

Time-odd distribution functions, breaking of long range correlations, and sudden entropy changes, in Drell-Yan high-energy processes

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Time-odd parton distribution functions in a Drell-Yan process are here studied by examining the evolution of the internal statistical properties of the interacting hadrons. Time-odd functions are shown to be a signature of the irreversible process in which a hadronic state characterized by long range correlation properties (hadronic phase) decays to produce a cloud of independent partons (partonic phase) because of initial/final state interactions. The relevant considered variable is the rate of increase of the entropy of the hadronic system. This quantity is shown to be roughly equal to the decay rate of the hadronic state. Conditions for getting a leading twist time-odd effect are established on this basis. Last, the relevant case of a large entropy increase associated with transverse-dominated initial/final state interactions is analyzed.

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I. INTRODUCTION

The field of time-reversal odd (T-odd) distribution functions in high-energy hadronic physics has known a rich development in the last years, and these distributions have become an essential tool in the interpretation of the phenomenology of transverse spin and azimuthal asymmetries in high-energy physics, and in the design of new experiments.

In the framework of QCD factorization the existence was predicted long ago [1,2] of twist-3 T-odd effects. The first T-odd leading twist distribution function was the Sivers function [3] in 1990, followed by the Boer-Mulders-Tangerman (BMT) function [4–6]. The Sivers function was used to explain single spin asymmetries [7], the BMT function to explain unpolarized Drell-Yan azimuthal asymmetries [6]. These phenomena could also be interpreted via soft mechanisms [8], despite still according with schemes that may be (qualitatively) considered “T-odd.” It has been recently shown [9] that in processes like Drell-Yan where the scale of the transverse momentum q_T is independent from the hard scale Q , the twist-3 T-odd structures predicted in [1,2] are not Q -suppressed at finite q_T , and they essentially coincide with the above quoted leading twist T-odd distributions. That work also shows that the details of the initial state may be of secondary relevance when the problem of producing leading twist T-odd distributions is addressed.

A time-reversal odd structure function, the so-called “fifth structure function,” was also long ago introduced [10] and more recently modeled [11,12] in nuclear physics to describe normal asymmetries in $A(e, e'p)(A-1)$ quasi-elastic scattering (for reviews see [13]). It was known [10] that this structure function could exist in the presence of final state interactions only, at the point that its vanishing was suggested [11] as an experimental signature for the

onset of color transparency. The generalization of nuclear physics experience to high-energy hadronic physics cannot be complete for the absence of a factorization framework in the former case. One interesting and perhaps more general point, however, is that, although the phenomenology of the fifth structure function is strictly related with spin asymmetries, models for it [11,12] do not need to involve spin-dependent interactions. Rather, the key point is in particle flux absorption.

In QCD, leading twist T-odd distribution functions were initially considered forbidden [14] by general invariance principles. After an explicit mechanism was shown to produce a nonzero T-odd distribution in a QCD framework [15], the existence problem was systematically fixed in Ref. [16], where Collins observed that the presence of a link operator necessary to restore gauge invariance for the two-point correlation function allowed for nonvanishing T-odd distributions in QCD. These and other works [17,18] also give conditions for having leading twist effects in k_T -dependent T-odd functions.

Models [19–23] or studies [24–29] around T-odd distribution functions have been produced and discussed by several authors. Recently several phenomenological parametrizations of the Sivers function for quarks [30–33] and gluons [34] have been deduced from available data [35–38]. An old parametrization [39] of the $\cos(2\phi)$ asymmetry has been translated into parametrizations [6,40] of the Boer-Mulders-Tangerman function. Several experiments aimed at the measurement of T-odd functions are planned for the next ten years [41–45].

In this work the problem of the existence of leading twist T-odd functions and of their interpretation is examined from an unusual point of view. They are supposed to originate in a phase transition of the statistical properties of the hadron ground state due to initial/final state interactions accompanying a hard electromagnetic process.

With “phase” we only mean a set of collective properties enjoying a relative stability. We may name “hadronic”

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phase the initial one, and “partonic” phase the final one. For having leading twist observables, the transition needs to take place over a time range that is singular in the infinite momentum limit.

We will here focus on the Drell-Yan process only, so “partonic distributions” means actually “partonic distributions measured in Drell-Yan.” On the ground of eclipse effects, one would expect Drell-Yan rates on nuclear targets to scale as $A^{2/3}$, possibly with shadowing effects on the shape of the measured distributions. The situation is rather different (see [46] for a detailed compared review of Drell-Yan proton and pion data on light to heavy targets). Data show that hadron/nuclear matter is practically transparent to individual partons, while hadronic states have a rather short survival path in them. The partonic distributions resulting from a Drell-Yan measurement are independent from the target mass number A from H to Pt, and the total Drell-Yan rate is $\propto A$, while the cross sections for the much more frequent hadronic processes resulting from the same collisions are $\sim A^{2/3}$ as obvious. Drell-Yan data recover the $A^{2/3}$ scaling law if the dilepton pair is on some resonant mass like ρ or J/ψ .

This suggests that any single parton is able to transport *probabilistic* information on its initial state through all the volume of a heavy nucleus. On the contrary, its initial state wave function is well known to be destroyed within 1 fm. To say that hadron-hadron initial state interactions cause a transition from a hadronic to a partonic phase, destroying the long range properties of the former, but without touching the short range ones associated with the latter, is just another way to say the same thing with other words.¹

However, this way of seeing the process will be useful in the following. In particular, it suggests a study of the entropy properties of the system, since any process where long range correlations are atomized is associated with a relevant increase of entropy. T-odd functions are here considered as a direct signature of the hadron to parton transition, and a quantitative measure of the time rate of increase of the entropy.

A weak point of the picture presented here can obviously be the difficulty in disentangling specific effects of QCD/strong interactions from general properties of a bound system subject to a hard external probe.

Section II is devoted to listing a few general definitions and relations that are systematically used in the following.

The presence of chaotic processes in association with hard hadronic interactions is obvious at a generic level. More specifically, however, we need to understand where the chaotic side of these processes enters formally into parton distribution functions, how this leads to T-odd ob-

¹The words “partonic phase” do not refer specifically to quark-gluon states like those studied in the physics of heavy ion collisions (see e.g. [47] and references therein). They refer more generically to any hadron state where long range correlations are not present.

servables, and at which conditions these observables are leading twist. To this point Secs. III and IV are devoted. In Sec. III the correlator is factorized into two parts. One contains those effects like S-P interference that are specific to the detection process, while the other one contains the statistical properties (“2-scalar correlator”). It is shown that the 2-scalar correlator may be decomposed with completeness in damped plane waves of the form $\exp(-ix\xi - \gamma(x)\xi)$, where $\xi \equiv P_+z_-$, and the condition of finiteness of $\gamma(x)$ is shown to be a signature of leading twist T-odd effects. In Sec. IV a review of some topics of statistical mechanics is given, focussing, in particular, on the decay properties of a self-correlation function, and on the correspondence with the relevant quantities of the Drell-Yan problem. The general criterion $\gamma \sim dS/d\xi \sim x$ relating the rate of change of entropy with the hadron state decay rate γ and with the existence of T-odd leading twist effect is qualitatively introduced.

In Sec. V the previous arguments are made more precise and specific. First, a criterion is given to somehow formalize the breaking of a long range correlation, i.e. to establish a borderline between long and short range correlations. Then, the relevant phase space entering the definition of entropy for long range and short range states is focussed. Initial state interactions causing the breaking are divided into two classes (single and multiple event processes) that are separately examined.

Section VI is devoted to a more restricted group of processes. Let $A(+)$ and $A(-)$ be the “blobs” graphically associated with factorization-separated areas of the full process. A peculiar side of linking T-odd effects with entropy changes is that it is natural to build models where an arbitrarily large increase of the entropy of $A(+)$ takes place in the absence of large energy exchanges between $A(+)$ and $A(-)$. Hard exchanges of transverse momentum between $A(+)$ and $A(-)$ may trigger large energy transfers among degrees of freedom all belonging to $A(+)$ alone. This is relevant since hadron-hadron interactions associated with large energy exchanges between the hadrons are statistically suppressed at large energies. So it is fair to imagine effective initial state interactions to be dominated by exchange of transverse momentum. In addition, energy conservation allows for a more precise calculation of the phase space and of the entropy.

II. SOME GENERAL DEFINITIONS AND NOTATIONS

We consider a Drell-Yan process taking place between two colliding hadrons with momenta P and P' . The parton we will examine in the following is a quark belonging to the former hadron.

To select leading twist terms, we will use the infinite momentum limit $P_+ \rightarrow \infty$, for P_+ measured in the center of mass frame of the two colliding hadrons. In other words, $(P + P')^2 \sim P_+ P'_-$ is arbitrarily large in this limit.

We name $A(+)$ and $A(-)$ those areas of the process/diagram associated with factorization-separated kinematical processes. The area of our interest is $A(+)$, where the relevant components are and k_+ (momentum) and z_- (spacetime). All those fields whose momentum component p_+ is $O(P_+)$ belong to $A(+)$. We will ignore completely what happens inside $A(-)$. We will however consider 4-momentum exchanges between $A(+)$ and $A(-)$, potentially undermining factorization.

The relevant process considered in this work is the one corresponding to the traditional cut amplitude associated by second order unitarity to the inclusive hadronic distribution functions:

In this process a hole is created in a hadron state by extracting a quark/antiquark in a point z'_μ , the “hadron + hole” set propagates through real states only, and in z''_μ the hole is filled and the original hadron state is restored by reinserting the quark in its place. During the propagation, the hole is subject to a gauge restoring field according to Ref. [16]. We will sometimes speak of “quark interactions” actually meaning “quark hole interactions.”²

The extracted quark has momentum k_μ with light cone $k_+ \equiv xP_+$ and negligible k_- . The impact parameter \vec{z}_T is the space vector conjugate to \vec{k}_T . The spacetime displacement of the hole from z'_μ to z''_μ is $z_\mu \equiv z''_\mu - z'_\mu \equiv (z_-, z_+, \vec{z}_T) \approx (z_-, 0, \vec{z}_T)$. The fourth coordinate z_+ plays no role and is not explicitly reported in the following.

Any relevant distribution function $q(x)$ is the Fourier transform with respect to z_- and z_T of a correlation function $g(z_-, \vec{z}_T)$. Leading twist effects are naturally selected by writing the Fourier transform with respect to the scaled variables x and ξ :

$$\xi \equiv P_+ z_-, \quad (1)$$

$$\begin{aligned} q(x, \vec{k}_T) &\equiv P_+ \int e^{-ixP_+ z_-} e^{ik_T z_T} g(z_-, \vec{k}_T) dz_- d\vec{k}_T \\ &\equiv \int e^{-ix\xi} e^{ik_T z_T} G(\xi, z_T) d\xi. \end{aligned} \quad (2)$$

$G(\xi, z_T)$, ξ , x , etc. will be named “scaled” quantities, $g(z_-, z_T)$, z_- , k_+ , etc. “nonscaled” ones. The range of useful values of ξ remains finite $\sim 1/x$ when $P_+ \rightarrow \infty$. So in this limit the function $G(\xi)$ does not need to be singular in the origin to produce a nonzero $q(x)$. So, in most of the following the discussion is in terms of the scaled quantities. On the other side, it will be useful sometimes to get back to the nonscaled z_- representation to evidenciate singularities.

²E.g., in spectator models the hole path starts from the quark-photon vertex, runs backwards along the quark propagator up to the hadron-quark-diquark vertex, follows the diquark across the cut up to the other hadron-quark-diquark vertex, and next a quark propagator up to the other photon-quark vertex.

In all cases, g or G are here assumed as complex numbers and not matrix objects, so they must be read as a projection of the spinor correlation matrix g_{ij} over a suitable operator, e.g. γ^+ . The γ^+ projection is the most comfortable one, since it selects simultaneously the main unpolarized quark distribution and the Sivers T-odd function. To get the former one needs to sum over opposite transverse polarization states, to get the latter one takes the difference, so e.g. in this case G means $G_+ \pm G_-$. More in general, G must be read as a linear combination of correlator projections, suitable to project interference terms between even and odd angular momentum waves.

For simplicity of the discussion we will assume that the transverse angular dependence is factorized in the correlator:

$$G(\xi, \vec{z}_T) \equiv G(\xi, z_T) G_A(\hat{z}_T), \quad (3)$$

$$q(x, \vec{k}_T) \equiv q(x, k_T) q_A(\hat{k}_T). \quad (4)$$

In general, for a given hadron spin along \hat{x} we have a double partial wave sum $G(z_\mu, 0) = \sum_{L,M,L'} g_{L,L'} Y_{L,M}(\theta_x, \phi_x) Y_{L,0}(0)$. Equations (3) and (4) refer to one term only in the sum.

In Eq. (3) the former term will be named the 2-scalar correlator, the latter the “angular” correlator. The word “2-scalar” only refers to 2-dimensional transverse rotations, not to general spacetime properties. Distribution functions are defined factorizing \hat{k}_T -dependence out of them, so they are in $q(x, k_T)$. Despite in general we have a double sum of partial wave contributions, for the following discussion one term (clearly an interference term between different angular momenta) is quite enough.

The role of the angular parts is discussed in next section. Apart for that section, in the rest of the paper the discussion regards the 2-scalar correlator and distributions only, and the word 2-scalar is omitted. Since in the 2-scalar correlator the explicit presence of z_T and k_T is often not necessary, we just write $G(\xi)$ and $q(x)$ in all the relations where k_T is not explicitly needed.

III. IMAGINARY PART OF THE 2-SCALAR CORRELATOR

Having defined $q(x, \vec{k}_T)$ as the Fourier transform of an amplitude that is projected on intermediate *real* states only, $q(x, \vec{k}_T)$ is the imaginary part of a complex amplitude, and as such it is real. Since however only $q(x, \vec{k}_T)$ is bound to be real, we may have

$$\begin{aligned} q(x, \vec{k}_T) &= \text{Re}[q(x, k_T)] \text{Re}[q_A(\hat{k}_T)] \\ &\quad - \text{Im}[q(x, k_T)] \text{Im}[q_A(\hat{k}_T)]. \end{aligned} \quad (5)$$

Clearly, if $q(x, k_T)$ represents a single partial wave contribution to the full distribution, only one of the two terms in the right-hand side will be nonzero. For an angular even-

odd interference term, the nonzero one should be the imaginary-imaginary one. Since this statement is not completely free from ambiguity, I take it as a simplifying assumption. Looking for T-odd contributions, we will focus on this term.

A leading twist T-odd effect manifests itself in a finite $\text{Im}[q(x, k_T)]$. Let us make this point more precise, following the scheme adopted in Ref. [12], and suppose that some of the Fourier components of a T-even $G(\xi)$ are substituted by damped waves

$$e^{ix\xi} \rightarrow e^{ix\xi - \gamma\xi}, \quad (6)$$

where γ in general depends on x . In other words, the integration path is shifted from the real axis and the Fourier transform becomes a Laplace transform. For the Laplace transform general theorems exist, but here we would like to rely on physics first, and demonstrate that the above waves are a proper and complete set of eigenfunctions for a decomposition of $G(\xi)$.

The above damped waves correspond to a nonconserved decaying probability at increasing times, and are eigenfunctions of a Hamiltonian operator that enjoys two relevant properties:

- (i) It is non-Hermitian and so it does not respect time inversion symmetry.
- (ii) It is invariant for ξ -translations.

In addition, we must notice that the above damped waves are the only possible eigenfunctions for a Hamiltonian that satisfies both (i) and (ii).

An amplitude $G(\xi', \xi'')$ describing the hole propagation under the action of the gauge restoring field is not invariant for ξ -translation, so it depends on both ξ' and ξ'' and not on their difference ξ only. Its Fourier transform depends on *two* x values corresponding to the incoming and outgoing momenta.

Since by definition only x -diagonal states may be present in a distribution function, these states must be eigenstates of a ξ -homogeneous and non-Hermitian Hamiltonian. Non-Hermiticity depends on the fact that it includes the gauge restoring field, but at the same time it excludes those eigenstates that, because of this field, do exist and contain violation of x -conservation.

Let us define \hat{H}_{true} as the true full Hamiltonian including the gauge restoring field. $\{F\}$ is the full set of eigenstates of \hat{H}_{true} , and $F \equiv \{XC\} + \{XNC\}$, where $\{XC\}$ is the set of x -conserving states and $\{XNC\}$ its complementary. The Hamiltonian \hat{H} that one is really using is the projection of H_{true} on $\{XC\}$ and such a projection is well known to have complex energy eigenvalues (all the theory of the nuclear optical potential is based on this fact and presents close analogies with what is discussed here, see e.g. [48]). On the other side, the projection is by definition x -diagonal and so invariant for ξ -translations.

Summarizing, $G(\xi)$ describes the hole propagation according to a Hamiltonian that is (i) non-Hermitian,

(ii) ξ -homogeneous. For these reasons it must admit a complete decomposition in states of the above form Eq. (6)

$$G(\xi) \equiv \int_0^1 dx' e^{ix'\xi} G(x') \rightarrow \int_0^1 dx' e^{ix'\xi - \gamma\xi} G(x'). \quad (7)$$

The Fourier transform of each damped wave with respect to $\exp(-ix\xi)$ introduces the factor

$$\frac{1}{x - i\gamma} \quad (8)$$

with imaginary part $2\gamma/(\gamma^2 + x^2)$ into the Fourier transform of the 2-scalar correlator. The ratio of the imaginary to the real part is γ/x .

The damping γ must be finite: an infinitesimal damping of plane waves is normally assumed to avoid convergence problems without introducing T-odd effects. In the infinite momentum limit $k_+ \equiv xP_+$, $P_+ \rightarrow \infty$, this requirement is rather strong, since it implies that in the nonscaled space-time we have a damped wave $\exp(ixP_+z_- - \Gamma z_-)$ with infinite Γ . This is however necessary, since the Fourier transform $\exp(-ixP_+z_-)$ effectively probes regions of size $\Delta z_- \sim 1/xP_+$, and within this z_- range the damping is negligible unless Γ is $O(xP_+)$.

So, the condition for a leading twist effect, largely used in the following, is

$$\gamma(x) \sim x, \quad \text{for } P_+ \rightarrow \infty \quad (9)$$

equivalent to $\Gamma = O(xP_+)$.

On the contrary, a finite Γ ($\rightarrow \gamma \sim 1/P_+$) allows for a finite T-odd effect as far as P_+ is finite. In other words, one gets a higher twist effect.

As above observed, $q(x, \vec{k}_T)$ must be real, so in Eq. (5) $\text{Im}[q(x, k_T)]$ must combine with a nonzero $\text{Im}[q_A(\hat{k}_T)]$. In all the models known to the author, T-odd properties become observable in interference terms between odd and even waves of orbital angular momentum. Typically, between S and P waves.

To avoid ambiguities, we remark that in the scheme adopted in this work, *two* kinds of interference effects have relevance:

- (i) interference between continuous sets of states (due to initial state interactions) producing an imaginary part in the Fourier transform of the 2-scalar correlator,
- (ii) interference between a few well-identified angular waves, producing an imaginary part in the Fourier transform of the angular correlator.

Missing one of the two, T-odd effects are not observable. Without interference terms the angular correlator is even with respect to \vec{z}_T , so its Fourier transform is real. A well-chosen interference between even and odd angular waves, quantized with respect to the \hat{y} axis, may imply the simultaneous presence of a z_x -even and a z_x -odd term in $G_A(\vec{z}_T)$. This produces both a real and an imaginary part in the \vec{k}_T -Fourier transform of $G_A(\vec{z}_T)$.

IV. NONADIABATIC DECAY OF A SELF-CORRELATION FUNCTION

A. Irreversibility

In the physics of the strong and electromagnetic interactions, no phenomena exist that may cause, at the fundamental level, a time asymmetry. This situation is conversely quite common when from the fundamental level one passes to (i) systems where a huge number degrees of freedom are involved, (ii) systems where some degree of freedom is hidden in a non-Hermitian Hamiltonian, (iii) degrees of freedom whose time frequency spectrum is continuous. The first class is a special case of the last one since strict irreversibility is obtained when the number of degrees of freedom tends to infinity. The second class also is a special case of the last one, since a non-Hermitian Hamiltonian has complex energy eigenvalues, implying a finite width for energy levels. So we will adopt the last and most general definition (seemingly introduced by N.S. Krilov, see [49] for a later recollection of his works in English) of an irreversible process.

B. Fok-Krilov theorem

In nonrelativistic quantum mechanics the decay of a self-correlation function can be present when a system A described by a stationary wave function $\psi_o(x, t)$ starts interacting with another system B , its wave function $\psi(x, t)$ deviates from the ψ_o form and loses its previous stationarity properties. It can be generically written as a sum over eigenstates of the full Hamiltonian (including interactions) with eigenvalues E . If the sum is over a continuous E set, the self-correlation function

$$C(t) = \int dx \psi^*(x, t) \psi_o(x, 0) \quad (10)$$

is a decaying function of t for $t \rightarrow \infty$. Otherwise, the correlator is periodic (Fok-Krilov theorem [50], see e.g. Ref. [51] for a simple introduction).

The meaning of the self-correlation function is just the overlap of a state with itself at a later time. If the correlator decays, it means that the evolved wave function has no resemblance with its initial form. Clearly, T-odd processes are associated with decaying correlators and irreversible processes.

C. Entropy

When a previously discrete energy spectrum $\{E_{\text{on}}\}$ is turned continuous, we may define a new fictitious set $\{E_n\}$ of energy levels such that any E_n is a function of E_{on} and the correspondence is one to one. Then we may write the true continuous energy eigenvalues E in the form $E \equiv E_n + a(E_{n+1} - E_n)$, $0 \leq a \leq 1$. Evidently, a new degree of freedom associated with the continuous variable a is now present. The energy shift $E_{\text{on}} \rightarrow E_n$ is by definition

adiabatical and reversible as far as the populations of E_{on} and E_n are equal. When a large fraction of the initial energy has been transferred to E levels not coinciding with any E_n , the process is not reversible anymore.

This irreversibility can be quantified by the increase of the total entropy S . As a rough estimate, an added continuous degree of freedom plays a role if the total entropy increases by about one Boltzmann constant unit k_B , since in the presence of n independent degrees of freedom, each associated with an effective phase space Φ_i we have $S = k_B \log(\prod \Phi_i/h) = k_B \sum \log(\Phi_i/h) \sim nk_B \langle \log(\Phi/h) \rangle$ where h is the Planck's constant and $\langle \log(\Phi/h) \rangle$ is an average phase space (since now, we will use $h/2\pi = k_B = 1$, and not write these constants explicitly anymore). "Effective" phase space means that this phase space is at least potentially able to get energy within the time scale for the process interesting us.

So, $\Delta S \sim 1$ may be considered a signature of the correlator decay, and a condition for a T-odd process to be present. This neither guarantees that the effect is a leading twist one, nor that its magnitude makes it observable. So this condition must be made more precise.

In addition, the entropy must be defined for the relevant cases interesting us. The entropy of a slowly evolving system is defined as the entropy of an equilibrium system instantaneously assuming the same configuration. For a quickly evolving and open system a general definition of entropy is often specific and largely heuristic (see e.g. [52–54]) since the general Boltzmann's definitions $S = \log(\Phi)$ or equivalently $S = -\sum p_i \log(p_i)$ face the problem of what must be meant by phase space Φ or subset probability p_i .

D. Time scale and nonadiabaticity

In practical applications, the time scale of the process is important: if the process is slow enough, the evolution is quasiadiabatic, and relevant entropy increase may be meaningless since it takes place over a time range that is too large to influence our experiment: If τ' is the time required for reaching $\Delta S \sim 1$, and τ is the relevant time scale for the considered process, $\Delta S(\tau) \approx \tau/\tau'$. If $\tau/\tau' \ll 1$, $\Delta S(\tau) \ll 1$.

So, the condition for the irreversibility of a process to play a role is $\Delta S(\tau) \geq 1$, or equivalently

$$\frac{dS}{dt} \geq \frac{1}{\tau}. \quad (11)$$

E. Hard inelastic collisions

Writing Eq. (10) in the interaction form

$$C(t) = \int dx \psi_o^*(x, t) \exp\left(-i \int V dt\right) \psi_o(x, 0), \quad (12)$$

the similarity between the time-dependent self-correlation function $C(t)$ and the correlation function entering the

definition of a partonic distribution function [16] is evident, with the gauge link operator substituting the interaction operator.

In a light cone formalism t is substituted by a light cone variable and the Hamiltonian by the corresponding conjugate light cone momentum. Since here we focus on the $A(+)$ area of the factorized diagram, z_- and k_+ , or equivalently ξ and x , play the role of time and energy.

Because of the Fourier transform $\exp(-ix\xi)$, ξ -ranges $\sim 1/x$ are relevant in the problem. So, in our case the parameter τ of Eq. (11) becomes $1/x$.

The condition $dS/dt \geq 1/\tau$ becomes

$$\frac{dS}{d\xi} = O(x), \quad \text{for } P_+ \rightarrow \infty, \quad (13)$$

or more simply, for the relevant valence region

$$\frac{dS}{d\xi} \geq 1, \quad \text{for } P_+ \rightarrow \infty. \quad (14)$$

If $dS/d\xi$ is e.g. 0.05 this will not create problems (but perhaps lead to too small asymmetries to be detected). The point is that we need a visibly *finite* limit for $dS/d\xi$ at infinite P_+ .

When expressed in terms of nonscaled variables, Eq. (14) means $dS/dt \sim P_+$, so it is a singular condition for $P_+ \rightarrow \infty$. If, however, this singularity is not present, we have entropy increase (that in a highly inclusive interaction is obvious) but at a rate that becomes adiabatic for $P_+ \rightarrow \infty$. In other words, we have a higher twist process, i.e. a process that contains an intrinsic finite time decay scale τ and is only visible up to a hardness scale $P_+ \lesssim 1/\tau$.

A comparison of Eq. (9) with Eq. (13) suggests that we have to expect

$$\frac{dS}{d\xi} \sim \gamma. \quad (15)$$

The parameter γ and the entropy S have been discussed, up to now, as unrelated variables. In specific contexts we will show later that they are normally related by equations that imply the previous Eq. (15).

F. Action of a probabilistic perturbation on a pure state

In the following we will often consider (partially) probabilistic processes destroying an initially pure quantum state. A relevant point is although the action of this perturbation produces a chaotic set of states, the evolution of the *initial* state is not chaotic. For an extensive and rigorous consideration of this problem in nuclear physics see Ref. [48]. At the simple level interesting us we may observe that e.g. a plane wave $\exp(-ipz)$ describing a particle crossing homogeneous nuclear matter is converted into a damped wave $\exp(-ipz - z/L)$. Despite that the damp-

ing factor $\exp(-z/L)$ is normally estimated by means of probabilistic considerations, the wave $\exp(-ipz - z/L)$ is a *coherent* overlap of plane waves $\int dk f(k) \exp(-ikz)$ with $f(k)$ relevant over a range $\Delta k \sim 1/L$.

V. BREAKING OF A LONG RANGE CORRELATION

A point that is peculiar to this work is the hypothesis that the breaking of a *long* range correlation is able to cause relevant *short* range modifications of the quark propagators. In other words, the γ in Eq. (9) represents the inverse lifetime of a long range hadronic state.

To avoid misunderstandings, we stress that short range processes are here considered decisive, but their main role is to destroy long range correlations. The author cannot exclude that they also influence short range correlations. It is more difficult to imagine processes where a correlation extending to a spacetime scale L is destroyed by a mechanism with a similar spacetime scale $L' \approx L$. Also at intuition level, such a mechanism is able to change the underlying structure, but not to introduce stochasticity within a range $L'' \leq L' \approx L$.

Apart from the above general considerations, this choice is born from experimental data, and from an analysis of available theoretical models.

A. Propagation of partonic and hadronic states through hadronic matter

As already discussed in the introduction section, for the Drell-Yan case there is full evidence that the average free path of parton distributions in nuclear matter is much longer than the corresponding path for hadronic states. This leads to two slightly paradoxical conclusions:

- (i) Short range correlations are softer than long range ones, when decay properties in hadronic matter are considered.
- (ii) Ψ is destroyed while $|\Psi|^2$ survives during crossing hadronic matter.

The former statement just translates the quoted experimental fact that both partonic distributions and their normalization seem to be reasonably untouched by nuclear matter. The second statement is necessary not to give up with the idea that parton distribution functions reproduce intrinsic, interaction independent, steady properties of the hadron structure.

Putting the above pieces together into a physical picture, we may imagine the full process through 3 stages (the real inclusive process, not the one of the unitarity diagram):

- (A) Long before the interaction, the wave function describing a single hadron has stationary features, and axial isotropy around the spin axis. We may speak of the “hadronic phase.”
- (B) Hadron-hadron interactions start filtering the different components of the initial state. Stationarity of

these components is lost, and they are projected onto a set of short range states that may be considered stationary in the crossed hadronic medium.

- (C) The initial hadron coherent state is reduced to a cloud of individual partons easily diffusing through the hadronic/nuclear matter. Several probabilistic features of the initial state are still present, but fast time evolution is over. We may speak of the “partonic phase.”

Seen this way, the evolution is similar to the sublimation of a piece of solid matter passing through a high-temperature region. In the following we will not appeal to this picture anymore since, as observed above, we work on the imaginary part of the forward amplitude and not directly on the inclusive process. However this picture is useful to understand the role that is here attributed to long range correlations.

B. Breaking of long range correlations in spectator models

The breaking of a long range correlation is a common point to several of the above quoted models for T-odd functions.

In particular we may consider spectator models [20–23] as an example.

In these models, long range correlations are introduced and tuned [55] by (i) form factors with soft cutoffs, (ii) unicity, compactness, and stability of the spectator state, (iii) quark/spectator masses. T-odd functions may then be obtained via inclusive production/exchange of extra particles, that break long range correlations.

In the absence of extra production mechanisms, a spectator or a quark may correlate two vertexes over a distance $\Delta\xi \sim 1/\Lambda$, if a form factor with soft cutoff Λ is present in one of the vertexes or in both. Typically this means $\Delta\xi \sim 1$, since this is what is needed to produce a valencelike x -distribution with size $\Delta x \sim 1/\Delta\xi \sim 1$.

When production/exchange of stable extra particles with pointlike vertex (e.g. gluons) is added to the basic spectator model, this introduces hard components in the quark or spectator propagator. These propagators must begin or end in a soft vertex, but the hard components cannot be reabsorbed by this vertex. So the projection of the initial hadronic state $|P\rangle$ onto the final state $\langle P|$ is suppressed by this evolution.

This suppression is equivalent to the previously discussed breaking of long range correlations. A similar pattern is also present in [19], where long range parameters are introduced by a bag model. If the extra particles are unstable or associated with further vertex form factors, the basic features of the hadron model may play a minor role and the relevant long range correlations may be those associated with these fields. But the general working principle is not different, if the introduced parameters (inverse lifetime, cutoffs) do not overcome a few hundred MeVs.

C. Hard scale loops and the Fock-Krilov theorem

In general, the reason why a loop going up to a hard scale Q destroys long range correlations is a peculiar application of the quoted Fock-Krilov theorem. A long range correlation is associated to a soft cutoff Λ on some momentum component k (e.g. k_T) of the set of particles crossing the cut, in the unperturbed diagram. When a factorization breaking exchange introduces a loop, momenta through this loop go up to $Q \gg \Lambda$. So the loop decomposes any wave packet into a set of components with momentum p , each with $p \lesssim Q$. These components propagate with a different phase, so the initial wave packet shape is soon lost. The decomposition is coherent, so sooner or later the wave packet will acquire again a shape that is similar to the original one. Introducing the quantum state size h we see that this time is finite since the decomposition is not really continuous. If however $Q \gg \Lambda$, the number of p -states available for each given k -state is very large, and it is proper to assume that the loop variable is continuous. This, however, means that the time needed to reform the original wave packet is infinite.

At a qualitative level the increase of phase space (associated with an increase of entropy) associated with this loop is of magnitude $(Q/\Lambda)^n$, where n is a number of order unity.

This argument, however, breaks down when the ratio Λ/Q is not $\ll 1$. In this case the preloop set of states composing the wave packet has the same extension of the in-loop one. This suggests that adding further loops would not change enormously the effect obtained via a single hard loop. If in addition we select peculiar initial states characterized by a hard scale (x very close to 1, or $q_T \sim Q$), decorrelation effects are suppressed. The case $x \approx 1$ is obvious, since we are selecting final states where decorrelation has not taken place at all. The large- q_T case deserves more discussion.

D. Large transverse momenta

In Drell-Yan at large enough q_T the electromagnetic probe is not testing initial state features, but features of the hard interactions themselves. As a consequence it is possible to calculate T-odd effects [9] in a fully perturbative scheme. As a further consequence, any correlation-destroying process is acting on a state that is short ranged $\sim 1/q_T$ also in the absence of factorization breaking interactions. So the real increase of phase space associated with an added factorization breaking loop is $(Q/q_T)^n$, not $(Q/\Lambda)^n$ as above stated. On the ground of the previous discussion, for $q_T \rightarrow Q$ we expect a suppression of T-odd effects. This means that in Eqs. (8) and (9) γ will not be $O(1)$ or $O(x)$ at large $q_T \sim Q$.

In Ref. [9] the transition from the soft- q_T to the hard- q_T regime is considered, by including real gluon emission already in diagrams where factorization breaking loops are absent. For small q_T the extra gluon is soft, and

although it enters the K-factor, it does not destroy the coherent features of the initial state. In this case the addition of a factorization breaking loop has the consequences described in the previous two subsections. For large $q_T \sim Q$ the soft features of the initial state are destroyed by the hard radiated gluon. In other words, also in diagrams with no factorization breaking interactions, initial state soft cutoffs play no role.

So, in the case of the models considered in the previous subsections V B and V C, and for small q_T we may speak of “soft to hard” transition. In the case of large q_T we may speak of “hard to hard” transition, with no decorrelation. This reflects in a q_T -suppression of the effect at large q_T , as predicted in [9].

E. Soft cutoffs

Let a variable Y be connected with the quark hole motion. It may represent the quark transverse momentum, or be related with gluons radiated by this quark, and so on.

Let Y have a maximum “soft” cutoff $Y < Y_M$ in its initial state. Let the quark hole undergo events associated with initial/final state interactions,³ that we simply name “events.”

We assume that most events lead to $Y < Y_M$ (soft events), but for a small and finite fraction of events $Y \gg Y_M$ (“hard” events).

To avoid ambiguities, we stress that in the following we will speak of soft and hard events referring, in both cases, to initial/final state interactions that accompany the electromagnetic Drell-Yan hard event, not to the electromagnetic event itself.

The meaning of the soft cutoff is a sharp way to represent a reasonable upper cutoff for all fields and degrees of freedom that may play a role in the hadron state when $P_+ \rightarrow \infty$. The assumption is that if $Y < Y_M$ the hadron is in its ground state, if $Y > Y_M$ it is in some excited state, and if $Y \gg Y_M$ it is fragmenting. Since the relevant correlation function is defined as $\langle P | \dots | P \rangle$, i.e. as a correlation operator sandwiched between equal hadron states, if some internal degree of freedom largely overcomes its soft cutoff the projection on the final $\langle P |$ state is very small. We may imagine some relevant cases: (i) Y_M coincides with the vacuum expectation value for Y , (ii) Y_M is assigned by the uncertainty principle (e.g. for k_T we have $\max(k_T) \sim 1/R_{\text{hadron}}$), (iii) Y_M is an infrared cutoff.

A relevant point is that

$$S \approx 0, \quad \text{for } Y < Y_M. \quad (16)$$

Indeed, as just observed, as far as the soft variables are inside their soft cutoffs the considered state is a quantum fluctuation of the hadron ground state.

³E.g., in a spectator model a gluon production from a quark propagator is an initial state interaction, a gluon production from a diquark line is a final state interaction.

F. Single and multiple step processes: hard and stretching events

A series of events may determine a transition of the bound hadronic state to the continuum. We expect that the entropy is then increased by some units (or more), and correspondingly that some more degrees of freedom have appeared. This may take place within a single or a multiple scattering scheme.

If this process is dominated by a single hard event, we need the final phase space of this single event to contain more degrees of freedom than the initial one. This can be obtained in several ways: (i) by infinitely expanding the phase space of an existing degree of freedom; (ii) by producing some new particle (that is a subclass of the previous case, since the associated phase space expands from zero to a finite value); (iii) for the case of an initially discrete set of values, by filling each range $Y_n - Y_{n-1}$ with an infinite set of new accessible values (that again is a subclass of (i)). So, essentially a single step process is dominated by an infinite expansion of the phase space.⁴

In a multistep process associated with a large number of uncorrelated events, it may be sufficient to have a regular increase of the phase space associated with a given variable at each step. This will lead to an exponential increase of the phase space with the event number, and the effect on S is still an increase of some units. In this case we speak of “stretching events” instead of “hard events.”

Single and multiple step processes are separately examined in the following. Hybrid features do normally appear, e.g. when a hard scattering leads to a multiparticle production, or when in a multiparticle production we may radiate a few hard particles and several soft ones as well. So the above two sets of processes should be considered just as opposite limits for a continuum of possibilities. In these limits approximations may be applied making statistical analysis easier.

In the format given by Ref. [16] and widely adopted as a formal framework for adding the effects of initial state interactions to partonic distributions, the gauge link factor incorporates interactions in a continuous and abstract form. So there is no difference in principle between the two classes of events. The problem rises when the gauge factor is expanded in a perturbative scheme, and one needs to decide whether low order diagrams, or a full resummation scheme, have more relevance.

For a specific model it may be easier to discuss low order perturbative schemes. Sometimes, however, resummation is hidden, in form factors, in finite widths for propagators, in the use of no-event probabilities. E.g. the factor $\exp(-\gamma\xi)$ used in several places of this work is a quasi-

⁴If the phase space is evaluated in classical sense, i.e. without counting the finite volume h of each state. This is justified since we reach quasiclassical situations, where h may be neglected.

classical all-order resummation of uncorrelated similar events.

G. Single scattering/radiation scheme: real and effective degrees of freedom

In the limit $P_+ \rightarrow \infty$, in the presence of a hard exchange of energy or transverse momentum between the factorization-separated areas of the diagram $A(+)$ and $A(-)$, a large amount of energy can be reversed on a set of soft variables $\{Y_j\}$ belonging to $A(+)$. As above anticipated, we may select three typical situations, that actually are specializations of the first one.

We use a single variable Y to represent a combination of all the involved ones. E.g., it may be their total energy. $Y < Y_M$ in soft conditions.

If hard events may lead to a value for Y distributed in a reasonably uniform way within a range ΔY with $\Delta Y/Y_M \rightarrow \infty$ for $P_+ \rightarrow \infty$, and if hard and soft events coexist, we need *two* continuous variables a and y to describe the final state.

Let $\inf(z)$ be a monotonously increasing function of z that is infinite for infinite z and positive for $z > 1$. $\log(z)$, z , or $\exp(z)$ are all good functions of this kind.

We define

$$Y \equiv aY_M + yY_M \inf(P_+), \quad 0 \leq a, y \leq 1. \quad (17)$$

In the limit $P_+ \rightarrow \infty$ we have

- (i) soft events: a has a precise value between 0 and 1, $y \equiv 0$.
- (ii) hard events: $0 \leq y \leq 1$, a may be arbitrarily assigned within 0 and 1.

For hard events a would be infinite if we had defined $Y = aY_M$ without introducing y . So, the joint presence of soft and hard events (i.e. of Y values ranging over two infinitely different scales of magnitude) implies the effective opening of a new degree of freedom.

When the soft cutoff coincides with the vacuum expectation for Y , we may roughly speak of a “new turned on” degree of freedom. In this case we may simply define:

$$Y \equiv Y_{\text{vacuum}} + yY_o, \quad y \sim 1, \quad (18)$$

where the constant Y_o keeps y dimensionless. This is a special case of the previous definition, with $Y_M \equiv 1/\inf(P_+)$ and the soft phase space reduced to unity.

Another case is the one of a variable with a discrete set of eigenvalues Y_n below the cutoff, and a continuous set over the cutoff. If the number of discrete eigenvalues is finite this case is equivalent to the previous one. If it is infinite we may map them one to one into a set of discrete Y'_n eigenvalues spanning all the final state space. Then we may define

$$Y \equiv Y'_n + y(Y'_n - Y'_{n+1}), \quad 0 < y < 1. \quad (19)$$

Other cases that one may imagine may be reduced to the discussed ones. In all cases one sees that a new continuous variable y has been introduced.

One important remark: In the previous cases two dimension scales are selected for the same set of events. Separating the associated phase space into 2 degrees of freedom a and y shows that, despite appearances, the final phase space is not infinitely large. It only has a different dimension. Since the two final degrees of freedom are roughly independent, we have $S \sim \log(\Phi(a)\Phi(y)) \sim S(a) + S(y)$, where $\Phi(a)$ and $\Phi(y)$ are the phase spaces associated with each variable. The former quantity is zero because of Eq. (16), and the latter is ~ 1 .⁵

H. $dS/d\xi$ in a single scattering/radiation scheme

Let us first consider a single scattering center in the unit volume. For the probability W of transitions of both soft and hard kinds, we have

$$\begin{aligned} W &= W_{\text{soft}} + W_{\text{hard}} \approx |M|^2(\Phi(a) + n\Phi(a, y)) \\ &\approx |M|^2(\Phi(a) + n\Phi(a)\eta_y), \end{aligned} \quad (20)$$

$$\eta_y \equiv \Phi(a, y)/\Phi(a). \quad (21)$$

In the previous relations n is the relative fraction of hard to soft scattering centers. “Scattering centers” means partons in $A(-)$ potentially able to cause one of the required processes. $\Phi(a)$ and $\Phi(a, y)$ are the phase space associated with soft transitions and hard transitions, respectively. $|M|^2$ is an average soft transition probability to a single final state. The corresponding $|M_{\text{hard}}|^2$ factor for hard transitions is supposed to differ mainly for selection rules (included in n) and for conservation rules (included in the phase space). So what remains in $|M|^2$ is supposed to be similar for soft and hard transitions.

We need to be inclusive with respect to the initial values of the soft variable a , so the overall transition probability must be divided by the soft phase space $\Phi(a)$. So we have

$$\begin{aligned} W &= W_{\text{soft}} + W_{\text{hard}} \approx |M|^2(1 + n\eta_y) \equiv |M|^2\Phi_{\text{tot}} \\ &\equiv |M|^2 \exp(S). \end{aligned} \quad (22)$$

The previous equation may be considered as the definition of entropy that is relevant here: the logarithm of the number of all the final states that directly determine the rate of hard interactions of the quark hole, starting from a soft state.

This excludes all those processes that are present but have scarce relation with the quark hole line. With the above definition of useful phase space, we have $S = 0$ for soft final states, as previously required. Of course, deciding what must be counted in the relevant phase space

⁵It may be much smaller or much larger, but neither zero nor infinite; its precise value depends on our ability in distinguishing different y values within a distance δy ; then $S(y) = -\log(\delta y)$.

is a matter of taste, but this problem is present in the calculation of any inclusive reaction rate in hadron physics.

Assuming that relevant values for S and $n\eta_y$ are ≤ 1 ,

$$e^S \approx 1 + S \approx 1 + n\eta_y, \quad (23)$$

$$W_{\text{hard}} \approx W_{\text{soft}}S, \quad S \approx n\eta_y. \quad (24)$$

If we consider events taking place with continuity along a ξ path, the previous equations need to be modified:

$$\frac{dW}{d\xi} = \left(\frac{dW}{d\xi}\right)_{\text{soft}} + \left(\frac{dW}{d\xi}\right)_{\text{hard}} \approx \left(\frac{dW}{d\xi}\right)_{\text{soft}} (1 + S). \quad (25)$$

If with $dW_{\text{soft}}/d\xi$ we mean the full set of processes that leave a soft state untouched after a unitary path $\Delta\xi = 1$, we may approximate it with 1 (since scattering and no-scattering events are not distinguishable). Then n (the *relative* number of hard to soft scatterers) must be substituted by $dn/d\xi$, where $dn/d\xi$ is the total *absolute* number of hard scatterers per unitary $\Delta\xi$, and consequently $S \rightarrow dS/d\xi$:

$$\left(\frac{dW}{d\xi}\right)_{\text{hard}} \approx \frac{dS}{d\xi}, \quad (26)$$

$$\frac{dS}{d\xi} \approx \frac{dn}{d\xi} \eta_y. \quad (27)$$

Now, the probability for the set of soft states suffers unitarity loss:

$$|\psi(x, Y < Y_M, \xi)|^2 \sim |\psi(x, Y < Y_M, 0)|^2 e^{-2\gamma\xi}, \quad (28)$$

since for the overall flux we have $dN/N = (dW_{\text{hard}}/d\xi)d\xi$, and dW_{soft} does not contribute to the decay. So we get the relation between $dS/d\xi$ and γ whose approximate form (Eq. (15)) was guessed in Sec. IV:

$$\gamma = \frac{1}{2} \frac{dS}{d\xi} \quad (29)$$

and

$$\frac{dS}{d\xi} \sim x \quad (30)$$

that selects leading twist T-odd effects.

Equation (27) suggests that a finite $dS/d\xi$ may be reached more than one way:

- (1) Both $dn/d\xi$ and η_y finite for $P_+ \rightarrow \infty$.
- (2) $dn/d\xi \rightarrow \infty$ and $\eta_y \rightarrow 0$ for $P_+ \rightarrow \infty$.

When ξ is converted to the nonscaled variable z_- , the less singular case may be the latter, since a unit ξ corresponds to an infinite z_- range, and a finite η_y implies infinite energy being acquired by Y in the limit $P_+ \rightarrow \infty$. However, in this limit it is quite natural to have to do with singularities.

In any case, both the above schemes are single scattering schemes, meaning that a quark hole flux moves in a me-

dium with several potential scatterers, but one event is sufficient for the destruction of the ground hadron state.

I. Stretching events and multiple scattering/radiation schemes for $dS/d\xi$

A further scheme is given by multiple uncorrelated scattering/radiation events.

A known example of this kind that could be generalized to our problem is a classical particle undergoing multiple scattering against randomly distributed spheres [49]. Then Y could be the angular deviation of the quark from the hadron direction. Next, this class of processes includes relevant cases like radiation of gluons/mesons by a quark line. Then $\{Y_i\}$ may be the set of kinematic variables of the radiated particles.

In a multiple scattering/radiation case, hardness is reached as a cumulative effect of several events that may be individually soft. This obliges us to modify the definition of $dn/d\xi$.

Let us redefine Y in such a way that it has the dimension of an action or, if a unitary cubic box has been assumed as normalization volume, of a momentum. We scale $y \equiv Y/Y_M$, with y that is soft (< 1) at the very beginning, but may reach any value during the evolution driven by initial/final state interactions. If the values of y at a certain time are randomly distributed within a mean quadratic fluctuation, $y \pm \Delta y$, the phase space that we may associate to y is a simple power m of Δy . Consequently,

$$S \equiv m \log(\Delta y), \quad m \sim 1. \quad (31)$$

Instead of defining $dn/d\xi$ as a density of scattering centers for hard events, we define now $dn/d\xi$ as a density of scattering centers for “stretching events.” A stretching event is defined by the condition:

$$\frac{\Delta y'}{\Delta y} \equiv 1 + \alpha_y, \quad \alpha_y > 0, \quad \text{for } P_+ \rightarrow \infty, \quad (32)$$

where Δy is the mean quadratic spread for y values before the event, and $\Delta y'$ the same after it. The phase-space factor α_y includes in itself the total probability of scattering in the presence of a single scattering center.

In several problems, $\alpha_y > 0$ is valid at finite P_+ , but it is lost in the $P_+ \rightarrow \infty$ limit. As an example we take y as the scattering angle in Rutherford scattering. Assuming an upper cutoff on the impact parameters $z_T < z_{\text{max}}$, we have $\alpha_y > 0$ for finite P_+ , but $\alpha_y \rightarrow 0$ in the $P_+ \rightarrow \infty$ limit.⁶

Assuming that a finite set of events respect the stretching condition, we may define S as

$$S = \log(\Delta Y/\Delta Y_o), \quad (33)$$

⁶If no upper limit is assumed for z_T the average scattering angle is zero at any energy.

where ΔY_o is a soft initial value. With this definition S is proportional to the number of scattering events. Using all quantities referring to one ξ unit, and neglecting the precise value of m in Eq. (31),

$$\begin{aligned} \frac{dS}{d\xi} &\approx \log(\Delta y)_{\Delta\xi=1} = \log[\Delta y_o(1 + \alpha_y)^{(dn/d\xi)}] \\ &= \frac{dn}{d\xi} \log(1 + \alpha_y). \end{aligned} \quad (34)$$

Assuming a small α_y , we have

$$\frac{dS}{d\xi} \approx \frac{dn}{d\xi} \alpha_y. \quad (35)$$

In the hadron ground state we have $S \approx 0$, as a result of having neglected $\log(\Delta Y_o)$ in Eq. (34). To estimate the relation between γ and $dS/d\xi$ we may use the known trick of considering a set of j independent emissions in a range ξ as Poisson-distributed with respect to j , and then $\exp(-2\gamma\xi)$ is identified as the probability of no emission. In the Poisson distribution this is $\exp(-\omega_1)$, where ω_1 is the probability of a single emission. Since here with ‘‘emission’’ we mean a process with an increase of phase space, we may identify $(1 + \alpha_y)dn/d\xi$ with the probability of an emission of whatever kind in a unit ξ range, and $\alpha_y dn/d\xi$ with the probability for a ‘‘useful’’ emission only, since the other processes do not change the initial situation. So

$$\exp(-2\gamma\xi) \sim \exp\left(-\alpha_y \frac{dn}{d\xi} \xi\right) = \exp\left(-\frac{dS}{d\xi} \xi\right). \quad (36)$$

So, again we have $\gamma \sim dS/d\xi$ and consequently the requirement $dS/d\xi \sim x$. In addition α_y plays a similar role as η_y in the single scattering scheme.

Also in this case we may have two possibilities:

- (1) Both $dn/d\xi$ and α_y finite for $P_+ \rightarrow \infty$.
- (2) $dn/d\xi \rightarrow \infty$ and $\alpha_y \rightarrow 0$ for $P_+ \rightarrow \infty$.

VI. T-ODD FUNCTIONS IN ENERGY-CONSERVING SCHEMES

In the title of this section ‘‘energy conserving’’ means that initial/final state interactions do not exchange energy between factorization-separated areas of the diagram. This is a natural consequence of relating T-odd effects with entropy nonconservation in $A(+)$. Entropy may increase also in energy-conserving systems. Since $O(P_+)$ energy exchanges are statistically suppressed in hadron-hadron collisions at high energies, the relevance of this possibility is evident.

As observed in Sec. III, T-odd functions describe the loss of flux due to events where x is not conserved in the propagation of the quark hole, with finite $x - x'$ for $P_+ \rightarrow \infty$. Since the colliding hadron is a stationary system if undisturbed, x nonconservation takes place because of hard interactions between $A(+)$ and $A(-)$. We must, how-

ever, distinguish between two very different classes of hard events:

- (1) Energy nonconserving events: $O(P_+)$ energy exchanges take place between $A(+)$ and $A(-)$, and the lost energy is not returned.
- (2) Events where the hard interaction between $A(+)$ and $A(-)$ is mainly transverse: it does not transfer relevant amounts of energy between the two, and only acts as a ‘‘trigger’’ for relevant energy exchanges between degrees of freedom all belonging to $A(+)$. In particular, energy is transferred from x to variables that consequently overcome the soft cutoff.

In this section, we focus on events of the latter kind. These necessarily increase the entropy of $A(+)$ and correspond to what, in classical mechanics, is energy degradation: energy is transferred from ‘‘mechanical’’ to ‘‘internal’’ degrees of freedom (from collective, ordered, translation, and rotation motion to thermal or chemical energy). Since the macroscopic dynamical evolution is associated with mechanical degrees of freedom only, energy degradation is equivalent to an energy nonconservation. In our problem, the mechanical degrees of freedom are ξ (conjugated to x), and the quark transverse motion as far as k_T remains within soft limits.

The above requirement of energy conservation inside $A(+)$ puts limits to the size of the final state phase space and allows for some approximate estimate.

A. Soft and hard cutoffs on the phase space

Specializing the assumptions of the previous Sec. V, we may assume that in $A(+)$ we have, for any given x , a set of m variables Y_1, Y_2, \dots, Y_m that in the hadron ground state satisfy one or more soft cutoffs of the form $f(Y_1, Y_2, \dots) < f_M$. In particular this set contains \vec{k}_T satisfying $k_x^2 + k_y^2 \lesssim \Lambda_T^2$, with Λ_T of magnitude 1 GeV/c. The set $\{Y_i\} \equiv Y$ only includes those degrees of freedom that may exchange energy with the quark hole within a time R_{hadron}/P_+ .

The soft cutoff will translate, in particular, in a soft cutoff for the *total* energy associated with Y :

$$\sum E(Y_i) \equiv E(Y) \lesssim f_M. \quad (37)$$

Since the total energy $E(x) + E(Y)$ is conserved, in the hadron ground state at large P_+ , also the longitudinal energy $E(x)$ may ordinarily fluctuate within this soft limit:

$$E(x) - E(x') \sim (x - x')P_+ \lesssim f_M, \quad (38)$$

so obviously soft events do not break x -conservation in the large P_+ limit.

Hard events break the soft cutoff. We assume that they lead to a reasonably nonzero distribution for the final values of Y up to a hard cutoff:

$$E(Y) \lesssim F_M(x) > f_M. \quad (39)$$

So, for soft events we have the phase space

$$x - x' < \frac{f_M}{xP_+} \quad (40)$$

and for hard events we have the phase space

$$\frac{f_M}{P_+} < x - x' < \frac{F_M}{xP_+} \quad (41)$$

that can be simply approximated by

$$x - x' < \frac{F_M}{xP_+}. \quad (42)$$

We also limit our discussion to the cases where

$$x - x' \ll x. \quad (43)$$

This means that $1/x$ and $1/x'$ are interchangeable in the right-hand side of the above equations. Clearly this is valid as far as $F_M \ll xP_+$ for $P_+ \rightarrow \infty$, and x is not too close to zero.

The total phase space $x' \leq x$ for one given x in the case of soft/hard events is

$$\Phi(E_M, x) = \int_0^x \frac{dx'}{x'} \int_{E(Y) < E_M} d^m \Phi(Y) \delta\left(x - x' - \frac{E(Y)}{x'}\right), \quad (44)$$

where E_M is the maximum for $E(Y)$, i.e. either f_M or F_M . We may evidentiate

$$d^m \Phi(Y) \equiv dE(Y) d^{m-1} \Phi(y) \quad (45)$$

with

$$y_i \equiv \frac{E(y_i)}{E_M}, \quad (46)$$

$$\begin{aligned} \int d^{m-1} \Phi(y) &\equiv [E(y)]^{m-1} \int \delta\left(1 - \sum y_i\right) f(x, y) \prod_i dy_i \\ &\equiv v(x) [E(y)]^{m-1}, \end{aligned} \quad (47)$$

where $f(x, y)$ is supposed to be reasonably flat through all the $m - 1$ above defined phase space, and to depend slowly on x as $v(x)$. Assuming

$$\frac{x - x'}{x} \lesssim 1/P_+ \quad (48)$$

(that is obvious but at very small x values) from the joint conditions $x = x' + E(Y)/xP_+$ and $E(Y) < E_M$ we have:

$$\begin{aligned} \int_0^x \frac{dx'}{x'} \delta\left(x - x' - \frac{E(Y)}{xP_+}\right) &\approx P_+ \int_{x-\Delta}^x dx' \\ &\times \delta(P_+ x(x - x') - E(Y)), \\ \Delta &= \frac{E_M}{xP_+}, \end{aligned} \quad (49)$$

and with some algebra

$$\Phi(E_M, x) = v(x) \frac{E_M^m}{m}. \quad (50)$$

Taking into account that we have neglected the lower cut-off in the calculation of the hard phase space (Eq. (41)) has been approximated by Eq. (42)),

$$\frac{\Phi_{\text{hard}}}{\Phi_{\text{soft}}} = \frac{\Phi(F_M, x)}{\Phi(f_M, x)} - 1 \approx \left(\frac{F_M}{f_M}\right)^m - 1. \quad (51)$$

In a single scattering scheme this may be identified with η_y of Eq. (21). In $dS/d\xi$ also $dn/d\xi$ appears, where this number is the density of scattering centers able to trigger the energy redistribution process.

As interesting limiting cases, we may imagine two situations:

- (i) F_M/f_M close to 1, large $mdn/d\xi$: a multiparticle production, or anyway a redistribution of energy among a large number of degrees of freedom.
- (ii) Large F_M/f_M , small $mdn/d\xi$: a single particle radiation or anyway few involved degrees of freedom.

B. Large number of degrees of freedom

If an average m is fixed we have

$$(F_M/f_M)^m \equiv (1 + \delta)^m \approx (1 + m\delta), \quad \delta \ll 1, \quad (52)$$

$$\frac{dS(x)}{d\xi} \sim \delta m \frac{dn}{d\xi}. \quad (53)$$

Otherwise, if m is Poisson-distributed, we may work as in the multiple scattering case (the two situations are now equivalent), and get the same results with the factor $1 + \alpha_y$ of Eq. (32) substituted by $1 + \eta_y|_{m=1}$:

$$\frac{dS(x)}{d\xi} \sim \delta \frac{dn}{d\xi}. \quad (54)$$

In both cases $dS/d\xi$ is proportional to the relative excess of released energy $\delta = (F_M - f_M)/f_M$ times the average number of new particles produced per unitary ξ path. The soft cutoff is linearly present in δ .

C. Single particle production

We neglect the “1” factor in Eq. (51).

In this case also single particle production in constituent or spectator models may enter, but then the previous treatment of the phase-space ratio must be corrected. Now the number of degrees of freedom is limited, and one needs to specify which ones play a role and where.

In a spectator model the effective number of degrees of freedom that can be associated with the initial state is roughly 2α , where $|1/Q^2|^\alpha$ is the asymptotic electromagnetic hadron form factor as it would result when calculated within the considered model. In a spectator model like the one of [55] this power law derives from form factors

in the hadron-quark-diquark vertexes of the kind $(k^2 - m_q^2)^a / |k_T^2 + \lambda^2|^b$.

If e.g. the diquark emits a gluon, the only degrees of freedom that may break softness are the two components of the recoiling diquark \vec{k}'_T . These are soft-limited by the final vertex form factor. As observed above, they are effectively treated as if they were 2α instead of two. On the other side, the effects of the vertex form factors disappear in the asymmetries, where form factors appear both in the numerator and in the denominator.

So in T-odd distributions we have

$$\eta_y \propto \lambda^{-2\alpha}. \quad (55)$$

As a result, T-odd distributions are $\sim 1/\lambda^m$ with relevant m factors.

In asymmetries, on the other side, we only have

$$\eta_y \propto \lambda^{-2} \quad (56)$$

as one may expect on the ground of the previous counting rule (see e.g. [22]).

This suggests that, despite in a hidden way, in models with this scaling law (asymmetry $\sim 1/\lambda^2$) gluons exchanged between factorization-separated areas of the diagram mainly carry transverse momentum.

As observed in Sec. V, at large q_T , Λ must be substituted by q_T . In addition, the power law must be reconsidered taking into account the number of poles constraining the real number of added degrees of freedom (see Ref. [9]).

VII. CONCLUSIONS

The initial state interactions affecting a Drell-Yan process have been reanalyzed. Assuming, on an experimental ground, that they cause a transition from a hadronic to a partonic phase, nonzero leading twist T-odd distribution functions have been shown to be closely associated to this phase transition.

A parallelism has been established between the time-dependent self-correlation function describing the time evolution of a state in an interacting quantum system, and the light cone correlator from which a distribution function is extracted via Fourier transform $\exp(ix\xi)$. The decay of the correlator is associated with entropy increase.

We have derived in a heuristic way the condition $dS/d\xi \sim x$. If it is respected in the infinite momentum limit, a leading twist T-odd structure is present in the correlator and it may be made detectable. In particular, the correlator is a sum of decaying functions of the kind $\exp[-ix\xi - \gamma(x)\xi]$, and $\gamma \sim dS/d\xi$.

Some general schemes for single and multiple scattering/radiation have been examined arriving, in each case, to relations of the kind $\gamma \approx dS/d\xi \approx \eta dn/d\xi$, where η is the relative gain of phase space associated with a single interaction event, and $dn/d\xi$ the density of scattering centers along the quark path.

Last, we have examined the special situation where the initial state interactions do not exchange energy between the two hadrons. For this case, that is likely to dominate initial state interactions in high-energy Drell-Yan, the entropy increase rate $dS/d\xi$ has been estimated, in the opposite cases of single and multiple radiation.

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