Baryon number conservation and enforced electric charge neutrality for bulk viscosity in quark matter

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General constraints on fluid velocity divergences for particles in quark matter are derived from baryon number conservation and enforced electric charge neutrality. A new oscillation pattern in three-flavor normal quark matter satisfying these conditions is found and its bulk viscosity is calculated. The result may have astrophysical implication for maximum rotation frequencies of compact stars.

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In the core of compact stars where the density could reach 5–10 times the normal nuclear matter density, the constituents of nucleons and hadrons can be squeezed out to form quark matter [1,2]. Quark matter in a normal state could exist in various forms, such as strangelets [3–5], mixed phases [6–9], etc. Searching for quark matter in stellar objects is very challenging. The cooling behavior dominated by neutrino emissions may be able to distinguish nuclear and quark matter [10–16]. But it is still subjected to many experimental and theoretical uncertainties [17–19]. Other transport properties such as shear and bulk viscosities are also of great interest to this end. The shear viscosity damps the differential rotation to make a uniform rigid body rotation of stars. The bulk viscosity is crucial for the damping of pulsations in compact stars. Such pulsations could take place during the formation of stars or by external perturbations. They could also be driven by instabilities of gravitational wave radiation such as the *r*-mode instability which arises in the absence of damping effects. There has been a lot of literature about calculations of viscosities in nuclear matter in various situations [20–26]. The shear viscosity for color-flavor-locking phase has been calculated [27]. The bulk viscosities for normal and color superconducting quark matter have been studied extensively [28–34]. In this paper we put emphasis on general constraints of baryon number conservation and charge neutrality on the bulk viscosity. Following these constraints, we will give new solutions to the bulk viscosity in normal quark matter different from previous results [28–30,32–34,31].

The bulk viscosity is associated with the damping of the baryon density oscillation denoted by $\delta n_B = \delta n_{B0} e^{i\omega t}$ with the amplitude δn_{B0} and the frequency ω in the range 10^3–10^4 s^{-1}. We assume that the amplitude of the oscillation is small and can be treated as perturbation to its equilibrium value. The perturbation in baryon density drives quark matter out of chemical equilibrium via following processes

$$\begin{aligned} u + d &\leftrightarrow u + s, && (d \leftrightarrow s \text{ transition}),\\ u + e &\leftrightarrow d + \nu, && (\text{Urca I}),\\ u + e &\leftrightarrow s + \nu, && (\text{Urca II}). \end{aligned} \tag{1}$$

Here we consider the normal quark matter with three flavors u, d and s. Light quarks u and d are almost massless, while s quarks have large mass. The relaxation to chemical equilibrium is related to deviations of chemical potentials from their equilibrium values for these processes:

$$\begin{aligned} \delta\mu_1 &\equiv \mu_s - \mu_d = \delta\mu_s - \delta\mu_d,\\ \delta\mu_2 &\equiv \mu_d - \mu_u - \mu_e = \delta\mu_d - \delta\mu_u - \delta\mu_e,\\ \delta\mu_3 &\equiv \mu_s - \mu_u - \mu_e = \delta\mu_s - \delta\mu_u - \delta\mu_e. \end{aligned} \tag{2}$$

The reaction rates for processes in (1) can then be written as $\lambda_k \delta\mu_k$ $(k = 1, 2, 3)$ for small $\delta\mu_k \ll T$, where λ_k are coefficients. In our calculations, λ_1 and λ_3 are taken from Ref. [33,34] except that we have included the mass effect of s quarks [30] and phase space reduction from Fermi liquid behavior [10,11,15]. The value of λ_2 is taken from Ref. [33].

With massive s quarks a purely three-flavor system cannot be electrically neutral, so there must be electrons in the system. In reactions (1), the electron number and flavors are not conserved since they can be created or destructed, however the baryon number and electric charge are conserved. The continuity equations for quark flavor, baryon and electron number read

$$\frac{dn_i}{dt} + n_i \nabla \cdot \mathbf{v}_i = J_i, \qquad (i = u, d, s, B, e), \tag{3}$$

where the substantial or material derivative is defined by $\frac{d}{dt} \equiv \frac{\partial}{\partial t} + \mathbf{v}_i \cdot \nabla$. The sources J_i are linear combinations of reaction rates $\lambda_k \delta\mu_k$ when $\delta\mu_k$ are small. It is obvious that $J_B = 0$ required by baryon number conservation. The following relations for baryon number conservation and

charge neutrality are widely used in literature:

$$n_B = \frac{1}{3}\sum_{i=u,d,s} n_i, \qquad n_e = \sum_{i=u,d,s} Q_i n_i,$$
$$J_B = \frac{1}{3}\sum_{i=u,d,s} J_i = 0, \qquad J_e = \sum_{i=u,d,s} Q_i J_i. \tag{4}$$

The charge neutrality condition in the above is a natural constraint since the accumulation of net charges would make the Coulomb energy density blow up and be less favorable in energy. With Eq. (4) and continuity equations (3), one obtains the following constraints on number densities and velocity divergences:

$$n_B \nabla\cdot\mathbf{v}_B = \sum_{i=u,d,s} \frac{1}{3} n_i \nabla\cdot\mathbf{v}_i,$$
$$n_e \nabla\cdot\mathbf{v}_e = \sum_{i=u,d,s} Q_i n_i \nabla\cdot\mathbf{v}_i, \tag{5}$$

where the first line is the result of baryon number conservation and the second one that of charge neutrality. Equation (5) was not presented and applied explicitly in previous literature. A simple solution to the above equations can be found at first glance [28–34],

$$\nabla\cdot\mathbf{v}_i = \nabla\cdot\mathbf{v}_B, \qquad (i = u, d, s, e), \tag{6}$$

with number densities satisfying Eq. (4). This solution corresponds to the homogeneous or one-component fluid with a single fluid velocity field. Inserting Eq. (6) back into continuity Eqs. (3), one finds

$$n_B \frac{dX_i}{dt} = \frac{dn_i}{dt} - X_i \frac{dn_B}{dt} = J_i, \tag{7}$$

where $X_i \equiv n_i/n_B$ are partial fractions for particles i in terms of the baryon number density. We have used the continuity equation for the baryon number in Eq. (3). Obviously Eq. (7) respects baryon number conservation and charge neutrality. A property of the solution (6) is that it will give a vanishing bulk viscosity in the case that all quarks are massless, where the system is always in chemical equilibrium. An example of this property in two-flavor normal quark matter can be found in Ref. [33].

In this paper we rigorously apply the constraints (5) and find a new solution to Eq. (5) different from Eq. (6). Our starting point is that the role of strange quarks is special since they are much heavier and may respond to the density oscillation more reluctantly than other particles. As an extreme case, we assume $\nabla\cdot\mathbf{v}_s = 0$. We also assume $\nabla\cdot\mathbf{v}_u = \nabla\cdot\mathbf{v}_d$. Then one obtains from Eq. (5) the following relations:

$$\nabla\cdot\mathbf{v}_e = \frac{n_B(2n_u - n_d)}{n_e(n_u + n_d)}\nabla\cdot\mathbf{v}_B,$$
$$\nabla\cdot\mathbf{v}_{u,d} = \frac{3n_B}{n_u + n_d}\nabla\cdot\mathbf{v}_B. \tag{8}$$

Generally the velocity divergence for a particle in a fluid depends on its mass. Rigorously the fluid with more than one particle species with different masses should be treated as a multicomponent fluid. The above solution to Eq. (5) is reasonable in the case that the masses of strange quarks are of the same order as the chemical potentials and much larger than light quark masses. If the strange quark masses are small, one can investigate many other solutions which are close to the solution (6). For example, one can assume velocity divergences of particles deviate from that of baryons by a small amount, $\nabla\cdot\mathbf{v}_i = \nabla\cdot\mathbf{v}_B + \varepsilon_i$ with ($i = u$, d, s, e). Following Eq. (5), these ε_i satisfy $\sum_{i=u,d,s}\frac{1}{3}n_i\varepsilon_i = 0$ and $\sum_{i=u,d,s} Q_i n_i \varepsilon_i = n_e\varepsilon_e$. Any small values of ε_i under these constraints denote a slightly different solution from Eq. (6). We will not consider these solutions in this paper and focus on the solution (8). With these relations in Eq. (8) for velocity divergences the bulk viscosity for a system of quarks and electrons can be derived. The deviation of the pressure from its thermodynamic value is related to the bulk viscosity [35],

$$\delta\mathcal{P} = -\zeta\nabla\cdot\mathbf{v}_B. \tag{9}$$

The above also provides a definition for the bulk viscosity. The variation $\delta\mathcal{P}$ can be expressed in terms of density variations $\delta n_i = \delta n_{i0} e^{i\omega t}$ for some quarks or electrons, which are linearly independent after using Eq. (4). Here δn_{i0} are complex amplitudes and can be solved by applying continuity equations. The number of continuity equations applied is equal to that of independent densities in order to close the system of equations. Normally the baryon density n_B is set to be one independent variable. The r.h.s of Eq. (9) is actually $\zeta(n_B)^{-1}dn_B/dt$. So a complex ζ can be obtained from Eq. (9), $\zeta = n_B\delta\mathcal{P}/(d\delta n_B/dt)$, whose real part gives the bulk viscosity [35].

We will first consider in reactions (1) two simplest cases, the $d \leftrightarrow s$ transition (by turning off Urca I and Urca II) and Urca II (by turning off $d \leftrightarrow s$ transition and Urca I), separately. Finally we will address a more realistic case, the three coupled processes together. The Urca processes with light quarks (Urca I) was already studied in Ref. [33]. The calculations for the $d \leftrightarrow s$ transition without charge neutrality can be found in Ref. [34]. We list all quantities needed in evaluating bulk viscosities in three cases in Table I. The second column is for independent variables. The third one is for continuity equations used in this paper to solve the variations of densities. For example, we use continuity equations for s quarks and electrons for the three coupled processes. With Eq. (8) it makes no difference to use other independent continuity equations. The fourth column lists other variables expressed in terms of independent ones. In the $d \leftrightarrow s$ transition and Urca II, there are additional constraints on electrons and d quarks, respectively, since they do not participate in the reactions. With

TABLE I. Quantities for bulk viscosities in three cases with $C \equiv 3n_d/(n_u + n_d)$.

	Independent variables	Continuity eqs.	Other variables	Special constraints
$d \leftrightarrow s$	δn_B, δn_s	s	$\delta n_u = (3 - C)\delta n_B$ $\delta n_d = C\delta n_B - \delta n_s$	$J_e = 0$ $\delta n_e = (2 - C)\delta n_B$
Urca II	δn_B, δn_s	s	$\delta n_u = (3 - C)\delta n_B - \delta n_s$ $\delta n_e = (2 - C)\delta n_B - \delta n_s$	$J_d = 0$ $\delta n_d = C\delta n_B$
Coupled	δn_B, δn_s, δn_e	s, e	$\delta n_u = \delta n_B + \delta n_e$ $\delta n_d = 2\delta n_B - \delta n_e - \delta n_s$	

$J_{e,s} = 0$ in Eq. (3) and (8), one can solve δn_e and δn_d in terms of δn_B in the last column.

For the $d \leftrightarrow s$ transition, following Eq. (9) and with the second row in Table I, one obtains the bulk viscosity,

$$\zeta_1 = \mathrm{Re}\left(\frac{n_B \delta \mathcal{P}}{d\delta n_B/dt}\right) = \frac{\lambda_1 n_B \frac{\partial \mu_1}{\partial n_B}\frac{\partial \mathcal{P}}{\partial n_s}}{\omega^2 + \lambda_1^2(\frac{\partial \mu_1}{\partial n_s})^2},$$

where

$$\frac{\partial \mu_1}{\partial n_B} \equiv -C\frac{\partial \mu_d}{\partial n_d},$$
$$\frac{\partial \mu_1}{\partial n_s} \equiv \frac{\partial \mu_d}{\partial n_d} + \frac{\partial \mu_s}{\partial n_s},$$
$$\frac{\partial \mathcal{P}}{\partial n_s} \equiv n_s\frac{\partial \mu_s}{\partial n_s} - n_d\frac{\partial \mu_d}{\partial n_d},$$

with $C \equiv 3n_d/(n_u + n_d)$. One can verify that ζ_1 is definitely positive since $\frac{\partial \mu_1}{\partial n_B} < 0$ and $\frac{\partial \mathcal{P}_B}{\partial n_s} < 0$ using the equation of state for degenerate Fermi gas for d and s quarks. The numerical results for ζ_1 are shown in Fig. 1. As shown in the figure that the bulk viscosity increases with decreasing frequency until it saturates below a critical frequency $\omega_{1c} \sim \lambda_1|\frac{\partial \mu_1}{\partial n_s}|$. One also sees that for low frequencies $\omega \ll \omega_{1c}$, ζ_1 is inversely proportional to the transport coefficient λ_1. This means the faster the reaction proceeds the smaller the bulk viscosity is.

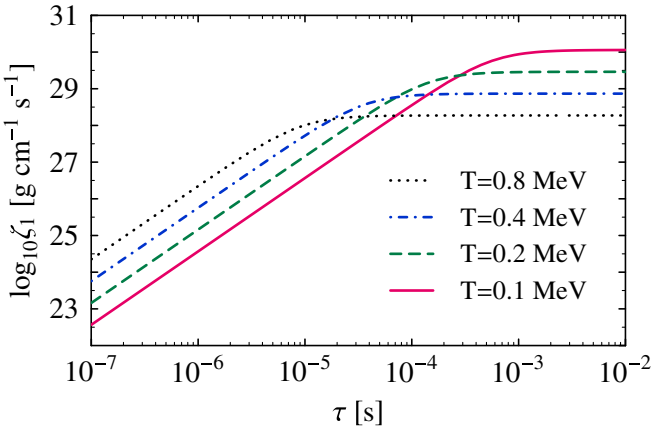

FIG. 1 (color online). The bulk viscosity ζ_1 for $d \leftrightarrow s$ transition. The masses and chemical potentials (in MeV) are set to $m_u = m_d = 0$, $m_s = 100$, $\mu_s = \mu_d = 500$, $\mu_u = 495$, $\mu_e = 5$. Note that these values satisfy charge neutrality and chemical equilibrium conditions.

Similarly, with the third row in Table I, the bulk viscosity for Urca II is

$$\zeta_3 = \frac{\lambda_3 n_B \frac{\partial \mu_3}{\partial n_B}\frac{\partial \mathcal{P}}{\partial n_s}}{\omega^2 + \lambda_3^2(\frac{\partial \mu_3}{\partial n_s})^2}, \tag{10}$$

where

$$\frac{\partial \mu_3}{\partial n_B} \equiv -(3 - C)\frac{\partial \mu_u}{\partial n_u} - (2 - C)\frac{\partial \mu_e}{\partial n_e},$$
$$\frac{\partial \mu_3}{\partial n_s} \equiv \frac{\partial \mu_s}{\partial n_s} + \frac{\partial \mu_u}{\partial n_u} + \frac{\partial \mu_e}{\partial n_e},$$
$$\frac{\partial \mathcal{P}}{\partial n_s} \equiv -n_u\frac{\partial \mu_u}{\partial n_u} + n_s\frac{\partial \mu_s}{\partial n_s} - n_e\frac{\partial \mu_e}{\partial n_e}.$$

One sees $\frac{\partial \mu_3}{\partial n_B} < 0$ due to $C < 2$, and $\frac{\partial \mathcal{P}}{\partial n_s} = -\frac{m_s^2}{3\mu_s} + \frac{m_u^2}{3\mu_u} < 0$ with $\mu_e \ll \mu_u$ and $m_s \gg m_u$. The numerical results are shown in Fig. 2. The behavior of ζ_3 is similar to ζ_1. For critical frequencies one sees $\omega_{3c} \sim \lambda_3|\frac{\partial \mu_3}{\partial n_s}| \ll \omega_{1c}$ because $\lambda_3 \ll \lambda_1$. The saturation value of ζ_3 is larger than that of ζ_1 for the same reason. At high frequencies $\omega \gg \omega_{3c}$ one observes $\zeta_3 \ll \zeta_1$.

With the fourth row in Table I, the bulk viscosity for three coupled processes can be expressed as

$$\zeta = \frac{n_B}{\omega}\left(D_s\,\mathrm{Im}\frac{\delta n_s}{\delta n_B} + D_e\,\mathrm{Im}\frac{\delta n_e}{\delta n_B}\right) = \frac{n_B}{F}(D_s N_s + D_e N_e), \tag{11}$$

where F and $N_{e,s}$ are quadratic and linear functions of ω^2

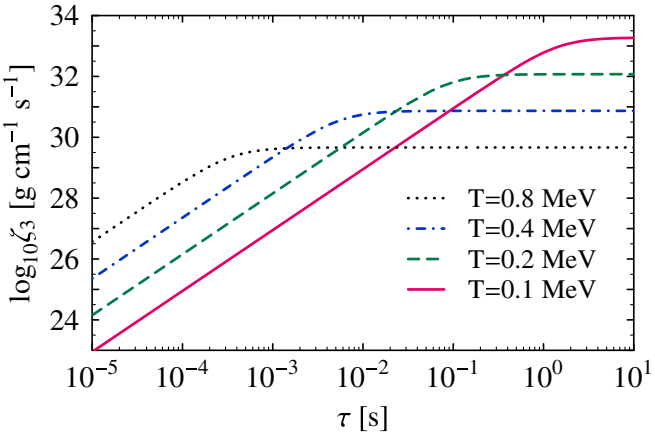

FIG. 2 (color online). The bulk viscosity ζ_3 for the Urca II. The parameters are the same as in Fig. 1.

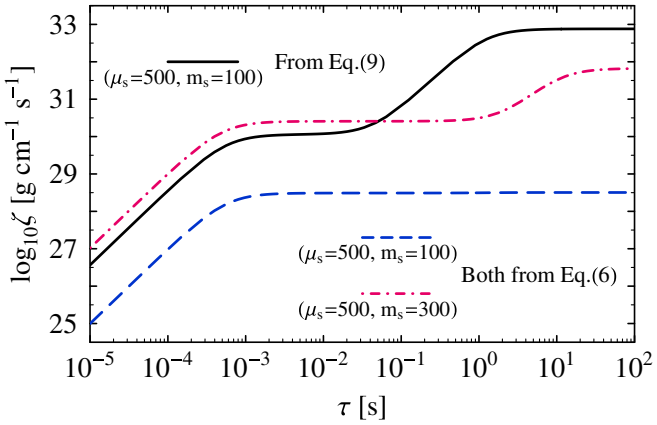

FIG. 3 (color online). The bulk viscosity ζ for the coupled processes at $T = 0.1$ MeV. The dashed and dash-dotted curves are the results from the solution of Eq. (6) with two sets of parameters. The solid curve is the result from the solution of Eq. (8). The parameters of the solid and dashed curves are the same as in Figs. 1 and 2, while those of dash-dotted curve are different and still satisfy neutrality and chemical equilibrium conditions, $m_s = 300$, $\mu_s = \mu_d = 500$, $\mu_u = 456$, $\mu_e = 44$, all in unit MeV.

respectively. They also depend on coefficients λ_k. The explicit form of these functions will be presented elsewhere [36]. The coefficients $D_{e,s}$ are given by

$$D_e \equiv n_u \frac{\partial \mu_u}{\partial n_u} - n_d \frac{\partial \mu_d}{\partial n_d} + n_e \frac{\partial \mu_e}{\partial n_e},$$

$$D_s \equiv n_s \frac{\partial \mu_s}{\partial n_s} - n_d \frac{\partial \mu_d}{\partial n_d}.$$

The numerical result for the bulk viscosity ζ is shown in Fig. 3 by solid curve. Also shown are dashed and dash-dotted curves from the method with Eq. (6). With the same parameters, the result from Eq. (8) (solid curve) is about 1 order of magnitude larger than that from Eq. (6) (dashed curve). When m_s is set to 300 MeV, the result from Eq. (6) (dash-dotted curve) is comparable to the solid curve at high frequencies but still differs from it substantially at low ones. Generally there can be up to 3 plateaus in the log-log plot of the bulk viscosity as function of frequency depending on parameters. In Fig. 3, the solid and dash-dotted curves have two plateaus, while the dashed curve has only one. It can be verified that ζ is dominated by the s quark part in Eq. (11). Since D_s is negative, the phase of δn_s should be delayed relative to δn_B manifested by the positivity of the bulk viscosity. The another calculation of the bulk viscosity for the coupled processes can be found in Ref. [37].

In summary, we have derived general constraints on fluid velocity divergences of quarks and electrons in normal quark matter from baryon number conservation and enforced electric charge neutrality. Under these constraints we find a new solution to velocity divergences leading to different bulk viscosities. As an extreme case, this new solution could be realistic in some circumstances where strange quarks respond to the oscillation in a very different way from light quarks. Other solutions are also allowed by these constraints. The new result for the coupled processes may have astrophysical implication for larger maximum rotation frequencies of compact stars. Similar constraints on fluid velocity divergences can also be obtained for nuclear matter.

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