the NEC/WEC and DEC-fulfillment. As discussed in Ref. [6], an interesting aspect of these panels is that they indicate that these energy conditions might have been violated by a considerable number of nearby SNe Ia, at $z \lesssim 0.2$. To better visualize that we show a closer look of the data points in this interval (panel 2(b)) and take as an example the cases of the SNe 1992bs,

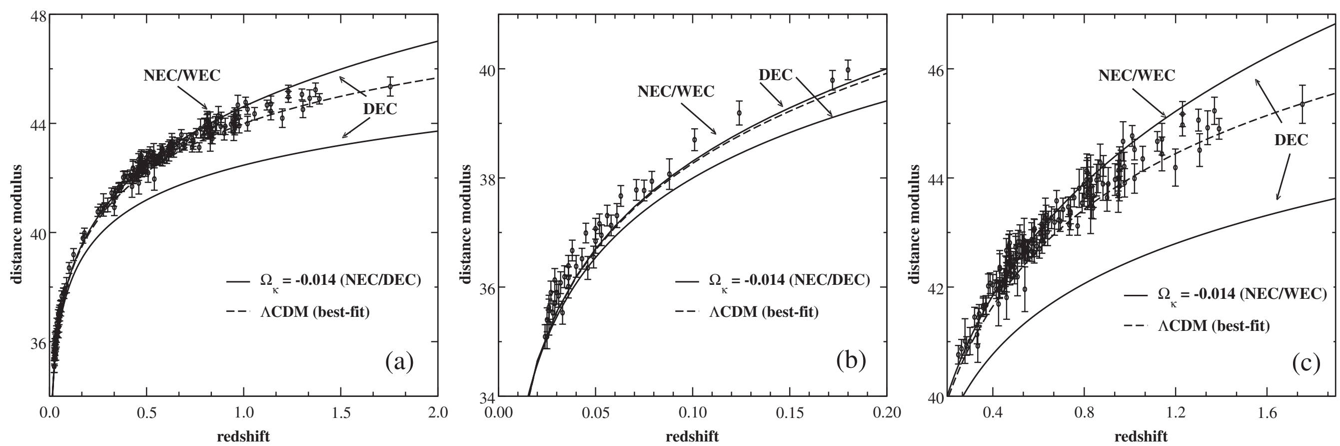

FIG. 2. NEC/WEC and DEC predictions for the distance modulus $\mu(z)$. The data points in the panels correspond to the new *gold* sample of 182 SNe Ia. Panel (a) shows the NEC/WEC and DEC bounds and the data points in the entire redshift interval $0.01 \lesssim z \lesssim 1.755$ while panels (b) and (c) show the curves and data for a smaller range of the redshift ($0.01 \lesssim z \lesssim 0.2$ and $0.2 \lesssim z \lesssim 1.755$, respectively). As discussed in the text, these panels indicate that these energy conditions seem to have been violated by a considerable number of nearby ($z \lesssim 0.2$) SNe Ia. For the sake of comparison, the best-fit ΛCDM model for the new *gold* sample of 182 SNe Ia is also shown.

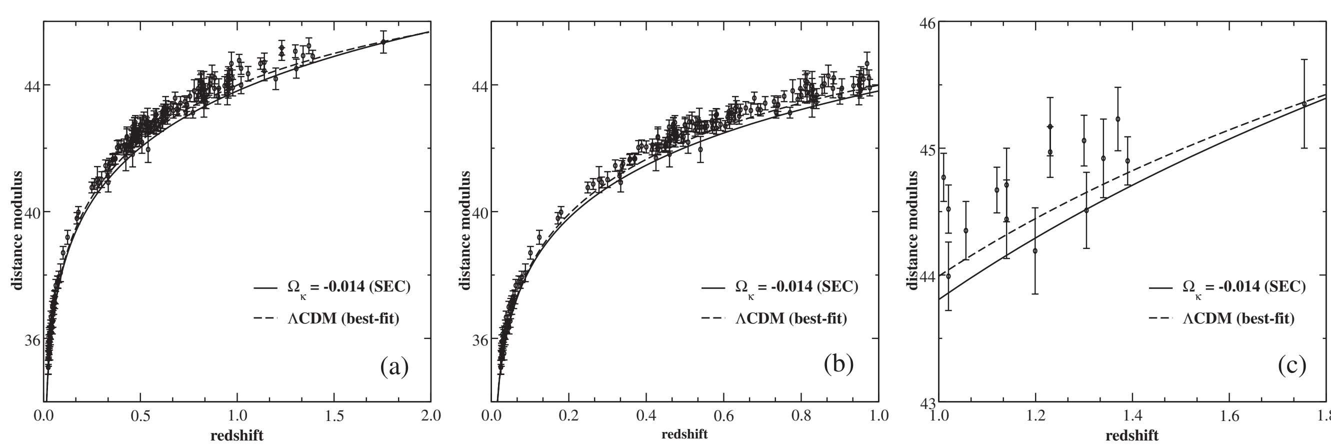

FIG. 3. The SEC upper-bounds predictions for the distance modulus $\mu(z)$ are shown in the panels (a) to (c) for three different redshift subintervals of $0.01 \lesssim z \lesssim 1.8$. As in the previous panels, the data points are from the new *gold* sample of 182 SNe Ia. This figure shows that apart from a few SNe events the SEC seems to have been violated in the whole redshift range. As discussed in the text, the panel (c) shows that even at high redshifts ($z \gtrsim 1$) a considerable number of SNe Ia points lie above the SEC-fulfillment curves. In all panels the best-fit ΛCDM model for the new *gold* sample of 182 SNe Ia is also shown.

1992aq, and 1996ab which are, respectively, at $z = 0.063$, $z = 0.101$, and 0.124. While their observed distance modulus are $\mu_{1992\mathrm{bs}} = 37.67 \pm 0.19$, $\mu_{1992\mathrm{aq}} = 38.70 \pm 0.20$, and $\mu_{1996\mathrm{ab}} = 39.19 \pm 0.22$, the upper-bound NEC predictions for the corresponding redshifts give, respectively, $\mu(z = 0.063) = 37.22$, $\mu(z = 0.101) = 38.33$, and $\mu(z = 0.124) = 38.82$. In the case of the SN 1992bs, for instance, we note that the discrepancy between the observed value and the NEC/WEC prediction is of 0.45 in magnitude or, equivalently, $\simeq 2.36\sigma$, which clearly indicates a violation of NEC/WEC at this redshift. The largest discrepancy, however, is associated with the observations of the SN 1999ef at $z = 0.038$ and $\mu_{1999\mathrm{ef}} = 36.67 \pm 0.19$. In this case, the upper-bound NEC/WEC prediction is $\mu(z = 0.038) = 36.08$, which is $\simeq 3.15$ off from the central value measured by the High-z Supernovae Team [9].

Concerning the above analysis it is also worth emphasizing three important aspects at this point. First, that the above results hold for the upper-bound DEC predictions, and that the lower bound of DEC is not violated by the current SNe Ia data. Also, neither NEC/WEC nor DEC are violated at higher redshifts, i.e., at $z > 0.2$ [Fig. 2(c)]. Second, the analysis is very insensitive to the values of the curvature parameter in that all the above conclusions are unchanged for values of Ω_k lying in the interval provided by the current CMB experiments, i.e., $\Omega_k = -0.014 \pm 0.017$ [10].[1] Finally, we note that, although our analyses and results are completely model independent, in the context of a FLRW model with a dark energy component parametrized by an equation of state (EoS) $w \equiv p/\rho$, violation of NEC/WEC and DEC is associated with the existence of the so-called *phantom* fields ($w < -1$), an idea that has been largely explored in the current literature [12]. By assuming this standard framework, the results above, therefore, seem to indicate a possible dominion of these fields over the conventional matter fields very recently, at $z \lesssim 0.2$.

SEC is the requirement (easily verified in our everyday experience) that gravity is always attractive. In an expanding FLRW universe, this amounts to saying that cosmic expansion must be decelerating regardless of the sign of the spatial curvature, as mathematically expressed by Eq. (7).[2] Similarly to the NEC/WEC/DEC analysis, one can also estimate the epoch of the first SEC violation by mapping the current SNe Ia Hubble diagram.

[1]For instance, by taking the upper and lower 1σ limit given by WMAP, i.e., $-0.031 \le \Omega_k \le 0.003$, the NEC/WEC predicted distance modulus at $z = 0.038$ ranges between $\mu(z = 0.038) =$ 36.08 and 36.07, respectively.

[2]As is well known, an early period of cosmic deceleration is strongly supported by the primordial nucleosynthesis predictions and the success of the gravitational instability theory of structure formation.

The upper-bound curves $\mu(z)$ for the SEC-fulfillment are shown in Figs. 3(a)–3(c) for three different redshift intervals. Note that SEC seems to be violated in almost the entire redshift range, with only very few SNe events in agreement with the theoretical upper-bound SEC prediction. In particular, we note that even at very high redshifts, i.e., $z \gtrsim 1$, when the Universe is expected to be dominated by normal matter, 11 out of 16 SNe Ia measurements provide magnitude at least 1σ higher than the theoretical value derived from Eq. (18). As an example, let us consider the cases of the SNe HST04Sas (the highest-z SN to violate SEC) at $z = 1.39$ and $\mu_{\mathrm{HST04Sas}} = 44.90 \pm 0.19$ and HST05Koe at $z = 1.23$ and $\mu_{\mathrm{HST05Koe}} = 45.17 \pm 0.23$. While the distance modulus of the former is at the limit of $\simeq 1\sigma$ higher than the SEC prediction [$\mu(z = 1.39) = 44.68$], the observed value of $\mu(z)$ for the latter is $\simeq 3.5\sigma$ far from the theoretical value of Eq. (18) [$\mu(z = 1.23) = 44.36$], a discrepancy that clearly indicates violation of SEC at $z > 1$. An interesting aspect worth mentioning is that if the redshift of these first SNe Ia events that violate SEC could be taken as the beginning of the epoch of cosmic acceleration (z_a), then our current concordance scenario (a flat ΛCDM model with $\Omega_m \simeq 0.3$), whose prediction is $z_a \simeq 0.67$, would be in disagreement with this estimate. In reality, for the current accepted dark matter-dark energy density parameter ratio (of the order of $\Omega_m/\Omega_x \sim 0.4$), the entire class of flat models with a constant EoS w, which predicts $z_a = [-(3w + 1) \times \frac{\Omega_x}{\Omega_m}]^{-1/3w} - 1$, also would be in disagreement with the first redshifts of SEC violation discussed above.

IV. CONCLUDING REMARKS

In this work, by extending and updating previous results [6], we have derived, from the classical energy conditions, model-independent bounds on the behavior of the distance modulus of extragalactic sources as a function of the redshift for any spatial curvature. We have also confronted these energy-condition-fulfillment bounds with the new *gold* sample of 182 SNe observed events provided recently by Riess *et al.* in Ref. [9]. On general grounds, our analyses indicate that the NEC/WEC and DEC are violated by a significant number of low-z ($z \lesssim 0.2$) SNe Ia, while for higher redshifts none of these energy conditions seem to have been violated. Another important outcome of our analyses is that the SEC, whose violation in the expanding FLRW model is ultimately related to the cosmic acceleration, seems to be violated in the entire redshift range covered by the new SNe Ia *gold* sample. A surprising fact from the confrontation between the SEC prediction and SNe Ia observations is that this energy condition seems to have been first violated at very high-z ($\simeq 1.3$), which is very far from the transition redshift predicted by most of the quintessence models and by the current standard concordance flat ΛCDM scenario ($z \simeq 0.67$). In agreement with our previous analysis [6], we emphasize that the results reported here reinforce the idea that, in the context of the standard cosmology, no possible combination of *normal* matter is capable of fitting the current observational data.

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