Causality and/or energy-momentum conservation constraints on QCD amplitudes in the small *x* regime

B. Blok*

Department of Physics, Technion—Israel Institute of Technology, 32000 Haifa, Israel

L. Frankfurt†

School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences, Tel Aviv University, 69978 Tel Aviv, Israel

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The causality and/or the energy-momentum conservation constraints on the amplitudes of high energy processes are generalized to QCD. The constraints imply that the energetic parton may experience at most one inelastic collision (an arbitrary number of elastic collisions) and that the number of the constituents in the light cone wave function of the projectile is increasing with the collision energy and the atomic number.

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I. INTRODUCTION

The high energy behavior of the QCD amplitudes has attracted a lot of attention, both experimental and theoretical. This interest is focused on the new QCD phenomena and on a necessity to evaluate reliably the QCD effects accompanying the new particle production. The aim of the present paper is to study the role of the causality and the energy-momentum conservation in the particle production at high energies in the perturbative QCD.

These constraints are absent in relativistic quantum mechanics, and appear only in a quantum field theory, in particular, in the perturbative QCD (pQCD). In this paper we establish the constraints imposed on the high energy scattering amplitudes in the hard QCD by the causality and by the energy-momentum conservation. The closely related constraints were studied in detail before the advent of QCD, in the framework of the Reggeon Calculus and ϕ^3 theories, see Refs. [1–4] and references therein. However, these constraints were mostly put aside afterwards, since the dominance of the leading twist (LT) approximation in hard processes at moderately small Bjorken x had made unnecessary investigation of multiple scattering processes.

It has been understood in literature, for the review and appropriate references see Ref. [5], that the rapid increase with the energy of the leading twist perturbative QCD (pQCD) amplitudes [6–12] leads to the problems with the probability conservation for the leading twist approximation in the kinematics covered by LHC and, probably, by the leading parton production at RHIC. Thus it seems necessary to develop an adequate theoretical treatment of the pQCD regime of strong interactions with a small running coupling constant to resolve the problem with the violation of the probability conservation in the LT approximation at high energies. Recently there was a number of attempts to generalize the Reggeon field theory to pQCD [13–16].

In the present paper we explore the decrease of the pQCD amplitude with the vacuum quantum numbers in the crossed channel with the virtuality of the parton. This observation helps to generalize to pQCD the famous S. Mandelstam-V. Gribov [1,3,4] proof of the cancellation of the contribution of planar diagrams into the total cross section. The eikonal graphs due to the s-channel iteration of the color singlet ladder exchanges form a subset of planar graphs. Hence their contribution is cancelled out.

The complimentary constraints follow from the analysis of the multiparticle cross-sections as determined by the s-channel cuts of the multiladder diagrams. We shall show that the account of the energy-momentum conservation leads to the complimentary explanation of why the contributions of the planar (in particular eikonal) diagrams are zero in the perturbative QCD at high energies.

In order to visualize constraints derived in the paper it is rather convenient to choose the reference frame where the projectile is energetic but the target is at rest.

The qualitative explanation of the cancellation of the eikonal diagrams due to the causality is that the projectile fragments into the number of the particles that is increasing with the energy. These particles have no time to form back an incident particle in the intermediate states due to the Lorenz dilatation [1].

We have mentioned above that the reasoning used in the paper applies only to the iteration of the ladders with the singlet color quantum numbers. In the case of the s-channel iteration of ladders with color octet quantum numbers, the constraints discussed above are inapplicable (see the discussion in Sec. II). The amplitudes with the octet quantum numbers 8_F in the crossed channel do not decrease with the parton virtuality since they have the strongly off shell intermediate state in the corresponding Feynman diagrams relevant for the gluon reggeization in the pQCD. So the contribution of the two gluon exchange in the negative signature is absent in the eikonal approximation where

*Electronic address:blok@physics.technion.ac.il
†Electronic address: frankfur@lev.tau.ac.il

all particles are on mass shell. At the same time eikonal diagrams where "potential" is given by reggeized gluon exchange are not forbidden by constraints discussed in the paper. The reasoning based on the energy-momentum conservation is also inapplicable because the intercept of the reggeized gluon is less than 1 and the amplitudes are predominantly real.

Let us stress here once again that our results apply only to the quantum field theory at sufficiently high energies, but not to the quantum mechanics or the low energy field theory. The main difference between the quantum mechanics and the quantum field theory is in the absence of particle creation. In the quantum mechanics one iterates the predominantly real amplitudes arising due to the one-particle (one-photon, one -gluon) exchange and obtains the solution of the problem; the high energy projectile scattering off the static center. On the contrary, in the high energy quantum field theory one has to iterate not the one-particle exchange diagrams, but the ladders in order to obtain the leading order (LO) and the next to leading order (NLO) terms that are logarithmically increasing with the energy. These terms are predominantly imaginary. It is in this case that the eikonal expansion fails. Formally, the reason why the eikonal expansion fails in the field theory is that the QFT amplitudes decrease with the invariant masses more rapidly than $1/M^2$, contrary to the relativistic quantum mechanics where the potential does not depend on the virtuality.

It follows from the above discussion that the sufficiently energetic projectile parton may undergo at most one inelastic collision, thus strongly constraining the Feynman diagrams relevant for the high energy processes. This observation is particularly important for resolving the challenge with the probability conservation in the small x-processes. In particular, we show that the wave function of the energetic parton relevant for the n-ladder exchange must contain at least n constituents (contrary to the eikonal approximation, where a number of constituents is always one).

The cancellation of the contribution of the eikonal diagrams to the total cross-section has been found also in Refs. [17–20]. These papers were focused on the generalization of the AGK cutting rules [21] to the pQCD and did not analyze the constraints due to the causality and the energy-momentum conservation, in particular, because the LO BFKL approximation does not respect the energy-momentum conservation.

We find that the nonplanar diagrams that take into account the bremstrahlung in the initial state and diffraction in the intermediate states dominate in the impact factors.

The organization of the paper is the following. In Sec. II we explain how the account of the causality leads to the cancellation of the planar (eikonal) diagrams in QCD. In Sec. III we discuss the constraints on the Feynman diagrams due to the energy-momentum conservation. In particular we explain that all the s-channel cuts of the eikonal diagrams are zero. Thus the nonzero contributions to the multiladder exchange of the amplitudes arise entirely due to the nonplanar diagrams for the impact factor (the Mandelstam cross diagrams). In Sec. IV we discuss the generalizations of the Mandelstam cross diagrams. The conclusions are given in the Sec. V. In the appendices we review some known properties of the high energy amplitudes, that are rarely discussed in the literature, in order to make the paper self-contained. In Appendix A we remind the reader of the Mandelstam-Gribov arguments and their derivation in the field theory. In Appendix B we review briefly the definition and the properties of the impact-factors.

II. THE PLANAR DIAGRAMS AND THE CAUSALITY FOR THE HIGH ENERGY PROCESSES

Historically, the eikonal approximation in the theoretical description of the photon(hadron)-nucleus collisions at large energies the eikonal (the Gribov-Glauber) approximation is one of the most successful phenomenological approaches [22–24]. The eikonal approximation gives the legitimate solution of the Schrodinger, Dirac and Klein-Gordon equations describing the interaction of a sufficiently energetic particle with a statical source. However, in the beginning of the sixties, long before the advent of QCD, it had been understood that the exchange of double Pomerons, even with the intercept $\alpha(t = 0) \geq 1$, rapidly decreases with the energy if the impact factors are dominated by the planar (and, in particular, eikonal) diagrams, (see Fig. 1). This result follows from the analytic properties of the amplitude (causality) and decrease of the amplitude with the virtuality of a colliding particle [1,3,4]. Thus the eikonal approximation, while is useful tool in the quantum mechanics fails in the quantum field theory.

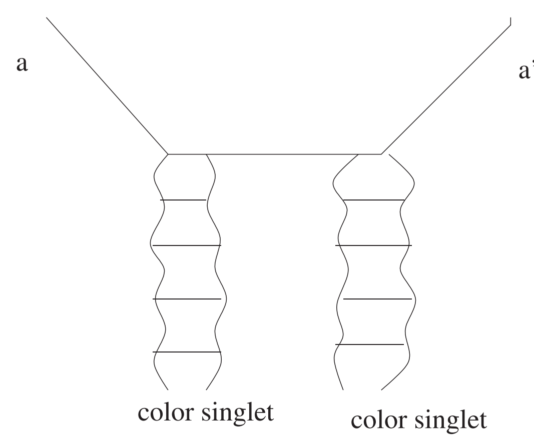

FIG. 1. Eikonal blob for the exchange of the two color singlet ladders in QCD.

Under the influence of the evident phenomenological success of the eikonal approximation in the description of the hadron-nucleus collisions this approximation is often used in gauge theories as an attempt to cure the rapid increase of the amplitudes with the energy and to restore the probability conservation (see e.g. Refs. [25,26] for a review of some recent eikonal-based models in QCD). The standard form of the eikonal models is [2,27–30]

$$\begin{aligned}\sigma_T &= 2\int d^2b(1-\exp(-a(s,b)))\\ \sigma_E &= \int d^2b(1-\exp(-a(s,b)))^2\\ \sigma_{\rm inel} &= \int d^2b(1-\exp(-2a(s,b))),\end{aligned} \tag{2.1}$$

where σ_T, σ_E, $\sigma_{\rm inel}$ are the total, elastic and inelastic cross-sections, respectively, and $a(s, b)$ is an eikonal phase for a given impact parameter $\vec{b}$, calculated perturbatively.

The aim of this section is to show that the eikonal approximation breaks down in QCD for sufficiently high energies. We show that the eikonal iterations of the amplitude with the vacuum quantum numbers in the crossed channel (e.g. the BFKL,DGLAP ladders) rapidly decrease with the energy. We discuss two complimentary reasons for the cancellation of the planar diagrams: one is the generalization to QCD of the Mandelstam [1,3,4] cancellation as the consequence of the causality, another is the impossibility to satisfy the energy-momentum conservation constraints for the planar (eikonal) graphs for the particle production. The dominant contribution is given by the nonplanar diagrams.

The cancellation of the eikonal diagrams was first proved in the ϕ^3 theory and generalized to the reggeon calculus, (see Appendix A for a short review of the proof). The origin of this cancellation is that in a field theory, contrary to the quantum mechanics, there exists the particle creation. As a result, for sufficiently high energies the amplitude is dominated by the exchange of the ladders. The eikonal representation breaks down because a the parton cannot have more than one inelastic scattering (i.e. one attachment to the ladder exchange). More intuitively, the ladder creation means the creation of a large number of particles, while the two ladder iteration in the QFT means these particles after being created, then come together into the same configuration after finite time. This clearly looks implausible.

In this section we somewhat generalize the causality (Mandelstam-Gribov) reasoning explaining such a cancellation.

Let us extend the Mandelstam-Gribov argument to QCD. Our starting point is a two body collision at high energies as given by the single-ladder exchange. Such construction arises in the LO and NLO logarithmic approximations in pQCD. In this case it is legitimate to neglect the longitudinal momentum transfer and the denominators in the propagators in the ladder are $\sim r_t^2$ where r is the momentum transverse to momenta of colliding particles in the line of the ladder (see Fig. 2). Then the calculation leads to the simple form for the collision amplitude A(s,t) [2,31]

$$\begin{aligned}A(s,t) = \int d^2k d^2k' \Phi_1(p_A, k, q-k)\\ \times f(s,k,q-k,k',k'-q)\Phi_2(p_B,k',q-k'),\end{aligned} \tag{2.2}$$

where f depends in pQCD on the s-channel energy squared s, and on the transverse momenta k, $k-q$, k', $k'-q$, $t=-q_t^2$. The $\Phi_{1,2}$ are the impact factors describing the upper and lower blobs in the Feynman diagrams of Fig. 2. These impact factors (see Appendix B for a short review of the impact factor formalism) as the consequence of the dominance of the single gluon polarization in the propagator of the exchanged gluon [1] have the form

$$\begin{aligned}\Phi_1 &= \int dM^2(1/s^2)p_B^\mu p_B^\nu f_{\mu\nu}\\ &= \int dM^2 k_t^\mu (q-k)_t^\nu f_{\mu\nu}/(M^2)^2\end{aligned} \tag{2.3}$$

where M^2 is the square of the mass of the diffractively produced state. In the last equation one uses the Ward identities and the two body kinematics. We use the Sudakov parametrization for the momentum of the exchanged gluon: $k = \alpha p_A + \beta p_B + k_t$. If the function f has the form of the Regge pole exchange

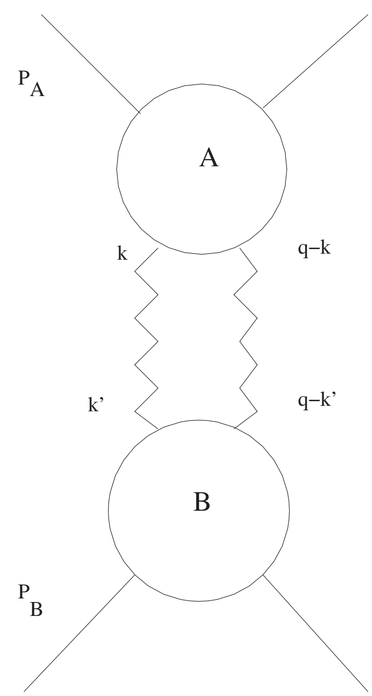

FIG. 2. One-ladder exchange in QCD.

$$f \sim s^{\alpha(-q_t^2)} F(k_t, k_t', q_t), \tag{2.4}$$

the amplitude acquires the form

$$A(s, t) \sim s^{\alpha}(t) G(t). \tag{2.5}$$

In the pQCD the leading singularity is the Regge cut, not a pole as in the ϕ^3 or the Reggeon field theory. Nevertheless, the one ''ladder'' exchange amplitude can be written as

$$A(s, t) = R(t)\kappa(s, t) \tag{2.6}$$

where

$$R(t) = \int ds_{12} \int ds_{34} d^2k_t d^2k_t' \Phi_1(s_{12}, k_t, q_t - k_t) \times F(k_t, k_t', q_t) \Phi_2(s_{34}, k_t', k_t' - q_t). \tag{2.7}$$

Here $\kappa(s, t)$ is the function calculable in pQCD. The dependence on the momenta k_t running in the ladder is factorized in the Particle-Particle-Reggeon (PPR) vertex [12,32].

Let us now proceed to the evaluation of two ladder exchange (Fig. 3). The general expression for the 2-''Reggeon'' exchange amplitude can be obtained similarly

$$A(s, t) = \int d^2r_t d^2k_t d^2r' d^2u d^2u' d\alpha s d\beta s \Phi_1(p_A, r, k, q, u) \times \Phi_2(p_B, r', k', q, u') \times f_1(r, r', k, s_1) f_2(q - k, u, u', s_2). \tag{2.8}$$

Here f_1 and f_2 are the functions corresponding to the two exchanged ladders, s_1 and s_2 are their invariant energy squared in the s-channel. For the eikonal diagrams $s_1 = s_2 = s$ while for the Mandelstam cross $s_1, s_2 \leq s$, and $\sqrt{s_1} + \sqrt{s_2} = \sqrt{s}$ (see below). The vectors r, u, r', u' are the momenta that propagate through these 2 ladders. The impact factors Φ_1, Φ_2 correspond now to the 6-point blobs. We made as above a Sudakov expansion: $u = \alpha_u p_A + \beta_u p_B + u_t$, $r = \alpha_r p_A + \beta_r p_B + r_t$, $k = \alpha_k p_A + \beta_k p_B + k_t$.

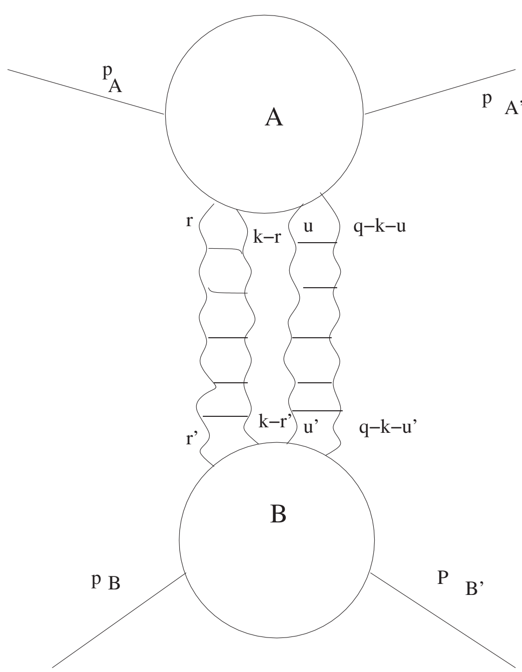

FIG. 3. Two ladder exchange in QCD

The invariant masses are $s_{\beta=1,2} = (p_A + r)^2 = \alpha_r s_1$, $s_{34} = (p_A + u)^2 = \alpha_u s_2$, $(p_A + k)^2 = \alpha s$. In the two body kinematics the integration d^4k factorizes between the upper and the lower blobs, like in ϕ^3 theory. Using $d\alpha = ds_a/s$, $d\beta = ds_b/s$ we obtain that the amplitude of the two-reggeon exchange is given by

$$A(s, t) = \int d^2r_t d^2u_t d^2k_t \int ds_a \Phi_1(p_A, u, r, k, q, s_a) \times \int d^2r' d^2u' d^2k' \int ds_b \Phi_2(p_B, u', r', k', q, s_b) \times f_1(r, r', k, s_1) f_2(q - k, u, u', s_2) \tag{2.9}$$

For high energies the functions f_i have the form of the product of $s_{1,2}$ in some power and the function depending only on the transverse components of the vectors. In particular, the dependence on the invariant masses s_{12}, s_{34}, s_a and s_{12}', s_{34}', s_b is factorized between the blobs like in the case of the ϕ^3 theory. Then we can use the Mandelstam reasoning (see Appendix A). For example, the integral over s_a still has the same analytical properties as the 4-point blob. The corresponding integration contour is depicted in Fig. 4 and is the same for the QCD and for ϕ^3 theory. Then one can deform the contour of the integration into the complex plane due to the absence of the left cut for for arbitrary planar diagrams for whom the spectral density $\rho_{s,u}$ in the Mandelstam representation of the amplitude $\rho_{s,u}$ is zero (in particular for eikonal diagrams). The only remaining point is that in the eikonal diagram we have $1/s_a$ dependence due to a single particle exchange. The additional dependence on $1/s_a \equiv 1/M^2$ of the blob follows from the dependence of the ladder on the invariant mass.

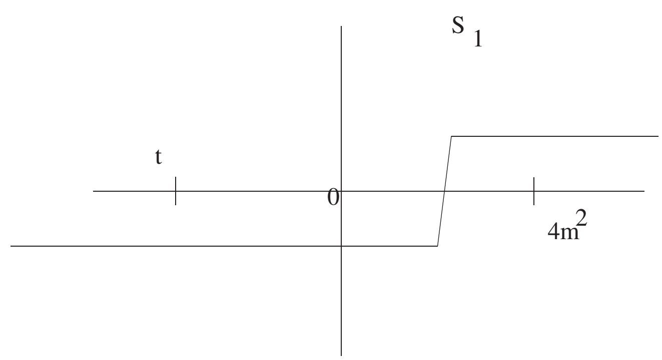

FIG. 4. The integration over the invariant masses. For the QCD case $m = 0$, and the integration contour can cross the x axis also to the right from the origin.

In the case of the Reggeon field theory this dependence is $\sim 1/(M^2)^{\alpha(t}$ with $\alpha(0) \geq 1$ [33], meaning that the impact factor decreases faster then $1/s_a$ as a function of the invariant mass. The similar dependence on the inavariant mass is valid in the perturbative QCD for the exchange of the color singlet ladder.

Indeed, let us consider the color singlet pQCD ladder amplitude for the case of two different invariant masses Q^2 and M^2. This amplitude is the weak function of M^2 in the area $M^2 \sim Q^2$. However, if $M^2 >> Q^2$ this amplitude decreases with M^2. In particular, amplitude of DIS evaluated at small x within the DGLAP approximation is

$$\sim 1/(Q^2 + M^2)^n$$

where $n \sim 3/4$ [34]. Similar behavior is expected for color singlet Generalized Parton Distributions (GPD) within the DGLAP approximation. Remember that at the achievable small x and Q^2 pQCD amplitudes evaluated within the NLO DGLAP and NLO BFKL approximations are rather close [11,12].

In contrast to the pre QCD approaches which conveniently assumed the fast decrease of any amplitudes with M^2, the pQCD gives two different patterns. The amplitude with the color octet quantum numbers in the crossed channel does not depend on M^2 for $s \geq M^2$ at least within the leading logarithmic approximation. This is because the virtuality of the interacting parton in the ladder is $\leq s$. Therefore such an amplitude does not decrease with the virtuality of a parton. Besides the color octet amplitude is predominantly real and decreases with the energy, so the analysis of the multiparticle states through s- channel cuts is unreliable in this case.

The above proof also makes clear why the eikonal expansion is valid in quantum mechanics, as it was mentioned in the introduction. Indeed, for the one-particle exchange, contrary to the ladder exchange, the impact factor decreases like $1/M^2$ at most, and the contour of Fig. 4 cannot be deformed, even if the left cut is absent.

In this paper we, however, are interested in the amplitude with the vacuum quantum numbers, where the virtuality of the interacting parton is $\approx \sqrt{Q}21$. Here Q_1^2 is the order of the maximum between Q^2 and M^2. In this kinematics, $Q^2 \ll M^2$, $Q_1^2 \sim M^2$. Then the amplitude with the vacuum quantum numbers in the crossed channel for the scattering of a parton is approximately proportional to $(1/M^2)S(M^2/Q_1^2)$, where S is the Sudakov form factor. Similar dependence is valid for the amplitude for the scattering of the dipole. Thus we have an additional M^2 dependence for the eikonal diagrams. Such a decrease is sufficient to justify the deformation of the contour of integration in Fig. 4 in the case of the planar diagrams. We have proved that in the multiRegge kinematics the 2-reggeon eikonal exchange amplitude is zero. The same arguments can be used for the multi- eikonal exchanges in the multiregge kinematics.

Note that in the proof it was essential to use Ward identities $k^\mu f_\mu = 0$ for the impact factor. It is known [35] that such form of the Ward identities is valid for the amplitudes where only one gluon is off mass shell, i.e. for a sum of all permutations of the gluon lines in the dipole. We also expect Ward identities to hold when only color singlet exchanges are considered, as in eikonal diagrams (see i.e. Fig. 1).

We can estimate the range of energies where the reasoning discussed in this paper applies. In fact, there are several relevant QCD regimes, depending on the problem under consideration.

In the deep inelastic scattering the one-ladder contribution is dominant in the whole kinematical region of x. The one-ladder (leading twist) contribution breaks down at $x \sim 10^{-5}$ [36], and the multiladder contributions become dominant. The area where the eikonal models fall under scrutiny is somewhere in the middle of this interval, i.e. $x \sim 10^{-3}$, where the multiladder contributions first appear. Our results show that the eikonal contribution should be zero.

III. CONSTRAINTS DUE TO THE ENERGY-MOMENTUM CONSERVATION

In the previous chapter we concluded that the eikonal contribution is zero if all the ladders are color singlets. In this chapter we shall argue that these results can be derived also from the requirement of the energy-momentum conservation. We shall first review the constraints due to energy-momentum conservation in ϕ^3 theory and then extend this reasoning to QCD. We shall see that the constraints due to energy-momentum conservation are sometimes even stronger than the Mandelstam one leading to the cancellation not only of the eikonal diagrams but also of all cut eikonal diagrams.

Let us start first from the ϕ^3 theory, from the eikonal graph of Fig. 5. This graph corresponds to the amplitude of the multiparticle creation resulting from the s-channel cut of the two ladder diagram. Then a total square of the energy of the created particles is $2s$, while the initial energy is s. Thus the initial parton releases in the two consequent scatterings the double of initial energy. Evidently as the consequence of the energy-momentum conservation law the contribution of this graph should be zero.

Note that the nonconservation of the energy by the cut eikonal diagrams is well known for the hadron-nucleus collision for some time [29,37,38]. However, while a prescription has been suggested how to include by hand the energy conservation law into the eikonal diagrams [38], this suggestion has no justification in the perturbation theory (cf. discussion in Ref. [39]).

Let us recall that the s-channel cut diagrams carry additional information as compared to the imaginary part of the total amplitude of the two body scattering. The s-channel cut amplitudes are relevant for the multiparticle cross-sections [21]. Indeed, if the average number of particles

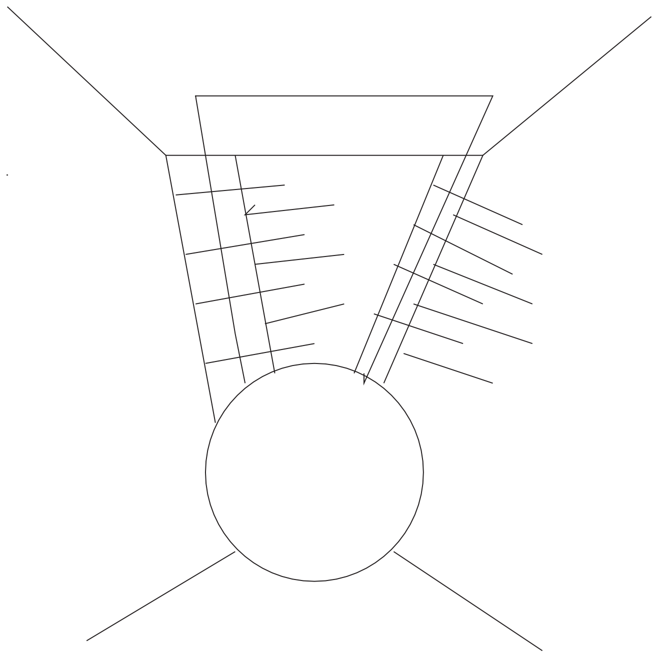

FIG. 5. Cut eikonal graph and energy-momentum conservation.

created in one reggeon exchange is $\sim\bar{n}$, then the contribution of the multiladder exchanges leads to the processes in which the number of particles produced is a multiple of $\bar{n}$. The cross-section of the creation of $n\bar{n}$ particles is dominated by the diagrams with n cut ladder exchanges. In other words the energy conservation law must be fulfilled separately for the diagrams with n cut ladders for each n. However for n cut ladders initiated by planar diagrams the square of invariant energy of the particles created is ns, while the original energy is s, the energy conservation law is violated and all the cut diagrams are zero. The related reasoning is to check that the momentum sum rule is violated [21].

The above reasoning can be directly translated to QCD without any changes since the imaginary parts of the ladders are significantly larger than the real parts in QCD, within both in the BFKL and DGLAP approximations. This means, that the contribution of eikonal diagrams with the NLO BFKL and DGLAP ladders is zero.

Let us note here that the energy-momentum conservation prohibits radiation of more than one inelastic ladder by a single parton. On the other hand the parton can have an arbitrary number of elastic rescattering on the target, since such rescatterings do not change the energy of the energetic parton.

IV. THE MANDELSTAM CROSS

We have explained that the eikonal iteration of the ladders gives zero in QCD. The obvious question is what is the dominant contribution. The simplest diagram that gives nonzero 2-ladder contribution is Mandelstam cross diagram of Fig. 6 [4]. The contribution of this cut can be easily derived in ϕ^3 theory, one just takes into account that the energy is split between the two ladders. One has [1]

$$A(s,t) = (i/4)\int d^2k_t N^2_{\gamma\gamma_1}(k_t,q_t)\xi_\gamma\xi_\gamma s^{\gamma+\gamma_1-1}. \quad (4.1)$$

Here the squared energy parameters of the ladders are different for 2 ladders

$$s_1 = \alpha s, \qquad s_2 = -s\beta \quad (4.2)$$

and $\xi_\gamma = -(\exp(-i\pi\gamma + 1)/\sin(\pi\gamma)$,

$$N_{\gamma\gamma_1} = \int ds_1 d^2k_t g_1 g_1^1 \beta_1^\gamma (1-\beta_1)^{\gamma_1}/(.....). \quad (4.3)$$

In the same way one can easily derive the contribution of the Mandelstam cross in the case of QCD. The answer is

$$A(s,t) = (i/4)\int \frac{d^2k_t}{(2\pi)^2}\int \frac{d^2u_t}{(2\pi)^2}\Phi_1(s,\alpha,\beta,k_t,u_t) \times \Phi_2(s,\alpha,\beta,k_t,u_t) f_1(\alpha s,k_t) f_2(\beta s,u_t) \times \delta(\alpha+\beta-1). \quad (4.4)$$

Here $\alpha + \beta = 1$, and the energy conservation law is fulfilled automatically. Evidently the contribution of the Mandelstam cross is nonzero. Moreover, one cannot add additional ladders to Mandelstam cross without increasing the number of constituents in the s-channel. Indeed, consider the diagrams of Fig. 7. It is easy to show using the arguments of the previous chapters that all these diagrams are equal to zero. The simplest nonzero diagrams with the three ladder exchange are the so called nested diagrams

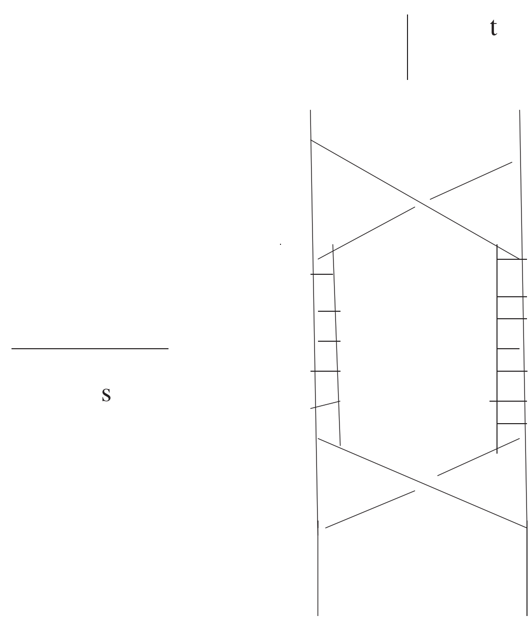

FIG. 6. The Mandelstam cut diagram.

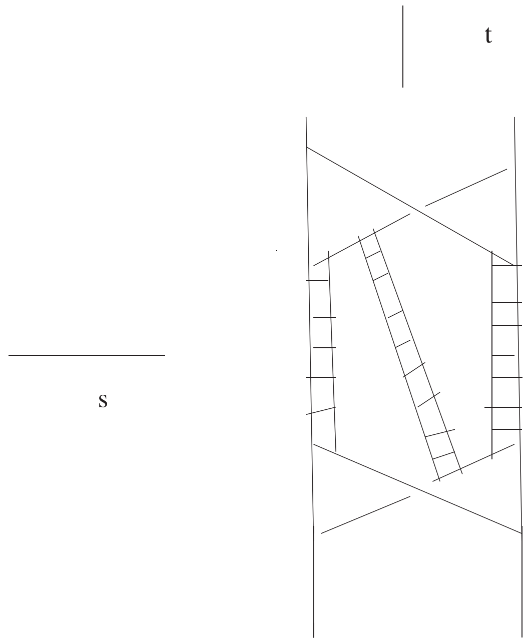

FIG. 7. Adding ladder to Mandelstam cut diagram.

[40]. These diagrams have 3 constituents in the intermediate state.

We conclude, that at high energies there is no universal impact factor that can be iterated. For n-ladder diagram to be nonzero, one needs to have at least n constituents in the wave function of the energetic projectile in the s-channel, i.e. not more than one ladder can be attached to a given line.

V. CONCLUSION

We have shown that the cancellation of the planar (in particular eikonal) diagrams found within the Reggeon Calculus by S. Mandelstam is valid in QCD also for the s-channel iterations of the color singlet ladder exchange. As a consequence the restriction by eikonal diagrams leads to the decrease with the energy of the shadowing effects which is the artifact of the eikonal approximation. The account of the energy-momentum constraints leads to the same conclusion. Moreover, the application of these constraints to the analysis of the iteration in the s-channel of the amplitudes evaluated within DGLAP approximation shows that such iterations are also decreasing as powers of energy. To obtain nonzero exchange by n ladders, incident parton should develop configuration of n constituents long before the collision. In other words, any given constituent can participate in the ladder exchange at most once.

The challenging question is how to take into account the nonplanar graphs, in particular, those generated by the Mandelstam crosses. At sufficiently large energies we showed that the number of exchanged ladders and therefore the number of constituents in the wave function of the incident particles is increasing with the energy, cf. Ref. [5]. The generalization of Gribov Reggeon Calculus [1] to the pQCD regime of strong interaction with the small coupling constant may lead to the solution of this problem.

Our considerations show that the application of the eikonal (Gribov-Glauber) approximation for the dipole interactions with a target have problems with causality and energy-momentum conservation. As a result one needs a different approximation in the reggeonlike approaches to the behavior of the QCD, when it approaches the black disk limit and the LT approximation breaks down.

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We thank L. Lipatov for the discussion of properties of amplitudes with negative signature (color octets)

APPENDIX A: THE EIKONAL DIAGRAMS CANCELLATION IN THE ϕ^3 THEORY

In this Appendix we briefly review the Mandelstam - Gribov explanation of the cancellation of the planar (eikonal) graphs in the ϕ^3 theory. We shall follow the very transparent derivation of this cancellation given in the Gribov's lectures [1], since such derivation can be directly generalized to QCD.

Let us write the expression for Fig. 8 , that describes the scattering in the ϕ^3 theory due to the exchange of two particles/reggeons in the t-channel. The blob A may be, for example, the diagram that corresponds to the eikonal interaction, a planar box diagram, or a Mandelstam cross. Suppose that as a function of the invariant mass $s_1 = (p_1 + k)^2$ the blob A decreases faster than s_1. Let us use the Sudakov variables

$$k = \alpha_q p_2' + \beta_{p_1'} + k_t, \tag{A1}$$

$$t = q^2 = q_t^2 + s\alpha_q\beta_q \sim q_t^2, \tag{A2}$$

$$s_1 = (p_1 + k)^2 \sim \alpha s, \qquad s_2 = (p_2 - k)^2 = -\beta s. \tag{A3}$$

Since the amplitudes $A(s_i)$ fall rapidly with increasing s_i, the essential values of $s_{1,2}$ are of the order of μ^2 where μ is the mass of the particles. Then $\alpha, \beta \sim s$, and for the product of particle propagators in Fig. 8 we have

$$\frac{1}{k^2 - m^2}\frac{1}{(q-k)^2 - m^2} = 1/(\alpha\beta s + k_t^2 - m^2)1/((\alpha - \alpha_q)(\beta - \beta_q)s + (q-k)_t^2 - m^2) = 1/(m^2 - k_t^2)1/(m^2 - (q-k)_t^2). \tag{A4}$$

The full amplitude B corresponding to Fig. 8 can be rewritten as

$$B = (i/4s)\int (d^2k_t)/(2\pi)^2 1/((m^2 - k_t^2)(m^2 - (k-q)_t^2)\int_\Gamma ds_1/(2\pi i)A(s_1)\int_\Gamma ds_2/(2\pi i)A(s_1). \tag{A5}$$

The integration contour is given in Fig. 4. It has evidently a cut in the s-channel starting from the mass of the first intermediary 2-particle state $s = (2m)^2 = 4m^2$. In the negative axis the cut in the s_1 plane starts from t. Since the function A as a function of s_1 falls rapidly, we can deform the contour. The integral will actually be zero if the left cut is absent. Indeed, in this case we can close the integral to the left cut, and it will be zero. As it is well known, the left cut corresponds to the nonzero Mandelstam double spectral density $\rho_{su}(s, t)$, i.e.

$$\mathrm{ImA} = (1/\pi)\int \rho_{su}(s', t)ds'/(s' - s). \tag{A6}$$

It is clear that the diagram that corresponds to the eikonal has no double spectral density—it is a tree diagram. In the same way the planar diagrams have no double spectral density ρ_{su}. The simplest diagram with the nonzero spectral density that contributes to the 2-particle exchange is a Mandelstam cross [4] of Fig. 6.

In quantum mechanics, or in the quantum field theory for not so high energies, when the dominant contribution to scattering comes from single particle exchange the blobs do not decrease with s_i. So it is impossible to deform the contour of integration to ∞. This is why eikonal approximation is applicable in the framework of the quantum mechanics.

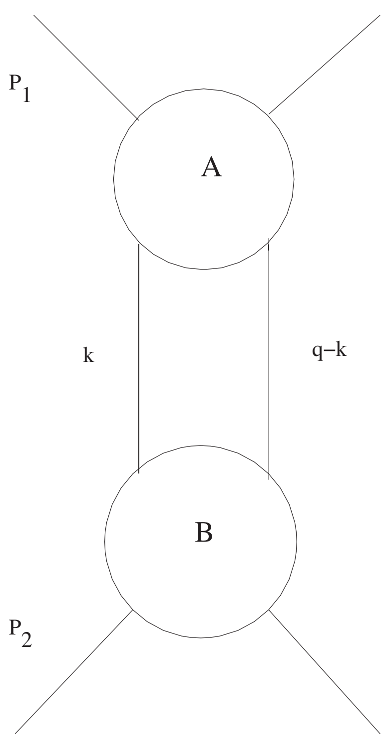

FIG. 8. Two particle and two-reggeon exchange in s-chanel in the ϕ^3 theory.

Let us now consider what happens when the dominant contribution to the scattering amplitude comes from ladder exchange. For the case of the ϕ^3 theory S. Mandelstam and V. Gribov substituted the one-particle exchange by the reggeon. The key is that the reggeon form factor has additional dependence on s_1, as it was proved for this theory by Mandelstam [3], and this dependence is $1/s_1$. After the substitution of the particles by the ladders, due to the invariant mass decrease of the reggeon form factors, the blob amplitude decreases now faster than $1/s_1$ and the eikonal graph (as well as all planar diagrams for which $\rho_{su} = 0$) is zero. The two ladder contribution decreases with the energy, contrary to the naive expectation that it rises as $s^{2\alpha(t)}$, where $s^\alpha(t)$ corresponds to the single-ladder exchange. We conclude that the planar (eikonal) contribution is decreasing as a function of energy for ϕ^3 theory.

APPENDIX B: THE IMPACT FACTORS

In this Appendix we shall briefly remind the reader the definition and the properties of impact-factors. The relevant formalism was developed by Cheng and Wu (see Ref. [2] and references therein and by Gribov, Lipatov and Frolov, [31], who studied the asymptotic behavior of the diagram of Fig. 8 in QED. The main result of Refs. [2,31] relevant for us is that the scattering amplitude with the exchange of vector particles of Fig. 8 also factorizes into the product of denominators of propagators in the intermediate states and two factors (so called impact-factors) that dependent only on the left and right blobs in the diagram separately. For completeness let us summarize here their beautiful proof. Indeed, let us once again use the Sudakov expansion for k. Suppose the blocks that correspond to left and right blobs are $f_1\mu_1\mu 2$ and $f_{2\nu_1\nu_2}$. Suppose also that these blocks do not increase as functions of invariant masses $s_1 = (p_1 - k)^2$, $(p_2 + k)^2 = s_2$. Then the relevant areas of integration are

$$s_1 \sim -\alpha s + k_t^2 \sim m^2, \qquad (p_2 + k)^2 \sim s\beta + k_t^2 \sim m^2, \tag{B1}$$

then for the photon propagator denominators we see that they are equal to $k_t^2, (q-k)_t^2$. Since the calculation is gauge invariant, we can use the Feynman gauge and then the amplitude has the factorized form

$$B = -i(1/(2\pi)^4)(1/2) \times \int d^2k_t \delta_{\mu_1\nu_1}\delta_{\mu_2\nu_2}\phi_{1\mu_1\mu_2}(k_t, p_1)\phi_{2,\nu_1\nu_2}(k_t, p_2) \tag{B2}$$

where

$$\phi_{\mu_1\mu_2} = \int_{-\infty}^{\infty} f^{1,2}_{\nu_1\nu_2}(s_{1,2}, k_t, Q). \tag{B3}$$

Here $s_1 = -s\alpha$, $s_2 = s\beta$. Let us rewrite the Eq. (B2) in a more convenient form. In order to do it, we can use the observation by Gribov, that for high energies the main contribution comes from the so called nonsense asymptotic states. The nonsense state of two virtual photons is a state where the total spin projection in the direction of motion in the center of mass (c.m.s.) reference frame of the t-channel is equal to two. In the physical region of the s-channel it is possible to prove (see Ref. [31] for details) that for the light cone components of the vector polarization of a photon are

$$e^- = p_1\sqrt{2/s}, \qquad e^+ = p_2\sqrt{2/s}. \tag{B4}$$

For each of the photons the propagator can be written as

$$\delta_{\mu\nu} - k_\mu k_\nu/k^2) = e^+_\mu e^-_\nu + e^+_\nu e^-_\nu + e^0_\mu e^0_\nu \tag{B5}$$

where e^0 is the longitudinal polarization vector of the photon in the t-channel,

$$e^0 = (1/\sqrt{k^2})(0, 0, |\vec{k}|, k_0). \tag{B6}$$

Then in the relevant integration area all of the external invariants of blocks 1 and 2 are of order m^2, i.e. $\beta_i \sim 1$, $\alpha_i \sim m^2/s$, $k^i_t \sim m$ for block 1 and $\alpha_i \sim 1, \beta_i \sim m^2/s, k^i_t \sim m$ for block 2. Hence all virtual momenta in block 1 have long components along the vector p_1 in the block 1 and those in 2 along the vector p_2. Therefore the largest contribution proportional to s in the equation for propagator will be given by the term $e^+_\mu \times e^-_\nu$ as compared to the term $e^0_\nu e^0_\mu \sim 1$, and $e^+_\nu e^-_\mu \sim 1/s$

$$\delta_{\mu\nu} - k_\mu k_\nu/k^2) = e^+_\mu e^-_\nu \sim (2/s)p_{2\mu}p_{1\nu}. \tag{B7}$$

We then obtain the explicit dependence of the amplitude B of Fig. 3 on s in the almost factorized form

$$F = -i(s/4)\int d^2k_t \frac{1}{k^2_t - \lambda^2} \times \frac{1}{(q-k)^2_t - \lambda^2}\Phi_1(k_t, Q)\Phi_2(k_t, Q) \tag{B8}$$

where

$$\Phi_1(k_t, q) = (2/s^2)\int_{-\infty}^{\infty}(ds_1/(2\pi))f^1_{\mu_1\mu_2}(s_1, k_t, q)p_{2\mu_1}p_{2\mu_2}$$
$$\Phi_2(k_t, q) = (2/s^2)\int_{-\infty}^{\infty}(ds_2/(2\pi))f^2_{\mu_1\mu_2}(s_1, k_t, q) \times p_{2\mu_1}p_{2\mu_2} \tag{B9}$$

We reproduced, in order to be self-contained, the first step of the GFL derivation. In fact, it is straightforward to obtain Eq. (B9) even simpler, just from the Gribov analysis of the vector particle exchange for high energies with the vertex $\Gamma_\mu(p, q)$. Then it is shown in Ref. [1] that the dominant contribution to the amplitude is due to the nonsense state and then the diagram can be written as a product of s-independent vertices, scalar particle propagator and s. From this we can straightforward obtain the result (B9). Now we can use the Ward identities

$$k_{\mu_1}f^{\mu_1\mu_2}_{1,2} = k_{\mu_1}f^{\mu_1\mu_2}_{1,2}. \tag{B10}$$

These Ward identities can be rewritten as

$$(\alpha p_2 + k_t)_{\mu_1}f^1_{\mu_1\mu_2} = (-\alpha p_2 + q - k_t)_{\mu_2}f^1_{\mu_1\mu_2} = 0$$
$$(\beta p_1 + k_t)_{\nu_1}f^2_{\nu_1\nu_2} = (-\beta p_2 + q - k_t)_{\nu_2}f^2_{\nu_1\nu_2} = 0. \tag{B11}$$

Accordingly, we can rewrite the expression for the amplitude of Fig. 8 with the exchange of two vector particles in a fully factorized form (B7), with the impact factors being

$$\Phi_1(k_t, Q) = (2/s^2)\int_{-\infty}^{\infty}(ds_1/(2\pi))f^1_{\mu_1\mu_2}(s_1, k_t, Q) \times k_{1\mu_1}(Q_t - k_{2t\mu_2 1})$$
$$\Phi_2(k_t, Q) = (2/s^2)\int_{-\infty}^{\infty}(ds_2/(2\pi))f^2_{\nu_1\nu_2}(s_1, k_t, Q) \times k_{2\nu_1}(Q-k)_{2t\nu_2}. \tag{B12}$$

We have the amplitude for the exchange of vector particles and for external particles with arbitrary spins, in the factorized form, like in the above treatment of ϕ^3 theory. One of the reasons we reproduced here the main points of [31], was to show that the proof is extended without any changes into pQCD. Indeed, all the points in the proof can be directly transferred to QCD except two. First, the Ward identities that were used in QED are not the same as in QCD. In order for Ward identities in QCD to become of practical use at most one external line must be out of mass surface-the condition fulfilled in the present case. Really all particles within ladder are on mass shell in the case of the amplitude of the positive signature. Therefore the cross section is determined by the amplitudes where only one gluon is off the mass shell.

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