Neutrino-electron scattering in a magnetic field with allowance for polarizations of electrons

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We present an analytic formula for differential cross section (DCS) of neutrino-electron scattering (NES) in a magnetic field (MF) with allowance for longitudinal polarizations of initial and final electrons (IAFE). The DCS of NES in a MF is sensitive to the spin variable of the IAFE and to the direction of the incident and scattered neutrinos (IASN) momenta. Spin asymmetries and field effects in NES in a MF enable us to use initial electrons having a left-hand circular polarization (LHCP) as polarized electron targets in detectors for detection of low-energy neutrinos or relic neutrinos and for distinguishing neutrino flavor (NF). In general, gas consisting of only electrons having a LHCP and gas consisting of only electrons having a right-hand circular polarization (RHCP) are heated by neutrinos asymmetrically. The asymmetry of heating (AH) is sensitive to NF, MF strength, energies (Landau quantum numbers and third components of the momenta) of IAFE, final electron chemical potential, the final temperature of gas consisting of only electrons having a LHCP (RHCP), polar angles of IASN momenta, the difference between the azimuthal angles of IASN momenta, the angle φ, and IASN energies. In the heating process of electrons by neutrinos the dominant role belongs to electron neutrinos compared with the contribution of muon (tauon) neutrinos. Electrons having a LHCP in NES in a MF are heated by ν_e and $\nu_\mu(\nu_\tau)$ unequally when both the IASN fly along or against the MF direction. For magnetars and neutrinos of 1 MeV energy, within the considered kinematics, the AH in an electron neutrino-electron scattering is 2.23 times that in a muon neutrino-electron scattering or in a tauon neutrino-electron scattering.

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I. INTRODUCTION

The neutrino-electron scattering

$$\nu + e^- \to \nu + e^- \tag{1}$$

plays a significant role in strongly magnetized stars or in supernova explosions.

This process is responsible for a significant fraction of the energy and momentum exchange between neutrinos and stellar matter [1–5]. Strong magnetic fields exist in compact objects in the Milky Way Galaxy (e.g., [6,7]). For example, magnetic fields of neutron stars can be as large as $H \geq H_0 = m_e^2 c^3/e\hbar = 4.41 \times 10^{13}$ G (e.g., [8]). Strong magnetic fields of $H \sim 10^{15}$–10^{17} G are generated inside astrophysical cataclysms such as a supernova explosion or a coalescence of neutron stars [6–10]. However, neutrinos and electrons are in thermodynamic equilibrium in the neutrino opaque core ($H \sim 10^{17}$ G). But in magnetars ($H \sim 10^{15}$ G), neutrinos and electrons are not in thermodynamic equilibrium. The effects of the magnetic field on neutrino-electron scattering become very strong for a field strength of the order of $H \sim 10^{15}$ G [11]. Such strong magnetic fields influence neutrino-electron scattering by modifying the motion of electrons. As indicated in [12], the presence of an external magnetic field provides a preferred direction in space and it opens the way for parity violating effects to produce an asymmetry in the cross section of neutrino-electron scattering.

Neutrino-electron scattering in a magnetic field and some aspects of polarization effects arising in this process were studied by numerous authors [11,13–21]. However, neutrino-electron scattering in hot stellar magnetic fields with allowance for longitudinal and transverse polarizations of initial and final electrons has not been investigated completely.

The motivation of the study of neutrino-electron scattering in a magnetic field with allowance for longitudinal polarizations of initial and final electrons is connected with the following reasons:

The presence of a strong magnetic field leads, on the one hand, to anisotropy and asymmetry in the heating of stellar matter and, on the other hand, to anisotropy and asymmetry of the subsequent explosion of outer layers of the collapsing stellar core [11]. To clarify anisotropy and asymmetry arising in astrophysical phenomena connected with neutrino-electron scattering, it is important to investigate the dependence of the differential cross section of the considered process in a magnetic field on the spin variables of initial and final electrons. The study of polarization properties of electrons and the investigation of the effect of parity violation in neutrino-electron scattering may help to distinguish the neutrino signal from the detector background [22]. This process also enables us to use the polarized electron target as a new neutrino detector [22]. As indicated in [23], polarization asymmetry provides a sen-

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sitive tool to investigate the flavor of incoming (anti)neutrinos. The scattering of an electron neutrino and an electron antineutrino on a polarized electron target was suggested as a test of the neutrino magnetic moment [24].

The main purpose of this paper is to present an analytic formula for the differential cross section of neutrino-electron scattering in a magnetic field with allowance for longitudinal polarizations of initial and final electrons, to analyze polarization effects, to calculate the asymmetry of the heating of electrons (electron gas) having a left-hand circular polarization and electrons (electron gas) having a right-hand circular polarization by neutrinos in a magnetic field, and to show possible applications of the obtained results.

Therefore, here we calculate the dependence of the differential cross section for neutrino-electron scattering on the spins of initial (final) electrons and on the polar and azimuthal angles of incident (scattered) neutrinos.

We consider that neutrino energies, transverse momenta of electrons, Landau energy levels of electrons, and the strength of a magnetic field are arbitrary. For future possible astrophysical applications formally we also take into account the temperature of matter (e.g., stellar matter) and the chemical potentials of electrons. To compare our results with the results obtained in [11], we keep the variables used in [11]. Because in the limiting case $E, E' \gg m_e$ [here $E(E')$ is the energy of an initial (final) electron, m_e is the electron mass] averaging (summation) over the initial (final) electron polarization of the result of this work ought to lead to a result similar to that obtained in [11].

Radiative corrections to neutrino-electron scattering are not included in the results of this work. Radiative corrections to neutrino-electron scattering have been investigated by Bahcall, Kamionkowski, Sirlin, and other authors [25–27].

II. MATRIX ELEMENT AND CROSS SECTION OF THE PROCESS

We use the standard Weinberg-Salam-Glashow electroweak interaction theory. When the momentum transferred is relatively small, $|q^2| \ll m_W^2, m_Z^2$ (m_W is the $W^{\pm}$-boson mass, m_Z is the Z-boson mass), the four-fermion approximation of the Weinberg-Salam-Glashow standard model can be used.

The gauge of a 4-potential is $A^{\mu} = (0, 0, xH, 0)$ and an external magnetic field vector $\mathbf{H}$ is directed along the axis Oz. The matrix element of the process in a magnetic field can be written in the form

$$M = \frac{4\sqrt{2}G_F\pi^3}{VL_yL_z(\omega\omega')^{1/2}}\delta(E' + \omega' - E - \omega) \times \delta(p'_y - p_y - q_y)\delta(p'_z - p_z - q_z) \times e^{-i\alpha_0}e^{i(n-n')\varphi'}[\bar{u}(k')\gamma_\alpha^L u(k)]J^\alpha, \tag{2}$$

where G_F is the Fermi constant, V is the normalization volume, L_y and L_z are the normalization lengths, $k^{\mu}(k'^{\mu})$ is the 4-momentum of the incident (scattered) neutrino with the energy $\omega(\omega')$, $k_y(k_z)$ and $k'_y(k'_z)$ are the $y(z)$-components of the 4-momenta of the incident and scattered neutrinos, $E(E')$ is the energy of the initial (final) electron, $p_y(p_z)$ and $p'_y(p'_z)$ are $y(z)$-components of the 4-momenta of the initial and final electrons, $n = 0, 1, \ldots$ ($n' = 0, 1, \ldots$) enumerates the Landau energy levels of the initial (final) electron, $\varphi' = \varphi - \pi/2$, $\tan\varphi = q_y/q_x$, $q = k - k'$, $\alpha_0 = (q_x/2h)(p_y + p'_y)$, $h = eH$, $\gamma_\alpha^L = \gamma_\alpha(1 + \gamma^5)/2$, γ_α are the Dirac matrices, $u(k)$ and $\bar{u}(k')$ are the Dirac bispinors, J^α is the transition amplitude of the 4-current for the neutrino-electron scattering. We have dealings with a massless neutrino. We use the pseudo-Euclidean metric with signature $(+ - - -)$ and the system of units $\hbar = c = 1$.

In the case of longitudinal polarization of electrons we have dealings with the generalized helicity operator [28]

$$\mathbf{\Sigma} \cdot \mathbf{P} = \gamma^5(m_e\gamma^0 - E), \tag{3}$$

$$(\mathbf{\Sigma} \cdot \mathbf{P})\psi = \zeta p\psi, \tag{4}$$

where $p = (E^2 - m_e^2)^{1/2}$ and the value $\zeta = +1(-1)$ corresponds to right-hand (left-hand) helicity. The general expressions for transition amplitude of the 4-current for neutrino-electron scattering with allowance for longitudinal polarization of electrons have the form

$$J^\alpha = \begin{bmatrix} (g_V s_0 - g_A s)f_0, \\ (g_V s - g_A s_0)f_i \end{bmatrix}, \qquad (i = 1, 2, 3), \tag{5}$$

$$\begin{aligned} f_{0,3} &= B_1B'_1I_{n-1,n'-1} \pm B_2B'_2I_{nn'}, \\ f_1 &= B'_1B_2e^{i\varphi}I_{n,n'-1} + B_1B'_2e^{-i\varphi}I_{n-1,n'}, \\ f_2 &= -iB'_1B_2e^{i\varphi}I_{n,n'-1} + iB_1B'_2e^{-i\varphi}I_{n-1,n'}, \end{aligned} \tag{6}$$

where

$$s_0 = \tfrac{1}{4}(A_1A'_1 + A_2A'_2), \tag{7}$$

$$s = \tfrac{1}{4}(A_1A'_2 + A'_1A_2), \tag{8}$$

$g_V = -0.5 + 2\sin^2\theta_w$, $g_A = -0.5$ for $\nu_\mu e^-(\nu_\tau e^-)$ scattering, $g_V = 0.5 + 2\sin^2\theta_w$, $g_A = 0.5$ for $\nu_e e^-$ scattering [29]. The spin coefficients in (5)–(8) have the following forms [28]:

$$A_1 = \left(1 + \frac{m_e}{E}\right)^{1/2}, \tag{9}$$

$$A_2 = \zeta\left(1 - \frac{m_e}{E}\right)^{1/2}, \tag{10}$$

$$B_1 = \left(1 + \frac{\zeta p_z}{\sqrt{E^2 - m_e^2}}\right)^{1/2}, \tag{11}$$

$$B_2 = \zeta\left(1 - \frac{\zeta p_z}{\sqrt{E^2 - m_e^2}}\right)^{1/2}. \tag{12}$$

In (6) $I_{nn'}$, $I_{n,n'-1}$, $I_{n-1,n'}$, $I_{n-1,n'-1}$ are the Laguerre functions, where

$$I_{nn'}(x) = \left(\frac{n'!}{n!}\right)^{1/2} e^{-x/2} x^{(n-n')/2} L_{n'}^{n-n'}(x) \tag{13}$$

and $L_{n'}^{n-n'}(x)$ is the Laguerre polynomial of the argument

$$x = \frac{q_x^2 + q_y^2}{2h}. \tag{14}$$

Taking into account components of the current J^α, we obtain the general expression for the differential cross section of neutrino-electron scattering in a magnetic field with an allowance for the longitudinal polarization of initial and final electrons

$$\frac{d\sigma}{d\omega' d\Omega'} = \frac{G_F^2 eH\omega'}{32\pi^4\omega} \sum_{n,n'=0}^{\infty} \int_{-\infty}^{\infty} dp_z \delta(E' + \omega' - E - \omega) \times f(1 - f')R, \tag{15}$$

where

$$R = \omega\omega' R_0, \tag{16}$$

$$R_0 = d_1(r_1 I_4^2 + r_2 I_3^2 + 2r_3 I_3 I_4) + 2d_2(r_4 I_1^2 + r_5 I_2^2) + d_3(r_6 I_4^2 + r_7 I_3^2 + 2r_8 I_3 I_4) + 4d_4 r_9 I_1 I_2 - 2d_5(r_{10} I_4^2 - r_{11} I_3^2) - 2d_6(r_4 I_1^2 - r_5 I_2^2) - 2d_7(r_{12} I_1 I_4 + r_{13} I_2 I_4 + r_{14} I_1 I_3 + r_{15} I_2 I_3) - 2d_8(r_{16} I_1 I_4 - r_{17} I_1 I_3 - r_{18} I_2 I_4 + r_{19} I_2 I_3) - 2d_9(r_{16} I_1 I_4 - r_{17} I_1 I_3 + r_{18} I_2 I_4 - r_{19} I_2 I_3) + 2d_{10}(r_{12} I_1 I_4 - r_{13} I_2 I_4 + r_{14} I_1 I_3 - r_{15} I_2 I_3). \tag{17}$$

Here $I_1 = I_{n,n'-1}$, $I_2 = I_{n-1,n'}$, $I_3 = I_{n-1,n'-1}$, $I_4 = I_{nn'}$ and the coefficients r_i $(i = \overline{1, 19})$ contain spins of the initial and final electrons

$$r_{1,2} = \tfrac{1}{4}[g_+(\mu_1 \mp \mu_2\zeta \mp \mu_3\zeta' + \mu_4\zeta\zeta') \pm 2g_\perp(\mu_5 \mp \mu_6\zeta \mp \mu_7\zeta' + \mu_8\zeta\zeta')],$$

$$r_3 = \tfrac{1}{4} g_- \delta\delta' \beta\beta' \zeta\zeta',$$

$$r_{4,5} = \tfrac{1}{8}[g_+(\mu_1' \mp \mu_2'\zeta \pm \mu_3'\zeta' + \mu_4'\zeta\zeta') - g_-\delta\delta'(1 \mp v\zeta \pm v'\zeta' - vv'\zeta\zeta') \mp 2g_\perp(-\mu_5' \pm \mu_6'\zeta \pm \mu_7'\zeta' + \mu_8'\zeta\zeta')],$$

$$r_{6,7} = \tfrac{1}{4} g_- \delta\delta'(1 \mp v\zeta \mp v'\zeta' + vv'\zeta\zeta'),$$

$$r_8 = \tfrac{1}{4}[g_+\beta\beta'(\sigma\sigma' + \zeta\zeta') - 2g_\perp(\nu_8\zeta' + \nu_8'\zeta)],$$

$$r_9 = \tfrac{1}{8}[g_+\beta\beta'(\sigma\sigma' + \zeta\zeta') - g_-\delta\delta'\beta\beta'\zeta\zeta' - 2g_\perp(\nu_8\zeta' + \nu_8'\zeta)],$$

$$r_{10} = \tfrac{1}{8}[g_+(-\mu_5 + \mu_6\zeta + \mu_7\zeta' - \mu_8\zeta\zeta') - 2g_\perp(\mu_1 - \mu_2\zeta - \mu_3\zeta' + \mu_4\zeta\zeta')],$$

$$r_{11} = \tfrac{1}{8}[g_+(\mu_5 + \mu_6\zeta + \mu_7\zeta' + \mu_8\zeta\zeta') - 2g_\perp(\mu_1 + \mu_2\zeta + \mu_3\zeta' + \mu_4\zeta\zeta')],$$

$$r_{12} = \tfrac{1}{8}[g_+(\nu_1' - \nu_5'\zeta - \nu_5\zeta' + \nu_2'\zeta\zeta') + 2g_\perp(\nu_7' - \nu_3'\zeta - \zeta' + \nu_4'\zeta\zeta')],$$

$$r_{13} = \tfrac{1}{8}[g_+(\nu_1 - \nu_6'\zeta - \nu_6\zeta' + \nu_2\zeta\zeta') + 2g_\perp(\nu_7 - \zeta - \nu_3\zeta' + \nu_4\zeta\zeta')],$$

$$r_{14} = \tfrac{1}{8}[g_+(\nu_1 + \nu_6'\zeta + \nu_6\zeta' + \nu_2\zeta\zeta') - 2g_\perp(\nu_7 + \zeta + \nu_3\zeta' + \nu_4\zeta\zeta')],$$

$$r_{15} = \tfrac{1}{8}[g_+(\nu_1' + \nu_5'\zeta + \nu_5\zeta' + \nu_2'\zeta\zeta') - 2g_\perp(\nu_7' + \nu_3'\zeta + \zeta' + \nu_4'\zeta\zeta')],$$

$$r_{16} = \tfrac{1}{8}[g_+(-\nu_7' + \nu_3'\zeta + \zeta' - \nu_4'\zeta\zeta') - g_-\delta\delta'\beta'\zeta'(1 - \zeta v) - 2g_\perp(\nu_1' - \nu_5'\zeta - \nu_5\zeta' + \nu_2'\zeta\zeta')],$$

$$r_{17} = \tfrac{1}{8}[g_+(\nu_7 + \zeta + \nu_3\zeta' + \nu_4\zeta\zeta') - g_-\delta\delta'\beta\zeta(1 + \zeta' v') - 2g_\perp(\nu_1 + \nu_6'\zeta + \nu_6\zeta' + \nu_2\zeta\zeta')],$$

$$r_{18} = \tfrac{1}{8}[g_+(-\nu_7 + \zeta + \nu_3\zeta' - \nu_4\zeta\zeta') - g_-\delta\delta'\beta\zeta(1 - \zeta' v') - 2g_\perp(\nu_1 - \nu_6'\zeta - \nu_6\zeta' + \nu_2\zeta\zeta')],$$

$$r_{19} = \tfrac{1}{8}[g_+(\nu_7' + \nu_3'\zeta + \zeta' + \nu_4'\zeta\zeta') - g_-\delta\delta'\beta'\zeta'(1 + \zeta v) - 2g_\perp(\nu_1' + \nu_5'\zeta + \nu_5\zeta' + \nu_2'\zeta\zeta')], \tag{18}$$

and the coefficients d_i $(i = \overline{1, 10})$ are

$$d_{1,2} = 1 \pm \cos\vartheta \cos\vartheta', \tag{19}$$

$$d_3 = \sin\vartheta \sin\vartheta' \cos(\alpha - \alpha'), \tag{20}$$

$$d_4 = \sin\vartheta \sin\vartheta' \cos(\alpha + \alpha' - 2\varphi), \tag{21}$$

$$d_{5,6} = \cos\vartheta \pm \cos\vartheta', \tag{22}$$

$$d_{7,8} = \sin\vartheta \cos(\alpha - \varphi) \pm \sin\vartheta' \cos(\alpha' - \varphi), \tag{23}$$

$$d_{9,10} = \cos\vartheta \sin\vartheta' \cos(\alpha' - \varphi) \pm \cos\vartheta' \sin\vartheta \cos(\alpha - \varphi), \tag{24}$$

where $\vartheta(\vartheta')$ and $\alpha(\alpha')$ are the polar angle and the azimuthal angle of the incident (scattered) neutrino momentum, the unprimed (primed) quantities belong to the initial (final) electron,

$$\begin{gathered}\mu_1(\mu_1') = 1 \pm \sigma\sigma' v v', \qquad \mu_2(\mu_2') = v \pm v'\sigma\sigma', \qquad \mu_3(\mu_3') = v' \pm v\sigma\sigma', \qquad \mu_4(\mu_4') = \sigma\sigma' \pm v v', \\ \mu_5(\mu_5') = \sigma v \pm \sigma' v', \qquad \mu_6(\mu_6') = \sigma \pm \sigma' v v', \qquad \mu_7(\mu_7') = \sigma' \pm \sigma v v', \qquad \mu_8(\mu_8') = \sigma v' \pm \sigma' v, \\ \nu_1 = \sigma\beta, \qquad \nu_1' = \sigma'\beta', \qquad \nu_2 = \sigma'\beta, \qquad \nu_2' = \sigma\beta', \qquad \nu_3 = \sigma\sigma'\beta, \qquad \nu_3' = \sigma\sigma'\beta', \qquad \nu_4 = v'\beta, \\ \nu_4' = v\beta', \qquad \nu_5 = \sigma v\beta', \qquad \nu_5' = \sigma' v\beta', \qquad \nu_6 = \sigma v'\beta, \qquad \nu_6' = \sigma' v'\beta, \qquad \nu_7 = \sigma\sigma' v'\beta, \\ \nu_7' = \sigma\sigma' v\beta', \qquad \nu_8 = \beta\beta'\sigma, \qquad \nu_8' = \beta\beta'\sigma'.\end{gathered} \tag{25}$$

In (15) $d\Omega'$ is a solid angle element along the momentum of the scattered neutrino, e is the elementary electric charge, $f = f(E) = [e^{(E-\mu)/T} + 1]^{-1}$ is the Fermi-Dirac distribution of the initial electrons, $f' = f'(E')$ is the Fermi-Dirac distribution of the final electrons, μ is the electron chemical potential, T is the temperature of the matter,

$$\begin{gathered}g_\pm = g_V^2 \pm g_A^2, \qquad g_\perp = g_V g_A, \qquad v = p_z/p, \qquad v' = p_z'/p', \\ \delta = m_e/E, \qquad \delta' = m_e/E', \qquad p = (E^2 - m_e^2)^{1/2}, \qquad p' = (E'^2 - m_e^2)^{1/2}, \\ \beta = (1 - v^2)^{1/2}, \qquad \beta' = (1 - v'^2)^{1/2}, \qquad \sigma = (1 - \delta^2)^{1/2}, \qquad \sigma' = (1 - \delta'^2)^{1/2}.\end{gathered} \tag{26}$$

In the limiting case $E, E' \gg m_e$ averaging (summation) over the initial (final) electron polarization in (15) leads to a result similar to that obtained in [11].

In (15) the energy conserving delta function can be written in the form [11]

$$\delta(E' + \omega' - E - \omega) = \sum_i \frac{E_i E_i'}{|E_i' p_{zi} - E_i p_{zi}'|} \delta(p_z - p_{zi}'), \tag{27}$$

where E_i, E_i' satisfy the energy conservation law and p_{zi}, p_{zi}' satisfy the conservation law $p_z + k_z = p_z' + k_z'$. Taking into account (15), (16), and (27) the differential cross section of the neutrino-electron scattering in a magnetic field can be written as

$$\frac{d\sigma}{d\omega' d\Omega'} = \frac{G_F^2 e H \omega'^2}{32\pi^4} \sum_{n,n'=0}^{\infty} \sum_i \frac{E_i E_i'}{|E_i' p_{zi} - E_i p_{zi}'|} f(1 - f') R_0. \tag{28}$$

III. ANALYSES OF THE CROSS SECTION OF THE PROCESS $\nu + e^- \to \nu + e^-$

The analyses show that the differential cross section for neutrino-electron scattering is sensitive to spin variables of the initial and final electrons and to polar angles of incident and scattered neutrinos.

When the incident neutrino flies along the magnetic field direction ($\vartheta = 0$), we have the following expression for R_0:

$$\begin{aligned}R_0(\vartheta = 0) = {} & d_+(h_1 I_4^2 + h_2 I_3^2 + 2r_3 I_3 I_4) + 4d_- r_5 I_2^2 \\ & - 4d_0(h_3 I_2 I_4 + h_4 I_2 I_3),\end{aligned} \tag{29}$$

where

$$\begin{gathered}d_+ = 1 + \cos\vartheta', \qquad d_- = 1 - \cos\vartheta', \\ d_0 = \sin\vartheta' \cos(\alpha' - \varphi), \qquad h_1 = r_1 - 2r_{10}, \\ h_2 = r_2 + 2r_{11}, \qquad h_3 = r_{13} + r_{18}, \qquad h_4 = r_{15} - r_{19}.\end{gathered} \tag{30}$$

When both the incident and scattered neutrinos fly along the magnetic field direction, R_0 has the form

$$\begin{aligned}R_0(\vartheta = 0, \vartheta' = 0) = {} & 2(1 + v)(1 + v') \\ & \times [g_L^2(1 - \zeta)(1 - \zeta') I_4^2 \\ & + g_R^2(1 + \zeta)(1 + \zeta') I_3^2],\end{aligned} \tag{31}$$

where

$$\begin{aligned} g_L &= \tfrac{1}{2}(g_V + g_A) = \tfrac{1}{2} + \sin^2\theta_w, \\ g_R &= \tfrac{1}{2}(g_V - g_A) = \sin^2\theta_w \end{aligned} \tag{32}$$

for $\nu_e e^-$ scattering and

$$g_L = -\tfrac{1}{2} + \sin^2\theta_w, \qquad g_R = \sin^2\theta_w \tag{33}$$

for $\nu_\mu e^-$ and $\nu_\tau e^-$ scatterings [29].

When neutrinos scatter on electrons with right-hand circular polarization ($\zeta = 1$), we have

$$R_0(\vartheta = 0, \vartheta' = 0, \zeta = 1) = 4g_R^2(1+v)(1+v')(1+\zeta')I_3^2. \tag{34}$$

This expression shows that, if $\vartheta = 0$, $\vartheta' = 0$ and neutrinos scatter on electrons with right-hand circular polarization, the final electrons can only have right-hand circular polarization. When $\vartheta = 0$, $\vartheta' = 0$, and $\zeta = 1$, the differential cross section only depends on g_R and it is not sensitive to the neutrino flavor.

When neutrinos scatter on electrons with left-hand circular polarization ($\zeta = -1$), we have

$$R_0(\vartheta = 0, \vartheta' = 0, \zeta = -1) = 4g_L^2(1+v)(1+v')(1-\zeta')I_4^2. \tag{35}$$

The obtained expression shows that, if $\vartheta = 0$, $\vartheta' = 0$, and neutrinos scatter on electrons with left-hand circular polarization ($\zeta = -1$), the final electrons can only have left-hand circular polarization. When $\vartheta = 0$, $\vartheta' = 0$, and $\zeta = -1$ the differential cross section depends on g_L and therefore it is sensitive to the neutrino flavor. When $\zeta = -1$, the differential cross section is different for ν_e and $\nu_\mu(\nu_\tau)$. This result enables us to come to the conclusion that initial electrons with left-hand circular polarization in neutrino-electron scattering in a magnetic field can be used as a polarized target in neutrino detectors. The polarized electron targets in neutrino-electron scattering in a magnetic field can also be used for distinguishing neutrino flavor.

Now let us consider the situation of propagation of neutrinos against the magnetic field direction ($\vartheta = \pi$). In this case we have the following expression for R_0:

$$\begin{aligned} R_0(\vartheta = \pi) = {} & d_-(h_1' I_4^2 + h_2' I_3^2 + 2r_3 I_3 I_4) + 4d_+ r_4 I_1^2 \\ & - 4d_0(h_3' I_1 I_4 + h_4' I_1 I_3), \end{aligned} \tag{36}$$

where

$$\begin{aligned} h_1' &= r_1 + 2r_{10}, \qquad h_2' = r_2 - 2r_{11}, \\ h_3' &= r_{12} - r_{16}, \qquad h_4' = r_{14} + r_{17}. \end{aligned} \tag{37}$$

When both the incident and scattered neutrinos fly against the magnetic field direction, the expression for R_0 has a simple form

$$\begin{aligned} R_0(\vartheta = \pi, \vartheta' = \pi) = {} & 2(1-v)(1-v') \\ & \times [g_R^2(1+\zeta)(1+\zeta')I_4^2 \\ & + g_L^2(1-\zeta)(1-\zeta')I_3^2]. \end{aligned} \tag{38}$$

When $\vartheta = \pi$, $\vartheta' = \pi$ and neutrinos scatter on electrons with right-hand (left-hand) circular polarization, we have the following expressions for R_0:

$$R_0(\vartheta = \pi, \vartheta' = \pi, \zeta = 1) = 4g_R^2(1-v)(1-v')(1+\zeta')I_4^2 \tag{39}$$

and

$$R_0(\vartheta = \pi, \vartheta' = \pi, \zeta = -1) = 4g_L^2(1-v)(1-v')(1-\zeta')I_3^2. \tag{40}$$

The last two expressions show that, if $\vartheta = \pi$, $\vartheta' = \pi$, and neutrinos scatter on electrons with right-hand (left-hand) circular polarization, the final electrons can only have right-hand (left-hand) circular polarization.

When $\vartheta = \pi$, $\vartheta' = \pi$, and $\zeta = 1$, the differential cross section only depends on g_R and it is not sensitive to the neutrino flavor. When $\vartheta = \pi$, $\vartheta' = \pi$, and $\zeta = -1$, the differential cross section depends on g_L and therefore it is sensitive to the neutrino flavor. In this case the differential cross section is different for ν_e and $\nu_\mu(\nu_\tau)$.

So, spin asymmetry in neutrino-electron scattering in a magnetic field enables us to use electrons with left-hand circular polarization as polarized electron targets for distinguishing neutrino flavor and for detection of neutrinos.

The expressions (34) and (39) show that electrons with right-hand circular polarization are heated by ν_e, ν_μ, and ν_τ equally. However, it is derived from expressions (35) and (40) that electrons with left-hand circular polarization in neutrino-electron scattering in a magnetic field are heated by ν_e and $\nu_\mu(\nu_\tau)$ unequally. This fact leads to asymmetry in the heating of the stellar matter.

IV. ASYMMETRY OF HEATING AND NUMERICAL ESTIMATIONS

Generally, it is derived from expressions (17), (18), and (28) that the differential cross section of the process $\nu + e^- \to \nu + e^-$ is sensitive to the spin variables of both initial and final electrons. This could lead to anisotropies and asymmetry in the heating of matter (electron gas). It happens due to asymmetric energy transfer from neutrinos to matter. Asymmetry of the heating of electrons by neutrinos can be determined by the general expression

$$A = \frac{d\sigma_R - d\sigma_L}{d\sigma_R + d\sigma_L}, \tag{41}$$

where $d\sigma_R = d\sigma(\zeta = 1, \zeta' = 1)$ and $d\sigma_L = d\sigma(\zeta = -1, \zeta' = -1)$.

Let us consider two different types of an electron gas: the gas consisting of only the electrons having a left-hand circular polarization, and the gas consisting of only the electrons having a right-hand circular polarization. We also assume that these two types of an electron gas are not mixed and the initial temperatures of both of the gases are equal. After the scattering of initial neutrinos at the electrons, the gases will be heated at the expense of energy transfer from the neutrinos to the electrons. However, the gas consisting of only electrons having a left-hand circular polarization and the gas consisting of only electrons having a right-hand circular polarization will be heated differently: $T_L \neq T_R$. Here T_L is the temperature of the gas (after scattering) consisting of only electrons having a left-hand circular polarization and T_R is the temperature of the gas (after scattering) consisting of only the electrons having a right-hand circular polarization. Within these conditions, we obtain the following expression for the asymmetry in the heating:

$$A = \frac{R_{0R}h_R - R_{0L}h_L}{R_{0R}h_R + R_{0L}h_L}, \tag{42}$$

where

$$R_{0R} = R_0(\zeta = 1, \zeta' = 1), \tag{43}$$

$$R_{0L} = R_0(\zeta = -1, \zeta' = -1), \tag{44}$$

$$h_R = 1 - f'_R = \exp[(E' - \mu)/T_R]/\{\exp[(E' - \mu)/T_R] + 1\}, \tag{45}$$

$$h_L = 1 - f'_L = \exp[(E' - \mu)/T_L]/\{\exp[(E' - \mu)/T_L] + 1\}, \tag{46}$$

$f'_R = f'_R(E', T_R)$ is the Fermi-Dirac distribution of the final electrons (electron gas) having a right-hand circular polarization and the temperature T_R, $f'_L = f'_L(E', T_L)$ is the Fermi-Dirac distribution of the final electrons (electron gas) having a left-hand circular polarization and the temperature T_L.

For magnetars we can take $H \simeq 4.41 \times 10^{15}$ G and for relativistic electrons ($E, E' \gg m_e$) we suppose that $p_{zi} \simeq 0$, $p'_{zi} \simeq 0$. On the other hand, at temperatures $T \simeq 10^{11}$ K the characteristic energy for electrons is $E \simeq 8.5$ MeV. It is obtained from the formula $E = \sqrt{m_e^2 + 2eHn + p_z^2}$ that the energy $E \simeq 8.5$ MeV corresponds to the Landau quantum number $n = 1$. Let us consider the transition of $n = 1 \rightarrow n' = 2$. At $n' = 2$ and $H \simeq 4.41 \times 10^{15}$ G the energy of the final electrons is $E' \simeq 12$ MeV. On the other hand, it is known that the chemical potential of electron gas in a very strong magnetic field is determined as (see, e.g., [21])

$$\mu = \frac{2\pi^2 n_0}{eH} \tag{47}$$

or

$$\mu \simeq 25.68\ \text{MeV}\left(\frac{n_0}{10^{33}\ \text{cm}^{-3}}\right)\left(\frac{10^{15}\ \text{G}}{H}\right). \tag{48}$$

At the electron density $n_0 \sim 10^{33}$ cm^{-3} and $H \sim 10^{15}$ G (e.g., $H \simeq 2.15 \times 10^{15}$ G) we have $\mu \simeq E' \simeq 12$ MeV. It means that $(E' - \mu)/T_R \ll 1$, $(E' - \mu)/T_L \ll 1$ and the factors $h_R = h_L \simeq 1/2$. In this case the asymmetry of heating is determined as

$$A = \frac{R_{0R} - R_{0L}}{R_{0R} + R_{0L}}. \tag{49}$$

The analyses of the formula (42) enable us to come to the conclusion that, in general, the asymmetry of heating is sensitive to neutrino flavor, magnetic field strength H, energies E, E' (or the Landau quantum numbers n, n', and z components of the momenta) of initial and final electrons, electron chemical potential μ, the final temperature of the gas consisting of only electrons having a left (right)-hand circular polarization T_L (T_R), the polar angle of the incident (scattered) neutrino momentum ϑ (ϑ'), the difference between the azimuthal angles of the incident neutrino momentum and the scattered neutrino momentum $\alpha - \alpha'$, the angle φ, and the parameter

$$x = \frac{1}{2eH}[\omega^2 \sin^2\vartheta + \omega'^2 \sin^2\vartheta' - 2\omega\omega' \sin\vartheta \sin\vartheta' \cos(\alpha - \alpha')]. \tag{50}$$

So, taking into account the expression (50) we can come to the conclusion that the asymmetry of heating is sensitive to neutrino flavor, magnetic field strength, energies (or Landau quantum numbers and third components of the momentum) of the initial and final electrons, electron chemical potential, the final temperature of the gas consisting of only electrons having a left (right)-hand circular polarization, the polar angle of the incident (scattered) neutrino momentum, the difference between the azimuthal angles of the incident and scattered neutrino momenta, the angle φ, and the incident (scattered) neutrino energy.

In a protoneutron star, the gas consisting of electrons having a left-hand circular polarization and the gas consisting of electrons having a right-hand circular polarization will be in thermodynamic equilibrium after the scattering of neutrinos at electrons because in a protoneutron star the gas consisting of electrons having a left-hand circular polarization and the gas consisting of electrons having a right-hand circular polarization are mixed. It means that for the final temperature of the electron gas (after scattering) we have $T_L = T_R = T'$. In this case $h_R = h_L$ and the asymmetry in the heating is determined by the formula (49). Let us consider the case of $\vartheta = 0$, $\vartheta' = \pi/2$, $\alpha' = \varphi$ for numerical estimations. In this case $x = \omega'^2/(2eH)$ and the following expression is obtained for the asymmetry in the heating:

$$A = \frac{g_R^2 I_3^2 - g_L^2 I_4^2 - (g_L^2 - g_R^2) I_2^2 - 2(g_R^2 I_2 I_4 - g_L^2 I_2 I_3)}{g_R^2 I_3^2 + g_L^2 I_4^2 + (g_L^2 + g_R^2) I_2^2 - 2(g_R^2 I_2 I_4 + g_L^2 I_2 I_3)}. \tag{51}$$

If we consider the transition $n = 1 \rightarrow n' = 2$, we obtain the following simple expression for the asymmetry of heating:

$$A = \frac{g_R^2 - 2g_L^2(1-\frac{x}{2})^2 - \frac{1}{2}(g_L^2 - g_R^2)x - 2[g_R^2 x^{1/2}(\frac{x}{2}-1) + 2^{-1/2}g_L^2 x^{1/2}]}{g_R^2 + 2g_L^2(1-\frac{x}{2})^2 + \frac{1}{2}(g_L^2 + g_R^2)x - 2[g_R^2 x^{1/2}(\frac{x}{2}-1) - 2^{-1/2}g_L^2 x^{1/2}]}. \quad (52)$$

For magnetars ($H \simeq 4.41 \times 10^{15}$ G) and the neutrinos of energy $\omega' \simeq 1$ MeV we obtain $x \simeq 0.019$ and $A_{\nu_e e^-} \simeq -0.89$ for the $\nu_e e^- \to \nu_e e^-$ process. Numerical estimations show that for the considered case of the magnetic field strength and the neutrino energy $A_{\nu_\mu e^-} = A_{\nu_\tau e^-} \simeq -0.4$, i.e., $A_{\nu_e e^-} \simeq 2.23 A_{\nu_\mu e^-} = 2.23 A_{\nu_\tau e^-}$.

These estimations enable us to come to the conclusion that, within the considered kinematics, electrons (electron gas) having a left-hand circular polarization and electrons (electron gas) having a right-hand circular polarization are heated by neutrinos asymmetrically and the asymmetry of heating is sensitive to neutrino flavor, magnetic field strength, and scattered neutrino energy.

The effect of asymmetrical heating could contribute to asymmetry and anisotropy of the subsequent explosion of the outer layers of the collapsing stellar core. But this is a topic of a separate scientific paper.

In conclusion, we assess the possibility of observing the effects of an external magnetic field. When E, $E' \gg m_e$, $p_z = 0$ and the initial neutrino momentum is directed along the magnetic field, the influence of the magnetic field on this process is determined by the parameter [17,20]

$$\eta = \frac{1}{2}\frac{H}{H_0}\frac{m_e}{\omega}, \quad (53)$$

where $H \ll H_0$. The field effects are essential when $\eta \geq 1$. When $H \simeq 4.41 \times 10^8$ G (pulsed magnetic fields or effective fields in single crystals), for low-energy neutrinos ($\omega \sim 1 \div 10$ eV) or relic neutrinos ($\sim 10^{-4}$ eV) we obtain $\eta \geq 1$. In this case the field effects are essential and neutrinos can be detected.

V. CONCLUSIONS

The cross section of neutrino-electron scattering in a magnetic field is sensitive to the spin variable of initial and final electrons and to the direction of the momenta of incident and scattered neutrinos. When neutrinos travel either parallel or antiparallel to the magnetic field direction, there is a significant asymmetry in the cross section depending on the longitudinal polarization state. Spin asymmetries and field effects in neutrino-electron scattering in a magnetic field enable us to use initial electrons having a left-hand circular polarization as polarized electron targets in detectors for the detection of low-energy neutrinos or relic neutrinos and for distinguishing neutrino flavor. In general, the gas consisting of only electrons having a left-hand circular polarization and the gas consisting of only electrons having a right-hand circular polarization are heated by neutrinos asymmetrically. The asymmetry of heating is sensitive to neutrino flavor, magnetic field strength, energies (Landau quantum numbers and third components of the momentum) of the initial and final electrons, electron chemical potential, the final temperature of the gas consisting of only electrons having a left (right)-hand circular polarization, the polar angle of the incident (scattered) neutrino momentum, the difference between the azimuthal angles of the incident and scattered neutrino momenta, the angle φ, and the incident (scattered) neutrino energy.

In the heating process of electrons by neutrinos, the dominant role belongs to electron neutrinos compared with the contribution of muon neutrinos or tauon neutrinos. In neutrino-electron scattering in a magnetic field electrons having a right-hand circular polarization are heated by ν_e, ν_μ, and ν_τ equally when both the initial and final neutrinos fly along or against the magnetic field direction. Electrons having a left-hand circular polarization in neutrino-electron scattering in a magnetic field are heated by ν_e and $\nu_\mu(\nu_\tau)$ unequally when both the initial and final neutrinos fly along or against the magnetic field direction. For magnetars and neutrinos of 1 MeV energy, within the considered kinematics, electrons (electron gas) having a left-hand circular polarization and electrons (electron gas) having a right-hand circular polarization are heated by neutrinos asymmetrically and the asymmetry of heating is sensitive to neutrino flavor, magnetic field strength, and scattered neutrino energy. The asymmetry of heating in an electron neutrino-electron scattering is 2.23 times more than that one in a muon neutrino-electron scattering or in a tauon neutrino-electron scattering. The effect of asymmetrical heating of electrons by neutrinos could contribute to asymmetry and anisotropy of the subsequent explosion of the outer layers of the collapsing stellar core.

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