

# Corrections to tribimaximal neutrino mixing: Renormalization and Planck scale effects

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We study corrections to tribimaximal (TBM) neutrino mixing from renormalization group (RG) running and from Planck scale effects. We show that while the RG effects are negligible in the standard model (SM), for quasidegenerate neutrinos and large  $\tan\beta$  in the minimal supersymmetric standard model (MSSM) all three mixing angles may change significantly. In both these cases, the direction of the modification of  $\theta_{12}$  is fixed, while that of  $\theta_{23}$  is determined by the neutrino mass ordering. The Planck scale effects can also change  $\theta_{12}$  up to a few degrees in either direction for quasidegenerate neutrinos. These effects may dominate over the RG effects in the SM, and in the MSSM with small  $\tan\beta$ . The usual constraints on neutrino masses, Majorana phases or  $\tan\beta$  stemming from RG running arguments can then be relaxed. We quantify the extent of Planck scale effects on the mixing angles in terms of “mismatch phases” which break the symmetries leading to TBM. In particular, we show that when the mismatch phases vanish, the mixing angles are not affected in spite of the Planck scale contribution. Similar statements may be made for  $\mu$ - $\tau$  symmetric mass matrices.

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## I. INTRODUCTION

Neutrino mixing [1] is a consequence of a nontrivial structure of the neutrino mass matrix. This mass matrix is generated by the following dimension five operator:

$$\mathcal{L}_5 = \frac{\mu_{\alpha\beta}}{\Lambda} (L_\alpha \Phi)(L_\beta \Phi) + \text{H.c.} \quad (1)$$

Here  $\mu_{\alpha\beta}$  is a coupling matrix,  $\Lambda$  some high energy scale,  $L_\alpha$  is the lepton doublet with  $\alpha \in \{e, \mu, \tau\}$ , and  $\Phi$  is the Higgs doublet [in the minimal supersymmetric standard model (MSSM),  $\Phi$  is the Higgs doublet that couples to the up-type fermions]. After the electroweak symmetry breaking ( $\langle\langle\Phi\rangle\rangle \equiv v/\sqrt{2} \simeq 174$  GeV for the SM,  $\langle\Phi\rangle \equiv v \sin\beta/\sqrt{2}$  for the MSSM), Eq. (1) gives rise to the neutrino mass matrix

$$(m_\nu)_{\alpha\beta} = \mu_{\alpha\beta} \frac{\langle\Phi\rangle^2}{\Lambda} = (U^* m_\nu^{\text{diag}} U^\dagger)_{\alpha\beta}, \quad (2)$$

where  $U$  is the leptonic mixing, or Pontecorvo-Maki-Nakagawa-Sakata (PMNS), matrix in the basis in which the charged lepton mass matrix is real and diagonal. The neutrino masses are contained in  $m_\nu^{\text{diag}} = \text{diag}(m_1, m_2, m_3)$ . The value of  $\Lambda$  may be taken to be the seesaw scale,  $\Lambda \sim 10^{12}$  GeV, where the underlying flavor structure of the theory is implemented. However, measurements take place at low scale, therefore the predictions of any neutrino mass model have to be evolved down to low energy through renormalization group (RG) running [2,3].

As the large majority of models is generated at a high energy, RG effects are a generic feature.

Another guaranteed correction to the dimension five operator in Eq. (1) is an additional operator of the same form but with  $\Lambda$  identified as the Planck mass  $M_{\text{Pl}} = 1.2 \times 10^{19}$  GeV. Since gravity does not distinguish between flavors, the operator is expected to be flavor democratic. The presence of such a term in the Lagrangian gives rise to additional contributions to the neutrino mass matrix after the electroweak symmetry is broken. While too small to be responsible for the leading structures of the neutrino mass matrix, those Planck scale effects can have observable consequences as well [4–6]. In the present paper we perform a comparative study of both renormalization and Planck scale effects.

For this analysis we will choose one particular and very interesting neutrino mixing scheme which is compatible with all data, the tribimaximal mixing (TBM), defined by [7]

$$U = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}) \times \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix} \text{diag}(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2}). \quad (3)$$

Here  $P \equiv \text{diag}(e^{-i\alpha_1/2}, e^{-i\alpha_2/2}, e^{-i\alpha_3/2})$  contains the Majorana phases, one of which can be rephased away. We further have an additional phase matrix  $Q \equiv \text{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ , which is usually phased away by a redefinition of the charged lepton fields in order to bring  $U$  in the standard form. Because of this, the entries in  $Q$  have

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received the (possibly unfair) title “unphysical phases.” However a complete theory of neutrino masses, especially like the one we are considering here which has more than one independent operator contributing to the neutrino mass, must be able to predict these phases. In the basis in which the flavor democratic Planck scale contribution to the neutrino mass matrix is completely real, the phases  $\phi_i$  of the TBM matrix above need not vanish. We call the phases  $\phi_i$  in this basis as the “mismatch phases” to stay clear of the connotation of the word “unphysical.”

TBM does by itself neither predict the Majorana or the unphysical phases nor the magnitudes or ordering of the neutrino masses. However, the magnitudes and ordering of the masses as well as the values of the Majorana phases are crucial for the size of renormalization group corrections [8–11] and the Planck scale corrections to this mixing scheme. Moreover, as we shall see in the course of the paper, in the analysis of Planck scale effects on tribimaximal mixing even the mismatch phases turn out to be crucial to modify any mixing angle. This is a feature specific to TBM, having to do with the structure of the corresponding neutrino mass matrix. In general, for a  $\mu$ – $\tau$  symmetric mass matrix [12], corrections to  $\theta_{13} = 0$  and  $\theta_{23} = \pi/4$  require mismatch phases to take nontrivial values.

Several interesting models giving rise to Eq. (3) have been proposed in the literature [13]. The predictions of TBM,  $\sin\theta_{13} = 0 = \cos 2\theta_{23}$  and  $\sin^2\theta_{12} = 1/3$ , may be compared with the current  $3\sigma$  ranges of the parameters [14]:

$$0.24 \leq \sin^2\theta_{12} \leq 0.40, \quad -0.36 \leq \cos 2\theta_{23} \leq 0.32, \\ |\sin\theta_{13}| \equiv |U_{e3}| \leq 0.20, \quad (4)$$

with best-fit values of 0.31 for  $\sin^2\theta_{12}$  and zero for  $\sin\theta_{13}$  and  $\cos 2\theta_{23}$ . As far as the neutrino masses are concerned, oscillations experiments are sensitive only to the mass-squared differences, which are measured to be

$$7.1 \times 10^{-5} \text{ eV}^2 \leq \Delta m_{\odot}^2 \equiv m_2^2 - m_1^2 \leq 8.9 \times 10^{-5} \text{ eV}^2, \\ 1.9 \times 10^{-3} \text{ eV}^2 \leq \Delta m_{\text{A}}^2 \equiv |m_3^2 - m_2^2| \leq 3.2 \times 10^{-3} \text{ eV}^2, \quad (5)$$

with the best-fit values of  $\Delta m_{\odot}^2 = 7.9 \times 10^{-5} \text{ eV}^2$  and  $\Delta m_{\text{A}}^2 = 2.5 \times 10^{-3} \text{ eV}^2$ . Cosmological observations give an upper limit on the neutrino mass of  $\lesssim 0.5 \text{ eV}$  [15], which is stronger than the limits obtained by direct searches,  $\leq 2.3 \text{ eV}$  [16]. It is still unknown whether neutrinos enjoy a normal hierarchy (NH:  $m_3^2 \approx \Delta m_{\text{A}}^2 \gg m_2^2 \approx \Delta m_{\odot}^2 \gg m_1^2$ ), an inverted hierarchy (IH:  $m_2^2 \approx m_1^2 \approx \Delta m_{\text{A}}^2 \gg m_3^2$ ) or are quasidegenerate (QD:  $m_3^2 \approx m_2^2 \approx m_1^2 \approx m_0^2 \gg \Delta m_{\text{A}}^2$ ).

Many future planned/proposed experiments are geared towards improving the precision of the mixing angles and mass-squared differences. Since this paper deals with the

deviations of the mixing angles from the presently favored TBM scenario, we list some of the future experimental proposals which can be particularly suitable for this purpose. The precision of  $\sin^2\theta_{12}$  can be considerably improved by a dedicated reactor neutrino experiment situated at  $\sim 60 \text{ km}$  corresponding to the survival probability minimum of  $\bar{\nu}_e$  [17–19]. For instance it was shown in [19] that with a statistics of  $\sim 60 \text{ GW kiloton year}$  and a systematic error of 2%,  $\sin^2\theta_{12}$  can be measured to within  $\sim 5\%$  at  $3\sigma$ . The parameter  $\sin^2\theta_{23}$  can be determined with an accuracy of  $\sim 10\%$  at  $3\sigma$  depending on the true value of  $\sin^2\theta_{23}$  in the T2K and NOVA experiments [20,21]. In what regards  $\theta_{13}$ , the Double Chooz experiment [22] (see also [20]) will improve the  $3\sigma$  limit on  $|U_{e3}|$  from its current value 0.04 to 0.01 (0.006) after 2 (6) years of data taking.

The outline of the paper is as follows: we start by discussing the RG running of TBM analytically in Sec. II. In Sec. III we then turn to Planck scale effects and point out, in particular, the importance of the mismatch phases. Both types of corrections are discussed as functions of the neutrino mass and the type of ordering. A numerical analysis and comparison of both effects is performed in Sec. IV. Section V summarizes our findings.

## II. RENORMALIZATION EFFECTS ON TRIBIMAXIMAL MIXING

In this section we will study the effect of renormalization group running of the mixing angles when at high scale they correspond to the tribimaximal mixing.

RG effects on tribimaximal mixing have been studied before in Refs. [23–26], but an analysis involving all possible mass values and schemes is still lacking. We focus here mostly on the *maximal* RG effects as a function of the neutrino mass and will not conduct a detailed analysis of the influence of the Majorana phases on the running. This will be performed in a separate work [27].

By inserting the neutrino mixing matrix from Eq. (3) in the definition of  $m_\nu = U^* m_\nu^{\text{diag}} U^\dagger$ , one finds

$$m_\nu = \begin{pmatrix} Ae^{2i\phi_1} & Be^{i(\phi_1+\phi_2)} & Be^{i(\phi_1+\phi_3)} \\ \cdot & \frac{1}{2}(A+B+D)e^{2i\phi_2} & \frac{1}{2}(A+B-D)e^{i(\phi_2+\phi_3)} \\ \cdot & \cdot & \frac{1}{2}(A+B+D)e^{2i\phi_3} \end{pmatrix}. \quad (6)$$

This is the most general mass matrix generating tribimaximal mixing. The parameters  $A$ ,  $B$  and  $D$  are given by

$$A = \frac{1}{3}(2m_1 e^{i\alpha_1} + m_2 e^{i\alpha_2}), \quad B = \frac{1}{3}(m_2 e^{i\alpha_2} - m_1 e^{i\alpha_1}), \\ D = m_3 e^{i\alpha_3}. \quad (7)$$

The neutrino mass matrix is subject to RG evolution. The running of this matrix may be described via [3]

$$m_\nu \rightarrow I_K I_\kappa m_\nu I_\kappa, \quad (8)$$

where  $I_K$  is a flavor independent factor arising from the gauge interactions and fermion-antifermion loops. This factor does not influence the mixing angles at all. The diagonal matrix  $I_\kappa$  is given by

$$I_\kappa = \text{diag}(e^{-\Delta_e}, e^{-\Delta_\mu}, e^{-\Delta_\tau}) \simeq \text{diag}(1, 1, 1 - \Delta_\tau), \quad (9)$$

where

$$\Delta_\tau \equiv \frac{m_\tau^2}{8\pi^2 v^2} (1 + \tan^2 \beta) \ln \frac{\Lambda}{\lambda}. \quad (10)$$

Here we have neglected the electron and muon mass (so that  $\Delta_e = \Delta_\mu = 0$ ) and used  $e^{-\Delta_\tau} \simeq 1 - \Delta_\tau$ , since  $\Delta_\tau \ll 1$ . For instance, for  $\tan \beta = 20$  and  $\Lambda/\lambda = 10^9$ , one has  $\Delta_\tau \simeq 0.0054$ . The results for the SM can be obtained by replacing the factor  $(1 + \tan^2 \beta)$  with  $(-3/2)$ .

Note that the RG effects correspond to multiplying every entry of the neutrino mass matrix with a real number. Consequently, the overall phase of the entries is not affected by the corrections. The corresponding ‘‘unphysical phases’’  $\phi_1$ ,  $\phi_2$  and  $\phi_3$  can therefore be rephased away by a redefinition of the charged lepton fields. Hence, for the analysis of RG effects it suffices to consider the mass matrix

$$m_\nu = \begin{pmatrix} A & B & B \\ \cdot & \frac{1}{2}(A + B + D) & \frac{1}{2}(A + B - D) \\ \cdot & \cdot & \frac{1}{2}(A + B + D) \end{pmatrix}. \quad (11)$$

Obviously, if the correction to the neutrino mass matrix is additive and not multiplicative, then the unphysical phases will play a role. We will show in the analysis of Planck scale effects to be presented in the next section that this indeed is the case.

Returning to the RG effects, the mass matrix in (11) is generated by some mechanism at a high scale  $\Lambda$ , which may be taken to be the typical seesaw scale  $\Lambda \sim 10^{12}$  GeV. If the mechanism involves right-handed neutrinos, some of them are expected to have masses above  $\Lambda$ , and we have to assume that the threshold effects [28] do not spoil the TBM relations till  $\Lambda$  (note that additional unknown parameters, namely, the entries of the Dirac neutrino mass matrix would enter the analysis). This will be a valid assumption, for example, when the heavy right-handed neutrinos are exactly degenerate. Anyway, the predictions of this mechanism will be modified by the RG evolution to the low energy scale  $\lambda$  at which measurements take place. We take  $\lambda = 10^3$  GeV, the typical scale of supersymmetry breaking. Note that the dependence on the actual values of the scales  $\Lambda$  and  $\lambda$  is only logarithmic, so our results are rather insensitive to the exact choice of scales. If the threshold effects indeed are sizable, our results can be considered to be only conservative estimates.

In order to study the effect of the RG corrections on the mixing angles, we employ the strategy presented in [11] to diagonalize the RG-corrected mass matrix and obtain simple expressions for the evolved mixing angles

$$\theta_{ij} \simeq \theta_{ij}^0 + k_{ij} \Delta_\tau + \mathcal{O}(\Delta_\tau^2), \quad (12)$$

where  $\theta_{12}^0 = \arcsin \sqrt{1/3}$ ,  $\theta_{13}^0 = 0$  and  $\theta_{23}^0 = -\pi/4$ . We only quote the result [27]:

$$\begin{aligned} k_{12} &= \frac{1}{3\sqrt{2}} \frac{|m_1 + m_2 e^{i\alpha_2}|^2}{\Delta m_\odot^2}, \\ k_{23} &= -\left( \frac{1}{3} \frac{|m_2 + m_3 e^{i(\alpha_3 - \alpha_2)}|^2}{m_3^2 - m_2^2} + \frac{1}{6} \frac{|m_1 + m_3 e^{i\alpha_3}|^2}{m_3^2 - m_1^2} \right), \\ k_{13} &= -\frac{1}{3\sqrt{2}} \left( \frac{|m_2 + m_3 e^{i(\alpha_3 - \alpha_2)}|^2}{m_3^2 - m_2^2} - \frac{|m_1 + m_3 e^{i\alpha_3}|^2}{m_3^2 - m_1^2} \right). \end{aligned} \quad (13)$$

Using the additional rephasing freedom we have set  $\alpha_1 = 0$ . Note that as long as the RG evolutions of the angles are much less than  $\mathcal{O}(1)$ , the difference in all the  $m_i/m_j$  ratios at the low and high scales is only  $\mathcal{O}(\Delta_\tau)$  [11]. Therefore for  $\mathcal{O}(\Delta_\tau)$  estimates, one may use the  $m_i$  values at the low scale. In the numerical calculations we have performed, the RG evolution of the  $m_i$  [8] are also taken into account.

It may be noted that for  $k_{12}$  there is no dependence on the third mass eigenstate or its Majorana phase  $\alpha_3$ . As expected,  $k_{13}$  and  $k_{23}$  are governed by the inverse of  $\Delta m_\oplus^2$  while  $k_{12}$  depends on  $1/\Delta m_\odot^2$ , which renders it typically larger. As  $k_{12}$  is always positive, the solar neutrino mixing angle always increases in the MSSM and decreases in the SM. In contrast,  $k_{23}$  is negative (positive) for a normal (inverted) mass ordering. Therefore  $|\theta_{23}|$  increases in the MSSM and decreases in the SM if the neutrino mass ordering is normal. If the ordering is inverted,  $|\theta_{23}|$  decreases in the MSSM and increases in the SM. From  $\theta_{ij} = \theta_{ij}^0 + k_{ij} \Delta_\tau$  it follows for  $|k_{ij} \Delta_\tau| \ll 1$  that

$$\begin{aligned} \sin \theta_{13} &\simeq k_{13} \Delta_\tau, & \cos 2\theta_{23} &\simeq 2k_{23} \Delta_\tau, \\ \sin^2 \theta_{12} - \frac{1}{3} &\simeq \frac{2\sqrt{2}}{3} k_{12} \Delta_\tau. \end{aligned} \quad (14)$$

In the next subsections we will discuss from these expressions the features of the running for the three main types of neutrino mass spectra: normal (NH) and inverted (IH) hierarchy, and quasidegeneracy (QD). In order to give numerical estimates of the extent of RG effects, we use the best-fit values of  $\Delta m_\odot^2 = 7.9 \times 10^{-5}$  eV<sup>2</sup>,  $\Delta m_\oplus^2 = 2.5 \cdot 10^{-3}$  eV<sup>2</sup>, and choose the phases such that the RG effect is maximized. Then we further choose  $\tan \beta = 20$  for illustration. The results for the SM can be obtained by replacing the factor  $(1 + \tan^2 \beta)$  with  $(-3/2)$ . The outcome of a full numerical analysis is plotted in Figs. 1–3: as we shall see, they are nicely reproduced by the analytical estimates to be presented below.

### A. Normal hierarchy

We start the estimates with the normal hierarchy. Defining the notation

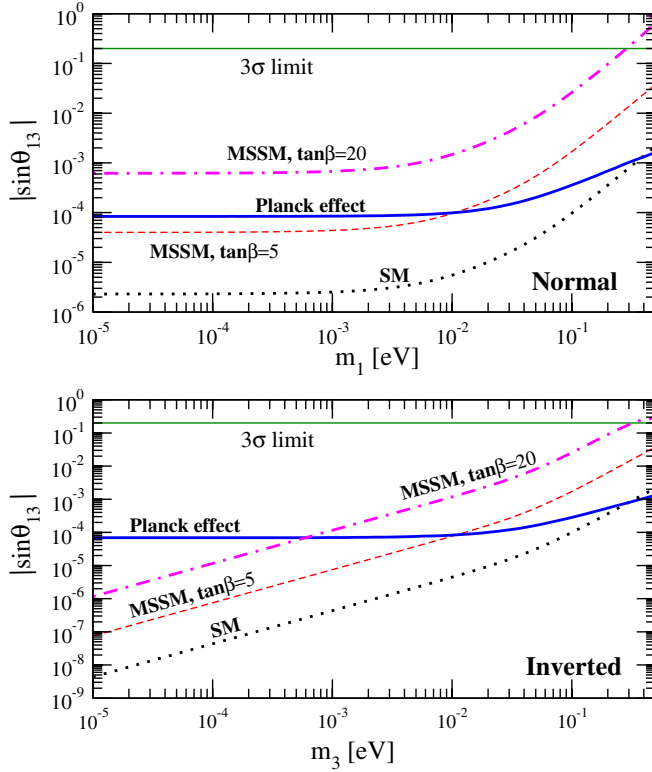


FIG. 1 (color online). Maximal generated value of the initially vanishing quantity  $|\sin\theta_{13}|$  as a function of the smallest neutrino mass for the normal (upper panel) and inverted (lower panel) mass ordering.

$$r \equiv \sqrt{\Delta m_{\odot}^2 / \Delta m_{\oplus}^2} \simeq 0.18, \quad (15)$$

we have  $m_1 \simeq 0$ ,  $m_2 \simeq r\sqrt{\Delta m_{\oplus}^2}$  and  $m_3 \simeq \sqrt{(1+r^2)\Delta m_{\oplus}^2}$ . From Eqs. (13) and (14) we get

$$\begin{aligned} |\sin\theta_{13}|_{\text{NH}} &\simeq \frac{\sqrt{2}}{3} \Delta_{\tau} |r \cos(\alpha_2 - \alpha_3) + r^2| \\ &\lesssim 1.4 \times 10^{-6} (1 + \tan^2\beta) \rightarrow 5.4 \times 10^{-4}, \end{aligned} \quad (16)$$

where the number indicated by the arrow is the value at  $\tan\beta = 20$ . Atmospheric neutrino mixing is now nonmaximal, the mixing angle being given by

$$\begin{aligned} (\cos 2\theta_{23})_{\text{NH}} &\simeq -\Delta_{\tau} \left(1 + \frac{4}{3} r \cos(\alpha_2 - \alpha_3)\right) \\ &\Rightarrow |\cos 2\theta_{23}|_{\text{NH}} \\ &\lesssim 1.7 \times 10^{-5} (1 + \tan^2\beta) \rightarrow 6.8 \times 10^{-3}. \end{aligned} \quad (17)$$

Note that  $(\cos 2\theta_{23})_{\text{NH}} < 0$ . The solar neutrino mixing angles increase from  $\sin^2\theta_{12} = \frac{1}{3}$  by

$$\begin{aligned} \left(\sin^2\theta_{12} - \frac{1}{3}\right)_{\text{NH}} &\simeq \frac{2}{9} \Delta_{\tau} \simeq 3.0 \times 10^{-6} (1 + \tan^2\beta) \\ &\rightarrow 1.2 \times 10^{-3}. \end{aligned} \quad (18)$$

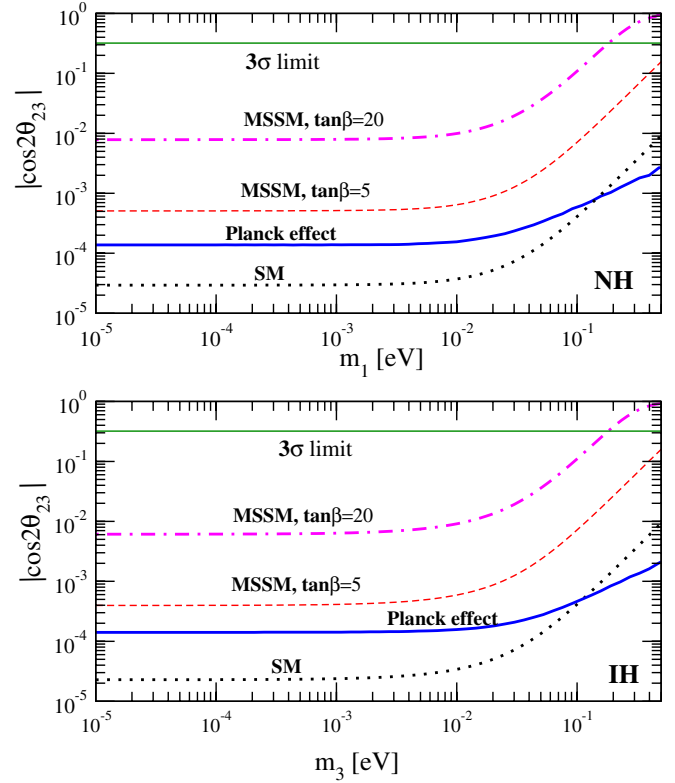


FIG. 2 (color online). Maximal generated value of the initially vanishing quantity  $|\cos 2\theta_{23}|$  as a function of the smallest neutrino mass for the normal (upper panel) and inverted (lower panel) mass ordering. In the MSSM the sign of  $\cos 2\theta_{23}$  is negative (positive) for normal (inverted) ordering. The signs are reversed in the SM.

Thus, we find that for NH, the deviations of the angles from tribimaximal values due to RG corrections are extremely small and virtually impossible to probe. Note that all the three deviations as plotted in the upper panels of Figs. 1–3 are independent of the value of  $m_1$  as long as  $m_1 \ll \sqrt{\Delta m_{\odot}^2} \simeq 9 \times 10^{-3}$  eV.

## B. Inverted hierarchy

Turning to the inverted hierarchy, using  $m_2 = \sqrt{m_3^2 + \Delta m_{\oplus}^2}$  and  $m_1 = \sqrt{m_3^2 + (1-r^2)\Delta m_{\oplus}^2}$ , it follows from Eqs. (13) and (14) that

$$\begin{aligned} |\sin\theta_{13}|_{\text{IH}} &\simeq \frac{m_3}{\sqrt{\Delta m_{\oplus}^2}} \frac{\sqrt{2}}{3} \Delta_{\tau} |\cos(\alpha_2 - \alpha_3) - \cos\alpha_3| \\ &\lesssim 2.6 \times 10^{-7} \left(\frac{m_3}{10^{-3} \text{ eV}}\right) (1 + \tan^2\beta) \\ &\rightarrow 1.0 \times 10^{-4} \left(\frac{m_3}{10^{-3} \text{ eV}}\right). \end{aligned} \quad (19)$$

Thus in the inverted hierarchy, the value of  $\sin\theta_{13}$  gener-

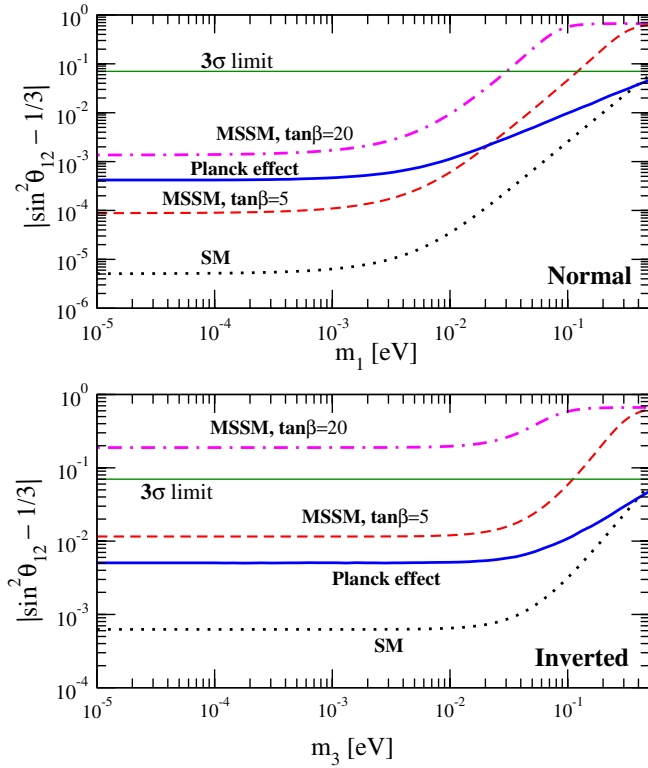


FIG. 3 (color online). Maximal generated value of the initially vanishing quantity  $|\sin^2\theta_{12} - 1/3|$  as a function of the smallest neutrino mass for the normal (upper panel) and inverted (lower panel) mass ordering. The sign of  $(\sin^2\theta_{12} - 1/3)$  is always positive in the MSSM and always negative in the SM.

ated through RG evolution is proportional to  $m_3$ , as can be seen in the lower panel of Fig. 1. The atmospheric neutrino mixing angle is given by

$$\begin{aligned} (\cos 2\theta_{23})_{\text{IH}} &\simeq \Delta_\tau \simeq 1.4 \times 10^{-5}(1 + \tan^2\beta) \\ &\rightarrow 5.4 \times 10^{-3}, \end{aligned} \quad (20)$$

which is independent of  $m_3$  as long as  $m_3 \ll \sqrt{\Delta m_A^2} \simeq 0.05$  eV. The dependence on the Majorana phases is introduced only at order  $m_3/\sqrt{\Delta m_A^2}$ . As alluded to before, in case of a normal ordering we have  $(\cos 2\theta_{23})_{\text{NH}} < 0$ , while for an inverted ordering  $(\cos 2\theta_{23})_{\text{IH}} > 0$ , i.e., RG effects increase  $|\theta_{23}|$  from its original TBM value of  $\pi/4$  for a normal hierarchy and decrease it for an inverted hierarchy. In the SM the effects have the opposite sign. Finally, the solar neutrino mixing angle increases for the MSSM and reads

$$\begin{aligned} \left(\sin^2\theta_{12} - \frac{1}{3}\right)_{\text{IH}} &\simeq \frac{4\Delta_\tau}{9r^2}(1 + \cos\alpha_2) \\ &\lesssim 3.9 \times 10^{-4}(1 + \tan^2\beta) \rightarrow 0.15. \end{aligned} \quad (21)$$

Even though Eq. (12) is strictly speaking no longer valid for such large values of  $|\theta_{ij} - \theta_{ij}^0|$ , the above result indi-

cates that the running of the solar neutrino mixing angle can be dramatic, namely, up to  $10^\circ$ . This will be confirmed in Sec. IV, where we will present figures quantifying the maximal RG effect, which are obtained by numerically solving the RG equations from Ref. [8].

The leading term in Eq. (21) is suppressed when  $\alpha_2 \simeq \pi$ . The measured value of  $|\theta_{12} - \theta_{12}^0| < 4^\circ$  (or  $\sin^2\theta_{12} - \frac{1}{3} \leq 0.07$ ) at  $3\sigma$  suggests that the observed value of  $\theta_{12}$  can be used to constrain the values of the absolute neutrino masses, the Majorana phase  $\alpha_2$  and  $\tan\beta$  in IH [23,27]. In contrast to this,  $\sin\theta_{13}$  and  $\cos 2\theta_{23}$  are too small to be observable.

### C. Quasidegeneracy

Finally, we shall consider running for quasidegenerate neutrinos. The RG running for QD is always the largest: indeed, the deviation of angles from their tribimaximal values grows quadratically with the common mass scale  $m_0$ . Using  $m_0^2 \gg \Delta m_A^2$  for the approximations and choosing  $m_0 = 0.2$  eV for illustration, it follows that

$$\begin{aligned} |\sin\theta_{13}|_{\text{QD}} &\simeq \frac{\sqrt{2}}{3} \Delta_\tau \frac{m_0^2}{\Delta m_A^2} |\cos(\alpha_2 - \alpha_3) - \cos\alpha_3| \\ &\lesssim 2.0 \times 10^{-4}(1 + \tan^2\beta) \rightarrow 0.08. \end{aligned} \quad (22)$$

Thus, testable values of  $\theta_{13}$  up to  $5^\circ$  may be generated. Turning to atmospheric mixing, one finds

$$\begin{aligned} (\cos 2\theta_{23})_{\text{QD}} &\simeq \mp \frac{2}{3} \Delta_\tau \frac{m_0^2}{\Delta m_A^2} (3 + 2\cos(\alpha_2 - \alpha_3) + \cos\alpha_3) \\ &\Rightarrow |\cos 2\theta_{23}|_{\text{QD}} \\ &\lesssim 8.0 \times 10^{-4}(1 + \tan^2\beta) \rightarrow 0.32, \end{aligned} \quad (23)$$

where the  $-$  sign is for normal ordering and the  $+$  sign for inverted ordering. The running is maximal when all the Majorana phases vanish. The value of  $\theta_{23}$  can deviate from its maximal value by up to  $10^\circ$ . This deviation is currently restricted by experiments to  $|\theta_{23} - \theta_{23}^0| < 10^\circ$  at  $3\sigma$ . Therefore, more accurate future measurements of  $\theta_{23}$  can be used to put bounds on  $m_0$ ,  $\alpha_2$  and  $\alpha_3$  [27]. The value of  $\alpha_2$  may be restricted to  $\alpha_2 \simeq \pi$  from the  $\theta_{12}$  measurements as we shall see next: the solar mixing angle can deviate strongly from its tribimaximal value:

$$\begin{aligned} \left(\sin^2\theta_{12} - \frac{1}{3}\right)_{\text{QD}} &\simeq \frac{4}{9} \Delta_\tau (1 + \cos\alpha_2) \frac{m_0^2}{\Delta m_A^2} \\ &\simeq 3.0 \times 10^{-3}(1 + \tan^2\beta)(1 + \cos\alpha_2). \end{aligned} \quad (24)$$

Since the maximum value of the quantity  $(\sin^2\theta_{12} - \frac{1}{3})$  can be 0.67 one can obtain a condition<sup>1</sup> for the validity of the analytic expressions as  $(m_0/\text{eV}) \tan\beta \lesssim 4$ . At large  $\tan\beta$ , the quantity  $\theta_{12}$  can be too large to be accommodated by

<sup>1</sup>The same comments as given after Eq. (21) apply here.

the data, unless  $\alpha_2 \approx \pi$ . This simplifies the predictions for the other two angles:

$$|\sin\theta_{13}|_{\text{QD}}^{\alpha_2=\pi} \simeq \frac{2\sqrt{2}}{3} \Delta_\tau \frac{m_0^2}{\Delta m_A^2} |\cos\alpha_3| \lesssim 2.0 \times 10^{-4} (1 + \tan^2\beta) \rightarrow 0.08, \quad (25)$$

$$|\cos 2\theta_{23}|_{\text{QD}}^{\alpha_2=\pi} \simeq 2\Delta_\tau \frac{m_0^2}{\Delta m_A^2} \left| 1 - \frac{1}{3} \cos\alpha_3 \right| \lesssim 5.4 \times 10^{-4} (1 + \tan^2\beta) \rightarrow 0.22, \quad (26)$$

with the sign of  $\cos 2\theta_{23}$  negative (positive) for normal (inverted) mass ordering. Note that  $\sin\theta_{13}$  can be zero if  $\alpha_3 = \pi/2$ , whereas  $\cos 2\theta_{23}$  is always nonzero.

Another phenomenological implication of  $\alpha_2 \approx \pi$  and quasidegenerate neutrinos is that the effective mass  $\langle m \rangle = |\sum U_{ei}^2 m_i|$  governing neutrinoless double beta decay [29] takes its minimal possible value:

$$\langle m \rangle_{\text{QD}}^{\alpha_2=\pi} \simeq m_0 \cos 2\theta_{12} \quad (27)$$

in the  $\theta_{13} \rightarrow 0$  limit. The same formula with  $m_0$  replaced by  $\sqrt{\Delta m_A^2}$  is valid for an inverted hierarchy and  $\alpha_2 \approx \pi$ , which suppresses the running also in this case [see Eq. (21)].

### III. PLANCK SCALE EFFECTS ON TRIBIMAXIMAL MIXING

In this section we will discuss the implications of Planck scale physics on tribimaximal mixing. The existence of the Planck scale implies the presence of higher dimensional nonrenormalizable interactions, among which the following dimension five operator is of interest for neutrino physics:

$$\mathcal{L}_{\text{Gr}} = \frac{\lambda_{\alpha\beta}}{M_{\text{Pl}}} (L_\alpha \Phi)(L_\beta \Phi) + \text{H.c.} \quad (28)$$

The coupling matrix  $\lambda_{\alpha\beta}$  will be assumed to be flavor democratic, since gravity does not distinguish between flavors. After electroweak symmetry breaking the above operator leads to a contribution to the low energy neutrino mass matrix  $m_\nu \rightarrow m_\nu + \delta m_\nu$  of the form

$$\delta m_\nu = \mu \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \equiv \mu \Delta \quad (29)$$

with  $\mu \simeq \frac{\langle \Phi \rangle^2}{M_{\text{Pl}}} \simeq 2.5 \times 10^{-6} \text{ eV}$ .

The implications of such a correction to  $m_\nu$  have been noted and analyzed for instance in Refs. [4–6]. With  $\mu \simeq 2.5 \times 10^{-6} \text{ eV} \ll \sqrt{\Delta m_\odot^2}$ , it would appear that only negligible corrections to the mixing phenomena can be expected. However, as will be seen later in this section, the Planck scale effects on  $\theta_{12}$  are governed by  $\mu m_0 / \Delta m_\odot^2$ ,

which can be substantial for quasidegenerate neutrinos. Moreover we stress that the presence of such a term is expected on general grounds and its implications are therefore model independent. In order to consider all possible corrections to a given mixing scheme, the perturbation (29) has to be included.

Note that the Planck scale contribution to the neutrino mass matrix,  $\delta m_\nu$ , has all its elements real only with a specific choice of the phases of the flavor eigenstates. In this basis, the so-called “unphysical” phases of the TBM matrix [see Eq. (3)] need not vanish. These “mismatch phases”—the phases  $\phi_i$  in this particular basis—turn out to be crucial in the context of the tribimaximal mixing, since without them there would be no effect on the mixing angles at all. To show this, consider the neutrino mass matrix giving rise to TBM in the absence of the  $\phi_{1,2,3}$ . It is given in Eq. (11) and can be written as:

$$m_\nu = \frac{m_1}{6} e^{i\alpha_1} \begin{pmatrix} 4 & -2 & -2 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_2}{3} e^{i\alpha_2} \begin{pmatrix} 1 & 1 & 1 \\ \cdot & 1 & 1 \\ \cdot & \cdot & 1 \end{pmatrix} + \frac{m_3}{2} e^{i\alpha_3} \begin{pmatrix} 0 & 0 & 0 \\ \cdot & 1 & -1 \\ \cdot & \cdot & 1 \end{pmatrix}. \quad (30)$$

The neutrino with mass  $m_2$  has a flavor democratic contribution to the total mass matrix. This is exactly the flavor-blind form that the Planck scale contribution  $\delta m_\nu$  in Eq. (29) possesses. Hence, adding  $\delta m_\nu$  to Eq. (30) is equivalent to a simple redefinition of  $m_2$  as

$$m_2 e^{i\alpha_2} \rightarrow m_2 e^{i\alpha_2} (1 + \epsilon_\mu e^{-i\alpha_2}), \quad (31)$$

where  $\epsilon_\mu = 3\mu/m_2 \ll 1$ . Hence, only the value of the second neutrino mass and its corresponding Majorana phase are modified, while the mixing angles and the other masses remain unchanged: with  $\mu \simeq 2.5 \times 10^{-6} \text{ eV}$  and  $m_2 \geq 8.4 \times 10^{-3} \text{ eV}$ , it holds that  $\epsilon_\mu \lesssim 8.9 \times 10^{-4}$  and the effect on  $m_2$  and on the mass-squared differences is at most of the order of 0.1%.

However, in general the situation is different since the most general mass matrix giving rise to TBM, as given in Eq. (6), contains the phases  $\phi_i$ . The new addition of a flavor democratic small perturbation to the neutrino mass matrix cannot be compensated anymore by a redefinition of  $m_2 e^{i\alpha_2}$ . This is because it is not possible to write  $m_\nu$  in terms of the individual masses as in Eq. (30), i.e., the elements of the matrix multiplying  $m_2 e^{i\alpha_2}$  are complex numbers. The corrections to TBM from the Planck scale effects are then nontrivial,<sup>2</sup> and henceforth we shall con-

<sup>2</sup>It has been noted [30] that the tribimaximal mixing scenario and Quark-Lepton Complementarity scenarios [31], which link the Cabibbo-Kobayashi-Maskawa and PMNS matrices, generate basically the same  $\sin^2\theta_{12}$ . We remark here that the quark-lepton complementarity scenarios will be affected by Planck scale effects even if the phases  $\phi_i$  vanish.

sider this general scenario. Note that though the vanishing of the mismatch phases is a special case, it may be relevant while postulating symmetries at the high scale that govern the structure of the mass terms.

We would like to make a few remarks at this stage:

- (i) The so-called unphysical phases, that are usually absorbed in the phases of charged leptons while constructing the leptonic mixing matrix, indeed are not well-defined outside the context of a theory of neutrino masses. However, a complete theory of neutrino masses has to predict the magnitudes and phases of all the terms in the neutrino mass matrix, and hence in the mixing matrix  $U$  from Eq. (3), once a choice for the phases of neutrino flavor eigenstates has been made. Such a choice has been made while writing the real democratic matrix  $\delta m_\nu$  in Eq. (29). Hence it should not be surprising that some of the predictions of the theory—like the values of the mixing angles—do indeed depend on the mismatch phases.
- (ii) The argument presented here regarding the vanishing of Planck scale effects on all three mixing angles and the masses  $m_{1,3}$  with vanishing mismatch phases is specific to the TBM scenario, where the mass matrix includes a flavor-blind term. However, similar results based on symmetries can be obtained in more general cases. For example, TBM mixing is a special case of  $\mu$ - $\tau$  symmetry of the mass matrix, which implies the form (including the mismatch phases) [12]:

$$m_\nu^{\mu-\tau} = \begin{pmatrix} Ae^{2i\phi_1} & Be^{i(\phi_1+\phi_2)} & Be^{i(\phi_1+\phi_3)} \\ \cdot & De^{2i\phi_2} & Ee^{i(\phi_2+\phi_3)} \\ \cdot & \cdot & De^{2i\phi_3} \end{pmatrix}, \quad (32)$$

where  $A, B, D, E$  define  $\theta_{12}$ , the neutrinos masses and Majorana phases. Because of the equality of the  $e\mu$  and  $e\tau$  elements, as well as of the  $\mu\mu$  and  $\tau\tau$  elements, maximal atmospheric mixing and vanishing  $U_{e3}$  results. Note now that the Planck scale contribution  $\delta m_\nu$  from Eq. (29) is also  $\mu$ - $\tau$  symmetric. Hence, adding  $\delta m_\nu$  to the  $\mu$ - $\tau$  symmetric matrix in case of vanishing  $\phi_{1,2,3}$  will keep the total mass matrix  $\mu$ - $\tau$  symmetric and the values  $U_{e3} = \cos 2\theta_{23} = 0$  will not be changed. In order for the Planck scale effects to change the values of these angles the mismatch phases are required not to vanish.

- (iii) The net order of magnitude of Planck scale effects should stay the same even if we take the elements of the democratic matrix  $\delta m_\nu$  from Eq. (29) to be  $\mathcal{O}(1)$ , and not necessarily exactly equal.

Now we will consider the general case of all possible phases in the neutrino mass matrix and study the resulting effect of the Planck scale contribution for the mixing

angles in case of TBM. Towards this end, we follow the formalism of [5] to calculate the deviations of the mixing angles from their TBM values. We use the shorthand notation

$$M_{ij} = \mu(U^T \Delta U)_{ij}, \quad (33)$$

such that  $M_{ij}$  incorporates all the  $\phi_i$  dependence and a part of the  $\alpha_i$  dependence. In general, the elements of  $M_{ij}$  are  $\mathcal{O}(\mu)$ . To a good approximation, the mass-squared difference  $m_3^2 - m_1^2$  does not change due to the Planck scale effects. In this limit one finds that [5]

$$\delta U_{e3} \simeq \sum_{i=1,2} U_{ei} \left( \frac{\Re(M_{i3})}{m_3 e^{i\alpha_3} - m_i e^{i\alpha_i}} - i \frac{\Im(M_{i3})}{m_3 e^{i\alpha_3} + m_i e^{i\alpha_i}} \right), \quad (34)$$

$$\delta U_{\mu 3} \simeq \sum_{i=1,2} U_{\mu i} \left( \frac{\Re(M_{i3})}{m_3 e^{i\alpha_3} - m_i e^{i\alpha_i}} - i \frac{\Im(M_{i3})}{m_3 e^{i\alpha_3} + m_i e^{i\alpha_i}} \right). \quad (35)$$

These quantities can be related to the Planck-corrected mixing angles after electroweak symmetry breaking through

$$|\sin \theta_{13}| = |\delta U_{e3}|, \quad (36)$$

$$\begin{aligned} \cos 2\theta_{23} &= 1 - 2\sin^2 \theta_{23} \simeq 1 - 2|U_{\mu 3} + \delta U_{\mu 3}|^2 \\ &\simeq -2\sqrt{2}|\delta U_{\mu 3}| \cos \chi_{\mu 3}, \end{aligned} \quad (37)$$

where  $\chi_{\mu 3}$  is the relative phase between  $U_{\mu 3}$  and  $\delta U_{\mu 3}$ . We see that the deviation from maximal atmospheric neutrino mixing can go in either direction.

We can estimate the size of the Planck scale contribution to be at most of the order  $\mu/(m_3 - m_1)$  for  $|\delta U_{e3}|$  and  $|\delta U_{\mu 3}|$ . For neutrinos with a normal hierarchy and  $m_1 = 0$ , this quantifies to  $\mu/\sqrt{\Delta m_A^2} \simeq 6 \times 10^{-5}$ , and is thus negligibly small. An inverted hierarchy with  $m_3 = 0$  gives the same result. The largest effect can be expected for quasi-degenerate neutrinos, in which case the corrections are at most of order  $\mu m_0/\Delta m_A^2 \simeq 5 \times 10^{-4}$ . We have inserted here for illustration a value of  $m_0 = 0.4$  eV for the common mass scale. Note that the corrections here are proportional to  $m_0$ , as opposed to  $m_0^2$  in the case of the RG running.

The modifications of the mixing angles depend on the values of the Majorana phases (as for radiative corrections) but, in particular, also on the mismatch phases. The full expressions for Eqs. (34) and (35) are rather lengthy and not very instructive. An example we give is for the normal hierarchy, in which case one can roughly estimate

$$|\delta U_{e3}| \simeq \sqrt{2} \left| \sin\left(\frac{\phi_2 - \phi_3}{2}\right) \right| \frac{\mu}{\sqrt{\Delta m_A^2}} \quad \text{and}$$

$$|\delta U_{\mu 3}| \simeq \frac{|\sin(\phi_2 - \phi_3)|}{\sqrt{2}} \frac{\mu}{\sqrt{\Delta m_A^2}}. \quad (38)$$

The Majorana phases appear only at  $\mathcal{O}(r\mu/\sqrt{\Delta m_A^2})$ . These expressions show explicitly that for vanishing mismatch phases  $\phi_i$  the corrections to the mixing angles vanish.

As in the case of the radiative corrections, the corrections to the mixing angles  $\theta_{ij}$  are approximately inversely proportional to the mass-squared difference  $\Delta m_{ij}^2$ , and therefore one may expect a more sizable correction to  $\theta_{12}$  than to  $\theta_{13}$  and  $\theta_{23}$ . The contribution from the Planck scale to  $U_{e2}$  is [5]

$$\delta U_{e2} \simeq \frac{U_{e1}}{\Delta m_\odot^2} (\Re(M_{12})|m_1 + m_2 e^{i\alpha_2}| - i\Im(M_{12})|m_1 - m_2 e^{i\alpha_2}|), \quad (39)$$

where there is only one term because for TBM  $U_{e3} = 0$  holds before the Planck scale effects are included. Note that in the above expression, the  $m_i$  and  $\alpha_i$  are defined before the Planck scale effects are switched on, whereas  $\Delta m_\odot^2$  is taken to be after the Planck scale effects are switched on. If  $\Delta m_\odot^2$  does not change much due to the Planck scale effects, then  $|m_1 \pm m_2 e^{i\alpha_2}|/\Delta m_\odot^2 \simeq 1/|m_1 \mp m_2 e^{i\alpha_2}|$  and  $\delta U_{e2}$  could be written in the same form as the expressions for  $\delta U_{e3}$  and  $\delta U_{\mu 3}$ . For the analytical estimates, we will assume that this is the case. The numerical analysis in the next section will not make this assumption. However, the expressions obtained here are quite close to the full result.

Since  $\sin\theta_{12} \simeq |U_{e2} + \delta U_{e2}|$ , the quantity in Eq. (39) can be related to the deviation of the solar mixing angle via

$$\begin{aligned} \sin^2\theta_{12} - 1/3 &\simeq |U_{e2} + \delta U_{e2}|^2 - 1/3 \\ &\simeq \frac{2}{\sqrt{3}} |\delta U_{e2}| \cos\chi_{e2}, \end{aligned} \quad (40)$$

where  $\chi_{e2}$  is the relative phase between  $U_{e2}$  and  $\delta U_{e2}$ . The deviation can thus have either sign as in the case of the atmospheric neutrino mixing angle.

If initially  $m_1 = 0$  (normal hierarchy), one can estimate  $|\delta U_{e2}| \lesssim \mu/\sqrt{\Delta m_\odot^2} \simeq 3 \times 10^{-4}$ . An inverted hierarchy leads to a larger correction,  $|\delta U_{e2}| \lesssim 2\mu\sqrt{\Delta m_A^2}/\Delta m_\odot^2 \simeq 4 \times 10^{-3}$ . Hence, in contrast to the corrections to  $\theta_{13}$  and  $\theta_{23}$ , the deviation of solar neutrino mixing from  $1/3$  is sensitive to whether neutrinos are normally or inversely ordered. For quasidegenerate neutrinos it follows from Eq. (39) that  $|\delta U_{e2}| \simeq \mu m_0/\Delta m_\odot^2 \simeq 10^{-2}$ , where we have again used  $m_0 = 0.4$  eV. With  $m_0^2 \gg \Delta m_\odot^2$ , we can write

$$\begin{aligned} |\delta U_{e2}| &\simeq \frac{2}{3\sqrt{3}} \frac{\mu m_0}{\Delta m_\odot^2} |2\cos 2\phi'_1 - \cos 2\phi'_2 + \cos(\phi'_1 + \phi'_2) \\ &\quad - \cos 2\phi'_3 + \cos(\phi'_1 + \phi'_3) - 2\cos(\phi'_2 + \phi'_3)|, \end{aligned} \quad (41)$$

where  $\phi'_i \equiv \phi_i - \alpha_2/4$ . This quantity, as it should, vanishes for  $\phi_{1,2,3} = 0$ . The function of the  $\phi_{1,2,3}$  inside the  $|\dots|$  above has a maximal value of  $\simeq 6.36$  and therefore  $|\delta U_{e2}| \lesssim 2.5\mu m_0/\Delta m_\odot^2 \simeq 0.03$  for  $m_0 = 0.4$  eV. Thus, with the maximal deviation being  $\sin^2\theta_{12} - \frac{1}{3} \simeq (2/\sqrt{3})|\delta U_{e2}|$  it follows that the deviation of  $\theta_{12}$  from its TBM value due to Planck scale effects can be up to 10% or  $2^\circ - 3^\circ$ . Moreover, as shown in Eq. (40), this deviation can be in either direction depending on the relative phase  $\chi_{e2}$ , which in turn depends on the Majorana phases  $\alpha_i$  as well as on the mismatch phases  $\phi_i$ .

If we now want to incorporate both the RG as well as the Planck scale effects on  $\theta_{12}$ , the predictions are thus uncertain by  $2^\circ - 3^\circ$  in the absence of any knowledge of the  $\phi_i$ . This would relax some of the constraints on  $m_0$ ,  $\alpha_2$  and  $\tan\beta$  that have been obtained in the literature by requiring the angles at the low scale to be compatible with the experiments. To be more quantitative, the maximal deviation from the initial value of  $\theta_{12} = \arcsin\sqrt{1/3}$  for a normal ordering is below  $0.1^\circ$  for  $m_1 < 0.02$  eV. For  $m_1 = 0.1$  eV the change can be up to  $0.6^\circ$  and then it increases linearly with the neutrino mass, for instance  $3^\circ$  for  $m_1 = 0.5$  eV. If neutrinos are inversely ordered, the change is up to  $0.3^\circ$  even for a vanishing  $m_3$ , and starts to increase linearly with the neutrino mass for  $m_3 \gtrsim 0.01$  eV. If radiative corrections are used to set constraints on the parameters, then one should take this uncertainty into account, which would weaken the corresponding limits.

One might wonder at this point whether one can generate successful phenomenology starting with bimaximal neutrino mixing [32], i.e., with Planck scale contributions perturbing an initial<sup>3</sup>  $\sin^2\theta_{12} = \frac{1}{2}$ . We checked numerically that in order to obey the current  $3\sigma$  limit of  $\sin^2\theta_{12} \leq 0.4$ , neutrinos should be heavier than 1.4 eV, i.e., in conflict with the already very tight neutrino mass limits from cosmology. Since the RG effects tend to increase  $\theta_{12}$ , even their addition cannot salvage the scenario.

#### IV. NUMERICAL RESULTS AND DISCUSSION

In Figs. 1–3 we show the maximal possible values of the initially vanishing quantities  $|\sin\theta_{13}|$ ,  $|\cos 2\theta_{23}|$  and  $|\sin^2\theta_{12} - \frac{1}{3}|$  generated from RG effects and from Planck scale effects, as a function of the smallest neutrino mass for normal and inverted mass ordering. The figures for MSSM and SM are generated by numerically solving the RG equations in the small  $\theta_{13}$  limit [8]. We have chosen in

<sup>3</sup>See also [6] for a discussion on Planck scale effects on bimaximal mixing.



case of the MSSM  $\tan\beta = 20$  and  $\tan\beta = 5$ . The relative magnitude of the deviations between these two cases should be  $(1 + 5^2)/(1 + 20^2) \approx 0.065$ , which is confirmed by the plot. Also given in the plots are the maximal RG effects in the SM. As far as the mixing angles are concerned, for all practical purposes the results for SM can be obtained from the ones of MSSM with  $\tan\beta = 20$  by multiplying them with  $|(-3/2)/(1 + 20^2)| \approx 0.0037$ . We also indicate the present  $3\sigma$  bounds on the mixing parameters.

The RG running in SM is found to be very small. Even for values of  $m_0$  in the QD regime the running stays within the present  $3\sigma$  limit for all three quantities. For MSSM the running is much stronger, especially for higher values of  $\tan\beta$  and in the QD regime. For  $|\sin^2\theta_{12} - \frac{1}{3}|$  there is a plateau at  $\sim 0.67$  in the QD regime which corresponds to the maximum possible deviation of  $2/3$  in this quantity. As we are plotting the absolute values of the observables, we stress again that  $\sin^2\theta_{12} - \frac{1}{3}$  is always larger than zero in the MSSM. The same holds for  $\cos 2\theta_{23}$  in case of a inverted ordering, whereas for a normal ordering  $\cos 2\theta_{23}$  is negative. For the SM the signs are reversed. For the inverted ordering and small masses ( $m_3 \lesssim 0.01$  eV), the deviation  $\sin^2\theta_{12} - \frac{1}{3}$  is roughly 2 orders of magnitude larger than that for the normal ordering.

Note that since we show the maximal effects, the RG running beyond the  $3\sigma$  limits in the figure does not imply that the corresponding neutrino mass values are ruled out. Rather, it implies that the Majorana phases can be constrained if the neutrino masses lie in the corresponding range.

The figures also show the maximal possible values of the above three quantities  $|\sin\theta_{13}|$ ,  $|\cos 2\theta_{23}|$  and  $|\sin^2\theta_{12} - \frac{1}{3}|$  due to Planck scale effects. They were obtained by numerically diagonalizing a mass matrix leading to TBM to which a flavor-blind Planck perturbation was added. It is to be noted that the values of  $\cos 2\theta_{23}$  and  $\sin^2\theta_{12} - \frac{1}{3}$  are symmetric about zero: depending on the values of the Majorana phases  $\alpha_i$  and the mismatch phases  $\phi_i$ , the Planck-corrected values of mixing angles can go in either direction. Thus these effects can either replenish or deplete the RG effects. The minimum values of the three quantities plotted in the figure can be zero for suitable choices of the phases  $\alpha_i$  and  $\phi_i$ .

Let us compare both the effects:

- (i)  $\sin\theta_{13}$ : the Planck scale effects can be larger than the RG effects in the SM unless the neutrino masses are above roughly 0.3 eV. Recall that the Planck corrections for QD neutrinos are proportional to  $m_0$ , whereas the RG corrections are proportional to  $m_0^2$ . For a normal ordering the Planck scale effects are always less than the maximal correction of  $\tan\beta = 20$  but can exceed the corrections for  $\tan\beta = 5$  if  $m_1 \lesssim 0.01$  eV. For an inverted ordering Planck scale effects can exceed the corrections for  $\tan\beta = 20$  (5)

if  $m_3 \lesssim 0.0005(0.01)$  eV;

- (ii)  $|\cos 2\theta_{23}|$ : the Planck scale effects can be larger than the RG effects in the SM unless the neutrino masses are above roughly 0.1 eV. They are always less than the maximal correction of  $\tan\beta = 20$  and  $\tan\beta = 5$ ;
- (iii)  $|\sin^2\theta_{12} - \frac{1}{3}|$ : the Planck scale effects can be larger than the RG effects in the SM unless the neutrino masses are above roughly 0.5 eV. They are always below the maximal correction of  $\tan\beta = 20$  and can exceed the corrections for  $\tan\beta = 5$  if neutrinos are normally ordered and  $m_1 \lesssim 0.01$  eV.

## V. CONCLUSIONS AND SUMMARY

We have studied the renormalization group and Planck scale corrections to neutrino mixing angles in the tribimaximal mixing scenario. Both these corrections need to be included while comparing the low energy neutrino mixing data with any postulated high scale mixing scenario.

We give approximate expressions for the values of mixing angles at low scale starting from tribimaximal mixing at high scale for NH, IH and QD scenarios with RG running in the SM and the MSSM. We also plot the maximum RG effects as a function of the smallest neutrino mass for these scenarios. We find that in the SM the RG running has a negligible effect on the mixing angles. In the MSSM with large  $\tan\beta$ , while NH still gives unobservably small deviations for all the mixing angles, IH is capable of generating significant running for  $\theta_{12}$ . In fact, matching  $\theta_{12}$  with the data requires constraining the Majorana phases and  $\tan\beta$  already at the present stage. For the QD scenario the running for all the three cases can be strong. The running depends crucially on the values of the Majorana phases and the neutrino mass scale: the corrections in the QD scenario grow as  $m_0^2$ . Precision measurements of neutrino mixing angles in future experiments should be able to put further constraints on the mass scale and Majorana phases if one assumes the TBM scenario.

For the Planck scale effect we assume a flavor democratic dimension five operator at the Planck scale that contributes to the neutrino mass after electroweak symmetry breaking. We show that the corrections to the mixing angles can be quantified in terms of the ‘‘mismatch phases’’  $\phi_i$ , which are the values of the so-called unphysical phases in the basis we have chosen. Because of the special structure of the neutrino mass matrix giving rise to tribimaximal mixing, nonzero values of these phases are required for any Planck effect on the mixing angles and the masses  $m_1$  and  $m_3$ . In general, if Planck scale effects are added to a  $\mu$ - $\tau$  symmetric mass matrix then corrections to vanishing  $U_{e3}$  and maximal  $\theta_{23}$  are only possible if the unphysical phases have nontrivial values.

In the most general case when the mismatch phases are nonvanishing, the Planck scale effects make the otherwise vanishing quantities  $\sin\theta_{13}$ ,  $\cos 2\theta_{13}$  and  $\sin^2\theta_{12} - \frac{1}{3}$  grow almost linearly with the neutrino mass scale  $m_0$  for quasi-

degenerate neutrinos. The effects are in general largest for  $\theta_{12}$ . Even with a large value of  $\mathcal{O}(\text{eV})$  for the neutrino masses,  $\sin\theta_{13}$  and  $\cos 2\theta_{23}$  hardly exceed  $10^{-3}$ , and hence are virtually impossible to probe experimentally. However, deviations of  $\sin^2\theta_{12}$  from  $1/3$  can be sizable, of the order of  $0.1(m_0/\text{eV})$ . Deviations of a few degrees are thus allowed when neutrinos are quasidegenerate. This deviation is larger than the resolution of the future precision  $\theta_{12}$  experiments, and can be measured.

An interesting possibility, though hardly realizable in practice, is the following: suppose one has measured deviations from tribimaximal mixing and knows the values of the Majorana phases and possibly of  $\tan\beta$ . In this case any additional correction beyond the RG effects to the mixing angles will stem from the Planck scale effects. As these depend on the mismatch phases, one could in principle extract some information on these phases. Moreover, if supersymmetry is not realized in nature, then the RG running is suppressed but the Planck scale effects can nevertheless inflict a sizable perturbation to the solar neutrino mixing angle, which can help us get a handle on these phases.

In the case of the RG running, the signs of the correction to the mixing angles are predictable. For example in MSSM,  $\theta_{12}$  always increases from its high scale value, whereas  $|\theta_{23}|$  increases (decreases) for normal (inverted)

hierarchy. In the case of the Planck scale effects, the sign of  $\cos 2\theta_{23}$  and  $\sin^2\theta_{12} - 1/3$  depends on all of the phases present, including the mismatch phases which will need a complete theory of neutrino masses for their prediction. Therefore the Planck scale effects can either enhance or compensate the RG running. Constraining neutrino parameters due to running might therefore be not as straightforward as is usually done. If the neutrino mass is  $0.5$  ( $0.2$ ) eV, then the modification from Planck scale effects to  $\theta_{12}$  can be nearly  $3^\circ$  ( $1^\circ$ ), which would weaken the constraints. Note that such a relaxation of constraints is applicable not only for TBM, but for any neutrino mixing scenario.

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