Bilepton effects on the WWV^* vertex in the 331 model with right-handed neutrinos via a $SU_L(2) \times U_Y(1)$ covariant quantization scheme

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In a recent paper [J. Montano, F. Ramírez-Zavaleta, G. Tavares-Velasco, and J. J. Toscano, Phys. Rev. D **72**, 055023 (2005).], we investigated the effects of the massive charged gauge bosons (bileptons) predicted by the minimal 331 model on the off-shell vertex WWV^* ($V = \gamma$, Z) using a $SU_L(2) \times U_Y(1)$ covariant gauge-fixing term for the bileptons. We proceed along the same lines and calculate the effects of the gauge bosons predicted by the 331 model with right-handed neutrinos. It is found that the bilepton effects on the WWV^* vertex are of the same order of magnitude as those arising from the standard model and several of its extensions, provided that the bilepton mass is of the order of a few hundred of GeVs. For heavier bileptons, their effects on the WWV^* vertex are negligible. The behavior of the form factors at high energies is also discussed as it is a reflection of the gauge invariance and gauge independence of the WWV^* Green function obtained via our quantization method.

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I. INTRODUCTION

Radiative corrections to the WWV ($V = \gamma$, Z) vertex have long received considerable attention. Apart from its sensitivity to new physics effects, this vertex has theoretical interest of its own as it may serve as a probe of the gauge sector of the standard model (SM). In this context, the one-loop contributions to the $WW\gamma$ vertex, which defines the static electromagnetic properties of the W boson, have been calculated in the SM [1,2] and several of is extensions such as two-Higgs doublet models (THDMs) [3], supersymmetric models [4], 331 models [5,6], etc. Similar attention has been paid to the study of the WWZ vertex. Even though the attention has focused mainly on the static properties of the W boson, a more comprehensive study of the W boson properties requires the analysis of the off shell WWV vertex, particularly when the neutral V boson is off shell and the W bosons are on shell as the resulting vertex could be tested with high precision at the CERN large hadron collider (LHC) or the planned future linear colliders. It is well known that, although the on-shell WWV vertex renders a gauge-independent amplitude by itself, gauge independence is lost once any external particle goes off shell, thereby requiring the use of a nonconventional calculation scheme. The difficulties to obtain a gauge-independent Green function for the WWV^* vertex have long been known. In particular, a careless calculation of the radiative corrections to the WWV^* vertex via a conventional gauge-fixing procedure irremediably leads to an ill-behaved gauge dependent Green function. For instance, one-loop corrections to the WWV^* vertex were first calculated in the SM by the authors of Ref. [2] using the Feynman-'t Hooft gauge via a conventional gauge-fixing scheme. The result was an infrared divergent Green function that violates unitarity. Aware of the fact that an off-shell Green function is generally gauge dependent, the authors of Ref. [7] invoked a diagrammatic method known as the pinch technique (PT) [8] to obtain a gauge-independent WWV^* Green function satisfying the requirements of gauge invariance and infrared finiteness. The PT exploits the fact that, although the off-shell Green functions are gauge dependent, the S-matrix elements to which they contribute are gauge independent. In this way, order by order in perturbation theory, one can construct a gauge-independent Green function for an off-shell vertex by combining its contribution with all other Feynman diagrams that also contribute to a particular physical process, thereby getting rid of any ξ dependent terms. In this respect, while the WWV^* vertex by itself does not represent a physical process, it can contribute to a gauge-invariant physical process such as $e^+e^- \to W^+W^-$ scattering. By this method, a gauge-invariant and gauge-independent WWV^* Green function can be constructed. The PT was also used in Ref. [9] to calculate the one-loop contributions to the WWV^* vertex from the unconstrained minimal supersymmetric standard model (MSSM).

The PT is thus a diagrammatic method that allows one to remove any unphysical gauge dependent term from an off-shell n-point function at the level of the Feynman graphs contributing to a certain physical process, thereby yielding a well-behaved gauge-independent and gauge-invariant Green function. The PT may turn itself, however, into a somewhat cumbersome task. There is also the alternative of removing any gauge dependent term at the level of the generating functional. In this respect, although the background field method (BFM) [10] renders a gauge-invariant quantum action, no mechanism has been found yet that allows one to obtain both gauge-invariant and gauge-independent Green functions from the generating functional. Eventually, a connection between the PT and the BFM has been found [11]: it turns out that the gauge-invariant and gauge-independent Green function obtained via the PT is exactly reproduced if it is calculated through the BFM using the Feynman rules given in the Feynman-'t Hooft gauge ($\xi = 1$). Although such a correspondence, which so far remains a puzzle, was first established at the one-loop level [12], it has been shown that it persists at all orders of perturbation theory [13]. Therefore, instead of using the PT, one can use the BFM Feynman-'t Hooft gauge to calculate gauge-independent off-shell amplitudes. Along these lines, in Ref. [14] we calculated the one-loop contributions to the WWV^* vertex from the gauge sector of the so-called minimal 331 model [15,16]. In order to construct a well-behaved gauge-independent Green function, rather than using the PT, we invoked an alternative method inspired in the BFM and the Becchi, Rouet, Stora, and Tyutin (BRST) symmetry [17], which is well suited to study the sensitivity of the WWV^* vertex to the virtual effects of the new gauge bosons predicted by 331 models. We will follow a similar approach here to calculate the one-loop contributions from the 331 model with right-handed neutrinos [18]. Although we will shortly describe the calculation scheme below, for more details we refer the reader to Ref. [14].

331 models have been the source of great interest lately. Recent studies within the framework of this class of theories have focused on neutrino physics [19], Z' physics [20], Higgs boson physics [21], bilepton physics [6,14,22,23], supersymmetric extensions [24], dark matter [25], *CP* violation [26], and theoretical aspects [27]. Models of this kind are based on the $SU_L(3) \times U_X(1)$ gauge symmetry [15,16] and are unique in the sense that they require that all 3 fermion families be summed up in order to cancel anomalies, in contrast with other models in which anomaly cancellation is achieved family by family. As a consequence, 331 models require that the number of fermion families be a multiple of 3, the quark color number. This may suggest a solution to the family replication problem. Apart from this feature, 331 models are interesting as they predict a pair of massive gauge bosons arranged in a doublet of the electroweak group. While the minimal 331 model predicts a pair of singly charged $Y^{\pm}$ and a pair of doubly charged $Y^{\pm\pm}$ gauge bosons, the model with right-handed neutrinos predicts a pair of neutral no self-conjugate gauge bosons Y^0 instead of the doubly charged ones. These new gauge bosons are called bileptons since they carry two units of lepton number, thereby being responsible for lepton-number violating interactions [18]. The neutral bilepton has been deemed a promising candidate in accelerator experiments since it may be the source of neutrino oscillations [28]. The reason why the effects of the bileptons on the WWV^* vertex are worth studying is because their couplings to the SM gauge bosons are rather similar to the SM gauge boson couplings. Current bounds put the bilepton masses in the range of a few hundred GeVs [29], which means that the bileptons may show up via their virtual effects in low-energy observables. This is an important reason to investigate the effect of these particles on the WWV^* vertex. Furthermore, due to the spontaneous symmetry-breaking (SSB) hierarchy, the splitting between the charged and neutral bilepton masses is bounded by m_W, so the bilepton masses might be almost degenerate since they are expected to be heavier than m_W. We will see below that this is particularly suited for our calculation method.

The rest of the paper is organized as follows. In Sec. II we present a brief introduction to the 331 model with right-handed neutrinos. A survey of the quantization method is presented in Sec. III along with a detailed discussion on the gauge-fixing procedure for the bileptons, whereas Sec. IV is devoted to the analytical results and the analysis. Finally, the conclusions are presented in Sec. V.

II. THE 331 MODEL WITH RIGHT-HANDED NEUTRINOS

The 331 model with right-handed neutrinos was first introduced in Ref. [18]. In a previous paper, we worked in the context of this model and calculated the static electromagnetic properties of the W boson [6]. Also, the one-loop contributions to the static electromagnetic properties of the neutral no self-conjugate Y^0 boson were calculated in Ref. [23]. It was shown that the main contributions to the $WW\gamma$ vertex arise from the gauge sector, i.e. from the bileptons, as the fermion sector does not contribute and the Higgs sector gives a negligible contribution quite similar to that arisen in a THDM [6]. Therefore, although for completeness we will present a short description of the main features of the model, the following discussion will be mainly focused on the gauge sector as it is the one which is expected to give the dominant contributions to the WWV^* vertex. Furthermore, we will see that our calculation scheme is suited to analyze the effects of the bileptons on the WWV^* Green function.

In the fermion sector of the 331 model with right-handed neutrinos, leptons of the same generation are arranged in left-handed triplets and right-handed singlets, the same being true for each quark generation. Apart from the introduction of right-handed neutrinos, there are three new quarks, D_1, D_2, and T, which have the following electric charge: $Q_{D_i} = -1/3e$ and $Q_T = 2/3e$. In order to cancel the $SU_L(3)$ anomaly, two quark families transform as $SU_L(3)$ antitriplets and the remaining one as a triplet. At the first stage of SSB, when the $SU_L(3) \times U_N(1)$ group is spontaneously broken into $SU_L(2) \times U_Y(1)$, the new quarks get their masses and emerge as singlets of the electroweak group. Consequently, they cannot interact with the W boson at the tree level. It follows that there is no contribution from the new quarks to the WWV^* vertex at the one-loop level.

As far as the scalar sector is concerned, several Higgs sectors have been proposed in the literature to achieve the

SSB in 331 models [30]. As for the 331 model with right-handed neutrinos, it requires the simplest Higgs sector of this class of theories [18], i.e. only three triplets of $SU_L(3)$ are required to reproduce the SM physics at the Fermi scale:

$$\chi = \begin{pmatrix} \Phi_Y \\ \chi'^0 \end{pmatrix} \sim (1, 3, -1/3), \quad \rho = \begin{pmatrix} \Phi_1 \\ \rho'^+ \end{pmatrix} \sim (1, 3, 2/3), \quad \eta = \begin{pmatrix} \Phi_2 \\ \eta'^0 \end{pmatrix} \sim (1, 3, -1/3), \tag{1}$$

where $\Phi_Y^\dagger = (G_Y^{0*}, G_Y^+)$, $\Phi_1^\dagger = (\rho^-, \rho^{0*})$, and $\Phi_2^\dagger = (\eta^{0*}, \eta^+)$ are $SU_L(2) \times U_Y(1)$ doublets with hypercharge -1, 1, and -1, respectively. The vacuum expectation values are $\langle\chi\rangle^T = (0, 0, w/\sqrt{2})$, $\langle\rho\rangle^T = (0, u/\sqrt{2}, 0)$, and $\langle\eta\rangle^T = (v/\sqrt{2}, 0, 0)$. The triplet χ breaks the $SU_L(3) \times U_N(1)$ group into $SU_L(2) \times U_Y(1)$ at the w scale, whereas ρ and η are meant to break $SU_L(2) \times U_Y(1)$ into $U_e(1)$.

In the gauge sector, the model predicts the existence of five new gauge bosons: two singly charged $Y^\pm$, two neutral no self-conjugate Y^0, and a neutral self-conjugate Z'. All these new gauge bosons and the new quarks as well acquire their masses at the w scale. At this stage of SSB, the Z' boson emerges as a singlet of the electroweak group and so no interaction between the Z' boson and the W boson arises. However, in the following stage of SSB, at the Fermi scale, the Z' boson couples with the W boson via the $Z - Z'$ mixing angle. As a consequence, the Z' contribution to the WWV^* vertex will be proportional to the square of the $Z - Z'$ mixing angle, which is highly suppressed according to the most recent experimental bounds [31,32]. We will thus ignore this contribution in this work. In sharp contrast, the bileptons arise as a doublet of the electroweak group at the w scale, which means that they couple to the SM gauge bosons in a rather peculiar way: due to the fact that the $SU_L(2)$ group is completely embedded in $SU_L(3)$, the bileptons couplings to the SM gauge bosons have similar strength and Lorentz structure as those of the SM gauge boson couplings themselves. These new interactions, which arise solely from the Yang-Mills sector, do not involve any mixing angle and are entirely determined by the $SU_L(2)$ coupling constant and the weak angle θ_W.

In the first stage of SSB, the bilepton masses are degenerate: $m_{Y^0} = m_{Y^\pm} = m_Y = gw/2$. However, when $SU_L(2) \times U_Y(1)$ is broken down to $U_e(1)$, the bilepton mass degeneration is also broken. Once the Higgs kinetic-energy sector is diagonalized, there emerge the following mass-eigenstate fields:

$$Y_\mu^0 = \frac{1}{\sqrt{2}}(A_\mu^4 - iA_\mu^5), \tag{2}$$

$$Y_\mu^- = \frac{1}{\sqrt{2}}(A_\mu^6 - iA_\mu^7), \tag{3}$$

$$W_\mu^+ = \frac{1}{\sqrt{2}}(A_\mu^1 - iA_\mu^2), \tag{4}$$

with masses $m_{Y^0}^2 = g^2(w^2 + u^2)/4$, $m_{Y^\pm}^2 = g^2(w^2 + v^2)/4$, and $m_W^2 = g^2(u^2 + v^2)/4$. The remaining three gauge fields A_μ^3, A_μ^8, and N_μ define the self-conjugate mass eigenstates. From the above expressions, it is straightforward to obtain the following upper bound on the bilepton mass splitting [18]:

$$|m_{Y^0}^2 - m_{Y^\pm}^2| \le m_W^2. \tag{5}$$

Therefore, the bilepton masses become nearly degenerate when they are much larger than m_W. While in the minimal 331 model an upper bound on the bilepton masses of the order of 1 TeV can be derived from the theoretical constraint $\sin^2\theta_W \le 1/4$ [16,32,33], which is obtained from matching the gauge couplings constants at the $SU_L(3) \times U_X(1)$ breaking scale, in the version with right-handed neutrinos the corresponding bound is highly relaxed as the theoretical constraint is $\sin^2\theta_W \le 3/4$ [18].

The Yang-Mills sector induces all the couplings we need to compute the bilepton contribution to the WWV^* vertex, so we will devote particular attention to it. The respective Lagrangian can be written as

$$\mathcal{L}_{\rm YM} = -\tfrac{1}{4}F_{\mu\nu}^a F_a^{\mu\nu} - \tfrac{1}{4}N_{\mu\nu}N^{\mu\nu}, \tag{6}$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc}A_\mu^b A_\nu^c$ and $N_{\mu\nu} = \partial_\mu N_\nu - \partial_\nu N_\mu$, with f^{abc} the structure constants associated with $SU_L(3)$. When the mass-eigenstate fields are introduced, the above Lagrangian can be split into three $SU_L(2) \times U_Y(1)$-invariant terms [18]:

$$\mathcal{L}_{\rm YM} = \mathcal{L}_{\rm YM}^{\rm SM} + \mathcal{L}_{\rm YM}^{\rm SM\text{-}NP} + \mathcal{L}_{\rm YM}^{\rm NP}. \tag{7}$$

While $\mathcal{L}_{\rm YM}^{\rm SM}$ stands for the usual SM Yang-Mills Lagrangian, $\mathcal{L}_{\rm YM}^{\rm SM\text{-}NP}$ comprises the interactions between the new gauge bosons and the SM ones. The latter Lagrangian is the only one relevant for the present discussion and we will get back to it later. As for the last term, $\mathcal{L}_{\rm YM}^{\rm NP}$, it induces the couplings of the Z' boson to the bileptons. At the one-loop level there are no contributions to the WWV^* vertex induced by this Lagrangian.

In order to calculate the one-loop correction to the WWV^* vertex, we will introduce a $SU_L(2) \times U_Y(1)$-covariant gauge-fixing procedure for the bileptons. This will be discussed in the following section.

III. A $SU_L(2) \times U_Y(1)$ COVARIANT GAUGE-FIXING PROCEDURE FOR THE BILEPTONS

A. Overview of the quantization method

We now turn to present an overview of the quantization method used to obtain a gauge-invariant and gauge-independent Green function for the WWV^* vertex in the 331 model with right-handed neutrinos. This method, which is inspired in the BFM and BRST formalism, was comprehensively discussed in our previous work [14] and we refrain from presenting a detailed discussion here.

The BFM [10] is a powerful tool that allows one to construct a gauge-invariant quantum action out of which gauge-invariant Green functions can be obtained that are free of pathologies and satisfy simple Ward identities. In a conventional quantization formalism, all the fields appearing in the action are quantized. In the BFM, the gauge fields are decomposed into a quantum field and a classical (background) field. While the quantum fields are integrated out, the background fields are treated as sources. As a result, the quantum fields can only appear as internal lines in loop diagrams, whereas the background fields appear as external lines. In principle, it is necessary to gauge fix both the quantum and the classical fields in order to define S-matrix elements, but it is enough to gauge fix the former to obtain off-shell Green functions. In this respect, while gauge invariance with respect to the quantum fields is broken when they are gauge fixed, this process leaves unaltered the gauge invariance with respect to the classical fields. The resulting Green functions will be gauge invariant but they will still maintain the dependence on the gauge parameter that characterizes the gauge-fixing scheme used for the quantum fields. However, we can exploit the connection between the BFM and the PT to obtain a gauge-invariant and gauge-independent Green function. We just need to use the BFM Feynman rules given in the Feynman-'t Hooft gauge. We will see below that our quantization method incorporates some features of the BFM.

Although gauge invariance with respect to the full theory is broken when a conventional quantization scheme is applied, one can still preserve the gauge invariance under a subgroup of the theory. This approach is well suited when the virtual effects of the gauge fields associated with the subgroup are expected to be considerably small. In this context, following the philosophy of the effective Lagrangian approach, where a $SU_L(2) \times U_Y(1)$ effective Lagrangian is constructed out of the light (SM) fields to assess the effects of the heavy fields on a low-energy observable, it would be convenient to analyze the virtual effects of the bileptons on the WWV^* Green function in a $SU_L(2) \times U_Y(1)$-covariant manner. A quantization scheme for the bileptons would only be required since the SM gauge fields would only appear as external lines. Thus, a $SU_L(2) \times U_Y(1)$-invariant effective Lagrangian can be constructed by introducing a $SU_L(2) \times U_Y(1)$-covariant gauge-fixing procedure for the bileptons, which then can be integrated out. The gauge-fixing procedure must involve the $SU_L(2) \times U_Y(1)$-covariant derivative given in the representation in which the heavy fields transform under this group, which is the reason why such gauges are known as nonlinear or covariant. This class of gauges has proved to be very useful for the calculation of radiative corrections in the SM and beyond [34].

Since we want to introduce a nonlinear gauge-fixing procedure for the bileptons, we need to be careful as the difficulties to implement the Faddeev-Popov method (FPM) [35] in such a case have long been known. More specifically, it is known that renormalizability becomes ruined when the FPM is applied to a nonlinear gauge. We can invoke instead the BRST formalism [17] to construct the most general renormalizable nonlinear gauge-fixing term [36]. We will thus introduce a $SU_L(2) \times U_Y(1)$-covariant gauge-fixing term for the bileptons, which will lead to an invariant quantum action out of which a gauge-invariant WWV^* Green function will be obtained. Invoking the connection between the PT and the BFM, the Feynman rules given in the Feynman-t' Hooft gauge will render a gauge-independent Green function.

Finally, we would like to point out that both our quantization method and the BFM are meant to construct gauge-invariant quantum actions. The main difference resides in the fact that, while the BFM allows one to analyze any new physics effects regardless the energy scale, ours is only appropriate to study heavy physics effects on low-energy (SM) Green functions. This stems from the fact that, while in the BFM gauge-invariance would be preserved with respect to the gauge group of a complete theory, in our quantization method there is only invariance under a subgroup of such a theory. In the case of the present paper, our quantization method preserves gauge invariance under the $SU_L(2) \times U_Y(1)$ group rather than $SU_L(3) \times U_N(1)$. While the calculation will be greatly simplified because of electroweak invariance, the price to be paid is that our result will only be approximate as it will only account for the bilepton effects on the WWV^* vertex at the w scale, when the bilepton masses are still degenerate. We will see below that this is indeed a good approximation.

B. Gauge-fixed Lagrangian and Feynman rules

The procedure to gauge fix the Yang-Mills Lagrangian of the 331 model with right-handed neutrinos is similar to that described in the case of the minimal 331 model. So, we refrain from presenting a detailed discussion here and refer the reader to Ref. [14].

After introducing the most general action for a Yang-Mills system consistent with BRST symmetry and renormalization theory [37], we integrate out the auxiliary fields to obtain an action defined by the following gauge-fixed $SU_L(2) \times U_Y(1)$-invariant Lagrangian:

$$\mathcal{L}_{\text{BRST}} = \mathcal{L}_{\text{YM}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}, \tag{8}$$

where $\mathcal{L}_{\text{GF}}$ and $\mathcal{L}_{\text{FP}}$ are the gauge-fixing term and the ghost sector Lagrangian, respectively. As for the Yang-Mills Lagrangian $\mathcal{L}_{\text{YM}}$, which is given in Eq. (7), for our calculation we only need the $\mathcal{L}_{\text{YM}}^{\text{SM-NP}}$ term as it is the only one that induces the interactions between the bileptons and the

SM gauge bosons. It can be expressed as

$$\begin{aligned}\mathcal{L}_{\rm YM}^{\rm SM\text{-}NP} = &-\frac{1}{2}(D_\mu Y_\nu - D_\nu Y_\mu)^\dagger (D^\mu Y^\nu - D^\nu Y^\mu) \\ &- iY_\mu^\dagger (g\mathbf{F}^{\mu\nu} + g'\mathbf{B}^{\mu\nu})Y_\nu \\ &- \frac{ig}{2}\frac{\sqrt{3-4s_W^2}}{c_W} Z'_\mu (Y_\nu^\dagger (D^\mu Y^\nu - D^\nu Y^\mu) \\ &- (D^\mu Y^\nu - D^\nu Y^\mu)^\dagger Y_\nu),\end{aligned} \tag{9}$$

where $Y_\mu^\dagger = (Y_\mu^{0*}, Y_\mu^+)$ is a doublet of the electroweak group with hypercharge -1, and $D_\mu = \partial_\mu - ig\mathbf{A}_\mu + ig'\mathbf{B}_\mu$ is the electroweak covariant derivative. We have also introduced the definitions $\mathbf{F}_{\mu\nu} = \sigma^i F_{\mu\nu}^i/2$, $\mathbf{A}_\mu = \sigma^i A_\mu^i/2$, and $\mathbf{B}_\mu = YB_\mu/2$, with σ^i the Pauli matrices.

As for the gauge-fixing term $\mathcal{L}_{\rm GF}$ and the ghost sector Lagrangian $\mathcal{L}_{\rm FP}$, they can be written as

$$\mathcal{L}_{\rm GF} = -\frac{1}{2\xi} f^{\bar a} f^{\bar a}, \tag{10}$$

and

$$\begin{aligned}\mathcal{L}_{\rm FP} = &-\bar{C}^{\bar a}(\delta f^{\bar a}) - \frac{2}{\xi} f^{\bar a bc} f^{\bar a} \bar{C}^b C^c \\ &- \frac{1}{2} f^{\bar a bc} f^{cde} \bar{C}^{\bar a} \bar{C}^b C^d C^e,\end{aligned} \tag{11}$$

where $f^{\bar a}$ are the gauge-fixing functions and $\bar{C}^{\bar a}$ stands for the antighost fields. In addition, C^a are the ghost fields associated with the A_μ^a fields, f^{abc} are the $SU_L(3)$ structure constants, and ξ is the gauge parameter.

According to Ref. [14], the most general $SU_L(2) \times U_Y(1)$-covariant gauge-fixing functions $f^{\bar a}$ consistent with renormalization theory is given by

$$\begin{aligned} f^{\bar a} &= (\delta^{\bar a b}\partial_\mu - g f^{\bar a b i} A_\mu^i) A^{\mu b} - \frac{\xi g}{\sqrt{3}} f^{\bar a b 8} \chi^\dagger \lambda^b \chi, \\ &\bar a = 4, 5, 6, 7; \qquad i = 1, 2, 3, 8. \end{aligned} \tag{12}$$

We can now insert this expression into the gauge-fixed Lagrangian. Apart from analyzing the dynamics induced by each term of the gauge-fixed Lagrangian, we would like to put special emphasis on the covariance under $SU_L(2) \times U_Y(1)$.

The covariant structure of the gauge-fixing term becomes manifest when the mass eigenstates $f_Y^0 = \frac{1}{\sqrt{2}}(f^4 - if^5)$ and $f_Y^- = \frac{1}{\sqrt{2}}(f^6 - if^7)$ are introduced in an $SU_L(2) \times U_Y(1)$ doublet,

$$\begin{aligned} f_Y &= \begin{pmatrix} f_Y^0 \\ f_Y^- \end{pmatrix} \\ &= \left(D_\mu - \frac{ig\sqrt{3-4s_W^2}}{2c_W} Z'_\mu\right) Y^\mu - \frac{ig\xi}{\sqrt{2}} \chi'^{0*} \Phi_Y. \end{aligned} \tag{13}$$

We will now decompose the gauge-fixing Lagrangian into three terms in order to analyze the dynamics it induces:

$$\mathcal{L}_{\rm GF} = \mathcal{L}_{\rm GF1} + \mathcal{L}_{\rm GF2} + \cdots, \tag{14}$$

where each term is separately $SU_L(2) \times U_Y(1)$ invariant:

$$\mathcal{L}_{\rm GF1} = -\xi^{-1}(D_\mu Y^\mu)^\dagger (D_\nu Y^\nu) - \frac{\xi g^2}{2}(\chi'^{0*}\chi'^0)(\Phi_Y^\dagger \Phi_Y), \tag{15}$$

$$\mathcal{L}_{\rm GF2} = \frac{ig}{\sqrt{2}}(\chi'^{0*}(D_\mu Y^\mu)^\dagger \Phi_Y - \chi'^0 \Phi_Y^\dagger (D_\mu Y^\mu)), \tag{16}$$

whereas the third term, denoted by $\cdots$, involves the Z' boson and is irrelevant here.

We note that the first term of $\mathcal{L}_{\rm GF1}$ defines the bilepton propagators and gives new contributions to the trilinear and quartic couplings involving the bileptons and the SM gauge bosons, which originally arise from the $\mathcal{L}_{\rm YM}^{\rm SM\text{-}NP}$ Lagrangian. After some algebra, we are led to the Feynman rules for these modified couplings, which are shown in Fig. 1. The Lorentz structure associated with the trilinear couplings is given by

$$\begin{aligned}\Gamma_{\alpha\mu\nu}(k, k_1, k_2) = &(k_2 - k_1)_\alpha g_{\mu\nu} + (k - k_2 - \xi^{-1}k_1)_\mu g_{\alpha\nu} \\ &- (k - k_1 - \xi^{-1}k_2)_\nu g_{\alpha\mu},\end{aligned} \tag{17}$$

whereas those of the quartic couplings are

$$\Gamma_{\alpha\beta\mu\nu}^{WWYY} = g_{\alpha\beta}g_{\mu\nu} - 2g_{\alpha\nu}g_{\beta\mu} + (1 + \xi^{-1})g_{\alpha\mu}g_{\beta\nu}, \tag{18}$$

and

$$\begin{aligned}\Gamma_{\alpha\beta\mu\nu}^{WVYY} = &(Q_{Y^-}^V + Q_{Y^0}^V) g_{\alpha\beta}g_{\mu\nu} \\ &+ [(1 + \xi^{-1})Q_{Y^0}^V - 2Q_{Y^-}^V] g_{\alpha\mu}g_{\beta\nu} \\ &+ [(1 + \xi^{-1})Q_{Y^-}^V - 2Q_{Y^0}^V] g_{\alpha\nu}g_{\beta\mu}.\end{aligned} \tag{19}$$

The reason why the trilinear couplings WYY and VYY have the same Lorentz structure is a consequence of $SU_L(2) \times U_Y(1)$ invariance. This is also reflected in the fact that a simple Ward identity is satisfied by these vertices:

$$k^\alpha \Gamma_{\alpha\mu\nu}(k, k_1, k_2) = \Pi_{\mu\nu}^{Y^\dagger Y^\dagger}(k_2) - \Pi_{\mu\nu}^{YY}(k_1), \tag{20}$$

where $\Pi_{\mu\nu}^{YY}(k)$ is the two-point vertex function

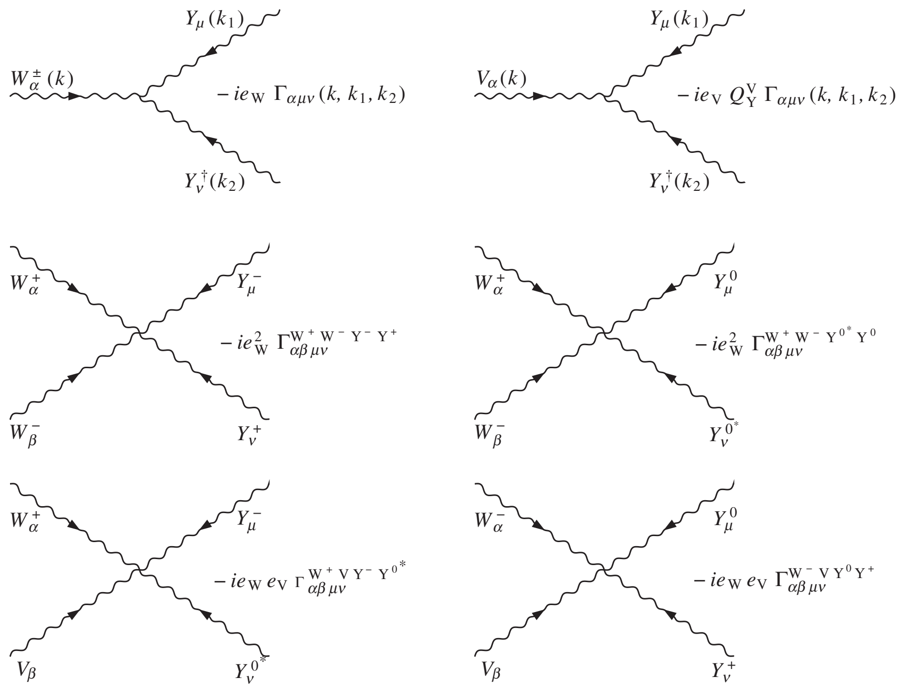

FIG. 1. Feynman rules for the trilinear and quartic vertices involving the bileptons and SM gauge fields in the $SU_L(2)\times U_Y(1)$-covariant R_ξ-gauge. $e_V = e$, $Q^V_{Y^-} = -1$, and $Q^V_{Y^0} = 0$ for $V = \gamma$, whereas $e_V = g/(2c_W)$, $Q^V_{Y^-} = 2s^2_W - 1$, and $Q^V_{Y^0} = 1$ for $V = Z$. In addition, $e_W = g/\sqrt{2}$. See Eqs. (17)–(19) for the Lorentz structures.

$$\Pi^{YY}_{\mu\nu}(k) = (-k + m^2_Y)g_{\mu\nu} - (\xi^{-1} - 1)k_\mu k_\nu. \tag{21}$$

Note that the unphysical masses for the pseudo-Goldstone bosons associated with the bileptons are determined by the scalar part of $\mathcal{L}_{\rm GF1}$, which also modifies various unphysical couplings arising from the Higgs potential. As for $\mathcal{L}_{\rm GF2}$, it allows one to remove various unphysical vertices that arise from the χ kinetic-energy sector $\mathcal{L}_{K\chi}$, which can be written as

$$\mathcal{L}_{K\chi} = \mathcal{L}_{K\chi 1} + \mathcal{L}_{K\chi 2} + \cdots, \tag{22}$$

where each term is $SU_L(2)\times U_Y(1)$-invariant by itself:

$$\mathcal{L}_{K\chi 1} = (D_\mu\Phi_Y)^\dagger(D^\mu\Phi_Y) + \partial_\mu\chi'^{0*}\partial^\mu\chi'^{0} + \frac{g^2}{2}(\chi'^{0*}\chi'^{0}Y^\dagger_\mu Y^\mu + (\Phi^\dagger_Y Y_\mu)(Y^{\mu\dagger}\Phi_Y)), \tag{23}$$

$$\mathcal{L}_{K\chi 2} = ie_W(\chi'^{0*}Y^\dagger_\mu(D^\mu\Phi_Y) + \Phi^\dagger_Y Y_\mu\partial^\mu\chi'^{0} - \text{H.c.}). \tag{24}$$

Once again, the terms denoted by $\cdots$ are irrelevant for our calculation as they involve the Z' boson.

From these expressions it is clear that $\mathcal{L}_{K\chi 1}$ induces the couplings of the pseudo-Goldstone bosons to the SM gauge bosons. The respective Feynman rules for these coupling can be extracted straightforwardly and are presented in Fig. 2. As for $\mathcal{L}_{K\chi 2}$, it induces the bilinear terms $Y^{0,0*}_\mu G^{0*,0}_Y$ and $Y^\pm_\mu G^\mp_Y$, together with the unphysical trilinear couplings $Y^{0,0*}_\mu W^{-,+}G^{+,-}_Y$, $Y^{+,-}_\mu W^{-,+}G^{0,0*}_Y$, and some quartic vertices irrelevant for our calculation. It turns out that all these couplings exactly cancel when $\mathcal{L}_{K\chi 2}$ and $\mathcal{L}_{\rm GF2}$ are combined:

$$\mathcal{L}_{K\chi 2} + \mathcal{L}_{\rm GF2} = ie_W(\chi'^{0*}\partial_\mu(Y^{\mu\dagger}\Phi_Y) + \Phi^\dagger_Y Y_\mu\partial^\mu\chi'^{0} - \text{H.c.}) + \cdots, \tag{25}$$

where $\cdots$ stands for irrelevant surface terms. This is the reason why this gauge-fixing procedure will simplify considerably our calculation as we can get rid of several Feynman diagrams involving the couplings $Y^{0,0*}_\mu W^{-,+}G^{+,-}_Y$ and $Y^{+,-}_\mu W^{-,+}G^{0,0*}_Y$.

Finally, we would like to show the covariant structure of the ghost sector and extract the Feynman rules necessary for our calculation. We introduce the mass eigenstates in $SU_L(2)\times U_Y(1)$ doublets:

$$C_Y = \begin{pmatrix} C^0_Y \\ C^-_Y \end{pmatrix} \qquad \bar{C}_Y = \begin{pmatrix} \bar{C}^0_Y \\ \bar{C}^-_Y \end{pmatrix}, \tag{26}$$

where the mass eigenstates are defined as $C^0_Y = \frac{1}{\sqrt{2}}(C^4 - iC^5)$ and $C^-_Y = \frac{1}{\sqrt{2}}(C^6 - iC^7)$, and similar expressions for the antighost field $\bar{C}_Y$. The $\mathcal{L}_{\rm FP}$ Lagrangian can thus be written as follows:

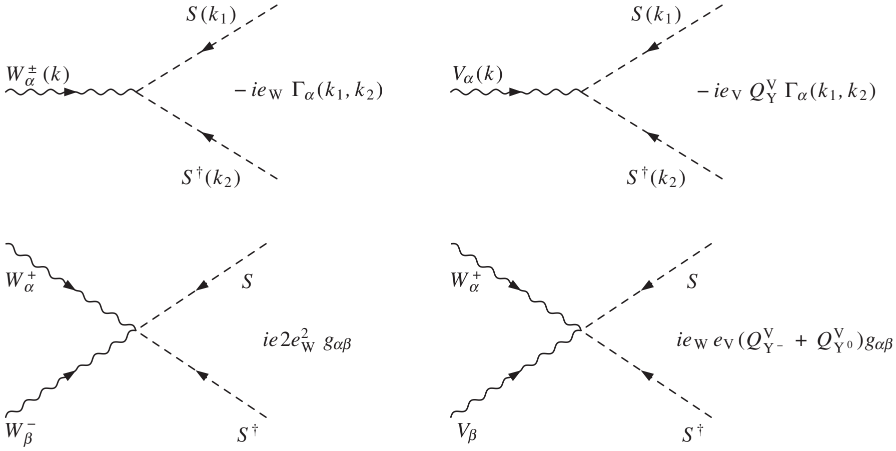

FIG. 2. Feynman rules for the trilinear and quartic vertices involving SM gauge fields and scalar unphysical particles (pseudo-Goldstone bosons and ghosts) in the $SU_L(2)\times U_Y(1)$-covariant R_ξ-gauge. In this gauge, the W and V couplings to pseudo-Goldstone bosons and ghosts coincide.

$$\begin{aligned}\mathcal{L}_{\rm FP} &= (D_\mu C_Y)^\dagger(D^\mu\bar{C}_Y) + \frac{g^2}{4}[(Y^\dagger_\mu\sigma^i Y^\mu)(C^\dagger_Y\sigma^i\bar{C}_Y) + 3(Y^\dagger_\mu Y^\mu)(C^\dagger_Y\bar{C}_Y) - 4(Y^\dagger_\mu C_Y)(Y^{\mu\dagger}\bar{C}_Y)] + \frac{ig}{\sqrt{2}}Y^\dagger_\mu M_C D^\mu\bar{C}_Y \\ &\quad + \frac{ig}{2}Y^\dagger_\mu\mathcal{M}_C\bar{C}_Y - \frac{\xi g}{2}[\chi'^{0*}\chi'^0 C^\dagger_Y\bar{C}_Y + \chi'^0\Phi^\dagger_Y M_C\bar{C}_Y - (C^\dagger_Y\Phi_Y)(\Phi^\dagger_Y\bar{C}_Y)] \\ &\quad + \frac{i\sqrt{2}}{\xi}[(\bar{M}_C C_Y + M_C\bar{C}_Y)^\dagger(D_\mu Y^\mu) - (D_\mu Y^\mu)(\bar{M}_C C_Y + M_C\bar{C}_Y)] \\ &\quad - g[\Phi^\dagger_Y(\bar{M}_C C_Y + M_C\bar{C}_Y)\chi'^0 + \chi'^{0*}(\bar{M}_C C_Y + M_C\bar{C}_Y)^\dagger\Phi_Y] + \text{H.c.} - \frac{1}{2}f^{\bar{a}bc}f^{cde}\bar{C}^{\bar{a}}\bar{C}^b C^d C^e, \end{aligned} \tag{27}$$

where

$$M_C = \begin{pmatrix} \frac{1}{\sqrt{2}}(C^3+\sqrt{3}C^8) & \frac{1}{\sqrt{2}}(C^1-iC^2) \\ \frac{1}{\sqrt{2}}(C^1+iC^2) & -\frac{1}{\sqrt{2}}(C^3-\sqrt{3}C^8)\end{pmatrix}, \tag{28}$$

$$\mathcal{M}_C = \begin{pmatrix} (\mathcal{D}^{3i}_\mu+\sqrt{3}\mathcal{D}^{8i}_\mu)C^i & (\mathcal{D}^{1i}_\mu - i\mathcal{D}^{21}_\mu)C^i \\ (\mathcal{D}^{1i}_\mu + i\mathcal{D}^{2i}_\mu)C^i & -(\mathcal{D}^{3i}_\mu-\sqrt{3}\mathcal{D}^{8i}_\mu)C^i\end{pmatrix}, \tag{29}$$

where $i = 1, 2, 3, 8$, $\mathcal{D}^{ij}_\mu = \delta^{ij}\partial_\mu - gf^{ija}A^a_\mu$ is the covariant derivative given in the adjoint representation of $SU_L(3)$, and $\bar{M}_C$ is obtained from M_C after replacing the ghost fields by the antighost fields. Under a $SU_L(2)\times U_Y(1)$ unitary transformation U, M_C transforms as $M_C\to UM_CU^\dagger$. The same is true for $\bar{M}_C$ and $\mathcal{M}_C$. It is thus clear that $\mathcal{L}_{\rm FP}$ is $SU_L(2)\times U_Y(1)$ invariant.

From $\mathcal{L}_{\rm FP}$, it is straightforward to show that the Feynman rules for the trilinear and quartic couplings involving the ghost fields and the SM gauge bosons have the same Lorentz structure than those involving the pseudo-Goldstone bosons and the SM gauge bosons, which stems from the fact that each sector is $SU_L(2)\times U_Y(1)$ invariant by itself. The Feynman rules for the ghost (antighost) fields are summarized in Fig. 2 together with the Feynman rules for the pseudo-Goldstone bosons. We see that the trilinear vertices $WS^\dagger S$ and $VS^\dagger S$, with S standing for a commutative (pseudo-Goldstone boson) or anticommutative (ghost) charged scalar, satisfy simple Ward identities:

$$k^\alpha\Gamma^{VS^\dagger S}_\alpha = \Pi^{S^\dagger S^\dagger}(k_2) - \Pi^{SS}(k_1), \tag{30}$$

where $\Gamma^{VS^\dagger S}_\alpha = (k_1-k_2)_\alpha$ and $\Pi^{SS}(k_i)$ stands for the two-point vertex functions $\Pi(k) = k^2 - \xi m^2_Y$.

IV. ANALYTICAL RESULTS AND DISCUSSION

A. A gauge-invariant and gauge-independent Green function for the WWV^* vertex

The most general transverse CP-even vertex function for the $W^+_\alpha(p-q)W^-_\beta(-p-q)V^*_\mu(q)$ coupling has the form [1,38]

$$\begin{aligned}\Gamma^V_{\alpha\beta\mu} = -ig_V\Big\{&A[2p_\mu g_{\alpha\beta} + 4(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu})] \\ &+ 2\Delta\kappa_V(q_\beta g_{\alpha\mu} - q_\alpha g_{\beta\mu}) \\ &+ \frac{4\Delta Q_V}{m^2_W}\Big(p_\mu q_\alpha q_\beta - \frac{1}{2}q^2 p_\mu g_{\alpha\beta}\Big)\Big\}. \end{aligned} \tag{31}$$

The longitudinal terms were ignored as they become negligibly small when V^* couples to a light fermion current, as in $e^-e^+\to WW$ scattering. The SM tree-level values are

$A = 1$ and $\Delta\kappa = \Delta Q = 0$. While $\Delta\kappa$, and ΔQ are ultraviolet finite at the one-loop level, A is divergent and requires renormalization. The coefficients $\Delta\kappa_\gamma$ and ΔQ_γ, obtained from the $WW\gamma$ vertex function, determine the static electromagnetic properties of the W boson, namely, its magnetic dipole moment μ_W and its electric quadrupole moment Q_W.

From the Feynman rules presented in Figs. 1 and 2, we can construct all the Feynman diagrams contributing to the WWV vertex at the one-loop level, which we show in Fig. 3, and extract the $\Delta\kappa_V$ and ΔQ_V coefficients. Although there is a similar set of Feynman diagrams for the pseudo-Goldstone bosons and the ghost fields, the unphysical particles only contribute to Eq. (31) via the triangle diagram. Moreover, it turns out that the ghost (antighost) contribution is minus twice that of the pseudo-Goldstone bosons, which is due to the separate $SU_L(2) \times U_Y(1)$ invariance of these sectors. We have evaluated the loop amplitudes in the Feynman-'t Hooft gauge via the Passarino-Veltman reduction method [39]. We have verified that the bilepton contribution can be written exactly as Eq. (31), which means the WWV^* Green function satisfies the Ward identity $\Gamma^V_{\alpha\beta\mu} q^\mu = 0$. It turns out that the form factors are the same regardless of V, which means that, just as occurs at the tree level, $\Gamma^\gamma_{\alpha\beta\mu}$ and $\Gamma^Z_{\alpha\beta\mu}$ only differ by the g_V coefficient. Thus, $SU_L(2) \times U_Y(1)$ invariance is preserved at the one-loop level. The g_V coefficient is given by $g_V = e_V(Q^V_{Y^0} - Q^V_{Y^-})$, which, after inserting the charge values, gives $g_V = e$ for $V = \gamma$ and $g_V = c_W g$ for $V = Z$.

We now introduce the following shorthand notation $Q = 2q$, $\hat{Q}^2 = Q^2/m_W^2$, $x_Y = m_Y^2/m_W^2$, $\beta(\hat{Q}^2) = 6a/(4 - \hat{Q}^2)^3$, $a = g^2/96\pi^2$, and express the $\Delta\kappa_V$ and ΔQ_V coefficients in terms of two- and three-point Passarino-Veltman scalar functions:

$$\Delta Q_V = 2\beta(\hat{Q}^2)\Big[(6x_Y G(Q^2) + 1)\hat{Q}^4 - 2(2(2x_Y + 1)F(m_W^2, Q^2) + 2x_Y F(0, Q^2) + 6(3x_Y - 1)G(Q^2) + 3)\hat{Q}^2 + 4((8x_Y - 5)F(m_W^2, Q^2) + 8x_Y F(0, Q^2) + 6(3x_Y - 1)G(Q^2) + 5) + 8(3F(m_W^2, Q^2) - 8x_Y F(0, Q^2) + 3(1 - 4x_Y)G(Q^2) - 6)\frac{1}{\hat{Q}^2}\Big] \tag{32}$$

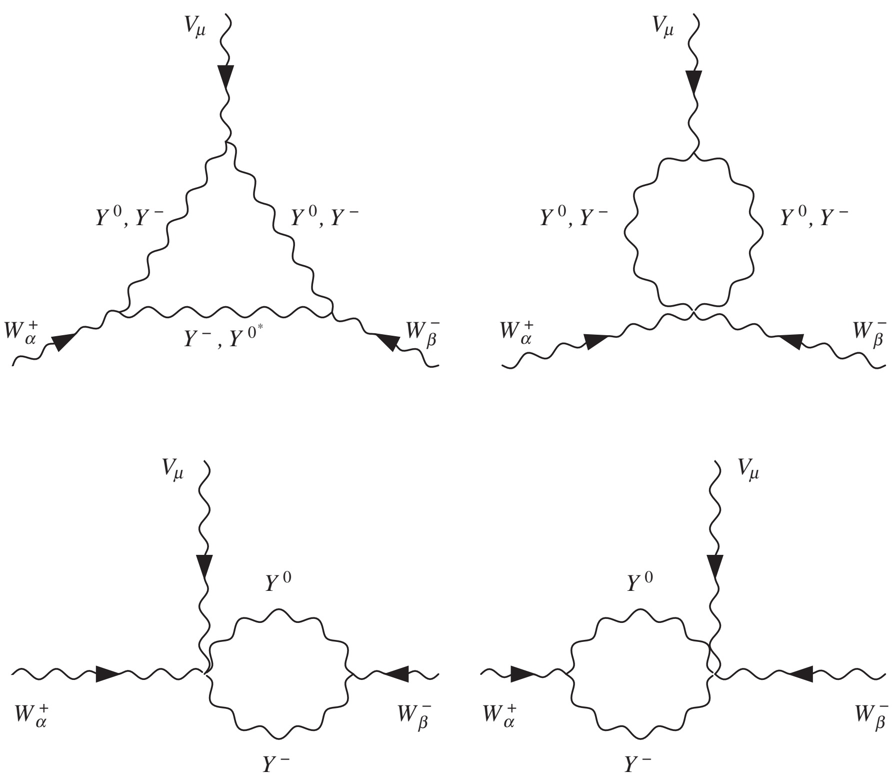

FIG. 3. Feynman diagrams for the WWV vertex in the $SU_L(2) \times U_Y(1)$-covariant gauge. The pseudo-Goldstone bosons and ghosts contribute through an identical set of diagrams, but only the triangle ones give a nonvanishing contribution to $\Delta\kappa_V$ and ΔQ_V.

$$\Delta\kappa_V = \beta(\hat{Q}^2)[(F(m_W^2, Q^2) - 4x_Y F(m_W^2, 0) - 6(1 + 3x_Y)G(Q^2) - 3)\hat{Q}^4 + 2(2 + 16x_Y F(m_W^2, 0) + (13 + 16x_Y)F(m_W^2, Q^2) - 3(1 - 3x_Y)G(Q^2))\hat{Q}^2 + 32(1 - 2(F(m_W^2, 0) + 2F(m_W^2, Q^2)))x_Y], \tag{33}$$

where $F(m^2, Q^2) = B_0(m^2, m_Y^2, m_Y^2) - B_0(Q^2, m_Y^2, m_Y^2)$ and $G(Q^2) = m_W^2 C_0(Q^2, m_W^2, m_W^2, m_Y^2, m_Y^2, m_Y^2)$, with the scalar two-point B_0 and three-point C_0 functions given in the usual notation. From here, it is clear that ultraviolet divergences cancel up in the W form factors.

Before the numerical evaluation, it is interesting to analyze the static electromagnetic form factors, i.e. the scenario when $Q^2 = 0$. Note that $\hat{Q}^2$ and $F(0, Q^2)$ vanish when $Q^2 = 0$, whereas $G(0) = (2 - F(m_W^2, 0))/(1 - 4x_Y)$ [40]. After taking the $Q^2 \to 0$ limit in the last two terms of Eq. (32), ΔQ_γ can be written as

$$\Delta Q_\gamma = \frac{3m_W^2 a}{2}\left(\frac{1}{6m_W^2(1 - 4x_Y)}(-21 + 48x_Y + 3(1 + (10 - 32x_Y)x_Y F(m_W^2, 0))) + (8x_Y - 3)\frac{\partial B_0(Q^2, m_Y^2, m_Y^2)}{\partial Q^2}\bigg|_{Q^2=0} + 3m_W^2(1 - 4x_Y)\frac{\partial C_0(Q^2, m_W^2, m_W^2, m_Y^2, m_Y^2, m_Y^2)}{\partial Q^2}\bigg|_{Q^2=0}\right), \tag{34}$$

where L'Hôpital rule has been used for the indeterminate limit. Since any three- and two-point scalar function and their derivatives can be written as a combination of two-point functions [40], we can write

$$\frac{\partial B_0(Q^2, m_Y^2, m_Y^2)}{\partial Q^2}\bigg|_{Q^2=0} = \frac{1}{6m_Y^2}, \tag{35}$$

and

$$\frac{\partial C_0(Q^2, m_W^2, m_W^2, m_Y^2, m_Y^2, m_Y^2)}{\partial Q^2}\bigg|_{Q^2=0} = \frac{1}{6m_W^2 m_Y^2}\frac{1 - (5 - 6x_Y)x_Y F(m_W^2, 0) - 5x_Y}{(1 - 4x_Y)^2}. \tag{36}$$

We thus obtain

$$\Delta Q_\gamma = \frac{4a}{1 - 4x_Y}(-1 + (1 - 3(2x_Y - 1)(B_0(m_W^2, m_Y^2, m_Y^2) - B_0(0, m_Y^2, m_Y^2)))x_Y), \tag{37}$$

whereas $\Delta\kappa_\gamma$ can be obtained immediately from (33)

$$\Delta\kappa_\gamma = \frac{3a}{2}(1 - 6x_Y(B_0(m_W^2, m_Y^2, m_Y^2) - B_0(0, m_Y^2, m_Y^2))). \tag{38}$$

In the limit of very large x_Y ($m_Y \gg m_W$) we obtain

$$B_0(m_W^2, m_Y^2, m_Y^2) - B_0(0, m_Y^2, m_Y^2) \simeq \frac{1}{6x_Y}, \tag{39}$$

which can be used to show that both ΔQ_γ and $\Delta\kappa_\gamma$ vanish in the large bilepton mass limit, i.e. these coefficients are of decoupling nature. Furthermore, it is evident that ΔQ_γ decreases more rapidly than $\Delta\kappa_\gamma$ as m_Y increases.

We emphasize that the above expressions account for the bilepton effects on the WWV^* vertex at the w scale, when the bilepton masses are degenerate and equal to m_Y. These results are thus approximate but can give a realistic estimate of this class of effects. We now proceed to the numerical evaluation and analysis.

B. Numerical evaluation and discussion

We now would like to analyze the behavior of ΔQ_V and $\Delta\kappa_V$ as functions of Q^2 and the bilepton mass, which are the only free parameters which they depend on.

As far as Q^2 is concerned, we will focus on the values that can be at the reach of the future planned linear colliders as this vertex has the chance of being studied through $e^- e^+ \to W^- W^+$ scattering. We thus consider the range 200 GeV $\leq \sqrt{Q^2} \leq$ 2 TeV [41].

As for the bilepton mass, it is convenient to give a short account on the existing bounds from both the theoretical and experimental sides. First of all, we already mentioned that the 331 model with right-handed neutrinos requires that $\sin^2\theta_W \leq 3/4$ because of the matching of the gauge coupling constants at the $SU_L(3) \times U_X(1)$ breaking scale, from which an upper bound on the bilepton mass can be derived after considering radiative corrections to $\sin\theta_W$ and the most recent experimental data [42]. The respective bound is somewhat weak and contrasts with the case of the minimal 331 model, which requires that $m_Y \lesssim 1$ TeV [16,32,33] for consistency with the theoretical bound $\sin^2\theta_W \leq 1/4$. Therefore, whereas the bileptons predicted by the minimal 331 model could be searched for via collider experiments in a near future, thereby confirming or ruling out the model, the 331 model with right-handed neutrinos would still leave open the door. We have also mentioned that, because of the symmetry-breaking hierarchy, the bilepton mass splitting is bounded by $|m_{Y^\pm}^2 - m_{Y^0}^2| \leq m_W^2$. This means that m_{Y^0} and $m_{Y^\pm}$ cannot be arbitrarily different. In fact, the heavier the bilepton mass, the closer its degeneracy. Our approximation of mass degeneracy has thus much sense. We now would like to examine the lower bounds on the bilepton masses.

In Ref. [29] it was argued that the data from neutrino neutral current elastic scattering give a lower bound on the mass of the new neutral gauge boson $m_{Z'}$ in the range of 300 GeV, which along with the symmetry-breaking hierarchy yield $m_{Y^\pm} \sim m_{Y_0} \sim 0.72 m_{Z'} \geq 220$ GeV. A similar bound was obtained in Ref. [28] from the observed limit on the wrong muon decay $R = \Gamma(\mu^- \to e^- \nu_e \bar{\nu}_\mu)/\Gamma(\mu^- \to e^- \bar{\nu}_e \nu_\mu) \leq 1.2\%$, which leads to $m_{Y^\pm} \geq 230 \pm 17$ GeV at 90% C.L. These lower bounds on $m_Y^\pm$ are in agreement with that obtained from the latest BNL measurement on the muon anomaly [29]. It is then reasonable to consider the range $200\ \text{GeV} \leq m_{Y^0} \leq 2000\ \text{GeV}$ for our numerical analysis. This will allow us to assess the behavior of the form factors and get an estimate of their order of magnitude. A word of caution is in order here: it has been pointed out that some of the existing bounds on the bilepton masses are too model dependent and thus can be relaxed by considering extra assumptions [43], so a relatively light bilepton cannot be ruled out yet.

In Fig. 4 we show the W form factors $\Delta\kappa_V$ and ΔQ_V vs the center-of-mass energy $E_{\rm CM} = \sqrt{Q^2}$, i.e. the momentum carried by the virtual V boson, for the bilepton mass values $m_Y = 200$, 600, and 1000 GeV. On the other hand, the dependence of the W form factors on the bilepton mass is shown in Fig. 5, where we have plotted $\Delta\kappa_V$ and ΔQ_V vs the bilepton mass for $E_{\rm CM} = 500$ and 1000 GeV.

As far as the energy dependence of the form factors, from Fig. 4 it is clear that both ΔQ_V and $\Delta\kappa_V$ are of the order of about 10^{-4} for a relatively light bilepton with a mass of 200 GeV and a center-of-mass energy around the energy threshold $E_{\rm CM} = 2m_Y$, where the form factors reach their extremum values and develop an imaginary part. Below this threshold, the form factors are purely real, which reflects the fact that the bileptons that couple to the V boson are necessarily virtual, whereas the direct production of a bilepton pair becomes possible for energies $E_{\rm CM} \geq 2m_Y$. Although the direct production of a bilepton pair would be preferred over the search for their virtual effects, the latter would be useful for a high precision test of the WWV vertex. We can also see that the form factors decrease by 1 order of magnitude when $m_Y = 500$ GeV and by 2 orders of magnitude when $m_Y = 1000$ GeV, which is depicted in Fig. 5. For very high energies, the form factors have a good behavior as they approach zero asymptotically after reaching an extremum value above $E_{\rm CM} = 2m_Y$. Therefore, unitarity is respected, which stems from the fact that the WWV Green function that we have obtained is gauge invariant. In Fig. 4 we can also observe that, except for the reversed sign, the curves for ΔQ_V and $\Delta\kappa_V$ are very similar. For a relatively light bilepton both form factors are of about the same order of magnitude, but the magnitude of ΔQ_V decreases more quickly as m_Y increases.

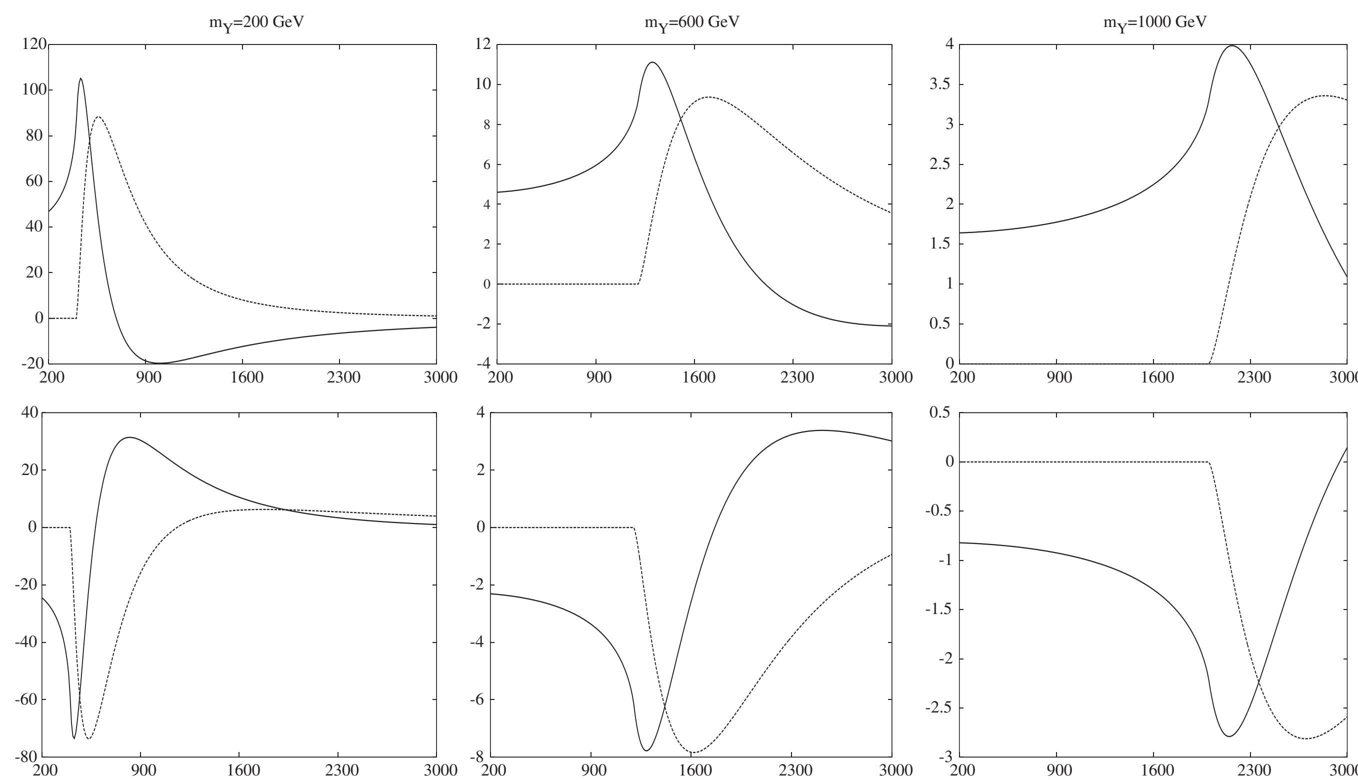

FIG. 4. The W form factors ΔQ_V (upper plots) and $\Delta\kappa_V$ (lower plots) vs the center-of-mass energy $E_{\rm CM} = \sqrt{Q^2}$ for various values of the bilepton mass. The form factors are in units of 10^{-6} and $E_{\rm CM}$ is in units of GeV. The solid (dashed) lines represent the real (imaginary) part of the form factors.

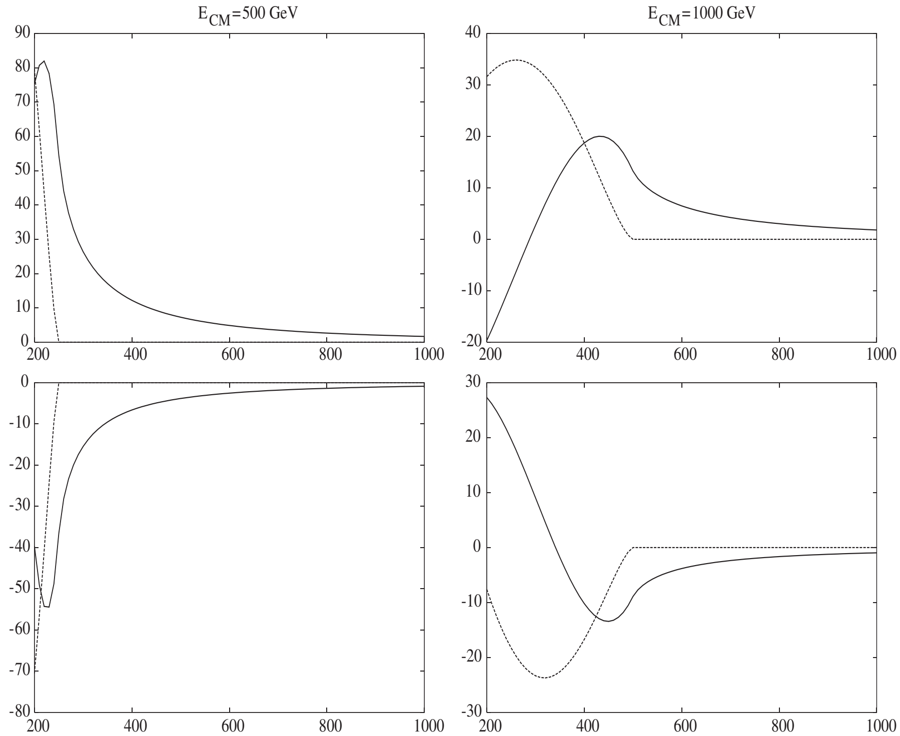

FIG. 5. The W form factors ΔQ_V (upper plots) and $\Delta\kappa_V$ (lower plots) vs the bilepton mass m_Y for $E_{\mathrm{CM}} = 500$, and 1000 GeV. The form factors are in units of 10^{-6} and m_Y is in units of GeV. The solid (dashed) lines represent the real (imaginary) part of the form factors.

It is interesting to compare our results with those obtained in the SM and some of its extensions. As far as the SM is concerned, the gauge boson contribution to the ΔQ_V form factor is of the order of 10^{-3}–10^{-4} for E_{CM} in the range 200–1000 GeV, whereas ΔQ_V is about 1 order of magnitude below [2,7]. These contributions are of the same order of magnitude as those of the fermion and scalar sectors of the theory [2]. As far as supersymmetric models are concerned, the total contribution from the unconstrained MSSM is about the same order of magnitude or larger than the SM total contribution for some set of the values of the parameters of the model [9]. Our results in the 331 model with right-handed neutrinos and relatively light bileptons, with a mass around 200 GeV, are of the same order of magnitude as the SM contribution. These results are similar to those obtained in the minimal 331 model [14] and other weakly coupled theories. Unless a very high precision is achieved in future particle colliders, it would be very hard to discriminate the contributions from different models to the WWV vertex. However, testing this vertex would still be useful for probing some particular model once it has been confirmed by experimental data.

V. FINAL REMARKS

We have used a nonconventional quantization method inspired on the BFM and BRST symmetry to analyze the effects of the new gauge bosons (bileptons) predicted by the 331 model with right-handed neutrinos on the off-shell WWV^* vertex. Hopefully, this class of effects might be searched for at a future linear collider through $e^-e^+ \to V \to W^-W^+$ scattering. In particular, it has been pointed out that CLIC would reach a sensitivity of about 10^{-4} in the measurement of the so-called form factors, ΔQ_V and $\Delta\kappa_V$, characterizing the WWV^* vertex [41]. Following the philosophy of the BFM, our method considers the bileptons as quantum fields and the SM gauge bosons as classical fields. A nonlinear $SU_L(2) \times U_Y(1)$ invariant gauge fixing term is then introduced for the bileptons, which are integrated out in the generating functional. The result is a quantum action defined by a $SU_L(2) \times U_Y(1)$ invariant Lagrangian[1] out of which a gauge-invariant and gauge-independent Green function can be obtained. To this end, we made use of the link between the diagrammatic method known as the PT and the BFM, which establishes that the gauge-invariant and gauge-independent Green function obtained via the PT coincides with that obtained via the BFM as long as the Feynman-'t Hooft gauge Feynman rules are used when calculating the latter. We emphasize that our method is only approximate as the quantum action

[1] Actually, this scheme is quite analogous to the effective Lagrangian approach, in which an $SU_L(2) \times U_Y(1)$ effective Lagrangian is constructed out of the SM fields to analyze the virtual effects of the heavy fields, which have been integrated out.

is only invariant under $SU_L(2) \times U_Y(1)$ rather than $SU_L(3) \times U_N(1)$. This method is suited to analyze the bilepton effects at the $SU_L(3) \times U_N(1)$ breaking scale, when their masses are still degenerate. The advantages of our calculation scheme are twofold: the introduced nonlinear gauge-fixing term for the bileptons allows one to remove several unphysical vertices, which in turn allows one to get rid of several Feynman diagrams; on the other hand, preserving the electroweak invariance turns the calculation into a simple task as each sector of the theory gives a gauge-invariant contribution that by itself satisfies simple Ward identities. Once our method was applied, we obtained the form factors ΔQ_V and $\Delta\kappa_V$, which were analyzed for several values of the bilepton mass and convenient values of the momentum carried by the virtual V boson. It was found that the bilepton effects on the WWV^* vertex are of the same order of magnitude as the SM and the minimal 331 model contributions, provided that the bilepton mass is of the order of a few hundred of GeVs. For very heavy bileptons, the respective contribution to the WWV^* is negligibly small. This indicates that it will be very hard to discriminate between different classes of effects on the WWV^* vertex arising from distinct models. However, if a high precision is achieved in future linear colliders, the WWV^* vertex might serve as a probe to some particular model by then confirmed by other experimental data. The good behavior of the form factors at high energies was also discussed, as it is an indication of the gauge invariance and gauge independence of the WWV^* Green function obtained via our quantization method.

ACKNOWLEDGMENTS

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