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Is there a paradox in CP asymmetries of $\tau^{\pm} \to K_{L,S}\pi^{\pm}\nu$ decays?

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Based on the description of unstable $K_{L,S}$ particles in quantum field theory (QFT), we compute the time-dependent probabilities for transitions between asymptotic states in $\tau^{\pm} \to [\pi^+\pi^-]_K\pi^{\pm}\nu$ decays, where the pair $[\pi^+\pi^-]_K$ is the product of (intermediate state) neutral kaon decays. Then we propose a definition of τ decays into K_L and K_S states, which reflects into the cancellation between their CP rate asymmetries, thus solving in a natural way the paradox pointed out in [I. I. Bigi and A. I. Sanda, Phys. Lett. B 625, 47 (2005).]. Since our definition of $K_{L,S}$ final states in τ decays is motivated on experimental grounds, our predictions for the integrated CP rate asymmetries can be tested in a dedicated experiment.

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I. INTRODUCTION

In a recent paper, Bigi and Sanda [1] have pointed out that $\tau^{\pm} \to K_{L,S}\pi^{\pm}\nu_{\tau}$ decays exhibit a CP asymmetry of the same size as the one measured in the charge asymmetry of semileptonic K_L decays. The "known" CP rate asymmetries for K_S and K_L final states [1]

$$\frac{\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}) - \Gamma(\tau^- \to K_S\pi^-\nu)}{\Gamma(\tau^+ \to K_S\pi^+\bar{\nu}) + \Gamma(\tau^- \to K_S\pi^-\nu)} = |p|^2 - |q|^2, \tag{1}$$

$$\frac{\Gamma(\tau^+ \to K_L\pi^+\bar{\nu}) - \Gamma(\tau^- \to K_L\pi^-\nu)}{\Gamma(\tau^+ \to K_L\pi^+\bar{\nu}) + \Gamma(\tau^- \to K_L\pi^-\nu)} = |p|^2 - |q|^2, \tag{2}$$

turn out to be identical. If true, this would indicate a paradox because the total rates of τ^+ and τ^- would be different, in contradiction with the CPT theorem.

Bigi and Sanda [1] proposed a solution to this contradiction by looking at the time evolution of the K^0 ($\bar{K}^0$) state produced in τ^+ (τ^-) decays at $t = 0$. They concluded that the sum of time integrated rates (from $t = 0$ to ∞) over all the final states that can be reached by neutral kaon decays is free from such CP asymmetries, restoring the equality of the $\tau^{\pm}$ lifetimes. In their discussion of the problem [1], the interference of the $K_{S,L}$ states in the time-dependent rates plays an essential role in the cancellation of the contributions of pure K_L and K_S exponential decays.

In this paper we approach this problem from a different point of view. Based on the description of unstable $K_{L,S}$ particles in quantum field theory (QFT), we compute the time evolution of transition amplitudes between physical (*in* and *out*) states. Let us point out that, in the evaluation of the matrix elements giving rise to the rates that enter Eqs. (1) and (2), neutral kaons ($K_{L,S}$) are assumed to be *asymptotic* physical states (defined as *outgoing* states at $t \to +\infty$). Since unstable particles in QFT enter as intermediate states of the physical amplitudes, the paradox contained in Eqs. (1) and (2) does not appear. We compute the time-dependent probabilities for transitions between asymptotic states in $\tau^{\pm} \to [\pi^+\pi^-]_K\pi^{\pm}\nu$, where $[\pi^+\pi^-]_K$ is the pair produced from neutral kaon decays. Then we propose a definition of τ decays into $K_{L,S}$ states, which reflects into a natural cancellation between the CP rate asymmetries defined in Eqs. (1) and (2).

Searches for CP violation effects in a double kinematical distribution of $\tau^{\pm} \to K_S\pi^{\pm}\nu_{\tau}$ decays have been pursued recently by the CLEO Collaboration [2]. Prospects for improved experimental searches are interesting in the light of the larger data samples of τ pairs accumulated at B-factories [3]. These exclusive decays can be used to provide further tests on the violation of the CP symmetry [1,2,4,5]. On the other hand, within the standard model, the CP rate asymmetry turns out to be negligibly small (of order 10^{-12}) in $\tau^{\pm} \to K^{\pm}\pi^0\nu_{\tau}$ decays [5], opening a large window to consider the effects of New Physics contributions. Indeed, virtual effects of supersymmetric particles may enhance this CP rate asymmetry to the level of $10^{-7} \sim 10^{-6}$ [6].

II. TIME-DEPENDENT AMPLITUDES IN τ LEPTON DECAYS

In this section we focus on calculation of the time evolution amplitudes of τ lepton decays into asymptotic

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physical states, as dictated by the S-matrix formalism of QFT.

Let us start by defining, at time $t = 0$, the τ lepton decays into virtual strange states of definite flavor ($\tau^+ \to K^0\pi^+\nu$ or $\tau^- \to \bar{K}^0\pi^-\bar{\nu}$). The decay amplitudes of these two processes are the same owing to the CPT invariance of weak interactions, and will be denoted by A. After being produced, initial kaon states of definite flavor evolve as a mixing of two *orthogonal* states of definite mass and width (K_L and K_S) and decay a later time $t = \tau$ into a final state f. For definiteness we will confine ourselves to $f = \pi^+\pi^-$ (see Fig. 1).

In order to introduce the CP violating effects, we change from the basis of definite flavor ($K^0, \bar{K}^0$) to the basis of definite CP parity (K_1, K_2) of neutral kaons

$$\begin{pmatrix} K_1 \\ K_2 \end{pmatrix} \equiv \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} K^0 \\ \bar{K}^0 \end{pmatrix} \tag{3}$$

with the convention $CP|K^0\rangle = |\bar{K}^0\rangle$.

In momentum space, the transition amplitudes for the $K_1 - \bar{K}_2$ system propagating with momentum p from $t = 0$ to $t = \tau$, are given by (see for example [7]):

$$D_R^{K_1-K_2}(p^2) = \frac{1}{1-\epsilon^2}\begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix}\begin{pmatrix} d_S^{-1} & 0 \\ 0 & d_L^{-1} \end{pmatrix}\begin{pmatrix} 1 & -\epsilon \\ -\epsilon & 1 \end{pmatrix}, \tag{4}$$

where $d_{S,L} \equiv p^2 - M_{S,L}^2 + iM_{S,L}\Gamma_{S,L}$. Equivalently, the matrix $D_D(p^2) = \mathrm{diag}(d_S^{-1}, d_L^{-1})$ describes the propagation of the two independent physical modes of definite mass ($M_{L,S}$) and width ($\Gamma_{L,S}$). The complex parameter ϵ that enters the (left and right) diagonalizing matrices of the $K_1 - K_2$ propagator system, describes the CP violation induced by the mixing of neutral kaons [8]. Such forms for the left and right diagonalizing matrices are identical to the ones appearing in the definition of the direct and reciprocal basis for $K_{L,S}$ states [9].

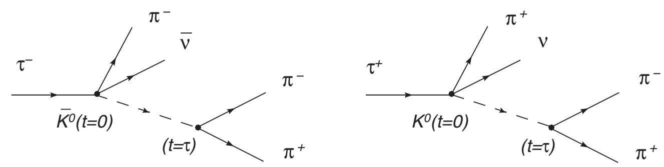

FIG. 1. τ lepton decay into neutral kaon states of definite flavor at time $t = 0$. These unstable kaon states evolve as a mixing of two orthogonal states of definite mass and width (K_L and K_S) and decay into a $\pi^+\pi^-$ pair at time $t = \tau$.

Following the procedure discussed in Ref. [7], we compute now the time-dependent transition amplitude of the whole $\tau^-(q) \to \bar{K}^0(p_1 + p_2; t = 0)\pi^-(k)\nu(q') \to \pi^+(p_1)\pi^-(p_2)\pi^-(k)\nu(q')$ decay process. The pair $\pi^+(p_1)\pi^-(p_2)$ is the product of kaon decay at time $t = \tau$, thus we define $E = p_1^0 + p_2^0$ as the total energy of the $\pi^+\pi^-$ pair and $E_{S,L} = \sqrt{(\vec{p}_1 + \vec{p}_2)^2 + M_{S,L}^2}$. The time-dependent amplitude becomes

$$\mathcal{T}_-(\tau) = (2\pi)^4\delta^{(4)}(q - k - q' - p_1 - p_2)\frac{A}{1-\epsilon}\mathcal{M}(K_1 \to \pi^+\pi^-) \times e^{iE\tau}\left\{\frac{1+\chi_{+-}\epsilon}{2E_S}e^{-iE_S\tau}e^{-(1/2)\Gamma_S(M_S/E_S)\tau} - \frac{\epsilon+\chi_{+-}}{2E_L}e^{-iE_L\tau}e^{-(1/2)\Gamma_L(M_L/E_L)\tau}\right\}. \tag{5}$$

Similarly, we can obtain the corresponding time-dependent amplitude for the $\tau^+(q) \to K^0(p_1 + p_2; t = 0)\pi^+(k)\bar{\nu}(q') \to \pi^+(p_1)\pi^-(p_2)\pi^+(k)\bar{\nu}(q')$ decay

$$\mathcal{T}_+(\tau) = (2\pi)^4\delta^{(4)}(q - k - q' - p_1 - p_2)\frac{A}{1+\epsilon}\mathcal{M}(K_1 \to \pi^+\pi^-) \times e^{iE\tau}\left\{\frac{1+\chi_{+-}\epsilon}{2E_S}e^{-iE_S\tau}e^{-(1/2)\Gamma_S(M_S/E_S)\tau} + \frac{\epsilon+\chi_{+-}}{2E_L}e^{-iE_L\tau}e^{-(1/2)\Gamma_L(M_L/E_L)\tau}\right\}. \tag{6}$$

In the above expressions we have defined the direct CP violation parameter [7]

$$\chi_{+-} \equiv \frac{\mathcal{M}(K_2 \to \pi^+\pi^-)}{\mathcal{M}(K_1 \to \pi^+\pi^-)}, \tag{7}$$

and $\mathcal{M}(K_{1,2} \to \pi^+\pi^-)$ denote the "instantaneous" decay amplitudes (at $t = \tau$) of the CP eigenstates into a $\pi^+\pi^-$ pair. Note that in Eqs. (5) and (6) we do not need to symmetrize the decay amplitudes because identical pions are produced at different space-time locations.

For further use in the following discussion we introduce the usual CP violation parameter

$$\eta_{+-} = \frac{\epsilon + \chi_{+-}}{1 + \chi_{+-}\epsilon} = |\eta_{+-}|e^{i\phi_{+-}}. \tag{8}$$

The time-dependent amplitudes given in Eqs. (5) and (6) are defined in an arbitrary reference frame [7]. Now, we choose the center of mass frame of the pion pair produced at $t = \tau$, which means $E_{S,L} = M_{S,L}$. The expressions for the time-dependent probabilities (up to the second order in the CP violation parameter and neglecting direct CP violation, i.e. $\chi_{+-} = 0$) are given by

$$|\mathcal{T}_-(\tau)|^2 \simeq \frac{B(1+2\,\mathrm{Re}[\epsilon])}{4M_S^2}[e^{-\Gamma_S\tau} + |\epsilon|^2 e^{-\Gamma_L\tau} - 2|\epsilon|e^{-(1/2)(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-})] \quad (9)$$

$$|\mathcal{T}_+(\tau)|^2 \simeq \frac{B(1-2\,\mathrm{Re}[\epsilon])}{4M_S^2}[e^{-\Gamma_S\tau} + |\epsilon|^2 e^{-\Gamma_L\tau} + 2|\epsilon|e^{-(1/2)(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-})], \quad (10)$$

where we have defined the common factor $B = (2\pi)^4\delta^{(4)}(q-k-q'-p_1-p_2)|A\mathcal{M}(K_1\to\pi^+\pi^-)e^{iE\tau}|^2$ and $\Delta m = M_L - M_S$. Such expressions for the time-dependent probabilities are similar to the ones obtained in the framework of the usual Lee-Oehme-Yang formalism [10] and used in several experimental analysis (see for example [11]).

With the above expressions we can calculate the time-dependent *CP* rate asymmetry for the processes under consideration. We get the result

$$A_{+-}(\tau) = \frac{|\mathcal{T}_-(\tau)|^2 - |\mathcal{T}_+(\tau)|^2}{|\mathcal{T}_-(\tau)|^2 + |\mathcal{T}_+(\tau)|^2} \simeq 2\,\mathrm{Re}[\epsilon]\left[\frac{-\frac{1}{\cos\phi_{+-}}e^{-1/2(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-}) + e^{-\Gamma_S\tau} + |\epsilon|^2 e^{-\Gamma_L\tau}}{e^{-\Gamma_S\tau} + |\epsilon|^2 e^{-\Gamma_L\tau}}\right]. \quad (11)$$

Note that this *CP* asymmetry, defined for an arbitrary decay time τ of neutral kaons, does not vanish.

III. A DEFINITION FOR THE $K_{L,S}$ STATES

Strictly speaking, within the *S*-matrix formalism we cannot define physical amplitudes for final states containing unstable particles (they are not asymptotic states). Instead, unstable particles appear as intermediate states of the *S*-matrix amplitude connecting the production and decay processes. However, we can resort to a definition of $\tau^\pm \to K_{L,S}\pi^\pm\nu$ decay rates by introducing a time scale T which allows to separate neutral kaon decays that occur at short and long decay times.

Thus, in order to define the *CP* asymmetry for τ decays into an specific K meson final state, one has first to adopt a definition of the amplitude with $K_{L,S}$ states. Because of the large difference between K_L (t_L) and K_S (t_S) lifetimes, it is possible to adopt a procedure to separate 'K_L' and 'K_S' events. We introduce a time scale T such that $t_S \ll T \ll t_L$. In this way, decays of neutral kaons that occur for $t \sim O(t_S)$ can be identified as K_S events, while those taking place for $t \sim O(t_L)$ would be identified as K_L events. As is usually done, to talk about a beam of K_L mesons we should look at $\pi^+\pi^-$ pairs produced after a time $t \geq T$. Inversely, at short decay times ($t \leq T$), our $\pi^+\pi^-$ pairs will originate mainly from decays of the K_S component.

Thus, in analogy with Ref. [1], we can define the *CP* rate asymmetry of τ leptons into $K_{S,L}\pi$ final states from our Eq. (11). The time integrated *CP* rate asymmetries for τ decays into K_S (respectively K_L) become

$$A^S_{CP} \approx \frac{\int_0^T(-2|\epsilon|e^{-1/2(\Gamma_S+\Gamma_L)\tau}\cos(\Delta m\tau - \phi_{+-}) + 2\,\mathrm{Re}[\epsilon]e^{-\Gamma_S\tau})d\tau}{\int_0^T e^{-\Gamma_S\tau}d\tau}, \quad (12)$$

$$A^L_{CP} \approx \frac{\int_T^\infty 2\,\mathrm{Re}[\epsilon]e^{-\Gamma_L\tau}d\tau}{\int_T^\infty e^{-\Gamma_L\tau}d\tau}, \quad (13)$$

where we have used the approximations $\Gamma_S \gg \Gamma_L$, $t_S \ll T \ll t_L$ and $\tan\theta_{+-} \approx 2\Delta m/\Delta\Gamma$, with $\Delta\Gamma = \Gamma_S - \Gamma_L \approx \Gamma_S$. Note also that in the approximation we are using of neglecting direct *CP* violation, $\mathrm{Re}[\epsilon] = |\epsilon|\cos\theta_{+-}$ (see Eq. (8)). A straightforward evaluation of the *CP* asymmetries from Eqs. (12) and (13) gives

$$A^L_{CP} \approx -A^S_{CP} \approx 2\,\mathrm{Re}[\epsilon]. \quad (14)$$

In other words, the *CP* asymmetry integrated over all decay times for a *given* physical final state does not exhibit the paradox discussed in the Introduction. Furthermore, it comes out that the "experimentally motivated" definitions of the *CP* asymmetries for $\tau^\pm$ decays into K_L and K_S proposed above cancel each other.

IV. CONCLUSIONS

In the framework of the *S*-matrix formalism of QFT, we compute the time evolution amplitudes for $\tau^\pm \to [\pi^+\pi^-]_{\mathcal{K}}\pi^\pm\nu_\tau$ decays, where $[\pi^+\pi^-]_{\mathcal{K}}$ denote the decay products of a unstable neutral kaon. By introducing a definition of decay rates for short and long decay times of neutral unstable kaons, we show that the integrated *CP* rate asymmetries of these decays cancels each other in a natural way. Therefore, in the description of unstable states based on the *S*-matrix formalism there is not a paradox like the one described in Ref. [1]. Since our definition of $K_{L,S}$ final states in τ decays is motivated on experimental grounds, our predictions for the integrated *CP* rate asymmetries (Eqs. (14)) can be tested in a dedicated experiment.

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