Causal field theory with an infinite speed of sound

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We introduce a model of scalar field dark energy, *Cuscuton*, which can be realized as the incompressible (or infinite speed of sound) limit of a scalar field theory with a noncanonical kinetic term (or *k*-essence). Even though perturbations of Cuscuton propagate superluminally, we show that they have a locally degenerate phase space volume (or zero entropy), implying that they cannot carry any microscopic information, and thus the theory is causal. Even coupling to ordinary scalar fields cannot lead to superluminal signal propagation. Furthermore, we show that the family of constant field hypersurfaces is the family of constant mean curvature hypersurfaces, which are the analogs of soap films (or soap bubbles) in Euclidian space. This enables us to find the most general solution in $1 + 1$ dimensions, whose properties motivate conjectures for global degeneracy of the phase space in higher dimensions. Finally, we show that the Cuscuton action can model the continuum limit of the evolution of a field with discrete degrees of freedom and argue why it is protected against quantum corrections at low energies. While this paper mainly focuses on interesting features of Cuscuton in a Minkowski space-time, a companion paper examines cosmology with Cuscuton dark energy.

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I. INTRODUCTION

There has been much recent interest in field theories with noncanonical kinetic terms. Many of these theories are inspired by purely phenomenological motivations. *k*-essence [1,2] and *k*-inflation [3] have been designed in order to solve the cosmological coincidence and inflationary fine-tuning problems, while Bekenstein's theory of gravity [4] attempts to accommodate apparent deviations from Newtonian gravity on galactic scales, within a relativistically covariant framework.

More theoretical motivations have led to theories such as the ghost condensate [5], which is considered as an analog of the Higgs boson for general relativity, or variations of the Born-Infeld action that describe nonperturbative objects in string theory [6,7].

Here we present a new class of actions with a noncanonical kinetic term which is characterized by the Hamiltonian symplectic structure of the theory degenerating in a cosmologically interesting homogeneous limit. In other words, in the limit that the field degree of freedom becomes homogeneous in the locally freely falling frame (where the metric is locally a Minkowski metric), the equation of motion does not have any second order time derivatives and the field becomes a nondynamical auxiliary field, which merely follows the dynamics of the fields that it couples to. Thus we call this field *Cuscuton* (pronounced käs-kü-tän), after the Latin name for the parasitic plant of dodder, Cuscuta [8].

Nonetheless, the Cuscuton action may *a priori* appear to have dynamical degrees of freedom because there is a nondegenerate conjugate momentum degree of freedom if the field configuration has a nonvanishing spatial gradient. It is this feature that justifies exploration of field theoretic aspects of the class of theories presented here.

For concreteness, we will focus on scalar field actions of the *k*-essence [1,10] form

$$S_\varphi = \int d^4x\sqrt{-g}\left[\frac{1}{2}F(X,\varphi) - V(\varphi)\right], \qquad (1)$$

where $X \equiv \partial_\mu\varphi\partial^\mu\varphi$ with a particular choice for F such as to satisfy our degeneration of the symplectic structure feature that will be defined in the next section.

There has been some controversy in the literature regarding the causality of dynamical *k*-essence models with $c_s > 1$. While the original literature on *k*-essence (e.g. [1,10]), as well as some follow-up studies [11,12], argue that superluminal modes of *k*-essence cannot carry information on closed loops, and thus do not break causality, others [13,14] take $c_s > 1$ at its face value, arguing that it cannot be realized as the IR limit of an inherently causal field theory. Superluminal scalar field models are also claimed to lead to signal propagation out of the horizon of a black hole [15], as well as large tensor-to-scalar ratios for inflationary perturbations [16].

In Sec. III, we show that, although at face value Cuscuton seems to possess dynamics, which allows superluminal signal propagation, it actually contains no local

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dynamical degree of freedom for $X > 0$. What that means is that the theory, even with a kinetic term, acts like a pure constraint system, modifying the dynamics of whatever it couples to. In that sense, this is a k-essence theory which behaves like modified gravity, since if it couples to gravity and other fields, it merely provides a means of changing those dynamical sectors through constraints without adding any new local degrees of freedom of its own. Moreover, the superluminal modes cannot carry any information due to this degeneration of dynamical degrees of freedom [17].

Section IV demonstrates that constant field hypersurfaces have constant mean curvature (CMC), making them Minkowski space analogs of soap bubbles/films in Euclidian space. This leads to exact solutions of theory in 1 + 1 dimensions in Sec. V, which enables us to make general conjectures about uniqueness and generic singularity of the solutions. We go on to briefly consider the coupling of Cuscuton to gravity (in the context of homogeneous cosmology), as well as ordinary scalar fields, in Sec. VI. The latter, in particular, shows that propagating degrees of freedom remain subluminal, even after coupling to Cuscuton.

In Sec. VII we discuss a possible physical model for the Cuscuton action, and argue why it may be protected against quantum corrections at low energies. Finally, Sec. VIII summarizes our results and concludes the paper. A companion paper [18] examines cosmology with a Cuscuton dark energy fluid.

II. DEFINING CUSCUTON ACTION

Here, we define the class of models of interest for this paper. Readers interested in only concrete examples should skip to Eq. (16). Consider the action of the form Eq. (1) in a Minkowski space-time and choose F such that in the homogeneous limit of the field the kinetic term becomes a total derivative for $\dot{\varphi} \neq 0$ (and thus would drop out of the equation of motion):

$$F(X, \varphi(x)) \to F(\dot{\varphi}^2, \varphi(t)) = \frac{d}{dt} J(\varphi, \dot{\varphi}, \ldots) = \dot{\varphi}\frac{\partial J}{\partial \varphi} + \ddot{\varphi}\frac{\partial J}{\partial \dot{\varphi}} + \ldots. \tag{2}$$

Since F does not contain any $\ddot{\varphi}$ or higher derivative functions, we conclude $J = J(\varphi)$ and

$$F(\dot{\varphi}^2, \varphi) = \sqrt{\dot{\varphi}^2}\frac{\partial J(\varphi)}{\partial \varphi}, \tag{3}$$

where we have absorbed the sign of $\dot{\varphi}$ into $\frac{\partial J(\varphi)}{\partial \varphi}$ and F is well defined with respect to the field variation as long as the sign of $\dot{\varphi}$ does not change. Hence, the action in the homogeneous limit is simply

$$S_{\varphi}^{\mathrm{homog}} = -\int d^4x V(\varphi) \tag{4}$$

(up to boundary terms) which when coupled to another field, for example χ, gives the total action

$$S_{\mathrm{example}\chi} = \int d^4x[\mathcal{L}_{\chi}(\chi, \varphi(t)) - V(\varphi(t))] \tag{5}$$

giving rise to a constraint equation

$$-\frac{\partial V}{\partial \varphi} + \frac{\partial \mathcal{L}_{\chi}(\chi, \varphi)}{\partial \varphi} = 0. \tag{6}$$

This modifies the dynamics of χ depending on the choice of function $V(\varphi)$. As long as χ has dynamics to make $\dot{\varphi} \neq 0$, the field φ acts like a nondynamical auxiliary field.

Just because the homogeneous limit with $\dot{\varphi} \neq 0$ is nondynamical does not mean *a priori* that φ does not have any dynamics, especially since the classical equation of motion in Minkowski space takes the form [for $X > 0$ with $F(X, \varphi) = \sqrt{X}\frac{\partial J(\varphi)}{\partial \varphi}$ coming from Eq. (3)]

$$\frac{1}{\sqrt{-g}}\partial_{\mu}\left[\frac{\sqrt{-g}}{2}\frac{\partial^{\mu}\varphi}{\sqrt{X}}\frac{\partial J}{\partial \varphi}\right] - \frac{\sqrt{X}}{2}\frac{\partial^2 J}{\partial \varphi^2} + V'(\varphi) = 0, \tag{7}$$

which clearly has a second time derivative as long as $\partial_i \varphi \neq 0$. More formally, the existence of dynamics can be studied in the Hamiltonian formalism

$$\{\mathsf{H}(\varphi, \Pi), \varphi(x)\} = \partial_0 \varphi(x), \tag{8}$$

$$\{\mathsf{H}(\varphi, \Pi), \Pi(x)\} = \partial_0 \Pi(x), \tag{9}$$

$$\{\Pi(t, \vec{x}), \varphi(t, \vec{y})\} = \delta^{(3)}(\vec{x} - \vec{y}), \tag{10}$$

where $\mathsf{H} \equiv \int d^3x \mathcal{H}$ is the Hamiltonian, $\{,\}$ are the functional Poisson brackets, and Π is the conjugate momentum to φ. This set of equations preserves the classical phase space whose volume element can be defined symplectically in the form $D\Pi \wedge D\varphi$. In the case of Eq. (3), the Hamiltonian density for the theory is

$$\mathcal{H} = \mathrm{sgn}\left(\frac{\partial J}{\partial \varphi}\right)|\nabla\varphi|\sqrt{\Pi^2 - \frac{1}{4}\left(\frac{\partial J}{\partial \varphi}\right)^2} + V(\varphi), \tag{11}$$

while the canonical momentum Π takes the form

$$\Pi = \frac{1}{2}\frac{\partial^0 \varphi}{\sqrt{X}}\frac{\partial J}{\partial \varphi}, \tag{12}$$

which should satisfy Eqs. (8)–(10), and have a normal phase space element $D\Pi \wedge D\varphi$. However, in the limit that $\partial_i \varphi \to 0$, Π becomes only dependent on φ, as long as $\dot{\varphi}(t)$ does not cross zero, i.e.

$$\Pi(t) = \frac{1}{2}\mathrm{sgn}(\dot{\varphi})\frac{\partial J(\varphi(t))}{\partial \varphi} \tag{13}$$

and the symplectic element $D\Pi \wedge D\varphi$ collapses (since $D\varphi \wedge D\varphi = 0$). That is simply a signature of φ becoming nondynamical in the homogeneous limit—i.e. has no phase space—except at the singular point $\dot{\varphi} = 0$.

To summarize, we define the Cuscuton action as the class of actions in which φ becomes nondynamical in the homogeneous limit. The most general such action, with a single real scalar field, and a covariant kinetic term with no more than first order gradients [as assumed in Eq. (1)], is

$$S_\varphi = \int d^4x\sqrt{-g}\left[\frac{1}{2}\frac{\partial J(\varphi)}{\partial\varphi}\sqrt{|g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi|} - V(\varphi)\right]. \tag{14}$$

It is easy to see that, as long as $\frac{\partial J}{\partial\varphi} > 0$, the field φ can be redefined to set

$$\frac{\partial J(\varphi)}{\partial\varphi} = 2\mu^2 = \text{const.}, \tag{15}$$

although the choice for the value of μ^2 is arbitrary. Thus, for the rest of the paper, we will use the following form for the Cuscuton action:

$$S_\varphi = \int d^4x\sqrt{-g}[\mu^2\sqrt{|g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi|} - V(\varphi)]. \tag{16}$$

Note that we have inserted the absolute value in the radicand of Eq. (16) to make the action well defined when $X \equiv g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi < 0$. However, when $X < 0$, $\partial_\mu\varphi$ is spacelike, implying that we cannot use a Lorentz boost to reach a locally homogeneous (and thus nondynamical) limit. Hence, the absolute value inside Eq. (16) is an ansatz that does not follow from our general definition of Cuscuton models and has been chosen merely for simplicity. Finally, note that, even with an absolute value in Eq. (16), the variation of the action is ill defined at $X = 0$. In such cases, the definition of the equation of motion from the direct consideration of the action is necessary. For example, the path integral of the form

$$Z = \int D\varphi e^{iS_\varphi} \tag{17}$$

is likely to be well defined even though the variation of $\delta S_\varphi/\delta\varphi$ does not exist at $X = 0$. We will not deal with this issue further in this paper.

III. WHY IS CUSCUTON CAUSAL?

At the linearized level, most dynamical theories are characterized by second order partial differential equations whose characteristic curves delimit the support for Green's functions which propagate Cauchy data. If the characteristic curves allow propagation of information outside of the light cone (defined by the local Lorentz group), one may worry that the theory is acausal, leading to an ill-defined initial value formulation. With this reasoning, the condition for a well-defined causal Cauchy data problem of linearized second order partial differential equations coming from actions of the form given in Eq. (1) is given by Aharonov, Komar, and Susskind [19]. The conditions for causal structure and energy positivity to be preserved are

$$F'(X) > 0, \tag{18}$$

$$F''(X) \geq 0, \tag{19}$$

where $'$ denotes the derivative with respect to X, while the stability of solutions with respect to small changes in Cauchy data requires

$$2XF''/F' > -1. \tag{20}$$

The latter condition is intimately related to the definition of the speed of sound, c_s, in scalar field theories with noncanonical kinetic terms (or k-essence) [1,10]:

$$c_s^2 = \frac{1}{1 + 2XF''/F'}. \tag{21}$$

Thus, the condition for stability of solutions [Eq. (20)] is equivalent to $c_s^2 > 0$, while the causality conditions (18) and (19) ensure $c_s \leq 1$.

With F coming from Eq. (3), Eq. (18) gives

$$\frac{1}{2}\frac{1}{\sqrt{X}}\frac{\partial J}{\partial\varphi} > 0, \tag{22}$$

while Eq. (19) gives

$$\frac{-1}{4}\frac{1}{X^{3/2}}\frac{\partial J}{\partial\varphi} \geq 0. \tag{23}$$

It is clear that both of these conditions cannot be satisfied. Hence, we would be naturally concerned that this class of theories are acausal, especially since substituting Eq. (3) in Eq. (21) yields $c_s = \infty$, or an infinite speed of sound. However, as we will now argue, upon closer inspection, the theory of Eq. (1) has no problems with causality from a local signal propagation point of view.

The first argument comes from a linearized analysis. The simplified equation of motion, Eq. (7), has a second order differential operator that can be rewritten as

$$\left(\eta^{\mu\nu} - \frac{\partial^\mu\varphi\partial^\nu\varphi}{X}\right)\partial_\mu\partial_\nu\varphi, \tag{24}$$

which means that the characteristic curves for the linearized equation (accounting only for the highest derivative operator) are governed by the effective metric

$$\tilde{g}_{\mu\nu} \equiv \eta_{\mu\nu} - \frac{\partial_\mu\varphi_0\partial_\nu\varphi_0}{\partial_\alpha\varphi_0\partial^\alpha\varphi_0}, \tag{25}$$

where the linearization is about a background field configuration φ_0: i.e. $\varphi = \varphi_0 + \delta\varphi$. Writing out a coordinate dependent expression for $\tilde{g}_{\mu\nu}$, we see

$$\tilde{g}_{\mu\nu} = \frac{1}{\dot{\varphi}^2 - (\vec{\nabla}\varphi)^2} \times \begin{pmatrix} -(\vec{\nabla}\varphi)^2 & -\partial_0\varphi_0\partial_i\varphi \\ -\partial_0\varphi_0\partial_i\varphi & -[\dot{\varphi}^2 - (\vec{\nabla}\varphi)^2]\delta_{ij} - \partial_i\varphi\partial_j\varphi \end{pmatrix}, \tag{26}$$

where the Latin indices denote the spatial components and run over 1–3. All diagonal components of the effective metric are manifestly negative, and by direct computation, one sees that the determinant of this matrix is 0. Hence, the theory is manifestly Euclidean at the linearized level (requiring only Dirichlet initial data on a 3-surface and not Cauchy data). Consequently, as far as the linear theory is concerned, there is no dynamics which means that $\delta\varphi$ merely satisfies a constraint equation. Even though the effective metric seems to allow characteristic curves to carry information about $\delta\varphi$ outside of the light cone, there is no information carried by $\delta\varphi$ independently of the fields to which $\delta\varphi$ couples. On the other hand, the fields to which φ couples, say χ, can change their causal properties due to the constraint equation. However, this is a model dependent problem requiring the analysis of the propagator for χ subject to the solution of the constraint equation. For example, we show in a companion paper [18] that metric perturbations coupled to a Cuscuton field evolve causally, as the scalar curvature does not change on superhorizon scales.

The second argument, which is a generalization of the argument above, is that, despite the generally nonvanishing Π, because of the underlying Lorentz symmetry, one can always go to a frame in which $D\Pi \wedge D\varphi = 0$ locally, and hence there is no local dynamics to this system.

To understand what we mean by the collapse of the phase space structure, let us consider a simple toy model particle dynamical system consisting of the Lagrangian

$$L(q_i, \dot{q}_i) = \sqrt{\dot{q}_1^2 + \dot{q}_2^2} - V(q_1, q_2). \tag{27}$$

Here, one can heuristically think of $\dot{q}_1$ as $\partial_0\varphi(t, \vec{x}_1)$ and $\dot{q}_2$ as $\partial_0\varphi(t, \vec{x}_2)$ with $\vec{x}_1 \neq \vec{x}_2$. In that sense, when $\dot{q}_2 \neq \dot{q}_1$, we have $\partial_i\varphi \neq 0$. Now, the conjugate momentum to q_i is

$$p_i = \frac{\dot{q}_i}{\sqrt{\dot{q}_1^2 + \dot{q}_2^2}}. \tag{28}$$

Hence one sees an analogous degenerate structure in which, when $\dot{q}_1 = \dot{q}_2$, the phase space structure $\prod_i dq_i \wedge dp_i$ collapses. However, this is secretly a theory with limited dynamics. To see this, consider making a change of integration variables of the phase space from (q_1, p_1, q_2, p_2) to $(q_1, \dot{q}_1, q_2, \dot{q}_2)$. The Jacobian for the transformation of the measure is

$$\det\left[\frac{\partial(q_1, p_1, q_2, p_2)}{\partial(q_1, \dot{q}_1, q_2, \dot{q}_2)}\right] = \det\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\dot{q}_2^2}{X^{3/2}} & 0 & \frac{-\dot{q}_1\dot{q}_2}{X^{3/2}} \\ 0 & 0 & 1 & 0 \\ 0 & \frac{-\dot{q}_1\dot{q}_2}{X^{3/2}} & 0 & \frac{\dot{q}_1^2}{X^{3/2}} \end{pmatrix} = 0, \tag{29}$$

where $X \equiv \dot{q}_1^2 + \dot{q}_2^2$. Hence, one sees that this is manifestly a theory with degenerate Hamiltonian dynamics, where, in fact, the solutions only occupy a 3D hypersurface of the full 4D phase space of (q_i, p_i).

It is straightforward to generalize the change of phase space measure in the field theory case. The Jacobian relating $D\varphi \wedge D\Pi$ to $D\varphi \wedge D\dot{\varphi}$ is

$$\det\frac{\partial(\varphi(\mathbf{x})\Pi(\mathbf{x}'))}{\partial(\varphi(\mathbf{y})\dot{\varphi}(\mathbf{y}'))} = \det\begin{pmatrix} \delta^{(3)}(\mathbf{x}-\mathbf{y}) & 0 \\ \frac{\Pi}{X}\nabla\varphi(\mathbf{x}')\cdot\nabla\delta^{(3)}(\mathbf{x}'-\mathbf{y}) & \frac{-\mu^2|\nabla\varphi(\mathbf{x}')|^2}{X^{3/2}}\delta^{(3)}(\mathbf{x}'-\mathbf{y}') \end{pmatrix}. \tag{30}$$

Now, because Lorentz symmetry allows us to locally rotate $\nabla\varphi(\mathbf{x}') = 0$ for any vector $\partial^\mu\varphi$ such that $\partial^\mu\varphi\partial_\mu\varphi > 0$, the symplectic structure of the phase space collapses without any local dynamical degrees of freedom. Note that this is in many ways just a corollary to the perturbative analysis of the theory.

The degeneration of the local phase space volume also implies that local perturbations do not carry any microscopic information, or equivalently Cuscuton fluid has zero entropy.

Here, we should point out that, although lack of internal dynamics prevents any transfer of information through the Cuscuton field, even superluminal propagation does not necessarily lead to a breakdown of causality [12]. However, coexistence of interacting k-essence fields, which allow superluminal signal propagations at different rest frames (and thus different chronologies), can generically yield the propagation of signals on closed timelike curves, which does imply a breakdown of causality [20]. In contrast, in Sec. VI B we will show that even (stable) coupling of Cuscuton to an ordinary field with propagating degrees of freedom cannot lead to superluminal signal propagation.

Let us close this section by commenting on the causal properties for $X < 0$. Note that in this case, plugging the kinetic term from Eq. (14), the conditions of Eqs. (18) and (19) become

$$F' = \frac{1}{2}\frac{\partial J}{\partial\varphi}\frac{\mathrm{sgn}X}{\sqrt{|X|}} > 0, \tag{31}$$

$$F'' = -\frac{1}{4}\frac{\partial J}{\partial\varphi}\frac{1}{|X|^{3/2}} \geq 0, \tag{32}$$

which merely require $\partial J/\partial\varphi < 0$ for causality. Hence, with an absolute value ansatz for the $\sqrt{|X|}$ part of the Lagrangian, it would be prudent to impose the $\partial J/\partial\varphi < 0$ condition to keep the theory causal in the regime $X < 0$ (see the Appendix for an explicit demonstration). Note that this will also keep the kinetic energy density positive definite in the $X < 0$ regime.

To summarize, we argued that since the Cuscuton field theory does not seem to have any internal dynamics, despite its apparently acausal structure, it cannot be used to send information. In the regime of $X < 0$, even though not directly relevant to our analysis, we can recover internal dynamics, a causal structure, and a positive definite energy, but only if $\partial J/\partial\varphi < 0$.

IV. CONSTANT MEAN CURVATURE SURFACES: ANALOGY TO SOAP FILMS

In this section, we will show that hypersurfaces of constant φ have constant mean curvature, and thus are Minkowski space analogs of soap films (or soap bubbles) in Euclidian space.

For the simplified system of Eq. (16), the equation of motion is

$$\frac{1}{\sqrt{-g}}\partial_\mu\left[\sqrt{-g}\frac{\partial^\mu\varphi}{\sqrt{|\partial^\mu\varphi\partial_\mu\varphi|}}\right] + \frac{V'(\varphi)}{\mu^2} = 0. \tag{33}$$

To get more insight into the geometrical nature of solutions, we should point out that

$$u^\mu \equiv \frac{\partial^\mu\varphi}{\sqrt{\partial^\nu\varphi\partial_\nu\varphi}} \tag{34}$$

are normal unit vectors to constant φ hypersurfaces. On the other hand, the trace of the extrinsic curvature tensor $K_{\mu\nu}$, or mean curvature of a surface, is defined as

$$K \equiv K^\mu_\mu = \nabla_\mu u^\mu, \tag{35}$$

which in combination with Eq. (33) simply implies that the mean curvature on constant φ hypersurfaces is only a function of φ and hence constant:

$$K(\varphi) = -\frac{V'(\varphi)}{\mu^2}. \tag{36}$$

CMC surfaces have been the subject of extensive study both in mathematics and physics, for their important and useful features. For example, we should note that a constant mean curvature surface in Euclidean space can be viewed as a surface where the exterior pressure and the surface tension forces are balanced. This can be seen explicitly by looking at the Cuscuton action itself [Eq. (16)], which can be rewritten as

$$\begin{aligned} S_\varphi &= \int d^4x\sqrt{-g}[\mu^2\sqrt{|g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi|} - V(\varphi)] \\ &= \int d^4x\sqrt{-g}\left[\mu^2\frac{|g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi|}{\sqrt{|g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi|}} - V(\varphi)\right] \\ &= \int d^4x\sqrt{-g}[\mu^2|u^\mu\partial_\mu\varphi| - V(\varphi)] \\ &= \mu^2\int_\varphi d\varphi\Sigma(\varphi) - \int d^4x\sqrt{-g}V(\varphi) \end{aligned} \tag{37}$$

where $\Sigma(\varphi)$ is the area of constant φ 2 + 1 hypersurfaces in 3 + 1 space-time.

For this reason, CMC surfaces in Euclidian space can be thought of as soap bubbles or films (depending on if they have boundaries), as their configuration is also determined by a similar balance between surface and volume terms in their energy. In fact, it is easy to show that our action exactly reproduces such solutions in the case of $\dot\varphi = 0$ and $X < 0$, and perturbations around $\dot\varphi = 0$ or constant φ are analogous to propagating waves on a bubble surface (see the Appendix).

V. EXACT SOLUTIONS, UNIQUENESS, AND SINGULARITIES

In general, the question of the existence and uniqueness of CMC surfaces (which constitute the classical solutions to the Cuscuton field equation) for a given boundary condition is of significant subtlety, and the subject of ongoing investigation [21].

Nevertheless, we can gain significant insight by studying the features of this problem in 1 + 1 dimensions, where the field equation (33) can be exactly solved. In this case, CMC surfaces of curvature K are, in general, hyperbolae of the form

$$(t - t_0)^2 - (x - x_0)^2 = K^{-2}, \tag{38}$$

where x_0 and t_0 are constants. Note that the hyperbolae degenerate into spacelike lines in the $K \to 0$ limit. Therefore, using Eq. (36), the general solution to the Cuscuton field equation is given by

$$[t - t_0(\varphi)]^2 - [x - x_0(\varphi)]^2 = \frac{\mu^4}{V'^2(\varphi)} \tag{39}$$

where $t_0(\varphi)$, $x_0(\varphi)$ can be multivalued functions of φ.

We can now argue that only a discrete set of possible solutions exists after imposing just the Dirichlet conditions and that general Cauchy data typically overconstrain the partial differential equation, resulting in no solution. Suppose $\varphi_0(x) \equiv \varphi(t = 0, x)$ corresponds to the initial Dirichlet data. Suppose there exists a set $S \equiv \{x_i\}$ which satisfies $\varphi_0(x_i) = v$ where v is a particular fixed field value occurring in the Dirichlet data. For any solution $\varphi(t, x)$ to the equation of motion consistent with the initial Dirichlet data at $t = 0$, Eq. (39) describes the set of constant φ

hyperbolae slices which *must* pass through *all* the points in $\{(t = 0, x_i)|x_i \in S\}$ [22]. Since the curvature K is fixed for each of the hyperbolae (because v is fixed), the sets of possible hyperbolae form a discrete set. (Clearly, the set need not be discrete if the curvatures of the hyperbolae can be adjusted.) This is remarkable because the set of possible solutions to the naively "hyperbolic" partial differential equation is a discrete set rather than a continuous set even when only Dirichlet conditions are imposed.

A corollary to this result is that a Cauchy initial condition, which fixes both φ_0 and $\dot{\varphi}_0$ (although locally consistent), typically overconstrains the system globally, and thus is inconsistent for $K(\varphi) \neq 0$ except for discrete exceptions. Of course, this is in contrast with regular field theories which require Cauchy initial conditions to fix their future evolution, and is reminiscent of local phase space degeneration of Cuscuton, discussed above (see Sec. III), since a discrete set of points forms a set of measure zero for a continuous measure.

Another corollary of the exact solution (39) is that, since the hyperbolae that thread the points in the set S generically intersect in the finite future or past, the normal derivatives at the intersections are generically not well defined. Even when single constant v hyperbolae do not intersect (for example, in the case of there being one hyperbola), for different values of $\varphi_0(x)$, the hyperbolae will have different curvatures and thus can intersect, this time making the field value ill defined at the intersection. In these cases, singularities or discontinuities will generically develop in the solutions within a finite time.

The development of the field discontinuities is in nature similar to the development of shocks in fluid mechanics, which suggests that they can be traced using appropriate jump conditions. In our case, the jump condition is simply a generalization of Eq. (36):

$$-\mu^2 K_{\rm dis} = \frac{\Delta V}{\Delta \varphi}, \tag{40}$$

where ΔV and $\Delta \varphi$ are changes in potential and field values, respectively, across the discontinuity, while $K_{\rm dis}$ is the mean extrinsic curvature of the discontinuous hypersurface.

We will close this section by commenting on the extension of these results to 3 + 1 dimensions. It is easy to construct a family of exact solutions by considering CMC hyperboloids of the form

$$[t - t_0(\varphi)]^2 - |\mathbf{x} - \mathbf{x_0}(\varphi)|^2 = \frac{9\mu^4}{V'^2(\varphi)}. \tag{41}$$

However, this is not the most general solution to the field equation, as it only accommodates spherical surfaces of constant φ in 3-space. Therefore, if the surfaces of constant φ are not spherical for initial conditions in configuration space, a more general solution should be sought.

Nevertheless, the statement that given Dirichlet initial/boundary conditions [plus $\mathrm{sgn}(\dot{\varphi})$] only admit a discrete set of solutions is still a reasonable conjecture. This conjecture could be more motivated through the analogy with soap films/bubbles in Euclidian space (see Sec. IV). For the case of soap films/bubbles, and assuming a fixed pressure difference, a given boundary condition only admits a discrete set of solutions [21], which suggests that the same may be true in Minkowski space. A generic singularity of the solutions naturally follows from this conjecture in a similar way to the 1 + 1 dimensional case.

One can gain another perspective on the solutions to the Cuscuton equation of motion, Eq. (33), in Minkowski space by rewriting Eq. (41) with x_0^μ set to a constant. One can identify $\varphi(x) = f(\Delta x^\mu \Delta x_\mu)$ (where $\Delta x^\mu \equiv x^\mu - x_0^\mu$) as the solution to the equation of motion with the potential

$$V(\varphi) = 3\mu^2 \int \frac{d\varphi}{\sqrt{f^{-1}(\varphi)}} \tag{42}$$

with any suitable choice of a single variable function $f(z)$. For example, with the choice $f(z) = Me^{-\lambda z^2}$, the potential is

$$V(\varphi) = 3\mu^2 M \lambda^{1/4} \Gamma\left(\frac{3}{4}, -\ln\frac{\varphi}{M}\right) \tag{43}$$

where $\Gamma(a, b)$ is the incomplete gamma function, and the solution is

$$\varphi = Me^{-\lambda(\Delta x_\mu \Delta x^\mu)^2}. \tag{44}$$

Notice that this solution is not singular, which is consistent with our discussion of singularities, because it does not have intersecting hyperbolae for two different field values nor does it have two different hyperbolae characterizing the same field value.

Another example of a nonsingular solution is in the case of $V(\varphi)$ being a constant. In that case, the solution to the equation of motion is any smooth function

$$\varphi(x) = F(k \cdot x) \tag{45}$$

for any constant one-form k_μ. Again, this solution can be nonsingular because the constant φ surfaces are parallel planes.

To summarize this section, we have demonstrated that Cauchy boundary conditions generically overconstrain the equation of motion of Cuscuton field theory in 1 + 1 dimensions. We have also demonstrated in the same theory that the classical solutions generically have singularities (or discontinuities) except in some special cases. While we can prove these theorems in 1 + 1 dimensions, they remain conjectures in higher dimensional space-time.

VI. COUPLED CUSCUTON

In this section, we provide two examples of how coupling to Cuscuton can modify the dynamics of physical systems. We first show how homogenous, or Friedmann-Robertson-Walker (FRW) cosmology is modified by the presence of the Cuscuton field, with only minimal coupling to gravity. Then we consider how the Klein-Gordon dispersion relation of perturbations in an ordinary scalar field is modified through Cuscuton coupling, and show that propagating degrees of freedom remain causal.

A. Homogeneous (FRW) Cuscuton cosmology

A comprehensive study of homogeneous cosmology as well as linear perturbations and observational constraints can be found in our companion paper [18]. Here, for completeness, we provide a brief treatment of homogeneous (FRW) Cuscuton cosmology.

The mean extrinsic curvature of comoving hypersurfaces has a particularly simple interpretation in a cosmological context. Equation (35) is also the definition of the expansion rate, or 3 times the Hubble constant, $H = \dot{a}/a$, where a is the scale factor in FRW cosmology. Therefore, Eq. (36) takes the form

$$3H\mathrm{sgn}(\dot{\varphi}) = -\frac{V'(\varphi)}{\mu^2}. \tag{46}$$

Moreover, in Sec. II [Eq. (11)], we saw that the Hamiltonian (or energy) density of Cuscuton approaches $V(\varphi)$ in the homogeneous limit. Therefore, the Friedmann equation will take the form

$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{\rho_m + V(\varphi)}{3M_p^2}, \tag{47}$$

where ρ_m is the mean matter density of the Universe, $M_p = (8\pi G)^{-1/2}$ is the reduced Planck mass, and $\mathcal{K}$ is the (constant) spatial curvature of the Universe. The Cuscuton field φ can be simply eliminated from Eqs. (46) and (47), to yield a modified Friedmann equation:

$$H^2 + \frac{\mathcal{K}}{a^2} = \frac{\rho_m + V[V'^{-1}(3\mu^2 H)]}{3M_p^2}, \tag{48}$$

where V'^{-1} is the inverse function of $V'(\varphi)$ and, without loss of generality, $\dot{\varphi} < 0$ is assumed. Equation (48) illustrates the auxiliary nature of Cuscuton, and shows that it simply modifies the dependence of the Hubble expansion rate on matter density and the spatial curvature of the Universe.

A simple example is the quadratic Cuscuton potential

$$V(\varphi) = \tfrac{1}{2}m^2\varphi^2, \tag{49}$$

which yields the modified Friedmann equation

$$H^2 + \frac{\mathcal{K}}{(1 - \frac{3\mu^4}{2m^2M_p^2})a^2} = \frac{\rho_m}{3(M_p^2 - \frac{3\mu^4}{2m^2})}, \tag{50}$$

implying that the quadratic Cuscuton renormalizes the spatial curvature and Planck mass to $\mathcal{K}'$ and M_p', where

$$M_p'^2 = M_p^2 - \frac{3\mu^4}{2m^2}, \tag{51}$$

$$\mathcal{K}' = \frac{\mathcal{K}}{1 - \frac{3\mu^4}{2m^2M_p^2}}. \tag{52}$$

This is a manifestation of why Cuscuton can be considered to be a theory of modified gravity [18].

Because of its infinite speed of sound, Cuscuton does not cluster on subhorizon scales, implying that the Planck mass approaches its "fundamental" value in the UV limit. Therefore, a signature for the quadratic Cuscuton model will be a running (or mismatch) of the Planck mass from the (cosmological) IR to the UV regime. Other cosmological features and observational constraints on the quadratic Cuscuton are discussed in [18].

B. Coupling to an ordinary scalar field

Let us study an ordinary scalar field, ψ, that is coupled to Cuscuton through its potential $V(\varphi, \psi)$:

$$\mathcal{L}(\varphi, \psi) = \tfrac{1}{2}\partial_\alpha\psi\partial^\alpha\psi + \mu^2\sqrt{|\partial_\alpha\varphi\partial^\alpha\varphi|} - V(\varphi, \psi). \tag{53}$$

We now study the dispersion relation for linear perturbations $\delta\psi(\mathbf{x}, t)$ and $\delta\varphi(\mathbf{x}, t)$, around a homogeneous background $\psi(t)$ and $\varphi(t)$, in the short-wavelength (or WKB) approximation. After decomposing perturbations into plane wave solutions, or their Fourier components, $\delta\varphi_{\omega,\mathbf{k}}$ and $\delta\psi_{\omega,\mathbf{k}}$, the linearized Cuscuton field equation (33) takes the following form [23]:

$$\delta\varphi_{\omega,\mathbf{k}} = -\left(\frac{V_{,\varphi\psi}}{k^2 + V_{,\varphi\varphi}}\right)\delta\psi_{\omega,\mathbf{k}}, \tag{54}$$

where, without loss of generality, we have locally redefined φ to have $|\dot{\varphi}| = \mu^2$, for the background $\varphi(t)$. This can be plugged into the linearized Klein-Gordon equation for ψ,

$$(\omega^2 - k^2)\delta\psi_{\omega,\mathbf{k}} = V_{,\psi\psi}\delta\psi_{\omega,\mathbf{k}} + V_{,\varphi\psi}\delta\varphi_{\omega,\mathbf{k}}, \tag{55}$$

to give the modified dispersion relation for $\delta\psi_{\omega,k}$ perturbations. After simple manipulations, the dispersion relation takes the form

$$\omega^2 = \frac{k^4 + (V_{,\varphi\varphi} + V_{,\psi\psi})k^2 + V_{,\varphi\varphi}V_{,\psi\psi} - V_{,\varphi\psi}^2}{k^2 + V_{,\varphi\varphi}}. \tag{56}$$

Notice that, again, φ acts as an auxiliary field and is dropped out of the field equation, without introducing any additional degree of freedom.

Assuming a positive determinate Hessian matrix, $V_{,ab}$, for the second derivatives of the potential $V(\varphi, \psi)$, we require that both the trace and the determinant of $V_{,ab}$ are positive:

$$V_{,\varphi\varphi} + V_{,\psi\psi} > 0, \tag{57}$$

$$V_{,\varphi\varphi}V_{,\psi\psi} - V^2_{,\varphi\psi} > 0, \tag{58}$$

which ensures that there will be no tachyonic solutions ($\omega^2 > 0$, i.e. no instability).

To demonstrate causal propagation of *dressed* $\delta\psi$ perturbations, we can calculate the group velocity in the short-wavelength limit:

$$v_g \equiv \frac{d\omega}{dk} = 1 - \frac{V_{,\psi\psi}}{2k^2} + O\left(\frac{V^2_{,\psi\varphi}}{k^4}\right) < 1. \tag{59}$$

Therefore, given that we require $V_{,\psi\psi} > 0$ from Eqs. (57) and (58), *signal propagation will always be subluminal/causal* in the regime of validity of the plane wave (WKB) approximation.

VII. CUSCUTON AS AN EFFECTIVE ACTION AND QUANTUM CORRECTIONS

In this section, we discuss whether the Cuscuton action may be derived from an ordinary field theory by integrating out degrees of freedom. In the process, we elucidate the physical intuition for this theory. We also discuss whether this type of action can be stable against quantum corrections.

One may wonder whether one can introduce auxiliary fields that lead to the Cuscuton action after the constraints involving the auxiliary fields are solved for. Such systems can certainly be written, but unless the auxiliary fields are made dynamical (by giving them kinetic terms), such theories are identical to the original theory. We have tried several actions with auxiliary fields whose solutions to the constraint equations lead to the Cuscuton action. Unfortunately, when dynamics (in the form of canonical kinetic terms) are given to the auxiliary fields, even after fine-tuning of couplings and scales, the would-be auxiliary fields settle to a field configuration different from the case when the fields were nondynamical.

Given that the action may have something to do with soap films/bubbles, we have also tried to interpret the theory in terms of integrating out short-wavelength degrees of freedom which include instanton transitions. Imagine a regular field theory with a tilted washboard potential that has an infinite number of discrete local minima at $\chi_1, \chi_2, \chi_3, \ldots,$ with the values $\mathsf{V}_1, \mathsf{V}_2, \mathsf{V}_3, \ldots,$ which monotonically decrease with the field value. While, classically, the field could have multiple vacua at each minimum, none of them is stable under quantum tunneling. Therefore, assuming a low tunneling probability, the full evolutionary history of the field consists of a series of

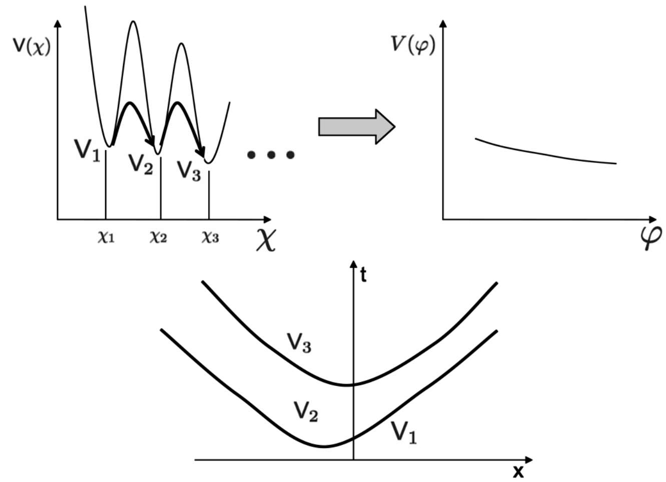

FIG. 1. Top: Correspondence between the potential of an ordinary scalar field $\mathsf{V}(\chi)$ and the effective Cuscuton potential $V(\varphi)$ which only passes through the minima of $\mathsf{V}(\chi)$. Bottom: Corresponding space-time diagram for consecutive tunneling events through minima of $\mathsf{V}(\chi)$.

tunnelings $\mathsf{V}_1 \to \mathsf{V}_2 \to \mathsf{V}_3 \to \ldots,$ as shown in Fig. 1, plus small oscillations around each minima following each tunneling event. The Euclidian instanton action for the successive tunnelings has the form

$$S_{\mathrm{E}} = \int d^4x\left[\frac{1}{2}\partial_a\chi\partial_a\chi + \mathsf{V}(\chi)\right], \tag{60}$$

where $a = 1, \ldots, 4$ counts over the 4-coordinates, including Euclidian time.

Now, let us focus on long-wavelength limit, $k \ll (\Delta V_{\max})^{1/2}/\Delta\chi$, which is equivalent to integrating out modes that are shorter than the tunneling time. In the thin-wall approximation limit, where $\mathsf{V}_i - \mathsf{V}_{i+1}$ is much smaller than the height of the potential barrier, $\Delta V_{\max}$ [24], one may naively guess an effective action of the form

$$S_{\mathrm{E,eff}} \simeq \sum_i J_i \int d\Sigma_i + \int d^4x \mathsf{V}(\chi), \tag{61}$$

where

$$J_i \equiv \int_{\chi_i}^{\chi_{i+1}} d\chi\sqrt{2[\mathsf{V}(\chi) - \mathsf{V}_i]}, \tag{62}$$

and $d\Sigma_i$ is the volume of the tunneling hypersurface for the V_i to V_{i+1} transition [24,25].

Now defining

$$\mu^2(\varphi_{i+1} - \varphi_i) \equiv J_i = \int_{\chi_i}^{\chi_{i+1}} d\chi\sqrt{2[\mathsf{V}(\chi) - \mathsf{V}_i]} \tag{63}$$

and rotating back to the Minkowski coordinates, one would end up with the action

$$S_{\mathrm{eff}} \simeq \mu^2\sum_i(\varphi_{i+1} - \varphi_i)\int d\Sigma_i - \int d^4x V(\varphi), \tag{64}$$

where we also define

$$V(\varphi_i) = \mathsf{V}(\chi_i). \tag{65}$$

This is interesting since Eq. (64) becomes equivalent to the Cuscuton action [Eq. (37)] in the continuous limit.

Unfortunately, because Eq. (61) is valid only for a somewhat ill-defined restricted class of field variations close to those mimicking thin walls, and because the action is based on analytic continuation into Euclidean space, the extent to which Eq. (61) can be interpreted as that due to integrating out tunneling transitions is unclear, at best. Note that the field path in the analytically continued saddle-point approximation does not have a simple connection with the space-time picture of the process since the former is simply an approximation scheme. Hence, although tantalizing, we cannot make any rigorous connection of Cuscuton to instanton induced bubble walls.

On the other hand, what we have learned from this exercise is that, if there is any quantum action which can be reduced to a set of discrete degrees of freedom described by the action of the form Eq. (64), then this dynamics can be encoded by the Cuscuton action in the limit that the number of discrete degrees of freedom is large.

Furthermore, notice that the action, Eq. (37) [or Eq. (64)], is the most general local action that remains invariant under the field transformations that preserve the area $\Sigma(\varphi)$ and volume $\mathcal{V}(\varphi)$ of constant field hypersurfaces, where

$$\Sigma(\varphi_0) = \frac{\partial}{\partial \varphi_0} \int d^4x \sqrt{-g} \sqrt{|\partial^\mu \varphi \partial_\mu \varphi|} \Theta(\varphi_0 - \varphi), \tag{66}$$

$$\mathcal{V}(\varphi_0) = \frac{\partial}{\partial \varphi_0} \int d^4x \sqrt{-g} \Theta(\varphi_0 - \varphi). \tag{67}$$

Therefore, it is reasonable to expect the quantum corrections to be under control at low energies, where higher order curvature effects could be neglected. This is also consistent with the absence of quantum corrections to the Cuscuton action, as the linear perturbations have a degenerate phase space.

The attempted instanton picture for the Cuscuton action involves a potential, $\mathsf{V}(\chi)$, which is qualitatively very similar to the recently proposed model known as devaluation [26] (or chain inflation [27]). It remains to be seen if their picture of rapid bubble nucleation has any relevance to our picture of coherent Cuscuton evolution [18], in an appropriate limit, where the radiation generated due to bubble collisions can be neglected.

VIII. CONCLUSIONS

In this paper, we have analyzed the flat space field theoretic aspects of the Cuscuton action, which we define as a scalar field theory whose kinetic term reduces to a total derivative in the homogenous limit. The theory was defined in this way and may be of interest to cosmology because it is equivalent to a *k*-essence fluid with an infinite speed of sound.

The surprise is that, even though the dynamical equations seem to admit superluminal signal propagation, there is *no* physical violation of causality, as perturbations have degenerate phase space and thus transport no information. That means this can be used to modify gravity and other field theories to which it couples in a novel manner since the fields have a kinetic term, unlike the usual Lagrange multipliers or auxiliary fields. We have also verified that this modification does not lead to superluminal propagation in ordinary scalar fields.

We have found some general, interesting features of classical solutions to this theory. Constant field hypersurfaces have constant mean curvatures, which makes them the Minkowski space analogs of soap films (or soap bubbles) in Euclidian space. We can also solve the theory completely in $1 + 1$ dimensions, and thereby prove nonlocal degeneracy of phase space (which is equivalent to the overconstraining behavior of Cauchy initial conditions), as well as the generic presence of singularities in the solutions. Extension of these results to higher dimensions is a plausible conjecture.

As far as the qualitative behavior of the physics is concerned, the Cuscuton action can be viewed as a continuum limit of an action of the form Eq. (64) governing a large number of discrete degrees of freedom. It is the most general (local) action that only depends on the area and volume of constant field hypersurfaces, which is why it can be protected against quantum corrections at low energies. This is consistent with the absence of quantum corrections to the Cuscuton action as a result of its degenerate phase space.

Cosmology literature does not lack in abundance of models of dark energy (or its alternatives). The features that still make Cuscuton interesting are the following:

(i) Even though it has a kinetic term and modifies the cosmic dynamics, it does not introduce any additional (perturbative) degree of freedom. Therefore, it can be considered a minimal modification of a cosmological constant, or a minimal model for evolving dark energy.

(ii) It is protected against quantum corrections at low energies.

A companion paper [18] examines cosmology with a Cuscuton dark energy fluid.

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APPENDIX: PROPAGATING WAVES IN $X < 0$ REGIME

In this section, we show that perturbations in an $X < 0$ background are similar to waves propagating on the surface of a bubble. To see this, consider $\varphi_0(x)$ to be a stationary solution; thus $u^i \equiv \nabla\varphi_0/|\nabla\varphi_0|$ represents the normal vectors to two dimensional constant φ_0 CMC surfaces in 3-space.

Now perturbing the field around this solution,

$$\varphi(x, t) = \varphi_0(x) + \delta\varphi(x, t), \tag{A1}$$

and substituting $\varphi(x, t)$ back into the equation of motion (33), in the short-wavelength limit where the changes in $|\nabla\varphi_0|$ can be neglected, we find

$$\partial_\mu \partial^\mu \delta\varphi(x, t) + u_i u_j \partial^i \partial^j \delta\varphi(x, t) + \frac{V''(\varphi_0)}{\mu^2} |\nabla\varphi_0| \delta\varphi(x, t) \simeq 0. \tag{A2}$$

Note that $\mu^2 < 0$ to ensure energy positivity, as discussed in Sec. III. In Fourier space $\delta\varphi(x, t) = \int \frac{d^4k}{(2\pi)^4} \varphi_k e^{k_\mu x^\mu}$, this equation imposes the following dispersion relation modes:

$$\omega^2 - |\mathbf{k}_\parallel|^2 + \frac{V''(\varphi_0)}{\mu^2} |\nabla\varphi_0| = 0, \tag{A3}$$

where $\omega = k^0$ and $\mathbf{k}_\parallel$ is the component of the spatial wave vector parallel to a constant field surface. Equation (A3) describes modes that propagate along the surfaces of constant φ and are thus analogous to waves on the surface of a bubble.

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