Ξ and Ω baryons in the Skyrme model

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The mass spectrum and magnetic moments of Ξ and Ω baryon resonances are investigated in the bound-state approach of the Skyrme model. The empirical hyperon spectrum shows that several hyperon resonances share a pattern of (approximately) equal mass spacings between the states of same spin but of opposite parity. It is found that this pattern can be explained mostly by the energy difference between the P-wave and S-wave kaons bound to the soliton. Although one cannot exclude the possibility that these states can be described as pion-hyperon resonances, the present approach predicts that $\Xi(1620)$ and $\Xi(1690)$ have $J^P = \frac{1}{2}^-$, while $\Xi(1950)$ has $J^P = \frac{1}{2}^+$. The differences with the quark model predictions for the Ξ and Ω baryon spectrum are pointed out. Several relations for the masses and magnetic moments of those resonances are also obtained.

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I. INTRODUCTION

In spite of the early efforts for studying Ξ and Ω resonances, our understanding on those resonances is still far from complete. Any new significant information on the multistrangeness baryons has not been accumulated during the past two decades [1] except for the measurement of the magnetic moment of $\Omega^-(1672)$ [2]. (See, e.g., Ref. [3] for the latest experimental study on Ξ resonances.) Recently, however, interest in cascade physics is increasing. New activities in measuring weak decays of Ξ^0 hyperon were reported by the KTeV Collaboration [4] and by the NA48/1 Collaboration [5]. Also, recent research activities on Ξ resonances in relativistic heavy-ion collisions can be found in Ref. [6]. In addition, the cascade physics program of the CLAS Collaboration at the Thomas Jefferson National Accelerator Facility has been launched and some preliminary results were already reported [7–11]. The purpose of this program includes searching for and confirming Ξ resonances as well as understanding their properties through electromagnetic production processes. Investigation of the production mechanisms of Ξ baryon production induced by photon-nucleon reaction was just started recently [11–13].

At present, there are eleven Ξ baryons and four Ω baryons listed in the review of the Particle Data Group (PDG) [1]. Among them only the octet and decuplet ground states, $\Xi(1318)$, $\Xi(1530)$, and $\Omega(1672)$, have four-star ratings with definite spin-parity,[1] and there are four Ξ baryons and one Ω baryon with three-star ratings. Among the three-star-rated baryons, $\Xi(1820)$ is the only state whose spin-parity quantum numbers are known and nothing is known about the spin-parity of all the other states. A summary on the Ξ and Ω spectrum of PDG is given in Table I.

Theoretically, there have been studies on the excited multistrangeness hyperons using phenomenological models. Those models could reproduce the octet and decuplet ground states, but have very different and even contradictory predictions on the spectrum of excited states. Early studies in quark models are summarized in Refs. [15–17]. A more detailed study including the quark dynamics was done by Chao, Isgur, and Karl [18] in a nonrelativistic quark model. In this model, the $\Xi(1820)\frac{3}{2}^-$ is well explained and the third lowest state following $\Xi(1318)$ and $\Xi(1530)$ is predicted to be at a mass of 1695 MeV with $J^P = \frac{1}{2}^+$. Therefore, this state would be a good candidate for the observed three-star resonance $\Xi(1690)$. However, this expectation is not valid in other models that predict a very different Ξ spectrum except for the ground states. For example, in the relativized quark model of Capstick and Isgur [19], the first excited state of $\Xi(\frac{1}{2}^+)$ would have a mass of around 1840 MeV.

TABLE I. Ξ and Ω baryons listed in the review of the Particle Data Group [1].

Particle	$I(J^P)$	Rating	Particle	$I(J^P)$	Rating
$\Xi(1318)$	$\frac{1}{2}(\frac{1}{2}^+)$	****	$\Omega(1672)$	$0(\frac{3}{2}^+)$	****
$\Xi(1530)$	$\frac{1}{2}(\frac{3}{2}^+)$	****	$\Omega(2250)$	$0(?^?)$	***
$\Xi(1620)$	$\frac{1}{2}(?^?)$	*	$\Omega(2380)$	$?(?^?)$	**
$\Xi(1690)$	$\frac{1}{2}(?^?)$	***	$\Omega(2470)$	$?(?^?)$	**
$\Xi(1820)$	$\frac{1}{2}(\frac{3}{2}^-)$	***			
$\Xi(1950)$	$\frac{1}{2}(?^?)$	***			
$\Xi(2030)$	$\frac{1}{2}(\geq \frac{5}{2}^?)$	***			
$\Xi(2120)$	$\frac{1}{2}(?^?)$	*			
$\Xi(2250)$	$\frac{1}{2}(?^?)$	**			
$\Xi(2370)$	$\frac{1}{2}(?^?)$	**			
$\Xi(2500)$	$\frac{1}{2}(?^?)$	*			

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[1]Some quantum numbers of the ground states have not been directly measured but assigned [1]. The most recent measurement on the spin of $\Omega^-(1672)$ was reported in Ref. [14].

Furthermore, the third lowest Ξ state would have a mass of 1755 MeV with $J^P = \frac{1}{2}^-$, which is 65 MeV larger than the mass of $\Xi(1690)$.

In the one-boson exchange model of Glozman and Riska [20], the third lowest state would have odd parity at a mass of 1758 MeV with $J = 1/2$ or $3/2$. This mass is in the middle of the masses of $\Xi(1820)$ and $\Xi(1690)$. Therefore, this model overestimates the mass of $\Xi(1690)$ and underestimates that of $\Xi(1820)$. (See also Refs. [21,22].) As a result, the nonrelativistic quark model [18], relativized quark model [19], and one-boson exchange model [20] give very different predictions on the Ξ spectrum.

The large N_c QCD predictions, where N_c is the number of colors, also lead to a quite different Ξ spectrum [23–27]. In this approach, the mass of $\Xi(1820)\frac{3}{2}^-$ is used as an input and the third lowest state would have $J^P = \frac{1}{2}^-$ at a mass of 1780 MeV. Although the lowest state with $J^P = \frac{1}{2}^-$ of this model lies close to the predictions on the same J^P states of the quark models [18,20], the explanation for the observed $\Xi(1690)$ is still uncertain. (A connection between the quark models and the large N_c expansion was recently discussed in Ref. [28].)

In the algebraic model of Bijker *et al.* [29], the third lowest state would have $J^P = \frac{1}{2}^+$ at a mass around 1730 MeV. It also predicts two $\Xi(\frac{3}{2}^-)$ states which lie close to the observed $\Xi(1820)$. This model predicts a richer hyperon spectrum than quark models and it has very different predictions especially on the $J^P = \frac{1}{2}^-$ states.

There also have been some efforts to construct QCD sum rules for Ξ baryons, but the results depend strongly on the approximations and assumptions made during the calculation [30,31]. The model of Ref. [31] prefers $\Xi(1690)$ as a $J^P = \frac{1}{2}^-$ state, while that of Ref. [30] predicts very low mass for this state although it can describe $\Xi(1820)\frac{3}{2}^-$. The lattice QCD calculation for multistrangeness baryons is still at the very early stage. But some progress has been reported, e.g., in Refs. [32,33].

One common characteristic of the quark models is that it is very difficult to accommodate $\Xi(1620)$ and $\Xi(1690)$ together. In particular, the low mass of one-star-rated $\Xi(1620)$ is puzzling in the quark models if its existence is confirmed by future experiments. In Ref. [34], Azimov *et al.* assigned $J^P = \frac{1}{2}^-$ to $\Xi(1620)$ so that it can form a narrow light baryon octet with $N'(\sim 1100)$, $\Lambda(1330)$, and $\Sigma(1480)$. Although this model satisfies the Gell-Mann-Okubo mass relation, it requires the existence of a very low mass nucleon and Λ resonances. In Ref. [35], Ramos *et al.* suggested to identify $\Xi(1620)$ as a dynamically generated S-wave Ξ resonance based on a unitary extension of chiral perturbation theory, which predicts a Ξ resonance at a mass around 1606 MeV. Then $\Xi(1620)$ has $J^P = \frac{1}{2}^-$ and it was suggested to form a unitary octet with $N(1535)$, $\Lambda(1670)$, and $\Sigma(1620)$, which, however, does not satisfy the Gell-Mann-Okubo mass formula. In a similar approach but with different details, García-Recio *et al.* [36] claimed that both $\Xi(1620)$ and $\Xi(1690)$ are S-wave resonances having $J^P = \frac{1}{2}^-$, while $\Xi(1690)$ was claimed to be a genuine resonance in Ref. [35]. (Some Ω baryon resonances are discussed in a similar approach in Ref. [37].)

The predictions of various models on the Ξ spectrum are summarized in Table II, which evidently shows strong model dependence of the resulting spectrum. (See also Refs. [38–40] for the other model predictions.) The strong model dependence can also be found in the Ω baryon spectrum as presented in the same table, which shows the degeneracy of $J^P = \frac{1}{2}^-$ and $J^P = \frac{3}{2}^-$ Ω baryons in the quark models of Refs. [18,20,29].

In this paper, we investigate strange hyperon resonances in the bound-state approach of the Skyrme model [41]. In this approach, strange hyperons are described as the bound states of the soliton and K/K^* meson(s). This model was successful to describe the ground state octet and decuplet hyperons [42,43]. It also has been applied to heavy-quark baryons [44–46] and the resulting heavy baryon spectrum was shown to respect the heavy quark spin symmetry once the heavy vector meson fields were treated in a consistent way [47–51]. Recent discussion on this model for the existence of exotic states can be found in Refs. [52,53]. In the beginning, this model was developed with Lagrangians which have the pseudoscalar K meson only by integrating out the K^* vector meson. However, as discussed in Ref. [53], although the K^* vector meson has a larger mass, it may have a nontrivial role, which cannot be correctly accounted for if it is integrated out in favor of the pseudoscalar meson. But our discussion in this paper does not depend on the choice of the degrees of freedom in the strangeness direction, and from now on by "kaons" we mean both K and K^* mesons.

One novel feature of the bound-state model is that it renders two kinds of bound kaons: one in P-wave and one in S-wave [41].[2] Therefore, whereas simple quark models have difficulties in describing $\Lambda(1405)$, this model gives a natural description of $\Lambda(1116)$ and $\Lambda(1405)$ on the same ground [55]. Namely, the even-parity $\Lambda(1116)$ is a bound state of the soliton and P-wave kaon, while the odd-parity $\Lambda(1405)\frac{1}{2}^-$ contains an S-wave kaon. The mass difference between these two Λ hyperons is 289 MeV. One can easily find similar mass gaps in hyperon mass spectrum, e.g., between $\Sigma(1385)\frac{3}{2}^+$ and $\Sigma(1670)\frac{3}{2}^-$ and between $\Xi(1530)\frac{3}{2}^+$ and $\Xi(1820)\frac{3}{2}^-$. Their mass differences are 285 MeV and 290 MeV, respectively, and are very close to the mass difference between $\Lambda(1405)$ and $\Lambda(1116)$. This observation gives us a strong motivation for the application of this model to low-lying hyperon resonances. The application to excited multistrangeness baryons was first tried in Refs. [45,56] by taking the simplest Skyrme model

[2] For soliton-heavy-meson systems, there are more bound states than the soliton-kaon system [54].

TABLE II. Low-lying Ξ and Ω baryon spectrum of spin 1/2 and 3/2 predicted by the nonrelativistic quark model of Chao *et al.* (CIK), relativized quark model of Capstick and Isgur (CI), Glozman-Riska model (GR), large N_c analysis, algebraic model (BIL), and QCD sum rules (QCD-SR). The mass is given in the unit of MeV.

State	CIK [18]	CI [19]	GR [20]	Large-N_c [23–27]	BIL [29]	QCD-SR [30] ([31])
$\Xi(\frac{1}{2}^+)$	1325	1305	1320		1334	1320 (1320)
	1695	1840	1798	1825	1727	
	1950	2040	1947	1839	1932	
$\Xi(\frac{3}{2}^+)$	1530	1505	1516		1524	
	1930	2045	1886	1854	1878	
	1965	2065	1947	1859	1979	
$\Xi(\frac{1}{2}^-)$	1785	1755	1758	1780	1869	1550 (1630)
	1890	1810	1849	1922	1932	
	1925	1835	1889	1927	2076	
$\Xi(\frac{3}{2}^-)$	1800	1785	1758	1815	1828	1840
	1910	1880	1849	1973	1869	
	1970	1895	1889	1980	1932	
$\Omega(\frac{1}{2}^+)$	2190	2220	2068	2408	2085	
	2210	2255	2166		2219	
$\Omega(\frac{3}{2}^+)$	1675	1635	1651		1670	
	2065	2165	2020	1922	1998	
	2215	2280	2068	2120	2219	
$\Omega(\frac{1}{2}^-)$	2020	1950	1991	2061	1989	
$\Omega(\frac{3}{2}^-)$	2020	2000	1991	2100	1989	

Lagrangian for the dynamics of the soliton-kaon system. However, the results were not so successful mainly because of the simplicity of the adopted Lagrangian. In this paper, we do not work with a specific Lagrangian for the system of the soliton and kaon. Instead, we focus on the general mass formula of this approach and find mass and magnetic moment sum rules among the even and odd parity baryons. For this purpose, we develop an improved mass formula which differs from that given in, e.g., Ref. [43]. We will also fit the mass parameters to some known baryon masses and make predictions on the hyperon spectrum. It should be mentioned, however, that the present approach cannot be applied to *all* hyperon resonances. There exist many low-lying hyperon resonances which potentially could be described as resonances in the pion-hyperon channel. In the Skyrme model, these resonances then may be investigated by including fluctuating pion fields [57]. In this paper we focus on the multikaon-soliton channel for describing several hyperon resonances leaving the issue on the pion-hyperon resonances to a future study.

This paper is organized as follows. In the next section, we briefly review the mass formula of the bound-state approach and develop a new formula. In Sec. III, we present our results for the hyperon mass spectrum. Mass sum rules and the "best fit" results for the hyperon mass spectrum will be given. Section IV is devoted to the magnetic moments of baryons and we derive several magnetic moment sum rules. We conclude in Sec. V.

II. QUANTIZATION IN THE BOUND-STATE MODEL

Referring the details to Refs. [50,58], we begin with a brief review on the derivation of the mass formula of the bound-state model. We first consider the case of only one bound kaon. Then the mass of a hyperon with isospin i and spin j in the soliton-kaon bound-state model reads

$$M(i, j) = M_{\rm sol} + \omega + E_{\rm rot}, \tag{1}$$

where $M_{\rm sol}$ is the soliton mass, ω the energy of the bound kaon, and $E_{\rm rot}$ the rotational energy arising from the collective rotation. The rotational energy is obtained as

$$E_{\rm rot} = \frac{1}{2\mathcal{I}}(\boldsymbol{R} - \boldsymbol{\Theta})^2, \tag{2}$$

where $\mathcal{I}$ is the moment of inertia of the soliton and $\boldsymbol{\Theta}$ is the isospin of the kaon embedded in the soliton field. Effectively, $\boldsymbol{R}$ has the role of the spin of the light u, d quarks. This mass formula respects $1/N_c$ expansion since $M_{\rm sol}$, ω, and $E_{\rm rot}$ are of the order of N_c, N_c^0, and $1/N_c$, respectively. The collective quantization gives

$$I^a = D^{ab} R^b, \qquad \boldsymbol{J} = \boldsymbol{R} + \boldsymbol{J}_K, \tag{3}$$

where D^{ab} is the SU(2) adjoint representation associated with the collective variables and $\boldsymbol{J}_K$ is the "grand-spin" of the kaon: $\boldsymbol{J}_K = \boldsymbol{L}_K + \boldsymbol{S}_K + \boldsymbol{T}_K$, with $\boldsymbol{L}_K$, $\boldsymbol{S}_K$, and $\boldsymbol{T}_K$ being the orbital angular momentum, spin, and isospin of the kaon. It is well known that the bound state of kaon is obtained only for $j_K = 1/2$ [41]. In order to represent $\boldsymbol{\Theta}$ in terms of good quantum numbers, one makes use of the relation

$$\boldsymbol{\Theta} = -c\boldsymbol{J}_K. \tag{4}$$

This defines the constant c which plays the role of the hyperfine splitting constant. Therefore, as pointed out by Ref. [59], the rotational energy is very similar to the magnetic moment interactions of quark models since $E_{\rm rot}$ contains the spin-spin interactions. In this sense, the constant c distinguishes the strength of the interactions with strange quarks from that of the light-quark-light-quark interactions. When there are n bound kaons of the same kind, the mass formula of Ref. [43] gives

$$M(i,j) = M_{\rm sol} + n\omega + \frac{1}{2\mathcal{I}}\{cj(j+1) + (1-c)i(i+1) + c(c-1)j_1(j_1+1)\}, \tag{5}$$

where $j_1 = nj_K$ as constrained by the Bose statistics.

If there are two kinds of bound mesons, such as the system including both the P-wave and S-wave kaons or the system of K meson and D meson, the rotational energy reads

$$E_{\rm rot} = \frac{1}{2\mathcal{I}}(\boldsymbol{R} - \boldsymbol{\Theta}_1 - \boldsymbol{\Theta}_2)^2. \tag{6}$$

By defining $\boldsymbol{J}_m = \boldsymbol{J}_1 + \boldsymbol{J}_2$, the mass formula of Ref. [44] gives

$$M(i,j,j_m) = M_{\rm sol} + n_1\omega_1 + n_2\omega_2 + \frac{1}{2\mathcal{I}}\Big\{i(i+1) + c_1c_2j_m(j_m+1) + (c_1-c_2)[c_1j_1(j_1+1) - c_2j_2(j_2+1)] + [j(j+1) - j_m(j_m+1) - i(i+1)]\Big[\frac{c_1+c_2}{2} + \frac{c_1-c_2}{2}\,\frac{j_1(j_1+1)-j_2(j_2+1)}{j_m(j_m+1)}\Big]\Big\}, \tag{7}$$

where ω_i, j_i, and c_i are the energy, grand spin, and hyperfine constant of the ith kind of bound meson whose number is represented by n_i.[3]

However, the assumptions made to arrive at the mass formula (7) require further consideration. First, it is assumed that

$$\boldsymbol{\Theta}^2 = c^2\boldsymbol{J}_K^2. \tag{8}$$

As mentioned before, $\boldsymbol{R}$ is the isospin of the kaon embedded in the soliton field. It was pointed out in Ref. [49] that $\boldsymbol{\Theta}^2$ can be exactly calculated in the infinite heavy mass limit thanks to the simplicity of the heavy meson (spatial) wave function. In this limit, $\boldsymbol{\Theta}$ is nothing but the isospin operator sandwiched by $\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}$. As a result, $\boldsymbol{\Theta}^2$ can be calculated exactly as

$$\boldsymbol{\Theta}^2 = \boldsymbol{J}_K^2 = \tfrac{3}{4}, \tag{9}$$

which evidently shows the failure of the approximation (8) in this limit. It should also be mentioned that the role of vector mesons is crucial to get the correct heavy quark limit. With this observation, one can show that the soliton-meson bound-state model is equivalent to the nucleon-meson bound-state model of Ref. [47] in the heavy quark limit. Another ambiguity in calculating $\boldsymbol{\Theta}^2$ is that the kaon-kaon interactions which have been neglected so far should contribute to this term. Including the kaon-kaon interactions would have an important role especially for multi-strangeness baryons and $\boldsymbol{\Theta}^2$ may depend on the strangeness of the baryon. However, this requires one to expand the soliton Lagrangian up to the kaon quartic terms, which is laborious [60] and beyond the scope of this work.[4] Instead of working with a specific model Lagrangian, we introduce another parameter $\bar{c}$,

$$\boldsymbol{\Theta}^2 = \bar{c}\boldsymbol{J}_K^2, \tag{10}$$

as suggested by the authors of Refs. [59,60]. Therefore the approximation (8) corresponds to $\bar{c} = c^2$, which is shown to be comparable to the quark model with magnetic moment interactions [59,60]. However, it should be kept in mind that, in the heavy quark limit, one has $\bar{c} \to 1$ and $c \to 0$, which shows that the relation $\bar{c} = c^2$ does not hold in general. By working with Eq. (10), the mass formula is still in the form of Eq. (5) but c^2 is replaced by $\bar{c}$.

Second, in order to arrive at the expression for the $(c_1 - c_2)$ term of Eq. (7), $\boldsymbol{J}_1 - \boldsymbol{J}_2$ is assumed to precess about the $\boldsymbol{J}_m$ direction, which is similar to the case of anomalous Zeeman effect in weak magnetic field. This assumption allows us to write

$$(\boldsymbol{J}_1 - \boldsymbol{J}_2)\cdot\boldsymbol{R} \approx \frac{(\boldsymbol{J}_1 - \boldsymbol{J}_2)\cdot\boldsymbol{J}_m}{\boldsymbol{J}_m^2}\boldsymbol{R}\cdot\boldsymbol{J}_m, \tag{11}$$

which then leads to the geometrical factor of the $(c_1 - c_2)$ term. However, this can be a good approximation only when the vector $\boldsymbol{J}_1 - \boldsymbol{J}_2$ precesses about the $\boldsymbol{J}_m$ axis. But, in fact, this term can be calculated directly from the

[3]The typographical errors committed in Refs. [44,45] were corrected in Ref. [46].

[4]The anharmonic corrections in the bound-state soliton model were first addressed by Björnberg *et al.* in Ref. [61] within a concise form of the effective Lagrangian. By treating the kaon quartic terms as a perturbation, the authors found that the anharmonic corrections would lead to an about 10% correction to the P-wave kaon energy.

wave functions without making any assumptions. In principle, this term can contribute only to the Ξ and Ω baryon masses but the expression of Eq. (11) vanishes in both cases since $j_1 = j_2$ for Ξ baryons and $r = 0$ for Ω baryons. However, although this term vanishes for those baryons, it causes a mixing between odd-parity Ξ resonances as will be shown in the next section. Therefore, we work with the mass formula,

$$M(i, j, j_m) = M_{\rm sol} + n_1\omega_1 + n_2\omega_2 + \frac{1}{2\mathcal{I}}\Big\{i(i+1) + c_1c_2 j_m(j_m+1) + (\bar{c}_1 - c_1c_2)j_1(j_1+1) + (\bar{c}_2 - c_1c_2)j_2(j_2+1) + \frac{c_1+c_2}{2}[j(j+1) - j_m(j_m+1) - i(i+1)] + \frac{c_1-c_2}{2}\boldsymbol{R}\cdot(\boldsymbol{J}_1 - \boldsymbol{J}_2)\Big\}. \quad (12)$$

When there is only one kind of bound kaon, this formula reduces to Eq. (5) by replacing c^2 by $\bar{c}$.

III. MASS SPECTRUM

In order to calculate baryon mass spectrum, we need to construct the baryon wave functions. We classify the states according to the quantum numbers, i, j^P, j_1, j_2, and j_m as shown in Table III. Since we are considering the P- and S-wave bound kaons, we can construct two Λ's, four Σ's, seven Ξ's, and six Ω's which lie below or close to the threshold energies. Table III also gives the names of each state of which the subscript shows the spin-parity quantum numbers explicitly. The number following the spin-parity in the subscript is introduced to distinguish the states of the same spin-parity quantum number. In particular, the subscript 0 means that the baryon belongs to the octet or decuplet ground state of the quark model.

Given in Table IV are the spin-up wave functions of each baryon in the basis of $|r, r_z\rangle|j_1, j_{1z}\rangle_{\ell=1}|j_2, j_{2z}\rangle_{\ell=0}$. It is then straightforward to write down the baryon masses and the results are given in Table V.

For a given model Lagrangian, the mass parameters, namely, $M_{\rm sol}$, $\mathcal{I}$, $\omega_{1,2}$, $c_{1,2}$, and $\bar{c}_{1,2}$, can be computed, and, therefore, these quantities are important to understand the dynamics of the soliton-meson system. However, our purpose is to test the mass formula without resorting to a specific model Lagrangian. We believe that by employing more realistic effective action for the soliton-meson system, one can calculate the mass parameters which converge to the values of our estimation. For this purpose, therefore, we fit the mass parameters to the known baryon

TABLE III. Particles with quantum numbers. The last column shows the particle identification with the octet and decuplet ground states as well as Λ(1405) and Ξ(1820).

Strangeness	$n_{\ell=1}$	$n_{\ell=0}$	j_m	i	j^P	Particle name	Particle Data Group
$S = 0$	0	0	0	1/2	$1/2^+$	N	$N(939)$
	0	0	0	3/2	$3/2^+$	Δ	Δ(1232)
$S = -1$	1	0	1/2	0	$1/2^+$	$\Lambda_{1/2^+,0}$	Λ(1116)
	0	0	1/2	0	$1/2^-$	$\Lambda_{1/2^-,1}$	Λ(1405)
	1	0	1/2	1	$1/2^+$	$\Sigma_{1/2^+,0}$	Σ(1193)
	1	0	1/2	0	$1/2^+$	$\Sigma_{3/2^+,0}$	Σ(1385)
	0	1	1/2	1	$1/2^-$	$\Sigma_{1/2^-,1}$	
	0	1	1/2	1	$3/2^-$	$\Sigma_{3/2^-,1}$	
$S = -2$	2	0	1	1/2	$1/2^+$	$\Xi_{1/2^+,0}$	Ξ(1318)
	2	0	1	1/2	$3/2^+$	$\Xi_{3/2^+,0}$	Ξ(1530)
	1	1	0	1/2	$1/2^-$	$\Xi_{1/2^-,1}$	
	1	1	1	1/2	$1/2^-$	$\Xi_{1/2^-,2}$	
	1	1	0	1/2	$3/2^-$	$\Xi_{3/2^-,1}$	Ξ(1820)
	0	2	1	1/2	$1/2^+$	$\Xi_{1/2^+,1}$	
	0	2	1	1/2	$3/2^+$	$\Xi_{3/2^+,1}$	
$S = -3$	3	0	3/2	0	$3/2^+$	$\Omega_{3/2^+,0}$	Ω(1672)
	2	1	1/2	0	$1/2^-$	$\Omega_{1/2^-,1}$	
	2	1	3/2	0	$3/2^-$	$\Omega_{3/2^-,1}$	
	1	2	1/2	0	$1/2^+$	$\Omega_{1/2^+,1}$	
	1	2	3/2	0	$3/2^+$	$\Omega_{3/2^+,1}$	
	0	3	3/2	0	$3/2^-$	$\Omega_{3/2^-,2}$	

TABLE IV. States of spin-up baryons in the bases of $|r, r_z\rangle |j_1, j_{1z}\rangle_{\ell=1} |j_2, j_{2z}\rangle_{\ell=0}$.

Particle	State
$\|N\rangle$	$\|\frac{1}{2}\,\frac{1}{2}\rangle_R$
$\|\Delta\rangle$	$\|\frac{3}{2}\,\frac{3}{2}\rangle_R$
$\|\Lambda_{1/2^+,0}\rangle$	$\|00\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_1$
$\|\Sigma_{1/2^+,0}\rangle$	$-\frac{1}{\sqrt{3}}\|10\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_1 + \sqrt{\frac{2}{3}}\|11\rangle_R\|\frac{1}{2}-\frac{1}{2}\rangle_1$
$\|\Sigma_{3/2^+,0}\rangle$	$\|11\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_1$
$\|\Lambda_{1/2^-,1}\rangle$	$\|00\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_0$
$\|\Sigma_{1/2^-,1}\rangle$	$-\frac{1}{\sqrt{3}}\|10\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_0 + \sqrt{\frac{2}{3}}\|11\rangle_R\|\frac{1}{2}-\frac{1}{2}\rangle_0$
$\|\Sigma_{3/2^-,1}\rangle$	$\|11\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_0$
$\|\Xi_{1/2^+,0}\rangle$	$\frac{1}{\sqrt{3}}\|\frac{1}{2}\,\frac{1}{2}\rangle_R\|10\rangle_1 - \sqrt{\frac{2}{3}}\|\frac{1}{2}-\frac{1}{2}\rangle_R\|11\rangle_1$
$\|\Xi_{3/2^+,0}\rangle$	$\|\frac{1}{2}\,\frac{1}{2}\rangle_R\|11\rangle_1$
$\|\Xi_{1/2^-,1}\rangle$	$\frac{1}{\sqrt{2}}\|\frac{1}{2}\,\frac{1}{2}\rangle_R[\|\frac{1}{2}\,\frac{1}{2}\rangle_1\|\frac{1}{2}-\frac{1}{2}\rangle_0 - \|\frac{1}{2}-\frac{1}{2}\rangle_1\|\frac{1}{2}\,\frac{1}{2}\rangle_0]$
$\|\Xi_{1/2^-,2}\rangle$	$\frac{1}{\sqrt{6}}\|\frac{1}{2}\,\frac{1}{2}\rangle_R[\|\frac{1}{2}\,\frac{1}{2}\rangle_1\|\frac{1}{2}-\frac{1}{2}\rangle_0 + \|\frac{1}{2}-\frac{1}{2}\rangle_1\|\frac{1}{2}\,\frac{1}{2}\rangle_0] - \sqrt{\frac{2}{3}}\|\frac{1}{2}-\frac{1}{2}\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_1\frac{1}{2}\,\frac{1}{2}\rangle_0$
$\|\Xi_{3/2^-,1}\rangle$	$\|\frac{1}{2}\,\frac{1}{2}\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_1\|\frac{1}{2}\,\frac{1}{2}\rangle_0$
$\|\Xi_{1/2^+,1}\rangle$	$\frac{1}{\sqrt{3}}\|\frac{1}{2}\,\frac{1}{2}\rangle_R\|10\rangle_0 - \sqrt{\frac{2}{3}}\|\frac{1}{2}-\frac{1}{2}\rangle_R\|11\rangle_0$
$\|\Xi_{3/2^+,1}\rangle$	$\|\frac{1}{2}\,\frac{1}{2}\rangle_R\|11\rangle_0$
$\|\Omega_{3/2^+,0}\rangle$	$\|00\rangle_R\|\frac{3}{2}\,\frac{3}{2}\rangle_1$
$\|\Omega_{1/2^-,1}\rangle$	$\frac{1}{\sqrt{3}}\|00\rangle_R[\sqrt{2}\|11\rangle_1\|\frac{1}{2}-\frac{1}{2}\rangle_0 - \|10\rangle_1\|\frac{1}{2}\,\frac{1}{2}\rangle_0]$
$\|\Omega_{3/2^-,1}\rangle$	$\|00\rangle_R\|11\rangle_1\|\frac{1}{2}\,\frac{1}{2}\rangle_0$
$\|\Omega_{1/2^+,1}\rangle$	$\frac{1}{\sqrt{3}}\|00\rangle_R[-\sqrt{2}\|\frac{1}{2}-\frac{1}{2}\rangle_1\|11\rangle_0 + \|\frac{1}{2}\,\frac{1}{2}\rangle_1\|10\rangle_0]$
$\|\Omega_{3/2^+,1}\rangle$	$\|00\rangle_R\|\frac{1}{2}\,\frac{1}{2}\rangle_1\|11\rangle_0$
$\|\Omega_{3/2^-,2}\rangle$	$\|00\rangle_R\|\frac{3}{2}\,\frac{3}{2}\rangle_0$

masses. First, the soliton mass and its moment of inertia can be obtained from the nucleon and Δ masses as

$$M_{\rm sol} = 866 \text{ MeV}, \qquad I = 1.01 \text{ fm}. \tag{13}$$

Second, the parameters for the P-wave kaon are obtained from $\Lambda(1116)$, $\Sigma(1385)$, and $\Xi(1318)$, which give

$$\omega_1 = 211 \text{ MeV}, \qquad c_1 = 0.754, \qquad \bar{c}_1 = 0.532. \tag{14}$$

Third, in order to determine the empirical values for the S-wave kaon parameters, we use $\Lambda(1405)$, $\Xi(1820)$, and $\Xi(2120)$.[5] Then we have

$$\omega_2 = 479 \text{ MeV}, \qquad c_2 = 0.641, \qquad \bar{c}_2 = 0.821. \tag{15}$$

This implies that the S-wave kaon is very slightly bound with the binding energy of about 15 MeV. It also shows that $\bar{c}_1$ ($= 0.532$) is close to the value of c_1^2 ($= 0.569$), while the difference between $\bar{c}_2$ ($= 0.821$) and c_2^2 ($= 0.411$) is rather large. This may indicate that the role of kaon quartic terms would be more important for the S-wave kaons than for the P-wave kaons.

The resulting mass spectrum is shown in Table VI.[6] By considering the simple structure of the mass formula, the resulting mass spectrum is quite impressive. In particular, it can explain many of the Ξ resonances reported in PDG. It also gives a very natural explanation for the low-lying (one-star resonance) $\Xi(1620)$ and (three-star resonance) $\Xi(1690)$ identifying their spin-parity as $\frac{1}{2}^-$.

From the mass formula, one can derive Lagrangian-independent mass relations. We found that the bound-state model does not obey the well-known Gell-Mann-Okubo mass relation and the decuplet equal spacing rule, which follow from the quark model including the leading order of the flavor symmetry breaking. Instead, it satisfies the modified relations,

[5]We assume that the spin-parity of $\Xi(2120)$ is $\frac{3}{2}^+$. This is based on the observation that its mass is higher than $\Xi(1820)$ by 300 MeV, which is the typical energy scale of the energy difference between the P-wave and S-wave kaon-soliton systems. The $\Sigma(1670)\frac{3}{2}^-$ cannot be used for the fitting process because it contains the same $\bar{c}_2$ term with $\Lambda(1405)$ and $\Xi(1820)$.

[6]With the assumption of $\bar{c} = c^2$, we can obtain the parameters from $\Lambda(1116)$, $\Sigma(1193)$, $\Lambda(1405)$, and $\Xi(1820)$. This gives $\omega_1 = 223$ MeV, $c_1 = 0.604$, $\omega_2 = 492$ MeV, and $c_2 = 0.80$. The resulting mass spectrum is similar to that given in Table VI within the difference of 40 MeV at most.

TABLE V. Baryon masses in terms of mass parameters. The quantities with the subscript 1 are for the P-wave kaon and those with the subscript 2 are for the S-wave kaon.

Particle	Mass
N	$M_{\rm sol} + \frac{3}{8I}$
Δ	$M_{\rm sol} + \frac{15}{8I}$
$\Lambda_{1/2^+,0}$	$M_{\rm sol} + \omega_1 + \frac{1}{2I}\frac{3}{4}\bar{c}_1$
$\Lambda_{1/2^-,1}$	$M_{\rm sol} + \omega_2 + \frac{1}{2I}\frac{3}{4}\bar{c}_2$
$\Sigma_{1/2^+,0}$	$M_{\rm sol} + \omega_1 + \frac{1}{2I}(2 + \frac{3}{4}\bar{c}_1 - 2c_1)$
$\Sigma_{3/2^+,0}$	$M_{\rm sol} + \omega_1 + \frac{1}{2I}(2 + \frac{3}{4}\bar{c}_1 + c_1)$
$\Sigma_{1/2^-,1}$	$M_{\rm sol} + \omega_2 + \frac{1}{2I}(2 + \frac{3}{4}\bar{c}_2 - 2c_2)$
$\Sigma_{3/2^-,1}$	$M_{\rm sol} + \omega_2 + \frac{1}{2I}(2 + \frac{3}{4}\bar{c}_2 + c_2)$
$\Xi_{1/2^+,0}$	$M_{\rm sol} + 2\omega_1 + \frac{1}{2I}(\frac{3}{4} + 2\bar{c}_1 - 2c_1)$
$\Xi_{3/2^+,0}$	$M_{\rm sol} + 2\omega_1 + \frac{1}{2I}(\frac{3}{4} + 2\bar{c}_1 + c_1)$
$\Xi_{1/2^-,1}$	$M_{\rm sol} + \omega_1 + \omega_2 + \frac{1}{2I}(\frac{3}{4} + \frac{3}{4}\bar{c}_1 - \frac{3}{2}c_1c_2 + \frac{3}{4}\bar{c}_2)$
$\Xi_{1/2^-,2}$	$M_{\rm sol} + \omega_1 + \omega_2 + \frac{1}{2I}(\frac{3}{4} + \frac{3}{4}\bar{c}_1 + \frac{1}{2}c_1c_2 + \frac{3}{4}\bar{c}_2 - c_1 - c_2)$
$\Xi_{3/2^-,1}$	$M_{\rm sol} + \omega_1 + \omega_2 + \frac{1}{2I}(\frac{3}{4} + \frac{3}{4}\bar{c}_1 + \frac{1}{2}c_1c_2 + \frac{3}{4}\bar{c}_2 + \frac{1}{2}c_1 + \frac{1}{2}c_2)$
$\Xi_{1/2^+,1}$	$M_{\rm sol} + 2\omega_2 + \frac{1}{2I}(\frac{3}{4} + 2\bar{c}_2 - 2c_2)$
$\Xi_{3/2^+,1}$	$M_{\rm sol} + 2\omega_2 + \frac{1}{2I}(\frac{3}{4} + 2\bar{c}_2 + c_2)$
$\Omega_{3/2^+,0}$	$M_{\rm sol} + 3\omega_1 + \frac{1}{2I}\frac{15}{4}\bar{c}_1$
$\Omega_{1/2^-,1}$	$M_{\rm sol} + 2\omega_1 + \omega_2 + \frac{1}{2I}(2\bar{c}_1 - 2c_1c_2 + \frac{3}{4}\bar{c}_2)$
$\Omega_{3/2^-,1}$	$M_{\rm sol} + 2\omega_1 + \omega_2 + \frac{1}{2I}(2\bar{c}_1 + c_1c_2 + \frac{3}{4}\bar{c}_2)$
$\Omega_{1/2^+,1}$	$M_{\rm sol} + \omega_1 + 2\omega_2 + \frac{1}{2I}(\frac{3}{4}\bar{c}_1 - 2c_1c_2 + 2\bar{c}_2)$
$\Omega_{3/2^+,1}$	$M_{\rm sol} + \omega_1 + 2\omega_2 + \frac{1}{2I}(\frac{3}{4}\bar{c}_1 + c_1c_2 + 2\bar{c}_2)$
$\Omega_{3/2^-,2}$	$M_{\rm sol} + 3\omega_2 + \frac{1}{2I}\frac{15}{4}\bar{c}_2$

$$3\Lambda + \Sigma - 2(N + \Xi) = \Sigma^* - \Delta - (\Omega - \Xi^*),$$
$$(\Omega - \Xi^*) - (\Xi^* - \Sigma^*) = (\Xi^* - \Sigma^*) - (\Sigma^* - \Delta), \tag{16}$$

where the symbols denote the masses of the corresponding octet and decuplet ground states. These modified mass relations are known to be obtained by including the second order of the symmetry breaking terms, i.e., m_s^2 in terms of the strange quark mass. Therefore, these modified relations work better than the original mass relations. (See, e.g., Ref. [62].) In fact, the mass formula of the bound-state model contains the $\boldsymbol{J}_K^2$ term. This term arises not only from the $\boldsymbol{\Theta}^2$ term but also from the $\boldsymbol{R} \cdot \boldsymbol{\Theta}$ term. Since j_K^2 contains the square of the strangeness, the mass formula (12) already takes into account the effect of the second order of the flavor symmetry breaking terms. As discussed, e.g. in Ref. [62], the hyperfine relation holds even with the m_s^2 term, and, therefore, our model satisfies

$$\Sigma^* - \Sigma + \tfrac{3}{2}(\Sigma - \Lambda) = \Delta - N. \tag{17}$$

Since the mass relations (16) and (17) are obtained for the hyperons with the P-wave kaons, the same relations should be true for the hyperons containing the S-wave kaons only. Therefore, those relations are valid by replacing Λ, Σ, Σ^*, Ξ, Ξ^*, and Ω by $\Lambda_{1/2^-,1}$, $\Sigma_{1/2^-,1}$, $\Sigma_{3/2^-,1}$, $\Xi_{1/2^+,1}$, $\Xi_{3/2^+,1}$, and $\Omega_{3/2^-,2}$, respectively. Note that those mass sum rules relate the mixed parity states of hyperons, i.e., odd-parity Λ and Σ, even-parity Ξ, and odd-parity Ω.

One can derive additional mass sum rules like

$$\Omega_{3/2^+,1} - \Omega_{3/2^-,1} = \Omega_{1/2^+,1} - \Omega_{1/2^-,1}, \tag{18}$$

for Ω resonances. This shows that the mass differences between the baryons of the same quantum numbers but of opposite parity, which we call "parity partners," are exactly identical for these baryons. Although the mass splitting of other parity partner hyperons are not exactly equal to the above formula, we observe that their mass differences are always close to ~290 MeV and the same pattern is clearly seen in the experimental data. This is due to the fact that their mass difference starts from the $O(N_c^0)$ term in our mass formula, which is the energy of the bound kaon. Although $E_{\rm rot}$ contributes to the mass differences of the parity partners, its effect is $O(N_c^{-1})$ and is about 10% of the $O(N_c^0)$ term in our model. As a result, the equal mass splitting rule between the parity partners becomes a good approximation. This is different from the mass splittings between different spin states of the same parity, which are governed by $E_{\rm rot}$, the $O(N_c^{-1})$ term.

TABLE VI. Mass spectrum of our model. The underlined values are used to determine the mass parameters. The values within the parenthesis are obtained by considering the mixing effect. The question mark after the particle name means that the spin-parity quantum numbers are not identified by PDG.

Particle name	Mass (MeV)	Assigned state
N	$\underline{939}$	
Δ	$\underline{1232}$	
$\Lambda_{1/2^+,0}$	$\underline{1116}$	$\Lambda(1116)$
$\Lambda_{1/2^-,1}$	$\underline{1405}$	$\Lambda(1405)$
$\Sigma_{1/2^+,0}$	1164	$\Sigma(1193)$
$\Sigma_{3/2^+,0}$	$\underline{1385}$	$\Sigma(1385)$
$\Sigma_{1/2^-,1}$	1475	$\Sigma(1480)$?
$\Sigma_{3/2^-,1}$	1663	$\Sigma(1670)$
$\Xi_{1/2^+,0}$	$\underline{1318}$	$\Xi(1318)$
$\Xi_{3/2^+,0}$	1539	$\Xi(1530)$
$\Xi_{1/2^-,1}$	1658(1660)*	$\Xi(1690)$?
$\Xi_{1/2^-,2}$	1616(1614)*	$\Xi(1620)$?
$\Xi_{3/2^-,1}$	$\underline{1820}$	$\Xi(1820)$
$\Xi_{1/2^+,1}$	1932	$\Xi(1950)$?
$\Xi_{3/2^+,1}$	$\underline{2}120$	$\Xi(2120)$?
$\Omega_{3/2^+,0}$	1694	$\Omega(1672)$
$\Omega_{1/2^-,1}$	1837	
$\Omega_{3/2^-,1}$	1978	
$\Omega_{1/2^+,1}$	2140	
$\Omega_{3/2^+,1}$	2282	$\Omega(2250)$?
$\Omega_{3/2^-,2}$	2604	

We now consider the $\boldsymbol{R}\cdot(\boldsymbol{J}_1-\boldsymbol{J}_2)$ term in the mass formula (12). As we have discussed before, this term only gives the mixing of the Ξ resonances, in particular, for $\Xi_{1/2^-,1}$ and $\Xi_{1/2^-,2}$ as both of them have the same quantum numbers except j_m. By using the state wave functions given in Table IV, we obtain

$$\langle\Xi_{1/2^-,2}|\boldsymbol{R}\cdot\boldsymbol{K}|\Xi_{1/2^-,1}\rangle = \langle\Xi_{1/2^-,1}|\boldsymbol{R}\cdot\boldsymbol{K}|\Xi_{1/2^-,2}\rangle = \frac{\sqrt{3}}{2}. \tag{19}$$

Thus the mixing term in the mass is estimated to be

$$\Delta M = \frac{\sqrt{3}}{4I}(c_1-c_2) \approx 9.6\ \text{MeV}. \tag{20}$$

Therefore, when we define $\Xi_{H,L}$ as

$$\begin{aligned}\Xi_H &= \cos\theta\,\Xi_{1/2^-,1} + \sin\theta\,\Xi_{1/2^-,2},\\ \Xi_L &= -\sin\theta\,\Xi_{1/2^-,1} + \cos\theta\,\Xi_{1/2^-,2},\end{aligned} \tag{21}$$

we have the masses of Ξ_H and Ξ_L as 1660 and 1614 MeV, respectively, with $\theta \approx 27.5°$. So the effect of this mixing on the mass spectrum is small and may be neglected.

IV. MAGNETIC MOMENTS

The magnetic moment operator can be written as [46]

$$\hat{\mu} = \hat{\mu}_s + \hat{\mu}_v, \tag{22}$$

where

$$\begin{aligned}\hat{\mu}_s &= \mu_{s,0}R^z + \mu_{s,1}J_1^z + \mu_{s,2}J_2^z,\\ \hat{\mu}_v &= -2(\mu_{v,0} + \mu_{v,1}n_1 + \mu_{v,2}n_2)D^{33},\end{aligned} \tag{23}$$

with $D^{33} = -I^zR^z/\boldsymbol{I}^2$. Here, $\mu_{s,0}$ and $\mu_{v,0}$ are the magnetic moment parameters of the SU(2) sector while $\mu_{s,1}$ and $\mu_{v,1}$ ($\mu_{s,2}$ and $\mu_{v,2}$) are those for the P-wave (S-wave) kaon. The number of P-wave and S-wave kaons is denoted by n_1 and n_2, respectively. There are totally six parameters and they can be calculated for a given model Lagrangian [46,51,55,63–65].

With the wave functions of baryons obtained in the previous section, it is straightforward to express the magnetic moments of baryons in terms of those six parameters. The results are given in Table VII.

The values of the magnetic moment parameters depend on the dynamics of the soliton-meson system. Unlike the mass spectrum, we do not have empirical data on the magnetic moments of odd-parity hyperons. Therefore, instead of making predictions on the values of baryon magnetic moments, we suggest several magnetic moment sum rules.

TABLE VII. Magnetic moments of baryons. Here, I^z represents the third component of the isospin.

Particle	Magnetic moment
N	$\frac{1}{2}\mu_{s0} + I^z\frac{4}{3}\mu_{v0}$
Δ	$\frac{3}{2}\mu_{s0} + I^z\frac{4}{5}\mu_{v0}$
$\Lambda_{1/2^+,0}$	$\frac{1}{2}\mu_{s1}$
$\Sigma_{1/2^+,0}$	$\frac{2}{3}\mu_{s0} - \frac{1}{6}\mu_{s1} + I^z\frac{2}{3}(\mu_{v0}+\mu_{v1})$
$\Sigma_{3/2^+,0}$	$\mu_{s0} + \frac{1}{2}\mu_{s1} + I^z(\mu_{v0}+\mu_{v1})$
$\Lambda_{1/2^-,1}$	$\frac{1}{2}\mu_{s2}$
$\Sigma_{1/2^-,1}$	$\frac{2}{3}\mu_{s0} - \frac{1}{6}\mu_{s2} + I^z\frac{2}{3}(\mu_{v0}+\mu_{v2})$
$\Sigma_{3/2^-,1}$	$\mu_{s0} + \frac{1}{2}\mu_{s2} + I^z(\mu_{v0}+\mu_{v2})$
$\Xi_{1/2^+,0}$	$-\frac{1}{6}\mu_{s0} + \frac{2}{3}\mu_{s1} - I^z\frac{4}{9}(\mu_{v0}+2\mu_{v1})$
$\Xi_{3/2^+,0}$	$\frac{1}{2}\mu_{s0} + \mu_{s1} + I^z\frac{4}{3}(\mu_{v0}+2\mu_{v1})$
$\Xi_{1/2^-,1}$	$\frac{1}{2}\mu_{s0} + I^z\frac{4}{3}(\mu_{v0}+\mu_{v1}+\mu_{v2})$
$\Xi_{1/2^-,2}$	$-\frac{1}{6}\mu_{s0} + \frac{1}{3}(\mu_{s1}+\mu_{s2}) - I^z\frac{4}{9}(\mu_{v0}+\mu_{v1}+\mu_{v2})$
$\Xi_{3/2^-,1}$	$\frac{1}{2}(\mu_{s0}+\mu_{s1}+\mu_{s2}) + I^z\frac{4}{3}(\mu_{v0}+\mu_{v1}+\mu_{v2})$
$\Xi_{1/2^+,1}$	$-\frac{1}{6}\mu_{s0} + \frac{2}{3}\mu_{s2} - I^z\frac{4}{9}(\mu_{v0}+2\mu_{v2})$
$\Xi_{3/2^+,1}$	$\frac{1}{2}\mu_{s0} + \mu_{s2} + I^z\frac{4}{3}(\mu_{v0}+2\mu_{v2})$
$\Omega_{3/2^+,0}$	$\frac{3}{2}\mu_{s1}$
$\Omega_{1/2^-,1}$	$\frac{2}{3}\mu_{s1} - \frac{1}{6}\mu_{s2}$
$\Omega_{3/2^-,1}$	$\mu_{s1} + \frac{1}{2}\mu_{s2}$
$\Omega_{1/2^+,1}$	$-\frac{1}{6}\mu_{s1} + \frac{2}{3}\mu_{s2}$
$\Omega_{3/2^+,1}$	$\frac{1}{2}\mu_{s1} + \mu_{s2}$
$\Omega_{3/2^-,2}$	$\frac{3}{2}\mu_{s2}$

For the ground state baryons, we have

$$\begin{aligned}\mu(\Sigma^{*+}) - \mu(\Sigma^{*-}) &= \tfrac{3}{2}\{\mu(\Sigma^+) - \mu(\Sigma^-)\},\\ \mu(\Sigma^+) + \mu(\Sigma^-) &= \tfrac{4}{3}\{\mu(p) + \mu(n)\} - \tfrac{2}{3}\mu(\Lambda),\\ \mu(\Sigma^{*+}) + \mu(\Sigma^{*-}) &= 2\{\mu(p) + \mu(n)\} + 2\mu(\Lambda),\\ \mu(\Xi^0) + \mu(\Xi^-) &= -\tfrac{1}{3}\{\mu(p) + \mu(n)\} + \tfrac{8}{3}\mu(\Lambda),\\ \mu(\Xi^{*0}) + \mu(\Xi^{*-}) &= \mu(p) + \mu(n) + 4\mu(\Lambda),\\ \mu(\Xi^{*0}) - \mu(\Xi^{*-}) &= -3\{\mu(\Xi^0) - \mu(\Xi^-)\},\\ \mu(\Omega) &= 3\mu(\Lambda),\end{aligned} \tag{24}$$

where the symbols represent the octet and decuplet ground state baryons. The above relations hold by replacing Σ, Σ^*, Ξ, Ξ^*, and Ω by $\Sigma_{1/2^-,1}$, $\Sigma_{3/2^-,1}$, $\Xi_{1/2^+,1}$, $\Xi_{3/2^+,1}$, and $\Omega_{3/2^-,2}$, respectively.

Other interesting sum rules include

$$\begin{aligned}&\mu(\Xi^0_{3/2^-,1}) - \mu(\Lambda_{1/2^-,1}) - \tfrac{1}{2}\{\mu(\Sigma^+_{1/2^-,1}) - \mu(\Sigma^-_{1/2^-,1})\} = \mu(\Xi^0_{3/2^+,0}) - \mu(\Lambda_{1/2^+,0}) - \tfrac{1}{2}\{\mu(\Sigma^+_{1/2^+,0}) - \mu(\Sigma^-_{1/2^+,0})\},\\ &\mu(\Xi^-_{3/2^-,1}) - \mu(\Lambda_{1/2^-,1}) + \tfrac{1}{2}\{\mu(\Sigma^+_{1/2^-,1}) - \mu(\Sigma^-_{1/2^-,1})\} = \mu(\Xi^-_{3/2^+,0}) - \mu(\Lambda_{1/2^+,0}) + \tfrac{1}{2}\{\mu(\Sigma^+_{1/2^+,0}) - \mu(\Sigma^-_{1/2^+,0})\},\\ &\mu(\Xi^0_{1/2^-,1}) + 3\mu(\Xi^0_{1/2^-,2}) = \mu(\Xi^-_{1/2^-,1}) + 3\mu(\Xi^-_{1/2^-,2}) = 2\{\mu(\Lambda_{1/2^+,0}) + \mu(\Lambda_{1/2^-,1})\},\end{aligned} \tag{25}$$

The relations for the excited Ω baryons are

$$\begin{aligned}\mu(\Omega_{1/2^-,1}) &= \tfrac{4}{3}\mu(\Lambda_{1116}) - \tfrac{1}{3}\mu(\Lambda_{1405}),\\ \mu(\Omega_{3/2^-,1}) &= 2\mu(\Lambda_{1116}) + \mu(\Lambda_{1405}),\\ \mu(\Omega_{1/2^+,1}) &= -\tfrac{1}{3}\mu(\Lambda_{1116}) + \tfrac{4}{3}\mu(\Lambda_{1405}),\\ \mu(\Omega_{3/2^+,1}) &= \tfrac{1}{3}\mu(\Lambda_{1116}) + 2\mu(\Lambda_{1405}),\end{aligned} \tag{26}$$

by using $\Lambda_{1/2^+,0} = \Lambda(1116)$ and $\Lambda_{1/2^-,1} = \Lambda(1405)$.

V. CONCLUSION

One of the successes of the bound-state approach of the Skyrme model is that it provides a unified way to explain both $\Lambda(1116)$ and $\Lambda(1405)$. The mass difference between the two particles is explained mainly by the energy difference between the *P*-wave kaon and the *S*-wave kaon. In this paper, we have shown that this feature can be extended to other low-lying excited states of hyperons. In particular, in the Ξ baryon spectrum, $\Xi(1820)$ can be explained by the bound state of the soliton and kaons of one in *P*-wave and one in *S*-wave. Then the existence of $\Xi(1690)$ and $\Xi(1620)$ baryons of $J^P = 1/2^-$ is required as they are parity partners of $\Xi(1318)\frac{1}{2}^+$, while $\Xi(1690)$ would have $J^P = 1/2^+$ in the nonrelativistic quark model of Chao *et al.* [18].

Although the bound-state model has a strong resemblance to the nonrelativistic quark model for the ground state baryons, our results show that the similarities may be lost for excited baryons. The presence of two low-lying Ξ resonances of $J^P = 1/2^-$, which we identify as $\Xi(1620)$ and $\Xi(1690)$, comes out naturally in our model and this is evidently different from other quark model predictions. In addition, unlike the assumption of Ref. [34], we do not need to require the existence of very low mass nucleon and Λ resonances. We also note that the spin-parity quantum numbers of $\Xi(1620)$ and $\Xi(1690)$ of our model are consistent with Ref. [35] and with Ref. [36]. We also found that the bound-state model predicts Ξ resonance with $J^P = 1/2^+$ at a mass of around 1930 MeV. This would be a good candidate for the observed three-star resonance $\Xi(1950)$. Therefore, it is quite different from the quark model prediction, e.g., of Ref. [66], where $\Xi(1950)$ was interpreted to have $J^P = 5/2^-$ as the Gell-Mann-Okubo mass relation for $J^P = 5/2^-$ states [with $N(1675)$, $\Sigma(1765)$, and $\Lambda(1830)$] requires a Ξ at a mass of around 1960 MeV. (See Ref. [67] for a recent discussion.) It would be interesting to note that the Skyrme model study for radially excited states of Ref. [68] suggests $J^P = \frac{1}{2}^+$ for the $\Xi(1950)$, although $\Xi(1690)$ was speculated to be a $J^P = \frac{3}{2}^-$ state.

The quark-based models [18,20,29] predict that the lowest $\Omega(\frac{1}{2}^-)$ is degenerate in mass with the lowest $\Omega(\frac{3}{2}^-)$. However, this degeneracy is no longer valid in the bound-state model which predicts that the mass difference between the two Ω resonances is around 140 MeV. But our model predicts that the first excited Ω resonance would have $J^P = 1/2^-$ as in the quark models.

In Σ hyperon resonances, we found that $\Sigma(1670)\frac{3}{2}^-$ fits well as the parity partner of $\Sigma(1385)$. Our result for $\Sigma(\frac{1}{2}^-)$ strongly implies that the one-star-rated $\Sigma(1480)$, which was recently confirmed by COSY experiment [69], would have $J^P = \frac{1}{2}^-$ being the parity partner of $\Sigma(1193)$. This should be compared with other model predictions on $\Sigma(1480)$ [34,36,70,71].

Of course, as mentioned before, the bound-state model cannot be applied to all hyperon resonances. In particular, in the Skyrme model, $Y\pi$ resonances should be explored by introducing fluctuating pion fields explicitly [57] and the resonances above the threshold should be treated in a different manner [53,72]. However, the pattern observed in the empirical mass spectrum of low-lying hyperon resonances, i.e., (approximately) equal spacings between parity partners, supports the bound-state approach of the Skyrme model, and distinguishes this model from the other phenomenological models.

Therefore, verifying the spin-parity quantum numbers of $\Xi(1690)$ and $\Xi(1950)$ is important to understand the structure of excited hyperons. In addition, confirming the existence of the one-star $\Xi(1620)$ resonance [73,74] is also highly desirable as well as searching for low-lying Ω baryons. Theoretically, more investigation on the role of the K^* vector mesons in the dynamics of the meson-soliton systems and on the anharmonic corrections to Ξ and Ω hyperon resonances is required. In particular, the hyperon resonances in the pion-hyperon channel should be examined for a more realistic description of the hyperon spectrum.

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