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Updated analysis of two-body charmed B meson decays

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The charmed B decays, $B \to DP$, D^*P, and DV, are reanalyzed using the latest experimental data, where P and V denote the pseudoscalar meson and vector meson, respectively. We perform global fits under the assumption of flavor SU(3) symmetry. The size of the decay amplitudes and the strong phases between the topologically distinct amplitudes are studied. Predictions of the related B_s decay rates are made based upon the fitted results. We also note a serious SU(3) symmetry breaking or inconsistency in the DV sector.

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I. INTRODUCTION

The hadronic decays of B mesons have provided us with a good place to study CP violation in particle physics. In particular, the detection of direct CP violation in a decay process requires that there exist at least two contributing amplitudes with different weak and strong phases. The direct CP-violating effect in the B system has finally been observed in the $B^0 \to K^+\pi^-$ decay at the B factories [1,2], proving the existence of nontrivial strong phases in B decays. It is therefore of consequence to find out the patterns of final-state strong phases for a wider set of decay modes.

Since the Cabibbo-Kobayashi-Maskawa (CKM) factors involved in charmed B meson decays are purely real to a good approximation, the phases associated with the decay amplitudes thus have the origin of strong interactions. Such final-state rescattering effects have been noticed from data in these decays [3,4], and estimated to be at 15%–20% level [5]. Unfortunately, no satisfactory first-principle calculations can yield such strong phases [6]. In Ref. [7], we performed an analysis based upon the experimental data available at that time. A few theoretical and experimental questions are left unanswered. As more decay modes have been observed and others are measured at higher precisions, it becomes possible for us to look at and answer those questions. In this paper, flavor SU(3) symmetry is employed to relate different amplitudes and strong phases of the same topological type. Moreover, we will take a different approach by fitting theoretical parameters to all available branching ratios simultaneously. An advantage of this analysis is that the parameters thus obtained are insensitive to statistical fluctuations of individual modes.

This paper is organized as follows: In Sec. II, we give the amplitude decomposition of modes under flavor SU(3) symmetry and the current branching ratio data. Theoretical parameters involved in our analysis are defined. In Sec. III, we consider three sets of charmed decay modes: DP, D^*P, and DV, where P and V denote charmless pseudoscalar and vector mesons, respectively. A summary of our findings is given in Sec. IV.

II. FLAVOR AMPLITUDE DECOMPOSITION AND DATA

In the decomposition of decay amplitudes, relevant meson wave functions are assumed to have the following quark contents, with phases chosen so that isospin multiplets contain no relative signs:

(i) *Beauty mesons*: $\bar{B}^0 = b\bar{d}$, $B^- = -b\bar{u}$, $\bar{B}_s = b\bar{s}$.
(ii) *Charmed mesons*: $D^0 = -c\bar{u}$, $D^+ = c\bar{d}$, $D_s^+ = c\bar{s}$, with corresponding phases for vector mesons.
(iii) *Pseudoscalar mesons P*: $\pi^+ = u\bar{d}$, $\pi^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\pi^- = -d\bar{u}$, $K^+ = u\bar{s}$, $K^0 = d\bar{s}$, $\bar{K}^0 = s\bar{d}$, $K^- = -s\bar{u}$, $\eta = (s\bar{s} - u\bar{u} - d\bar{d})/\sqrt{3}$, $\eta' = (u\bar{u} + d\bar{d} + 2s\bar{s})/\sqrt{6}$, assuming a specific octet-singlet mixing [8,9] in the η and η' wave functions.
(iv) *Vector mesons V*: $\rho^+ = u\bar{d}$, $\rho^0 = (d\bar{d} - u\bar{u})/\sqrt{2}$, $\rho^- = -d\bar{u}$, $\omega = (u\bar{u} + d\bar{d})/\sqrt{2}$, $K^{*+} = u\bar{s}$, $K^{*0} = d\bar{s}$, $\bar{K}^{*0} = s\bar{d}$, $K^{*-} = -s\bar{u}$, $\phi = s\bar{s}$.

The amplitudes contributing to the decays discussed here involve only three different topologies [8–11]:

(1) *Tree amplitude T*: This is associated with the transition $b \to cd\bar{u}$ (Cabibbo-favored) or $b \to cs\bar{u}$ (Cabibbo-suppressed) in which the light (color-singlet) quark-antiquark pair is incorporated into one meson, while the charmed quark combines with the spectator antiquark to form the other meson.
(2) *Color-suppressed amplitude C*: The transition is the same as in the tree amplitudes, namely $b \to cd\bar{u}$ or $b \to cs\bar{u}$, except that the charmed quark and the $\bar{u}$ combine into one meson while the d or s quark and the spectator antiquark combine into the other meson.
(3) *Exchange amplitude E*: The b quark and spectator antiquark exchange a W boson to become a $c\bar{u}$ pair,

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TABLE I. Branching ratios and flavor amplitude decomposition for $B \to DP$ decays. Data are quoted from Refs. [12–19].

Decay	m_B (GeV)	Branching ratio (in units of 10^{-4})	p^* (GeV)	$\lvert\mathcal{A}\rvert$ (10^{-7} GeV)	Representation
$B^- \to D^0\pi^-$	5.2791	47.5 ± 2.1	2.308	7.61 ± 0.17	$-V_{cb}V_{ud}^*(T + C)$
$\to D^0K^-$		4.08 ± 0.24	2.281	2.24 ± 0.07	$-V_{cb}V_{us}^*(\xi_T T + \xi_C C)$
$\bar{B}^0 \to D^+\pi^-$	5.2793	29 ± 2	2.306	6.11 ± 0.21	$-V_{cb}V_{ud}^*(T + E)$
$\to D^+K^-$		2.0 ± 0.6	2.279	1.63 ± 0.24	$-V_{cb}V_{us}^*\xi_T T$
$\to D^0\pi^0$		2.61 ± 0.24	2.308	1.85 ± 0.09	$V_{cb}V_{ud}^*(E - C)/\sqrt{2}$
$\to D^0\eta$		2.0 ± 0.2	2.274	1.62 ± 0.08	$V_{cb}V_{ud}^*(C + E)/\sqrt{3}$
$\to D^0\eta'$		1.25 ± 0.23	2.198	1.31 ± 0.12	$-V_{cb}V_{ud}^*(C + E)/\sqrt{6}$
$\to D^0\bar{K}^0$		0.52 ± 0.07	2.280	0.83 ± 0.05	$-V_{cb}V_{us}^*\xi_C C$
$\to D_s^+K^-$		0.27 ± 0.05	2.242	0.61 ± 0.06	$-V_{cb}V_{ud}^*E$
$\bar{B}_s^0 \to D^+\pi^-$	5.3696		2.357		$-V_{cb}V_{us}^*\xi_E E$
$\to D^0\pi^0$			2.359		$V_{cb}V_{us}^*\xi_E E/\sqrt{2}$
$\to D^0K^0$			2.332		$-V_{cb}V_{ud}^*C$
$\to D^0\eta$			2.326		$V_{cb}V_{us}^*(\xi_E E - \xi_C C)/\sqrt{3}$
$\to D^0\eta'$			2.251		$-V_{cb}V_{us}^*(2\xi_C C + \xi_E E)/\sqrt{6}$
$\to D_s^+\pi^-$		$38 \pm 3 \pm 13$[a]	2.321	7.30 ± 1.28	$-V_{cb}V_{ud}^*T$
$\to D_s^+K^-$			2.294		$-V_{cb}V_{us}^*(\xi_T T + \xi_E E)$

[a]Reference [13].

which then hadronizes into two mesons by picking up a light quark-antiquark pair out of the vacuum.

After factoring out the CKM factors explicitly, we obtain the flavor amplitude decomposition of the charmed B decay modes in Tables I, II, and III. In these tables, we introduce positive ξ's to parametrize the flavor SU(3) breaking effects. This symmetry is respected between strangeness-conserving and strangeness-changing amplitudes when ξ's are taken to be unity. As we will discuss in the next section, ξ's will be allowed to change in order to test the assumption. Using the Wolfenstein parameters [20], the relevant CKM factors are

$$V_{cb} = A\lambda^2, \qquad V_{ud} = 1 - \frac{\lambda^2}{2}, \quad \text{and} \quad V_{us} = \lambda, \tag{1}$$

none of which contain a weak phase to the order we are concerned with. In the following analysis, we take the

TABLE II. Branching ratios and flavor amplitude decomposition for $B \to D^*P$ decays. Data are quoted from Refs. [12–19].

Decay	m_B (GeV)	Branching ratio (in units of 10^{-4})	p^* (GeV)	$\lvert\mathcal{A}\rvert$ (10^{-7} GeV)	Representation
$B^- \to D^{*0}\pi^-$	5.2791	50 ± 4	2.256	7.87 ± 0.32	$-V_{cb}V_{ud}^*(T_V + C_P)$
$\to D^{*0}K^-$		3.7 ± 0.4	2.227	2.16 ± 0.12	$-V_{cb}V_{us}^*(\xi_{T_V}T_V + \xi_{C_P}C_P)$
$\bar{B}^0 \to D^{*+}\pi^-$	5.2793	28.5 ± 1.7	2.255	6.17 ± 0.19	$-V_{cb}V_{ud}^*(T_V + E_P)$
$\to D^{*+}K^-$		2.14 ± 0.20	2.226	1.70 ± 0.08	$-V_{cb}V_{us}^*\xi_{T_V}T_V$
$\to D^{*0}\pi^0$		1.7 ± 0.3	2.256	1.52 ± 0.12	$V_{cb}V_{ud}^*(E_P - C_P)/\sqrt{2}$
$\to D^{*0}\eta$		1.8 ± 0.6	2.220	1.55 ± 0.24	$V_{cb}V_{ud}^*(C_P + E_P)/\sqrt{3}$
$\to D^{*0}\eta'$		1.23 ± 0.35	2.141	1.32 ± 0.19	$-V_{cb}V_{ud}^*(C_P + E_P)/\sqrt{6}$
$\to D^{*0}\bar{K}^0$		0.36 ± 0.12[a]	2.227	0.70 ± 0.12	$-V_{cb}V_{us}^*\xi_{C_P}C_P$
$\to D_s^{*+}K^-$		$0.20 \pm 0.05 \pm 0.04$[b]	2.185	0.53 ± 0.08	$-V_{cb}V_{ud}^*E_P$
$\bar{B}_s^0 \to D^{*+}\pi^-$	5.3696		2.306		$-V_{cb}V_{us}^*\xi_{E_P}E_P$
$\to D^{*0}\pi^0$			2.308		$V_{cb}V_{us}^*\xi_{E_P}E_P/\sqrt{2}$
$\to D^{*0}K^0$			2.279		$-V_{cb}V_{ud}^*C_P$
$\to D^{*0}\eta$			2.273		$V_{cb}V_{us}^*(\xi_{E_P}E_P - \xi_{C_P}C_P)/\sqrt{3}$
$\to D^{*0}\eta'$			2.195		$-V_{cb}V_{us}^*(2\xi_{C_P}C_P + \xi_{E_P}E_P)/\sqrt{6}$
$\to D_s^{*+}\pi^-$			2.267		$-V_{cb}V_{ud}^*T_V$
$\to D_s^{*+}K^-$			2.238		$-V_{cb}V_{us}^*(\xi_{T_V}T_V + \xi_{E_P}E_P)$

[a]Reference [14], [b]Reference [15].

TABLE III. Branching ratios and flavor amplitude decomposition for $B \to DV$ decays. Data are quoted from Refs. [12–19].

Decay	m_B (GeV)	Branching ratio (in units of 10^{-4})	p^* (GeV)	$\lvert\mathcal{A}\rvert$ (10^{-7} GeV)	Representation
$B^- \to D^0\rho^-$	5.2791	134 ± 18	2.237	13.0 ± 0.9	$-V_{cb}V_{ud}^*(T_P + C_V)$
$\to D^0K^{*-}$		5.3 ± 0.4	2.213	2.60 ± 0.11	$-V_{cb}V_{us}^*(\xi_{T_P}T_P + \xi_{C_V}C_V)$
$\bar{B}^0 \to D^+\rho^-$	5.2793	75 ± 12	2.235	10.1 ± 0.8	$-V_{cb}V_{ud}^*(T_P + E_V)$
$\to D^+K^{*-}$		4.5 ± 0.7	2.211	2.48 ± 0.19	$-V_{cb}V_{us}^*\xi_{T_P}T_P$
$\to D^0\rho^0$		3.2 ± 0.5	2.237	2.07 ± 0.16	$V_{cb}V_{ud}^*(E_V - C_V)/\sqrt{2}$
$\to D^0\omega$		2.6 ± 0.3	2.235	1.87 ± 0.11	$-V_{cb}V_{ud}^*(C_V + E_V)/\sqrt{2}$
$\to D^0\bar{K}^{*0}$		0.42 ± 0.06	2.212	0.76 ± 0.06	$-V_{cb}V_{us}^*\xi_{C_V}C_V$
$\to D_s^+K^{*-}$		<8	2.172	<3	$-V_{cb}V_{ud}^*E_V$
$\bar{B}_s^0 \to D^+\rho^-$	5.3696		2.288		$-V_{cb}V_{us}^*\xi_{E_V}E_V$
$\to D^+K^{*-}$			2.264		$-V_{cb}V_{us}^*(\xi_{T_P}T_P + \xi_{E_V}E_V)$
$\to D^0\rho^0$			2.289		$V_{cb}V_{ud}^*E_V/\sqrt{2}$
$\to D^0K^{*0}$			2.265		$-V_{cb}V_{ud}^*C_V$
$\to D^0\omega$			2.288		$-V_{cb}V_{us}^*\xi_{E_V}E_V/\sqrt{2}$
$\to D^0\phi$			2.237		$-V_{cb}V_{us}^*\xi_{C_V}C_V$
$\to D_s^+\rho^-$			2.250		$-V_{cb}V_{ud}^*T_P$
$\to D_s^+K^{*-}$			2.226		$-V_{cb}V_{us}^*(\xi_{T_P}T_P + \xi_{E_V}E_V)$

central values $\lambda = 0.2272$ and $A = 0.809$ quoted by the CKMfitter group [21].

Since only the relative strong phases are physically measurable, we fix the tree (T, T_P, and T_V) amplitudes to be real and pointing in the positive direction. We then associate the color-suppressed and exchange amplitudes with the corresponding strong phases explicitly as follows:

$$C = |C|e^{i\delta_C}, \qquad E = |E|e^{i\delta_E}, \tag{2}$$

$$C_P = |C_P|e^{i\delta_{C_P}}, \qquad E_P = |E_P|e^{i\delta_{E_P}}, \tag{3}$$

$$C_V = |C_V|e^{i\delta_{C_V}}, \qquad E_V = |E_V|e^{i\delta_{E_V}}. \tag{4}$$

The magnitude of invariant decay amplitude $\mathcal{A}$ for a decay process $B \to M_1M_2$ is related to its partial width via the following relation:

$$\Gamma(B \to M_1M_2) = \frac{p^*}{8\pi m_B^2}|\mathcal{A}|^2, \tag{5}$$

with

$$p^* = \frac{1}{2m_B}\sqrt{\{m_B^2 - (m_1 + m_2)^2\}\{m_B^2 - (m_1 - m_2)^2\}}, \tag{6}$$

where $m_{1,2}$ are the masses of $M_{1,2}$, respectively. To relate partial widths to branching ratios, we use the world-average lifetimes $\tau^+ = (1.638 \pm 0.011)$ ps, $\tau^0 = (1.530 \pm 0.009)$ ps, and $\tau_s = (1.466 \pm 0.059)$ ps computed by the Heavy Flavor Averaging Group (HFAG) [22].

III. STRONG PHASES

In our analysis, we take the amplitude sizes and the strong phases as theoretical parameters, and perform χ^2 fits to all the branching ratios in each category ($B_{u,d} \to DP$, D^*P, and DV). We consider three schemes to test the flavor SU(3) assumption:

(1) $\xi_T = \xi_C = \xi_{T_V} = \xi_{C_P} = \xi_{T_P} = \xi_{C_V} = 1$. This is the exact flavor SU(3)-symmetric case.

(2) $\xi_T = \xi_C = \xi_{T_V} = \xi_{C_P} = f_K/f_\pi \simeq 1.22$, and $\xi_{T_P} = \xi_{C_V} = f_{K^*}/f_\rho \simeq 1.00$. This takes into account the difference in the decay constants for the charmless meson in the final states.

(3) All ξ_T's and ξ_C's are taken as free parameters and determined by the χ^2 fit in each individual category.

Here we have taken the decay constants $f_\pi = 130.7$ MeV, $f_K = 159.8$ MeV [12], $f_{K^*} = 210.4$ MeV, and $f_\rho = 210.4$ MeV [23]. For scheme 3 in the DV sector, it turns out that this scheme does not work well with the present available experimental data. We will discuss this issue in Sec. III C.

Among all $B_{u,d}$ decays considered in this work, no Cabibbo-suppressed decay involves the exchange diagram. The only place to test this is the B_s decays, of which we know very little at the moment. We thus assume ξ_E's $= 1$ when we predict the branching ratios of those decays.

The strong phases given in the following results are subject to a twofold ambiguity. This is because only the cosines of the relative strong phases are involved in the branching ratios. Therefore, it is allowed to flip the signs of all the phases simultaneously without changing the fitting quality and our predictions. In view of this, we will restrict the strong phase associated with the color-suppressed amplitudes to the $[-180°, 0°]$ range in our analysis.

A. $B \to DP$ decays

In Table IV, we see that $\chi^2_{\rm min}$ is greatly reduced by the introduction of the SU(3) breaking factors ξ_T and ξ_C. The smallness of $\chi^2_{\rm min}$ in schemes 2 and 3 also shows the consistency of input observables.

The values of $|T|$ and $|C|$ can be directly obtained from the $\bar{B}^0 \to D^+ K^-$ and $D^0 \bar{K}^0$ decays via the U-spin symmetry, i.e., exchange between d quark and s quark. They are, respectively, 14.0 ± 2.1 and 7.17 ± 0.46 in units of 10^{-6} GeV. Here we take $\xi_T = \xi_C = f_K/f_\pi$. Likewise, $|E|$ is inferred from the $\bar{B}^0 \to D_s^+ K^-$ mode to be $(1.49 \pm 0.14) \times 10^{-6}$ GeV. These values directly extracted from individual modes are consistent with those given in Table IV and in general have larger errors except for $|E|$. The SU(3) breaking parameter ξ_T can also be extracted from $\bar{B}^0 \to D^+ K^-$ and $\bar{B}_s^0 \to D_s^+ \pi^-$. It leads to $\xi_T = 0.96 \pm 0.22$. This is smaller than the fitted value of ξ_T in Table IV.

According to our wave functions for η and η', the ratio $\mathcal{B}(D^0\eta)/\mathcal{B}(D^0\eta')$ is predicted to be 2, in comparison with 1.58 ± 0.33 given by the current data. From these decays, we determine $|C + E| = (7.07 \pm 0.30) \times 10^{-6}$ GeV. On the other hand, $|C - E| = (6.42 \pm 0.30) \times 10^{-6}$ GeV is inferred from the $D^0\pi^0$ mode. Therefore, one can form the combination $|C|^2 + |E|^2 = (45.6 \pm 2.9) \times 10^{-12}$ GeV^2 from these three modes, consistent with $(53.6 \pm 6.6) \times 10^{-12}$ GeV^2 that is derived from the $D^0\bar{K}^0$ and $D_s^+K^-$ modes assuming $\xi_C = f_K/f_\pi$.

In Fig. 1, we show the $\Delta\chi^2 = 1$ and 2.30 contours on the δ_C-$|C/T|$, δ_E-$|E/T|$, and $|E/T|$-$|C/T|$ planes in scheme 3, respectively, showing the correlations between each pair of parameters. The projections of the $\Delta\chi^2 = 1$ contours to individual axes give the 68.3% confidence level (C.L.) ranges of the corresponding quantities. In particular, we find that $|C/T| = 0.48 \pm 0.02$ and $|E/T| = 0.11 \pm 0.01$. Our result shows an enhancement in the color-suppressed amplitude. This can be explained by nonfactorizable effects or final-state interactions. The three flavor amplitude sizes fall into a hierarchy: $|T| > |C| > |E|$, with $|E|$ being about 1 order of magnitude smaller than $|T|$. This is the

TABLE IV. $B \to DP$ decays. Theoretical parameters are extracted from global χ^2 fits in different schemes explained in the text. The amplitude sizes are given in units of 10^{-6} GeV. Predictions of branching ratios are made with $\xi_E = 1$ and given in units of 10^{-4} unless otherwise noted.

	Scheme 1	Scheme 2	Scheme 3
$\lvert T\rvert$	$16.26^{+0.61}_{-0.68}$	13.74 ± 0.45	13.71 ± 0.46
$\lvert C\rvert$	$6.77^{+0.20}_{-0.21}$	6.67 ± 0.20	6.57 ± 0.22
$\lvert E\rvert$	$1.47^{+0.13}_{-0.15}$	$1.48^{+0.13}_{-0.15}$	$1.49^{+0.13}_{-0.15}$
δ_C (degrees)	$-69.0^{+9.2}_{-7.5}$	$-47.0^{+9.5}_{-8.2}$	$-48.7^{+9.8}_{-8.5}$
δ_E (degrees)	$-146.2^{+13.9}_{-12.0}$	$30.4^{+11.6}_{-11.8}$	$28.9^{+11.9}_{-12.1}$
ξ_T	1 (fixed)	f_K/f_π (fixed)	1.24 ± 0.02
ξ_C	1 (fixed)	f_K/f_π (fixed)	1.33 ± 0.02
$\chi^2_{\rm min}$	45.28	3.53	1.41
$\chi^2_{\rm min}$/dof	11.32	0.88	0.71
$B^- \to D^0\pi^-$	52.8 ± 5.3	48.6 ± 3.7	47.5 ± 3.8
$\to D^0K^-$	2.84 ± 0.28	3.91 ± 0.30	4.08 ± 0.34
$\bar{B}^0 \to D^+\pi^-$	29 ± 3	29 ± 2	29 ± 2
$\to D^+K^-$	1.8 ± 0.1	1.9 ± 0.1	2.0 ± 0.1
$\to D^0\pi^0$	2.76 ± 0.37	2.68 ± 0.35	2.61 ± 0.36
$\to D^0\eta$	2.2 ± 0.3	2.1 ± 0.2	2.1 ± 0.2
$\to D^0\eta'$	1.06 ± 0.12	1.03 ± 0.11	1.00 ± 0.12
$\to D^0\bar{K}^0$	0.31 ± 0.02	0.45 ± 0.03	0.52 ± 0.04
$\to D_s^+K^-$	0.27 ± 0.05	0.27 ± 0.05	0.27 ± 0.05
$\bar{B}_s^0 \to D^+\pi^-$ (in units of 10^{-6})	1.4 ± 0.3	1.4 ± 0.3	1.4 ± 0.3
$\to D^0\pi^0$ (in units of 10^{-6})	0.7 ± 0.1	0.7 ± 0.1	0.7 ± 0.1
$\to D^0K^0$	5.4 ± 0.3	5.3 ± 0.3	5.1 ± 0.3
$\to D^0\eta$	0.09 ± 0.01	0.14 ± 0.01	0.16 ± 0.02
$\to D^0\eta'$	0.20 ± 0.02	0.29 ± 0.02	0.33 ± 0.03
$\to D_s^+\pi^-$	31 ± 2	22 ± 1	22 ± 1
$\to D_s^+K^-$	1.8 ± 0.1	2.0 ± 0.1	2.0 ± 0.1

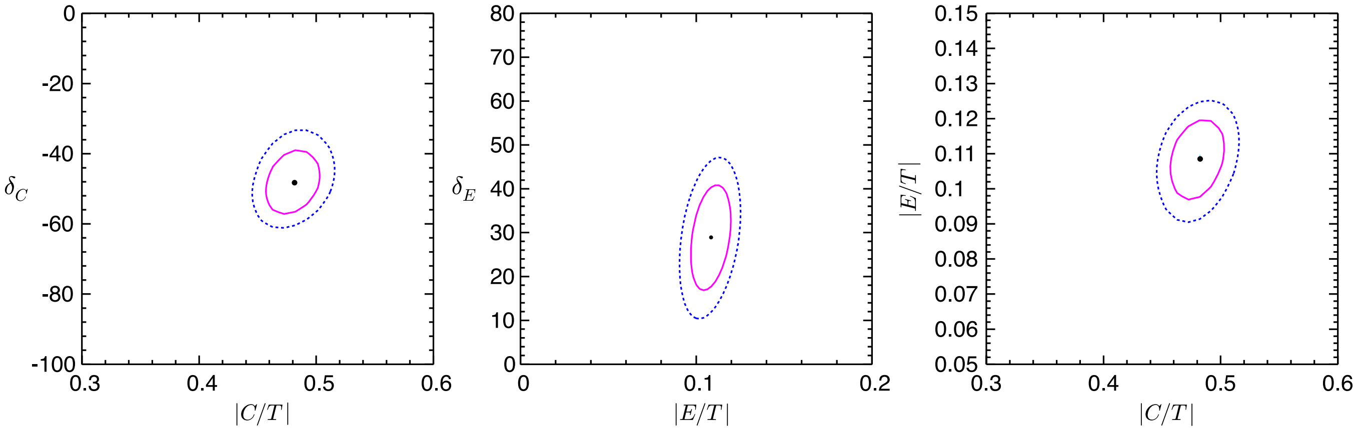

FIG. 1 (color online). $\Delta\chi^2 = 1$ (pink, solid) and 2.30 (blue, dotted) contours on the δ_C-$|C/T|$, δ_E-$|E/T|$, and $|E/T|$-$|C/T|$ planes in scheme 3.

reason why the 1σ bounds on δ_E are relatively loose. Moreover, we observe nontrivial strong phases δ_C and δ_E. These results are consistent with previous studies [7,24,25].

We note in passing that in our contour plots, the planes are scanned by minimizing χ^2, keeping all the other parameters free to vary. Therefore, our results are different from those given in Ref. [25]. In addition, their formalism corresponds to our scheme 1.

Based upon our fit results, we give predictions for all the $B_{u,d,s}$ meson decays in this category in the lower part of Table IV. For the B_s decays involving the exchange diagram, we take $\xi_E = 1$. The predicated branching ratios of those modes could be changed if we take into account SU(3) breaking in E. Conversely, measurements of those modes can provide useful information about the magnitude of the SU(3) breaking effect in the exchange diagram.

$\bar{B}_s^0 \to D_s^+\pi^-$ is a Cabibbo-favored decay involving the tree amplitude. Therefore, it has the largest decay rate among the channels in this group. Our preferred value for its branching ratio is $(22 \pm 1) \times 10^{-4}$. On the other hand, a recent measurement of this mode by CDF gives $(38 \pm 3 \pm 13) \times 10^{-4}$ [13]. The discrepancy is 1.2σ. Further measurements of this and other B_s decay modes with better precision will help in settling the question of whether or not flavor SU(3) symmetry can be reliably extended to the sector of B_s meson decays.

From the naive factorization (NF) approximation, the SU(3) breaking parameters are given by

$$\xi_T^{\rm NF} = \frac{f_K F_0^{BD}(m_K^2)}{f_\pi F_0^{BD}(m_\pi^2)} \simeq 1.23,$$
$$\xi_C^{\rm NF} = \frac{(m_B^2 - m_K^2) F_0^{BK}(m_D^2)}{(m_B^2 - m_\pi^2) F_0^{B\pi}(m_D^2)} \simeq 1.37, \qquad (7)$$

where the form factors are calculated using the covariant light-front model [26]: $F_0^{BD}(m_\pi^2) = 0.67$, $F_0^{BD}(m_K^2) = 0.67$, $F_0^{B\pi}(m_D^2) = 0.28$, $F_0^{BK}(m_D^2) = 0.38$. These theoretical predictions are very close to our fitted values: $\xi_T = 1.24 \pm 0.02$ and $\xi_C = 1.33 \pm 0.02$.

The ratio of the two effective Wilson coefficients $a_{1,2}^{\rm eff}$ for these decay processes can be extracted as

$$\left|\frac{a_2^{\rm eff}}{a_1^{\rm eff}}\right|_{DP} = \left|\frac{C}{T}\right| \frac{(m_B^2 - m_D^2) f_\pi F_0^{BD}(m_\pi^2)}{(m_B^2 - m_\pi^2) f_D F_0^{B\pi}(m_D^2)} = 0.59 \pm 0.03, \qquad (8)$$

where $|C/T| = 0.48 \pm 0.02$ as obtained from the χ^2 analysis in scheme 3, and $f_D = 222.6$ MeV [12] is used. In Ref. [24], $|a_2^{\rm eff}/a_1^{\rm eff}|_{DP}$ is found to be 0.54–0.70 at the 1σ level using the data of $B^- \to D^0\pi^-$, $\bar{B}^0 \to D^+\pi^-$, and $\bar{B}^0 \to D^0\pi^0$ modes, which is consistent with our result. In the perturbative QCD (pQCD) calculation [27], it is found that $|a_2^{\rm eff}/a_1^{\rm eff}|_{DP} = 0.42$–$0.51$, and the relative phase between $a_1^{\rm eff}$ and $a_2^{\rm eff}$ is estimated to be $-65.3° < \arg(a_2^{\rm eff}/a_1^{\rm eff})_{DP} < -61.5°$ without the exchange diagram.

B. $B \to D^*P$ decays

We see again in Table V that $\chi^2_{\rm min}$ is significantly lowered by the introduction of the SU(3) breaking factors ξ_{T_V} and ξ_{C_P}.

In this category, $|T_V| = (14.7 \pm 0.7) \times 10^{-6}$ GeV, $|C_P| = (6.0 \pm 1.0) \times 10^{-6}$ GeV, and $|E_P| = (1.29 \pm 0.21) \times 10^{-6}$ GeV can be directly extracted from the $D^{*+}K^-$, $D^{*0}\bar{K}^0$, and $D_s^{*+}K^-$ modes, respectively, taking $\xi_{T_V} = \xi_{C_P} = f_K/f_\pi$. Another way to constrain $|C_P|$ is to deduce from the $D^{*0}(\pi, \eta, \eta')$ and $D_s^{*+}K^-$ modes. Using this method, we find $|C_P| = (6.2 \pm 0.5) \times 10^{-6}$ GeV.

In Fig. 2, we show the $\Delta\chi^2 = 1$ and 2.30 contours on the δ_{C_P}-$|C_P/T_V|$, δ_{E_P}-$|E_P/T_V|$, and $|E_P/T_V|$-$|C_P/T_V|$ planes in scheme 3, respectively. We find that $|C_P/T_V| = 0.40^{+0.07}_{-0.04}$ and $|E_P/T_V| = 0.08^{+0.02}_{-0.01}$. As in the DP category, there can be sizable nonfactorizable contributions to the color-suppressed amplitude or final-state interactions. The

TABLE V. $B \to D^*P$ decays. Theoretical parameters are extracted from global χ^2 fits in different schemes explained in the text. The amplitude sizes are given in units of 10^{-6} GeV. Predictions of branching ratios are made with $\xi_{E_P} = 1$ and given in units of 10^{-4} unless otherwise noted.

	Scheme 1	Scheme 2	Scheme 3
$\lvert T_V\rvert$	$16.45^{+0.55}_{-0.61}$	$14.85^{+0.60}_{-1.00}$	$15.34^{+0.84}_{-1.70}$
$\lvert C_P\rvert$	$6.03^{+0.43}_{-0.46}$	$6.21^{+0.39}_{-0.43}$	$6.14^{+0.46}_{-0.50}$
$\lvert E_P\rvert$	$1.37^{+0.18}_{-0.20}$	$1.26^{+0.20}_{-0.23}$	$1.29^{+0.19}_{-0.22}$
δ_{C_P} (degrees)	$-63.4^{+13.2}_{-10.8}$	$-54.5^{+24.9}_{-12.0}$	$-57.3^{+30.0}_{-12.6}$
δ_{E_P} (degrees)	$-126.8^{+21.4}_{-19.3}$	$-84.7^{+84.7}_{-27.8}$	$-100.9^{+100.9}_{-30.0}$
ξ_{T_V}	1 (fixed)	f_K/f_π (fixed)	$1.17^{+0.03}_{-0.02}$
ξ_{C_P}	1 (fixed)	f_K/f_π (fixed)	1.20 ± 0.05
$\chi^2_{\rm min}$	12.00	1.39	0.72
$\chi^2_{\rm min}/{\rm dof}$	3.00	0.35	0.36
$B^- \to D^{*0}\pi^-$	52 ± 6	49 ± 8	50 ± 10
$\to D^0K^-$	2.8 ± 0.3	3.9 ± 0.6	3.7 ± 0.8
$\bar{B}^0 \to D^{*+}\pi^-$	30.3 ± 2.8	27.9 ± 5.4	28.4 ± 7.3
$\to D^{*+}K^-$	1.80 ± 0.13	2.19 ± 0.24	2.14 ± 0.37
$\to D^{*0}\pi^0$	1.9 ± 0.5	1.6 ± 0.6	1.7 ± 0.9
$\to D^{*0}\eta$	1.9 ± 0.4	2.2 ± 0.4	2.1 ± 0.6
$\to D^{*0}\eta'$	0.89 ± 0.17	1.05 ± 0.21	1.00 ± 0.29
$\to D^{*0}\bar{K}^0$	0.24 ± 0.04	0.38 ± 0.05	0.36 ± 0.06
$\to D_s^{*+}K^-$	0.23 ± 0.06	0.19 ± 0.06	0.20 ± 0.06
$\bar{B}_s^0 \to D^{*+}\pi^-$ (in units of 10^{-6})	1.2 ± 0.3	1.0 ± 0.3	1.1 ± 0.3
$\to D^{*0}\pi^0$ (in units of 10^{-7})	6.0 ± 1.6	5.0 ± 1.7	5.3 ± 1.7
$\to D^{*0}K^0$	4.2 ± 0.6	4.5 ± 0.6	4.4 ± 0.7
$\to D^{*0}\eta$	0.06 ± 0.02	0.09 ± 0.02	0.09 ± 0.04
$\to D^{*0}\eta'$	0.16 ± 0.03	0.27 ± 0.03	0.25 ± 0.05
$\to D_s^{*+}\pi^-$	31 ± 2	25 ± 3	27 ± 4
$\to D_s^{*+}K^-$	1.8 ± 0.1	2.1 ± 0.3	2.2 ± 0.5

hierarchy among $|T_V|$, $|C_P|$, and $|E_P|$ is also very similar to those in the DP category in Sec. III A. However, the strong phase δ_{E_P} can be zero within the 68.3% C.L. region.

Predictions for all the $B_{u,d,s}$ meson decays according to our fit results are listed in the lower part of Table V. The most dominant mode $D_s^{*+}\pi^-$ of B_s decays is predicted to have a branching ratio of $(27 \pm 4) \times 10^{-4}$, similar to that of the $D_s^+\pi^-$ mode.

From the naive factorization approximation, the SU(3) breaking parameters are given by

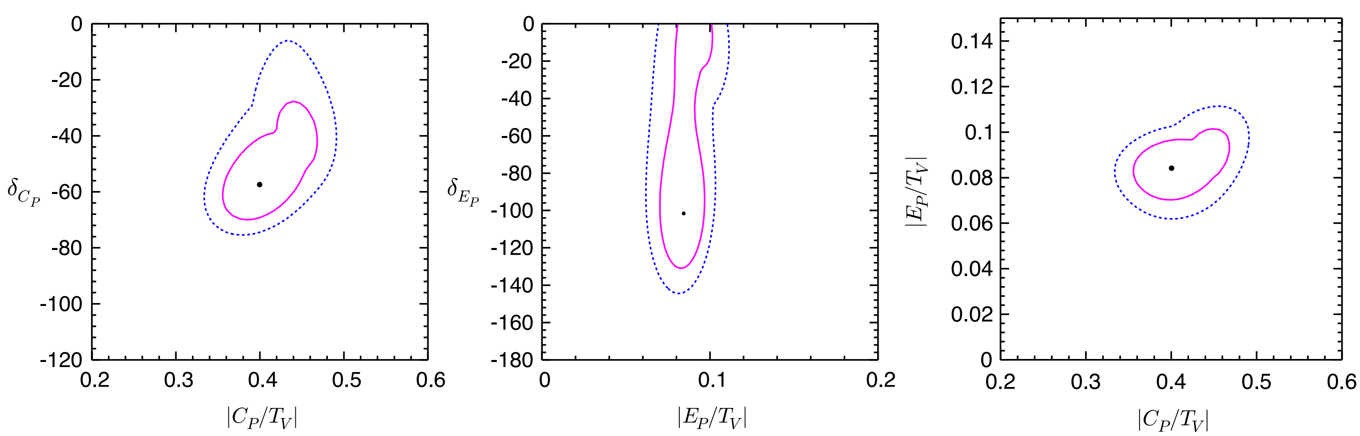

FIG. 2 (color online). $\Delta\chi^2 = 1$ (pink, solid) and 2.30 (blue, dotted) contours on the δ_{C_P}-$|C_P/T_V|$, δ_{E_P}-$|E_P/T_V|$, and $|E_P/T_V|$-$|C_P/T_V|$ planes in scheme 3.

$$\xi_{T_V}^{\rm NF} = \frac{p^*_{D^*K} f_K A_0^{BD^*}(m_K^2)}{p^*_{D^*\pi} f_\pi A_0^{BD^*}(m_\pi^2)} \simeq 1.22, \qquad \xi_{C_P}^{\rm NF} = \frac{p^*_{D^*K} F_1^{BK}(m_{D^*}^2)}{p^*_{D^*\pi} F_1^{B\pi}(m_{D^*}^2)} \simeq 1.36, \tag{9}$$

where $A_0^{BD^*}(m_\pi^2) = 0.64$, $A_0^{BD^*}(m_K^2) = 0.65$, $F_1^{B\pi}(m_{D^*}^2) = 0.31$, and $F_1^{BK}(m_{D^*}^2) = 0.43$ [26]. Unlike the DP sector, our fitted SU(3) breaking factors are somewhat smaller than the naive factorization expectations.

The ratio of the two effective Wilson coefficients can be extracted as

$$\left|\frac{a_2^{\rm eff}}{a_1^{\rm eff}}\right|_{D^*P} = \left|\frac{C_P}{T_V}\right| \frac{f_\pi A_0^{BD^*}(m_\pi^2)}{f_{D^*} F_1^{B\pi}(m_{D^*}^2)} = 0.42 \pm 0.04, \tag{10}$$

where $|C_P/T_V| = 0.40^{+0.07}_{-0.04}$ as obtained from the χ^2 analysis in scheme 3, and $f_{D^*} = 256.0$ MeV [12,28] is used. In the pQCD approach [27], $|a_2^{\rm eff}/a_1^{\rm eff}|_{D^*P}$ is found to be 0.47–0.55 and their relative phase is estimated to be $-64.8° < \arg(a_2^{\rm eff}/a_1^{\rm eff})_{D^*P} < -61.4°$ without the exchange diagram. These are consistent with our results at the 1σ level.

At this point, it is useful to compare our results of the $B \to DP$ and D^*P modes with the calculations in the soft-collinear effective field theory [29]. In their analysis, the decay amplitudes of the $D^{(*)}\pi$ modes are decomposed in the isospin amplitudes $\mathcal{A}_{1/2}$ and $\mathcal{A}_{2/3}$, which are related to our flavor amplitudes via

$$\mathcal{A}_{1/2} = \sqrt{\frac{2}{3}} T_{(V)} + \sqrt{\frac{3}{2}} E_{(P)} - \frac{1}{\sqrt{6}} C_{(P)}, \tag{11}$$

$$\mathcal{A}_{3/2} = \frac{1}{\sqrt{3}} (T_{(V)} + C_{(P)}). \tag{12}$$

They define the ratio $R_I = \mathcal{A}_{1/2}/(\sqrt{2}\mathcal{A}_{3/2}) \equiv |R_I| e^{i\delta_I}$. In the heavy quark limit, $|R_I|$ and δ_I are predicted to be the same for both $D\pi$ and $D^*\pi$ decays at the leading order in α_s [29]. From the experimental data, they extract $(|R_I|, \delta_I) = (0.70 \pm 0.07, 28.1° \pm 3.3°)$ for the $D\pi$ modes and $(0.76 \pm 0.07, 31.9° \pm 4.5°)$ for the $D^*\pi$ modes. Even though using a larger and newer data set, we obtain consistent results, in scheme 2, for example, with $(|R_I|, \delta_I) = (0.73 \pm 0.04, 23.6° \pm 4.2°)$ for the DP modes and $(0.69 \pm 0.14, 20.6° \pm 7.5°)$ for the D^*P modes.

TABLE VI. $B \to DV$ decays. Theoretical parameters are extracted from global χ^2 fits in different schemes explained in the text. The amplitude sizes are given in units of 10^{-6} GeV. Predictions of branching ratios are made with $\xi_{E_V} = 1$ and given in units of 10^{-4} unless otherwise noted.

	Scheme 1	Scheme 2	Scheme 3
$\lvert T_P\rvert$	$25.60^{+1.56}_{-1.62}$	$25.60^{+1.56}_{-1.62}$	$25.87^{+1.61}_{-1.72}$
$\lvert C_V\rvert$	$7.07^{+0.29}_{-0.33}$	$7.07^{+0.29}_{-0.33}$	$6.95^{+0.29}_{-0.37}$
$\lvert E_V\rvert$	$0.57^{+1.32}_{-0.43}$	$0.57^{+1.32}_{-0.43}$	$0.77^{+1.53}_{-0.66}$
δ_{C_V} (degrees)	$-75.1^{+19.1}_{-15.8}$	$-75.1^{+19.1}_{-15.8}$	$-79.2^{+18.0}_{-14.9}$
δ_{E_V} (degrees)	$143.4^{+36.6}_{-108.8}$	$143.4^{+36.6}_{-108.8}$	$158.6^{+21.4}_{-128.5}$
ξ_{T_P}	1 (fixed)	f_{K^*}/f_ρ (fixed)	$\xi_{T_P}^{\rm NF}$ (fixed)
ξ_{C_V}	1 (fixed)	f_{K^*}/f_ρ (fixed)	$\xi_{C_V}^{\rm NF}$ (fixed)
$\chi^2_{\rm min}$	5.91	5.91	4.18
$\chi^2_{\rm min}$/dof	2.96	2.96	2.09
$B^- \to D^0\rho^-$	105 ± 18	105 ± 18	103 ± 18
$\to D^0K^{*-}$	5.7 ± 1.0	5.7 ± 1.0	5.6 ± 1.0
$\bar{B}^0 \to D^+\rho^-$	78 ± 11	78 ± 11	78 ± 12
$\to D^+K^{*-}$	4.3 ± 0.5	4.3 ± 0.5	4.4 ± 0.6
$\to D^0\rho^0$	3.5 ± 0.8	3.5 ± 0.8	3.4 ± 1.0
$\to D^0\omega$	2.7 ± 0.7	2.7 ± 0.7	2.7 ± 0.9
$\to D^0\bar{K}^{*0}$	0.33 ± 0.03	0.33 ± 0.03	0.38 ± 0.04
$\to D_s^+K^{*-}$	0.04 ± 0.12	0.04 ± 0.12	0.07 ± 0.20
$\bar{B}_s^0 \to D^+\rho^-$ (in units of 10^{-7})	2.1 ± 6.3	2.1 ± 6.3	3.8 ± 10.7
$\to D^+K^{*-}$	4.2 ± 0.6	4.2 ± 0.6	4.2 ± 0.6
$\to D^0\rho^0$ (in units of 10^{-6})	1.9 ± 5.8	1.9 ± 5.8	3.5 ± 9.8
$\to D^0K^{*0}$	5.8 ± 0.5	5.8 ± 0.5	5.6 ± 0.5
$\to D^0\omega$ (in units of 10^{-7})	1.0 ± 3.2	1.0 ± 3.2	1.9 ± 5.4
$\to D^0\phi$	0.31 ± 0.03	0.31 ± 0.03	0.35 ± 0.03
$\to D_s^+\rho^-$	75 ± 9	75 ± 9	77 ± 10
$\to D_s^+K^{*-}$	4.1 ± 0.6	4.1 ± 0.6	4.2 ± 0.6

C. $B \to DV$ decays

The decays in this category render a very different pattern from the previous two in the χ^2 fitting (Table VI). First, scheme 1 and scheme 2 yield the same result. This is because $f_{K^*}/f_\rho \simeq 1.00$. Furthermore scheme 3 does not work well, unlike the DP and D^*P sectors. It is found that $\chi^2_{\rm min} = 0.045$, $\xi_{T_P} = 0.83$, $\xi_{C_V} = 4.58$, $|T_P| = 31.49$, $|C_V| = 1.75$, and $|E_V| = 6.64$ in units of 10^{-6} GeV, if we take ξ_{T_P} and ξ_{C_V} as free fitting parameters. Theoretically, we do not expect $|C_V| < |E_V|$. This unreasonable result is partly caused by the fact that $|E_V|$ is less constrained by the experiment $\bar{B}^0 \to D_s^+ K^{*-}$. Therefore we here adopt another prescription in which ξ_{T_P} and ξ_{C_V} are fixed by the naive factorization calculation, i.e.,

$$\xi_{T_P} = \xi_{T_P}^{\rm NF} = \frac{p^*_{DK^*} f_{K^*} F_1^{BD}(m^2_{K^*})}{p^*_{D\rho} f_\rho F_1^{BD}(m^2_\rho)} \simeq 1.00,$$
$$\xi_{C_V} = \xi_{C_V}^{\rm NF} = \frac{p^*_{DK^*} A_0^{BK^*}(m^2_D)}{p^*_{D\rho} A_0^{B\rho}(m^2_D)} \simeq 1.09, \qquad (13)$$

where $F_1^{BD}(m^2_\rho) = 0.69$, $F_1^{BD}(m^2_{K^*}) = 0.69$, $A_0^{B\rho}(m^2_D) = 0.35$, and $A_0^{BK^*}(m^2_D) = 0.38$ [26].

$|T_P| = (26.1 \pm 2.0) \times 10^{-6}$ GeV can be extracted from the D^+K^{*-} mode using the U-spin symmetry and taking $\xi_{T_P} = f_{K^*}/f_\rho$. This is slightly larger than our fit result in scheme 2. Directly from the $\bar{B}^0 \to D_s^+ K^{*-}$ mode, we have only a poor upper bound of 8.2×10^{-6} GeV on $|E_V|$.

The observable $\mathcal{B}(D^0\rho^-)$ has the largest contribution to the total $\chi^2_{\rm min}$. From Table III, we observe that the area of the triangle formed from the $B^- \to D^0\rho^-$, $\bar{B}^0 \to D^+\rho^-$, and $\bar{B}^0 \to D^0\rho^0$ decays is very small, while that of the triangle formed from the $B^- \to D^0 K^{*-}$, $\bar{B}^0 \to D^+K^{*-}$, and $\bar{B}^0 \to D^0\bar{K}^{*0}$ modes is not. This is the reason why the global χ^2 fits in the DV sector are not as satisfactory as those in the DP and D^*P sectors.

In Ref. [7], we noted that $|C_V|$ extracted from $D^0\bar{K}^{*0}$ was inconsistent with $\sqrt{|C_V|^2 + |E_V|^2}$ extracted from a combination of the $D^0\rho^0$ and $D^0\omega$ modes. Currently, the former is $(8.01 \pm 0.60) \times 10^{-6}$ GeV if we take $\xi_{C_V} = f_{K^*}/f_\rho$, and the latter is $(6.86 \pm 0.34) \times 10^{-6}$ GeV. There is still a discrepancy at the 1.7σ level, or this discrepancy implies that the SU(3) breaking factor ξ_{C_V} should be greater than about 1.17. A determination of $\mathcal{B}(D_s^+K^{*-})$ and better measurements of related modes will be very useful in providing further insights into this problem.

In Fig. 3, we show the $\Delta\chi^2 = 1$ and 2.30 contours on the δ_{C_V}-$|C_V/T_P|$, δ_{E_V}-$|E_V/T_P|$, and $|E_V/T_P|$-$|C_V/T_P|$ planes in scheme 3, respectively. We find that $|C_V/T_P| = 0.27 \pm 0.02$ and $|E_V/T_P| = 0.03^{+0.06}_{-0.03}$. We see that the magnitude of T_P is larger than T and T_V, resulting in a more hierarchical structure among $|T_P|$, $|C_V|$, and $|E_V|$. Another result of the large T_P is reflected in the bigger branching ratio prediction for the most dominant $\bar{B}_s^0 \to D_s^+\rho^-$ mode. As in the D^*P sector, the central value of δ_{E_V} is nonzero, but is still consistent with zero within the 68.3% C.L. region.

The ratio of the two effective Wilson coefficients can be extracted as

$$\left|\frac{a_2^{\rm eff}}{a_1^{\rm eff}}\right|_{DV} = \left|\frac{C_V}{T_P}\right| \frac{f_\rho F_1^{BD}(m^2_\rho)}{f_D A_0^{B\rho}(m^2_D)} = 0.50 \pm 0.04, \qquad (14)$$

where $|C_V/T_P| = 0.27 \pm 0.02$ as obtained from the χ^2 analysis in scheme 3. In Ref. [24], it is estimated that $|a_2^{\rm eff}/a_1^{\rm eff}|_{DV} = 0.24$–$0.42$ at the 1σ level using the data of the $B^- \to D^0\rho^-$, $\bar{B}^0 \to D^+\rho^-$, and $\bar{B}^0 \to D^0\rho^0$ modes.

Finally, we summarize our findings in Fig. 4. These diagrams are constructed by taking the central values of the fitted parameters in each category using scheme 3. They illustrate the sizes and relative phases among the tree, color-suppressed, and exchange amplitudes.

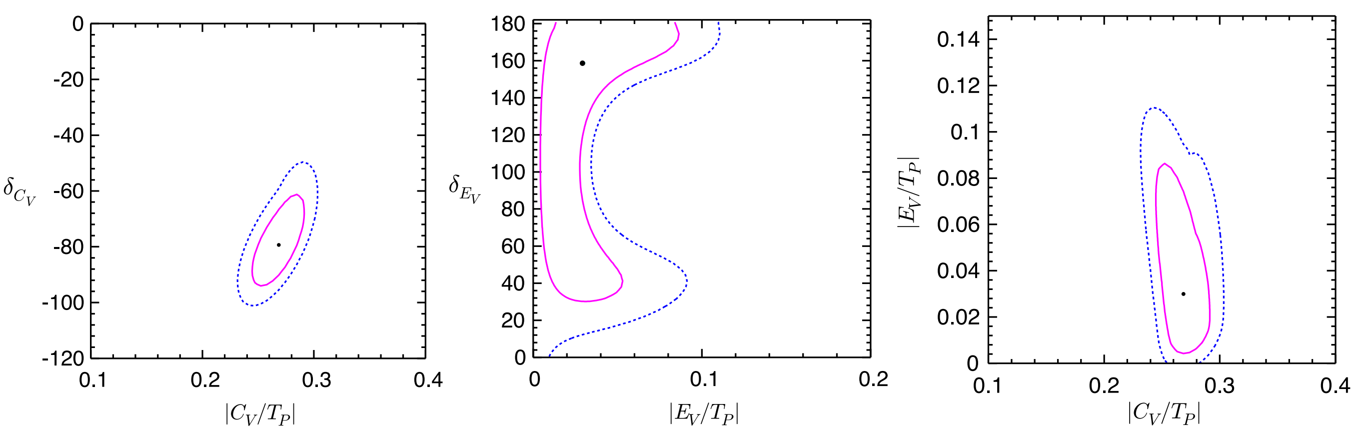

FIG. 3 (color online). $\Delta\chi^2 = 1$ (pink, solid) and 2.30 (blue, dotted) contours on the δ_{C_V}-$|C_V/T_P|$, δ_{E_V}-$|E_V/T_P|$, and $|E_V/T_P|$-$|C_V/T_P|$ planes in scheme 3.

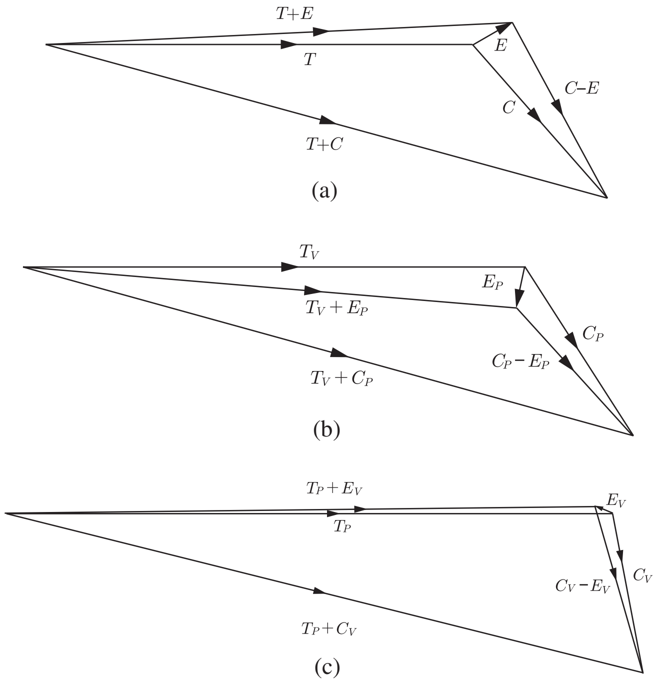

FIG. 4. Amplitude diagrams of (a): DP decays; (b): D^*P decays; and (c): DV decays.

IV. CONCLUSIONS

We have used the χ^2 fit approach to reanalyze the two-body charmed B meson decays in the flavor SU(3)-symmetric formalism, taking into account different symmetry breaking schemes as well. In the DP and D^*P decays, there is significant improvement in the χ^2 minimum between schemes 1 and 2, but not much between schemes 2 and 3. This shows that the major SU(3) breaking effect can be accounted for by the decay constant ratio f_K/f_π, as demanded, for example, by naive factorization. The same feature, however, is not observed in the DV sector, where the corresponding decay constant ratio is approximately one.

In our analysis, the fit results are generally consistent with those extracted from individual modes. We have found that the color-suppressed amplitudes are enhanced in the DP and D^*P sectors, but not in the DV sector. This strongly suggests that nonfactorizable effects or final-state rescattering effects cannot be neglected in the former two sectors.

In the DV sector, it is observed that the Cabibbo-suppressed $D^0\bar{K}^{*0}$ yields a $|C_V|$ that exceeds the bound $\sqrt{|C_V|^2 + |E_V|^2}$ given by a combination of the $D^0\rho^0$ and $D^0\omega$ branching ratios at 1.7σ level, or ξ_{C_V} should be greater than about 1.17. We urge the measurement of $\mathcal{B}(\bar{B}^0 \to D_s^+ K^{*-})$ for a direct determination of the exchange amplitude, which may provide a possible solution to this problem.

Finally, we note that the exchange diagrams are at least an order of magnitude smaller than the dominant tree topologies in these decays. Consequently, it is difficult to determine their phases, particularly in the D^*P and DV sectors, unless data precision can be significantly improved in the future.

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