# Weak radiative corrections to the Drell-Yan process for large invariant mass of a dilepton pair

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The weak radiative corrections to the Drell-Yan process above the $Z$-peak have been studied. The compact asymptotic expression for the two heavy boson exchange—one of the significant contributions to the investigated process—has been obtained, the results expand in the powers of the Sudakov electroweak logarithms. At the quark level we compare the weak radiative corrections to the total cross section and forward-backward asymmetry with the existing results and achieve a rather good coincidence at $\sqrt{s} \gtrsim 0.5$ TeV. The numerical analysis has been performed in the high energy region corresponding to the future experiments at the CERN Large Hadron Collider (LHC). To simulate the detector acceptance we used the standard CMS detector cuts. It was shown that double Sudakov logarithms of the WW boxes are the dominant contributions in hadronic cross section. The considered radiative corrections are significant at high dilepton mass $M$ and change the dilepton mass distribution up to $\sim +3(-12)\%$ at the LHC energies and $M = 1(5)$ TeV.

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## I. INTRODUCTION

Despite the fact that the standard model (SM) for more than 20 years has been considered a consistent and experimentally confirmed theory, the search of new physics (NP) manifestations have still been continued. As the possible (and, at least, most popular) phenomena of NP, we can name the supersymmetry, the large extra dimensions [1], and extra neutral gauge bosons [2]. The light on this paramount problem of modern physics can be shed by the coming in the near future experiments at the collider LHC.

The experimental investigation of the continuum for the Drell-Yan production of a dilepton pair, i.e. data on the cross section and the forward-backward asymmetry of the reaction

$$pp \to \gamma, Z \to l^+ l^- X, \tag{1}$$

at large invariant mass of a dilepton pair (see Ref. [3] and references therein) is considered to be one of the powerful tool in the experiments at the LHC from the NP exploration standpoint. The cause of this is high sensitivity of the dilepton continuum to any modification of SM induced by NP. The research of the NP effects is impossible without the exact knowledge of the SM predictions including higher-order QCD and electroweak radiative corrections.

There have been numerous publications on this problem; let us now determine the place of the present paper (further we will discuss only the electroweak corrections, the QCD contributions are out of the scope of the present paper). So, all of the electroweak corrections to reaction (1) can be divided conventionally into three categories:

(i) (I) electroweak corrections induced by gauge boson self-energies (BSE),
(ii) (II) the other QED corrections (i.e. radiative corrections induced by at least one additional photon: virtual or real),
(iii) (III) the other weak radiative corrections (WRC) (i.e. radiative corrections induced by additional heavy bosons: $Z$ or $W$),

The first and the second contributions have already been studied (see papers on pure QED corrections [4], and the QED and electroweak corrections in the $Z$-peak region and above in Ref. [5], and numerous papers cited there). Naturally the contribution (II) also requires the consideration of the diagrams with real bremsstrahlung (to cancel the infrared divergence) and, therefore, causes the experimentally-motivated kinematical cuts for observed photon; all of that has been taken into account in paper [5].

To describe all contributions to electroweak corrections, it is convenient to use the terminology of the so-called electroweak Sudakov logarithms [6], i.e. the expressions which are growing with the scale of energy, and thus giving one of the main effects in the region of large invariant dilepton mass. By now extensive studies have been done in this area. For instance, the weak Sudakov expansion for general four-fermion processes has been studied in detail (see, for example, Ref. [7] and the recent paper [8] along with the extensive list of references therein). Let us note that the (I) contribution contains single Sudakov logarithms, and in the (II) contribution (in $Z\gamma$ boxes) there are also double Sudakov logarithms (DSL) but they are mutually cancelled out and reduced to single ones (see Sec. IV). Finally, the (III) contribution contains the DSL, and as it will be shown below, they predominate in the region $M \gg m_Z$. Besides, all of the contributions (I–III)

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contain zero power of Sudakov logs, i.e. terms without weak boson masses.

Obviously, the collinear logarithms of (II) QED radiative corrections can compete with the DSL in the investigated region. This important issue has been studied at the one-loop level in Ref. [5], where both the QED and weak corrections have been calculated for $M \leq 2$ TeV, but it has yet remained unsolved in the region of $M > 2$ TeV. Other important contribution in the investigated reaction at high invariant masses are the higher-order corrections (two-loop electroweak logarithms, at least), which also have been studied in the works [8,9] (see also the numerous papers cited therein). Weak boson emission contribution has been recently calculated in Ref. [10] and the contribution of higher-order corrections due to multiple photon emission has been computed in Ref. [11], these contributions are beyond the presented calculations.

As has already been mentioned, for the future experiments at LHC aimed at the searches of NP in the reaction (1) it is important to know exactly the SM predictions, including the radiative background, i.e. the processes, which are experimentally indistinguishable from reaction (1). The important task is the insertion of this background into the CMS Monte Carlo generators, which should be both accurate and fast. For them to be fast it is necessary to have a set of compact formulas for the WRC which have been obtained here. Though we use the asymptotic approach (AA) (the Sudakov logarithms as the parameters of expansion in the investigated region of large $M$), retaining the first and zero powers in the expansion we get good accuracy of calculation.

Thus, we further introduce basic notations (Sec. II), show how to calculate the cross section corresponding to the (III) and (I) contribution diagrams [see Fig. 1(b)–1(h)]: the heavy vertices (Sec. III), the heavy boxes (Sec. IV), and the boson self-energies (Sec. V), we compare our results with those available at the quark level, and we give the numerical estimations of weak radiative corrections to the Drell-Yan cross section in the region $0.5 \text{ TeV} \leq M < 10$ TeV (Sec. V).

## II. NOTATIONS AND THE BORN CROSS SECTION OF THE DRELL-YAN PROCESS

The Born cross section for the inclusive hadronic reaction $AB \to l^+ l^- X$ is given in the quark parton model by formula

$$\sigma_0(M, y, \zeta) = \frac{1}{3} \frac{2\pi\alpha^2}{SM} \operatorname{Re} \sum_{i,j=\gamma,Z} D^{is} D^{js*} \times \sum_{\chi=+,-} (t^2 + \chi u^2) \lambda_{l\chi}{}^{i,j} \times \sum_{q=u,d,s,\ldots} F_\chi^q(x_+, x_-) \lambda_{q\chi}{}^{i,j}. \quad (2)$$

Our notations are the following [see Fig. 1(a)]: $p_1$ is the four momentum of the first unpolarized (anti)quark with the flavor $q$ and mass $m_q$; $p_2$ is the four momentum of the second (anti)quark with the flavor $q$ and mass $m_q$; $k_1(k_2)$ is the four momentum of the final charged lepton $l^+(l^-)$ with the mass $m$; $q = k_1 + k_2$ is the four momentum of the $i$ boson with the mass $m_i$; $P_{A(B)}$ is the four momentum of initial nucleons A(B). The invariant mass of dilepton is $M = \sqrt{q^2}$. We use the standard set of Mandelstam invariants for the partonic elastic scattering $s$, $t$, $u$:

$$s = (p_1 + p_2)^2, \qquad t = (p_1 - k_1)^2, \qquad u = (k_1 - p_2)^2, \quad (3)$$

and the invariant $S = (P_A + P_B)^2$ for hadron scattering. The first summation runs over two possible intermediate bosons: $\gamma$, $Z$. The third summation runs over all contributing parton configurations. The number $1/3$ is a color factor, $\alpha$ is the electromagnetic fine structure constant. The propagator for $j$ boson has the form

$$D^{js} = \frac{1}{s - m_j^2 + i m_j \Gamma_j}, \quad (4)$$

where $\Gamma_j$ is the $j$-boson width. The combinations of parton density functions look like

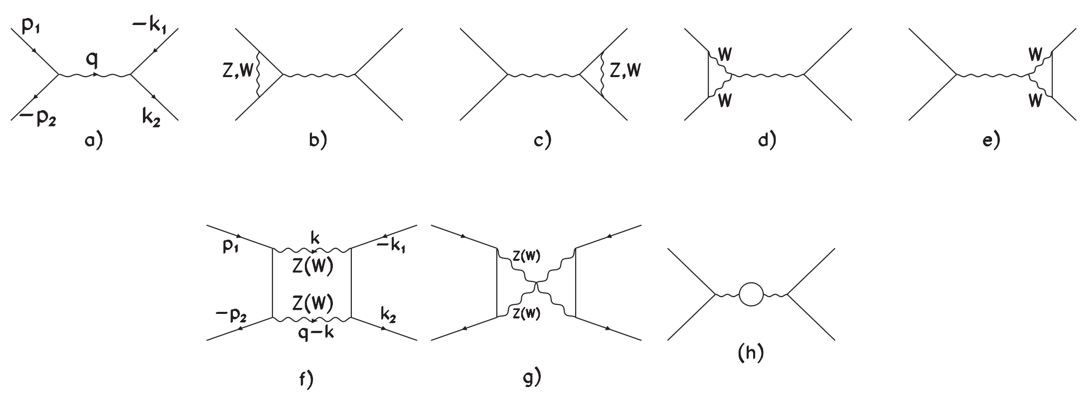


FIG. 1. Feynman graphs for the Born (a) and one-loop (b)–(h) diagrams with additional virtual heavy bosons corresponding to the WRC. Unsigned helix lines mean $\gamma$ or $Z$.

$$F_{\pm}^{q}(x_1, x_2) = f_q^A(x_1) f_{\bar{q}}^B(x_2) \pm f_{\bar{q}}^A(x_1) f_q^B(x_2), \tag{5}$$

where $f_q^H(x)$ is the probability of constituent $q$ with the fraction $x$ of the hadron's momentum in hadron $H$ finding.

The combinations of coupling constants for $f$-fermion with $i$- (or $j$-) boson have the form

$$\lambda_{f+}{}^{i,j} = v_f^i v_f^j + a_f^i a_f^j, \qquad \lambda_{f-}{}^{i,j} = v_f^i a_f^j + a_f^i v_f^j, \tag{6}$$

where

$$v_f^\gamma = -Q_f, \qquad a_f^\gamma = 0,$$
$$v_f^Z = \frac{I_f^3 - 2s_W^2 Q_f}{2s_W c_W}, \qquad a_f^Z = \frac{I_f^3}{2s_W c_W}, \tag{7}$$

$Q_f$ is the charge of fermion $f$, $I_f^3$ is the third component of the weak isospin of fermion $f$, and $s_W(c_W)$ is the sine (cosine) of the weak mixing angle: $s_W = \sqrt{1 - c_W^2}$, $c_W = m_W/m_Z$.

Reducing the phase space of reaction (1) to the variables $M$ and $y$—rapidity of dilepton (we need also the variable $\tau$ which is determined by $\tau^2 = q^2/S$) we get a three-, two-, and one-fold cross sections (without any experimental restrictions yet)

$$\sigma(M, y, \zeta) \equiv \frac{d^3\sigma}{dMdyd\zeta}, \tag{8}$$

$$\sigma(M, y) \equiv \frac{d^2\sigma}{dMdy} = \int_{-1}^{+1} d\zeta \sigma(M, y, \zeta), \tag{9}$$

and

$$\sigma(M) \equiv \frac{d\sigma}{dM} = \int_{-\log(\sqrt{S}/M)}^{+\log(\sqrt{S}/M)} dy \int_{-1}^{+1} d\zeta \sigma(M, y, \zeta), \tag{10}$$

here the invariants $s$, $t$, and $u$ mean

$$s = M^2, \qquad t = -\tfrac{1}{2}s(1-\zeta), \qquad u = -\tfrac{1}{2}s(1+\zeta), \tag{11}$$

$\zeta$ is cosine of the angle $\theta$ between $\vec{p}_1$ and $\vec{k}_1$ in the center mass system of hadrons ($\zeta = \cos\theta$) and the arguments of parton distribution functions in Eq. (2) have the form

$$x_{\pm} = \tau e^{\pm y}. \tag{12}$$

Integrating over the whole region of $\zeta$ we get much simpler expression

$$\sigma_0(M, y) = \frac{8\pi\alpha^2 M^3}{9S} \operatorname{Re} \sum_{i,j=\gamma,Z} D^{is} D^{js*} \lambda_{l+}{}^{i,j} \times \sum_{q=u,d,s,\ldots} F_+^q(x_+, x_-) \lambda_{q+}{}^{i,j}. \tag{13}$$

## III. HEAVY VERTICES

To construct the contribution of heavy vertices (HV) to the radiative corrections (see Fig. 1(b)–1(e)), we used the t'Hooft-Feynman gauge and an on-mass renormalization scheme which uses $\alpha$, $m_W$, $m_Z$, Higgs boson mass $m_H$, and the fermion masses as independent parameters. Appropriate (for ultrarelativistic limit we are interested in) results can be taken from Ref. [12], where renormalized gauge boson fermion vertices for on shell fermions have been obtained. These results are presented as the form factor set to the Born vertices, so we can easily use them to construct the cross section; hence we replace the coupling constants in the Born vertex for the corresponding form factors:

$$v_f^j \to F_V^{jf}, \qquad a_f^j \to F_A^{jf}. \tag{14}$$

Then the cross section of heavy vertices contribution will look like

$$\sigma_V(M, y, \zeta) = \frac{4\pi\alpha^2}{3SM} \operatorname{Re} \sum_{i,j=\gamma,Z} D^{is} D^{js*} \sum_{\chi=+,-} (t^2 + \chi u^2) \times \sum_{q=u,d,s,\ldots} F_\chi^q(x_+, x_-) \times (\lambda_{q\chi}^{Fi,j} \lambda_{l\chi}{}^{i,j} + \lambda_{q\chi}{}^{i,j} \lambda_{l\chi}^{Fi,j}), \tag{15}$$

where the combinations of couplings constants and form factors are

$$\lambda_{f+}^{Fi,j} = F_V^{if} v_f^j + F_A^{if} a_f^j, \qquad \lambda_{f-}^{Fi,j} = F_V^{if} a_f^j + F_A^{if} v_f^j, \qquad (f = q, l). \tag{16}$$

Electroweak form factors $F_{V,A}^{if}$ in ultrarelativistic limit have the form (there is the perfect coincidence with the formulas of Appendix C of Ref. [13]):

$$\begin{aligned}
F_V^{\gamma l} &= \frac{\alpha v_l^\gamma}{4\pi}\left[((v_l^Z)^2 + (a_l^Z)^2)\Lambda_2(m_Z) + \frac{3}{4s_W^2}\Lambda_3(m_W)\right],\\
F_A^{\gamma l} &= \frac{\alpha v_l^\gamma}{4\pi}\left[2v_l^Z a_l^Z\Lambda_2(m_Z) + \frac{3}{4s_W^2}\Lambda_3(m_W)\right],\\
F_V^{\gamma d} &= \frac{\alpha v_d^\gamma}{4\pi}\left[((v_d^Z)^2 + (a_d^Z)^2)\Lambda_2(m_Z) - \frac{1}{2s_W^2}\Lambda_2(m_W) + \frac{9}{4s_W^2}\Lambda_3(m_W)\right],\\
F_A^{\gamma d} &= \frac{\alpha v_d^\gamma}{4\pi}\left[2v_d^Z a_d^Z\Lambda_2(m_Z) - \frac{1}{2s_W^2}\Lambda_2(m_W) + \frac{9}{4s_W^2}\Lambda_3(m_W)\right],\\
F_V^{\gamma u} &= \frac{\alpha v_u^\gamma}{4\pi}\left[((v_u^Z)^2 + (a_u^Z)^2)\Lambda_2(m_Z) - \frac{1}{8s_W^2}\Lambda_2(m_W) + \frac{9}{8s_W^2}\Lambda_3(m_W)\right],\\
F_A^{\gamma u} &= \frac{\alpha v_u^\gamma}{4\pi}\left[2v_u^Z a_u^Z\Lambda_2(m_Z) - \frac{1}{8s_W^2}\Lambda_2(m_W) + \frac{9}{8s_W^2}\Lambda_3(m_W)\right].
\end{aligned} \tag{17}$$

$$\begin{aligned}
F_V^{Zl} &= \frac{\alpha}{4\pi}\left[v_l^Z((v_l^Z)^2 + 3(a_l^Z)^2)\Lambda_2(m_Z) + \frac{1}{8s_W^3c_W}\Lambda_2(m_W) - \frac{3c_W}{4s_W^3}\Lambda_3(m_W)\right],\\
F_A^{Zl} &= \frac{\alpha}{4\pi}\left[a_l^Z(3(v_l^Z)^2 + (a_l^Z)^2)\Lambda_2(m_Z) + \frac{1}{8s_W^3c_W}\Lambda_2(m_W) - \frac{3c_W}{4s_W^3}\Lambda_3(m_W)\right],\\
F_V^{Zd} &= \frac{\alpha}{4\pi}\left[v_d^Z((v_d^Z)^2 + 3(a_d^Z)^2)\Lambda_2(m_Z) + \frac{1-2Q_us_W^2}{8s_W^3c_W}\Lambda_2(m_W) - \frac{3c_W}{4s_W^3}\Lambda_3(m_W)\right],\\
F_A^{Zd} &= \frac{\alpha}{4\pi}\left[a_d^Z(3(v_d^Z)^2 + (a_d^Z)^2)\Lambda_2(m_Z) + \frac{1-2Q_us_W^2}{8s_W^3c_W}\Lambda_2(m_W) - \frac{3c_W}{4s_W^3}\Lambda_3(m_W)\right],\\
F_V^{Zu} &= \frac{\alpha}{4\pi}\left[v_u^Z((v_u^Z)^2 + 3(a_u^Z)^2)\Lambda_2(m_Z) - \frac{1+2Q_ds_W^2}{8s_W^3c_W}\Lambda_2(m_W) + \frac{3c_W}{4s_W^3}\Lambda_3(m_W)\right],\\
F_A^{Zu} &= \frac{\alpha}{4\pi}\left[a_u^Z(3(v_u^Z)^2 + (a_u^Z)^2)\Lambda_2(m_Z) - \frac{1+2Q_ds_W^2}{8s_W^3c_W}\Lambda_2(m_W) + \frac{3c_W}{4s_W^3}\Lambda_3(m_W)\right].
\end{aligned} \tag{18}$$

Here it is necessary to note that in the on-mass renormalization scheme the self-energies of $u$-quarks' diagrams also give a nonzero contribution to the cross section. However, the results for $u$-self-energy are factorized in the same manner as the vertices. It gives us a possibility to sum both contributions in one formula, where the $u$-self-energy contribution is completely cancelled with the corresponding terms of heavy vertices. So, it could be said that the formula (15) gives the cross section induced by both the heavy vertices and self-energy of $u$ quarks.

Exact expressions for $\Lambda_{2,3}$ can be found in Ref. [12], and there we give the real part of expressions for functions $\Lambda_{2,3}(m_i)$ that provide a good accuracy in large dilepton mass region (see Sec. I):

$$\Lambda_2(m_i) = \frac{\pi^2}{3} - \frac{7}{2} - 3l_{i,s} - l_{i,s}^2, \qquad \Lambda_3(m_i) = \frac{5}{6} - \frac{1}{3}l_{i,s}. \tag{19}$$

The Sudakov logarithm that we denote as $l_{i,x}$ has the form

$$l_{i,x} = \log\frac{m_i^2}{|x|}, \qquad (i = Z, W; x = s, t, u). \tag{20}$$

## IV. HEAVY BOXES

The calculation of a two heavy boson (box) contribution is more complicated procedure since it requires the integration of four-point functions with the complex masses in unlimited from above kinematical region of invariants. The difficulties of such a procedure have been pointed out as far back as a pioneer paper, Ref. [14]. Fortunately there is a way to avoid many troubles with the integration all of the terms in the box contribution using a rather simple algebra. Let us explain this way.

First of all we construct the ZZ-box cross section for $q\bar{q} \to l^+l^-$ (see diagrams in Fig. 1(f) and 1(g),) using the standard Feynman rules:

$$d\sigma_{ZZ} = -\frac{4\alpha^3}{\pi s}d\Gamma_q \,\mathrm{Re}\frac{i}{(2\pi)^2}\int d^4k \sum_{k=\gamma,Z} D^{ks*}(D^{ZZ} + C^{ZZ}), \tag{21}$$

here the two-particle phase space element is read

$$d\Gamma_q = \frac{d^3k_1}{2k_1^0}\frac{d^3k_2}{2k_2^0}\delta(q-p_1-p_2),$$

and $D^{ZZ}(C^{ZZ})$ is the direct (crossed) diagram contribution.

Neglecting the fermion masses we present the direct contribution in the form:

$$D^{ZZ} = \Pi^D_{ZZ}\,\mathrm{Tr}[\gamma^\alpha \hat{p}_2\gamma_\mu(\hat{p}_1-\hat{k})\gamma_\nu\rho_q^{ZZ,k}(p_1)] \times \mathrm{Tr}[\gamma_\alpha\hat{k}_2\gamma^\mu(\hat{k}-\hat{k}_1)\gamma^\nu\rho_l^{ZZ,k}(k_1)], \tag{22}$$

where the integrand of four-point scalar function looks like

$$\Pi^D_{ZZ} = \frac{1}{((q-k)^2-m_Z^2)(k^2-m_Z^2)(k^2-2k_1k)(k^2-2p_1k)}. \tag{23}$$

Combinations of the density matrices $\rho(p)$ and the coupling constants can be reduced to the production of $\lambda$-factors [see formulas (6)] and $\hat{p}\equiv\gamma_\mu p^\mu$ (here $p = p_1, p_2, k_1, k_2$, or $k$)

$$\rho_f^{ZZ,k}(p) = (v_f^{ZZ}-a_f^{ZZ}\gamma_5)\rho(p)(v_f^k+a_f^k\gamma_5) = \tfrac{1}{2}(\lambda_{f+}^{ZZ,k}-\lambda_{f-}^{ZZ,k}\gamma_5)\hat{p}, \tag{24}$$

$$v_f^{ZZ} = (v_f^Z)^2+(a_f^Z)^2, \qquad a_f^{ZZ} = 2v_f^Z a_f^Z. \tag{25}$$

To extract the part of cross section which predominates in the region $|x|\gg m_Z^2$ [see $x$ in formula (20)] we should make the equivalent transformation of the cross section based on the close connection of infrared divergency cross section terms and the Sudakov leading-log terms:

$$D^{ZZ} = (D^{ZZ}|_{k\to 0}+D^{ZZ}|_{k\to q}) + (D^{ZZ}-D^{ZZ}|_{k\to 0}-D^{ZZ}|_{k\to q}) = D_1^{ZZ}+D_2^{ZZ}. \tag{26}$$

The integral over $k$ of the first term in Eq. (26) is

$$\frac{i}{(2\pi)^2}\int d^4kD_1^{ZZ} = \frac{4t}{q^2-m_Z^2}B^{ZZ,k}\frac{i}{(2\pi)^2}\int\frac{d^4k}{k^2-m_Z^2} \times\left[\frac{1}{(k^2-2k_1k)(k^2-2p_1k)} + \frac{1}{(k^2-2k_2k)(k^2-2p_2k)}\right], \tag{27}$$

here we used the trivial correlation typical for the Born cross section

$$B^{ZZ,k} = \tfrac{1}{2}\mathrm{Tr}[\gamma^\alpha\hat{p}_2\gamma_\mu\rho_q^{ZZ,k}(p_1)]\,\mathrm{Tr}[\gamma_\alpha\hat{k}_2\gamma^\mu\rho_l^{ZZ,k}(k_1)] \approx b_+^{ZZ,k}t^2+b_-^{ZZ,k}u^2, \tag{28}$$

where

$$b_\pm^{n,k} = \lambda_{q+}{}^{n,k}\lambda_{l+}{}^{n,k} \pm \lambda_{q-}{}^{n,k}\lambda_{l-}{}^{n,k}. \tag{29}$$

Then we integrate one of the terms in Eq. (27) over $k$ using the standard method of Ref. [14] (another term is integrated similarly):

$$\begin{aligned}\frac{i}{(2\pi)^2}&\int\frac{d^4k}{(k^2-m_Z^2)(k^2-2k_2k)(k^2-2p_2k)}\\ &= \frac{1}{4}\int_0^1 dx\int_0^x\frac{dy}{m_Z^2(x-y)+(p_2(1-x)+k_2y)^2} =\\ &= \frac{1}{4}\int_0^1 dx\frac{1}{t(x-1)-m_Z^2}\log\frac{t(x-1)}{m_Z^2}\\ &= -\frac{1}{4t}\left(\frac{\pi^2}{3}+\frac{1}{2}\log^2\frac{-t-m_Z^2}{m_Z^2}+\mathrm{Li}_2\frac{m_Z^2}{m_Z^2+t}\right),\end{aligned} \tag{30}$$

here $\mathrm{Li}_2$ denotes the Spence dilogarithm. Retaining the terms which are proportional to the second ($\sim l_{i,x}^2$), first ($\sim l_{i,x}^1$) and zero ($\sim l_{i,x}^0 = 1$) power of the Sudakov logarithms and thereby neglecting the terms which are inessential in the region of large dilepton mass $|x|\gg m_Z^2$ we get the asymptotic expression

$$\frac{i}{(2\pi)^2}\int d^4kD_1^{ZZ} \approx -\frac{1}{s}B^{ZZ,k}\left(\frac{2\pi^2}{3}+l_{Z,t}^2\right). \tag{31}$$

Surely in these (and subsequent) expressions we can retain only the leading $\sim l_{i,x}^2$ term as it has been done, for example, in Ref. [15]. Naturally in such an approximation our results coincide with the results of that paper. However, we are able to retain the first and zero powers of expansion leading-log parameter and we will use this opportunity to improve the accuracy of radiative correction estimation (one remark more—we retain the $l_{i,x}^1$ and $l_{i,x}^0$ also in the heavy vertices part).

The asymptotic expression (at $|x|\gg m_Z^2$) for the second term of the formula (26) can be presented as

$$D_2^{ZZ} \approx D^{ZZ}|_{\Pi^D_{ZZ}\to\Pi^D_{\gamma\gamma}} - 4\Pi^D_{\gamma\gamma}\frac{(q-k)^2+k^2}{q^2}tB^{ZZ,k}. \tag{32}$$

Reducing the vectorial and tensor integrals to the scalar ones we get

$$\begin{aligned}\frac{i}{(2\pi)^2}\int d^4kD_2^{ZZ} \approx{}& -2b_-^{ZZ,k}\left[(G_0^m+G_0^M)(t^2+u^2) - 2(R-N_0)u - X_0t\frac{t^2+u^2}{s}\right]\\ &- 4b_+^{ZZ,k}t^2\left[G_0^m+G_0^M-X_0\frac{t}{s}\right],\end{aligned} \tag{33}$$

where scalar integrals are

$$
\begin{aligned}
G_0^m &= \frac{i}{(2\pi)^2}\int\frac{d^4k}{(k^2-2k_1k)k^2(k-q)^2}\\
&= -\frac{1}{8q^2}\Big(\log^2\frac{q^2}{m^2}+\frac{\pi^2}{3}\Big),\\
G_0^M &= \frac{i}{(2\pi)^2}\int\frac{d^4k}{(k^2-2p_1k)k^2(k-q)^2}\\
&= -\frac{1}{8q^2}\Big(\log^2\frac{q^2}{m_q^2}+\frac{\pi^2}{3}\Big),\\
R &= \frac{i}{(2\pi)^2}\int\frac{d^4k}{k^2(k-q)^2}=\frac{1}{4}\Big(\log\frac{q^2}{L}-1\Big),\\
N_0 &= \frac{i}{(2\pi)^2}\int\frac{d^4k}{(k^2-2k_1k)(k^2-2p_1k)}\\
&= \frac{1}{4}\Big(\log\frac{m^2}{L}-1-\log\frac{m}{m_q}+\log\frac{|2k_1p_1|}{mm_q}\Big),\\
X_0 &= \frac{i}{(2\pi)^2}\int d^4k\frac{2k(q-k)}{(k^2-2k_1k)(k^2-2p_1k)k^2(k-q)^2}\\
&= -\frac{1}{8p_1k_1}\Big(\frac{1}{2}\log\frac{|2p_1k_1|m^2}{q^4}\log\frac{|2p_1k_1|}{m^2}\\
&\quad+\frac{1}{2}\log\frac{|2p_1k_1|m_q^2}{q^4}\log\frac{|2p_1k_1|}{m_q^2}-\frac{\pi^2}{3}\Big).
\end{aligned}
\tag{34}
$$

These integrals have been calculated on the analogy with the corresponding ones in paper of J. Kahane [16] (with the important distinctions due to different reaction channel: $t$ channel in Ref. [16], and $s$ channel in this paper). The expressions (34) contain the parameters of two sorts: masses of fermions and $L$ parameter regulating the ultraviolet divergence, both of them are completely cancelled out in the final expression

$$
\frac{i}{(2\pi)^2}\int d^4kD_2^{ZZ}\approx b_-^{ZZ,k}l_su+(b_-^{ZZ,k}(t^2+u^2)+2b_+^{ZZ,k}t^2)\times\frac{1}{2s}l_s^2,\qquad l_s=\log\frac{s}{|t|}. \tag{35}
$$

The contribution of crossed part [Fig. 1(g)] to the cross section can be calculated on the analogy with the direct one. Besides, there is an interesting symmetry between the direct and crossed parts (see, for example, Ref. [5]), namely

$$
C^{ZZ}=-D^{ZZ}|_{t\leftrightarrow u,b_+^{ZZ,k}\leftrightarrow b_-^{ZZ,k}}. \tag{36}
$$

And now let us present the total final expression for the contribution of ZZ box into cross section of the Drell-Yan process at large invariant dilepton mass:

$$
\begin{aligned}
\sigma_{ZZ}(M,y,\zeta)=\frac{2\alpha^3}{3SM}\,\mathrm{Re}\sum_{k=\gamma,Z}D^{ks*}\sum_{q=u,d,s,\ldots}&[f_q^A(x_+)f_{\bar q}^B(x_-)(\delta^{ZZ,k}(t,u,b_+,b_-)-\delta^{ZZ,k}(u,t,b_-,b_+))\\
&+f_{\bar q}^A(x_+)f_q^B(x_-)(\delta^{ZZ,k}(u,t,b_+,b_-)-\delta^{ZZ,k}(t,u,b_-,b_+))],
\end{aligned}
\tag{37}
$$

where

$$
\delta^{ZZ,k}(t,u,b_+,b_-)=\frac{B^{ZZ,k}}{s}\Big(\frac{2\pi^2}{3}+l_{Z,t}^2\Big)-b_-^{ZZ,k}ul_s-(b_-^{ZZ,k}(t^2+u^2)+2b_+^{ZZ,k}t^2)\frac{l_s^2}{2s}. \tag{38}
$$

Integration over $\zeta$ of the formula (38) gives

$$
\begin{aligned}
\int_{-1}^{+1}\delta^{ZZ,k}(t,u,b_+,b_-)d\zeta&=\int_{-1}^{+1}\delta^{ZZ,k}(u,t,b_+,b_-)d\zeta\\
&=\frac{s}{9}\Big(12\lambda_{q+}^{ZZ,k}\lambda_{l+}^{ZZ,k}\Big(\frac{2\pi^2}{3}+l_{Z,s}^2\Big)+(22b_-^{ZZ,k}+4b_+^{ZZ,k})l_{Z,s}+27b_-^{ZZ,k}\Big)
\end{aligned}
\tag{39}
$$

(it is convenient to use it for the construction of twofold cross sections $\sigma_{ZZ,WW}(M,y)$).

To obtain the WW-box contribution into the Drell-Yan cross section using the expressions (37) and (38) one should: (1) do the trivial substitution $Z\to W$ in all indices of coupling constants and boson masses, (2) take into consideration that some quark diagrams with two $W$ bosons are forbidden by the charge conservation law. Then WW-box contribution will look like

$$
\begin{aligned}
\sigma_{WW}(M,y,\zeta)=\frac{2\alpha^3}{3SM}\,\mathrm{Re}\sum_{k=\gamma,Z}D^{ks*}\Big(&\sum_{q=u,c}[f_q^A(x_+)f_{\bar q}^B(x_-)\delta^{WW,k}(t,u,b_+,b_-)+f_{\bar q}^A(x_+)f_q^B(x_-)\delta^{WW,k}(u,t,b_+,b_-)]\\
&-\sum_{q=d,s,b}[f_q^A(x_+)f_{\bar q}^B(x_-)\delta^{WW,k}(u,t,b_-,b_+)+f_{\bar q}^A(x_+)f_q^B(x_-)\delta^{WW,k}(t,u,b_-,b_+)]\Big).
\end{aligned}
\tag{40}
$$

This second feature of WW boxes explains the domination of WW contribution into the Drell-Yan cross section in comparison to ZZ contribution (see the section on numerical analysis). The point is that the ZZ-box cross section is proportional to the difference

$$\delta^{\mathrm{ZZ,k}}(t, u, b_+, b_-) - \delta^{\mathrm{ZZ,k}}(u, t, b_-, b_+) \sim l^2_{Z,t} - l^2_{Z,u}$$
$$= \log\frac{u}{t}(l^1_{Z,t} + l^1_{Z,u}), \quad (41)$$

(see also Ref. [15]) i.e. it $\sim l^1_{Z,x}$, whereas the leading parts of the WW cross section do not contain the difference (41) and are proportional to $l^2_{W,x}$. Let us note here that the factorization property (41) is absent in heavy vertex part and is present in infrared finite part of $\gamma Z$-box contribution, which can be easily obtained on analogy with ZZ box using

$$\delta^{\gamma Z,k}(t, u, b_+, b_-) = \frac{B^{\gamma Z,k}}{s}\left(\frac{2\pi^2}{3} + l^2_{Z,t}\right) - 2b^{\gamma Z,k}_{-}ul_s - (b^{\gamma Z,k}_{-}(t^2 + u^2) + 2b^{\gamma Z,k}_{+}t^2)\frac{l^2_s}{s}. \quad (42)$$

We attribute the $\gamma Z$ box to the "other QED corrections" [see (II) contribution in Sec. I] and do not consider it here any more.

## V. DISCUSSION OF NUMERICAL RESULTS

### A. Numerical estimation at the parton level and comparison with the available results

Now we discuss the observable quantities at the parton level. The reason of it is twofold: to study the characteristics of the WRC in the absence of parton distribution functions and (what is much more important) to compare them with the available results. Except our paper [17], where the WRC to Drell-Yan production have been calculated by different methods: exact integration, numerical integration over Feynman parameters [18], and the AA considered here, there is the pioneer paper [5] and the calculation of SANC group [19].

At the parton level there is a possibility to compare the results with Ref. [5], since in this paper the set of relative non-QED corrections has been presented at high parton center of mass energies for the total cross section (Fig. 3 of Ref. [5]) and for the parton forward-backward asymmetry (Fig. 4 of Ref. [5]). Further we present our variant of corresponding WRC in the [5] region (up to 1 TeV) and in the region of higher energies (up to 10 TeV). For the correct and tuned comparison we used in this subsection the same set of the SM parameters which was used in Ref. [5] and all the rest prescriptions of this paper. We also add the contribution of boson self-energies to the numerical estimation [see Fig. 1(h)]

$$d\sigma^{q\bar{q}}_{\mathrm{BSE}} = -\frac{8\alpha^2}{s}d\Gamma_q\Big[\sum_{i,j=\gamma,Z}\Pi^i D^{is}D^{js*} \times \sum_{\chi=+,-}\lambda^{i,j}_{q\chi}\lambda^{i,j}_{l\chi}(t^2 + \chi u^2) + \Pi^{\gamma Z}D^{Zs} \times \sum_{i=\gamma,Z}D^{js*}\sum_{\chi=+,-}(\lambda^{\gamma,j}_{q\chi}\lambda^{Z,j}_{l\chi} + \lambda^{Z,j}_{q\chi}\lambda^{\gamma,j}_{l\chi}) \times (t^2 + \chi u^2)\Big]. \quad (43)$$

Here $\Pi^{\gamma,Z,\gamma Z}$ are connected with the renormalized photon-, $Z$-, and $\gamma Z$ self-energies [12] as

$$\Pi^\gamma = \frac{\hat{\Sigma}^\gamma}{s}, \qquad \Pi^Z = \frac{\hat{\Sigma}^Z}{s - m_Z^2}, \qquad \Pi^{\gamma Z} = \frac{\hat{\Sigma}^{\gamma Z}}{s}.$$

Thus, Fig. 2 shows the relative corrections to the total cross sections (the left picture) and forward-backward asymmetry at the parton level (the right picture) for $u\bar{u} \to \mu^+\mu^-$ as the functions of $\sqrt{s}$. As in Ref. [5], for $e^+e^-$ final

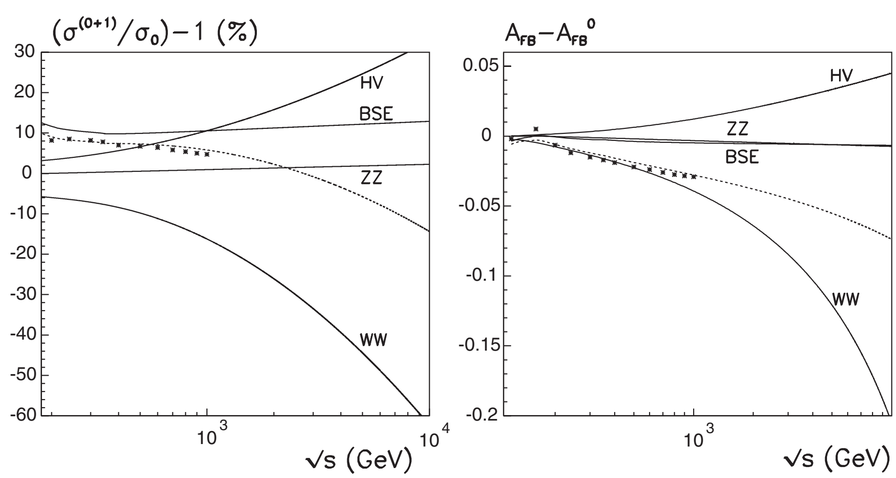


FIG. 2. The relative corrections to the total cross section (the left picture) and the forward-backward asymmetry at the parton level (the right picture) for $u\bar{u} \to \mu^+\mu^-$ as a functions of $\sqrt{s}$. The symbols BSE, HV, ZZ, WW denote the boson self-energy, heavy vertex, ZZ-box, and WW-box contributions correspondingly. The dashed line corresponds to the sum of all contributions. The asterisks are the points taken from Fig. 3 and Fig. 4 of paper [5].

state we obtain the identical results. The distinction between the $d\bar{d}$ and $u\bar{u}$ scattering is the same as in Ref. [5], therefore we do not present the graphs for $d\bar{d}$ case here. The symbols BSE, HV, ZZ, WW in Fig. 2 denote the boson self-energy, heavy vertex, ZZ-box, WW-box contributions correspondingly, the dashed line corresponds to the sum of all contributions. The asterisks are the points taken from Fig. 3 and 4 of paper [5].

A rather good coincidence with the presented results and the results available from Ref. [5] takes place: we can see in Fig. 2 almost the same scale and behavior of relative corrections (the decrease with energy, the convexity and $t$-quark threshold kink) and the tendency of the results to converge with the increase of energy. We remind that the approaches in both calculations are not quite similar. Although for the calculation of the BSE and HV parts here and in Ref. [5] the same BSE insertions and form factors have been used, but the heavy box contribution has been calculated in different ways: via the direct four-point functions integration in Ref. [5], and using asymptotic property of heavy boxes expressed through the Sudakov logarithms powers in this paper. It is obvious that the difference of the corrections in different approaches strongly depends on the scale of energy: the asymptotic Sudakov expansion works well at large values of $\sqrt{s}$ only. Figure 2 shows that at $\sqrt{s} = 1$ TeV this difference is rather small both for total $u\bar{u}$ cross section ( $\sim 1.5\%$) and for quark forward-backward asymmetry ( $\sim 0.001$) and with the increase of $\sqrt{s}$ is found to become still smaller. It is a pity, but in Ref. [5] the parton observables in the region $\sqrt{s} > 1$ TeV have not been investigated.

To compare our results with the available in the region of high invariant masses $\sqrt{s} > 1$ TeV we contacted to SANC Group [19] and to the authors of Ref. [5] (see also program ZGRAD [20]). Having used the same sets of electroweak parameters we got excellent coincidence for all WRC (see Table I), which shows the same (as in Fig. 2) corrections (in per cents) to $u\bar{u}$ cross section as a functions of $\sqrt{s}$ in the region 0.1 TeV $\leq \sqrt{s} \leq$ 10 TeV.

Let us analyze the ZZ box: we can see that in the region of not so large values of $\sqrt{s}$ (0.1–0.2 TeV) the results are rather different (although the corrections hold the similar behavior and scale), but starting with $\sqrt{s} \sim 0.5$ TeV the difference is small and tends to zero with the growth of $\sqrt{s}$. Of course, this fact confirms the precision of both calculations and estimates the region where the asymptotic approach suggested here works well. I suppose this point is 0.5 TeV, here the relative error of asymptotic (to exact SANC) calculation is $\sim$0.1161, whereas at 1 TeV (2 TeV) the error is $\sim$0.0284 ( $\sim$ 0.0056).

There is no special need to compare separately the direct and crossed parts of the ZZ-box correction (the same way as WW box) containing DSL, since having the good agreement for the whole ZZ box—which is proportional merely to the first power of Sudakov logarithms [see Eq. (41)]—we will obtain the agreement of the same order. Nevertheless, we compare the results for the direct WW box: for the point 0.5 TeV the relative error of asymptotic (to SANC) calculation is $\sim$0.0620, and at 1 TeV (2 TeV) this error is $\sim$0.0251 ( $\sim$ 0.0076). Table I also shows good agreement for the HV part of the radiative correction: the first HV column is the result of SANC, the second one is obtained using the formulas of Sec. III and exact expressions for $\Lambda_{2,3}$ from Ref. [12], and the third column corresponds to the approximation (19). We can see a very good coincidence of the first and second columns and a rather good agreement between the first and third columns; the relative error is 0.040 at $\sqrt{s} = 1$ TeV and $8.44 \times 10^{-4}$ at $\sqrt{s} = 10$ TeV. For the BSE part the agreement is not so good as for the HV part; there is a small constant difference $\sim$0.5% obviously caused by the use of various gauges.

Let us pass to the numerical analysis of hadron observables.

### B. Numerical estimation for future CMS program

Futher the scale of weak radiative corrections and their effect on the observables of the Drell-Yan processes for future CMS experiments will be discussed. We used the following set of electroweak parameters (such as in Ref. [20]): $\alpha = 1/137.035\,999\,11$, $m_W = 80.37399$ GeV, $m_Z = 91.1876$ GeV, the energy of LHC $\sqrt{S} = 14$ TeV and the CTEQ6 set of unpolarized parton distribution functions (PDF) [21] (with the choice $Q^2 = M^2$).

TABLE I. The relative corrections (in percents) to the ZZ, WW, HV, and BSE cross sections at the parton level for $u\bar{u} \rightarrow \mu^+\mu^-$ as functions of $\sqrt{s}$, calculated by different groups: SANC [19], program ZGRAD [20] and using the results of this paper.

| $\sqrt{s}$, TeV | ZZ [19] | ZZ [20] | ZZ, AA | WW [19] | WW, AA | HV [19] | HV, $\Lambda$ | HV, (19) | BSE [19] | BSE (43) |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.1 | −0.0186 | | 0.0683 | −0.329 | −1.2690 | −1.5549 | | 2.2972 | 6.0777 | 8.1119 |
| 0.2 | −0.0908 | −0.0907 | −0.0073 | −3.107 | −4.8739 | −1.6883 | −1.6688 | 2.4481 | 11.2259 | 12.2144 |
| 0.5 | −0.2144 | −0.2145 | −0.1895 | −10.777 | −10.1087 | 4.2958 | 4.2978 | 4.0943 | 11.1526 | 11.9455 |
| 1.0 | −0.3346 | −0.3346 | −0.3251 | −16.998 | −16.5720 | 6.2447 | 6.2451 | 5.9910 | 12.2096 | 12.9793 |
| 2.0 | −0.4638 | −0.465 | −0.4612 | −25.442 | −25.2497 | 8.5534 | 8.5535 | 8.4247 | 13.1993 | 13.9634 |
| 3.0 | −0.5423 | −0.543 | −0.5410 | −31.468 | −31.3554 | 10.1709 | 10.1710 | 10.0935 | 13.7682 | 14.5314 |
| 5.0 | −0.6432 | −0.643 | −0.6416 | −40.185 | −40.1306 | 12.4894 | 12.4895 | 12.4512 | 14.4811 | 15.2437 |
| 10.0 | −0.7787 | −0.779 | −0.7782 | −53.989 | −53.9695 | 16.1160 | 16.1160 | 16.1024 | 15.4456 | 16.2080 |

We impose the experimental restriction conditions on the detected lepton angle $-\zeta^* \leq \zeta \leq \zeta^*$ and on the rapidity $|y(l)| \leq y(l)^*$. For CMS detector the cut values of $\zeta^*$ and $y(l)^*$ are determined as

$$y(l)^* = -\log\ \tan\frac{\theta^*}{2} = 2.4, \qquad \zeta^* = \cos\theta^* \approx 0.9837. \tag{44}$$

Then the expression (10) with consideration of the experimental restrictions (44) and the use of theta functions can be modified to

$$\sigma(M) = \int_{-\log(\sqrt{S}/M)}^{+\log(\sqrt{S}/M)} dy \int_{-\zeta^*}^{\zeta^*} d\zeta \sigma(M, y, \zeta)\theta(\cos\alpha + \zeta^*) \times \theta(-\cos\alpha + \zeta^*), \tag{45}$$

where $\alpha$ is the scattering angle of the lepton with the 4-momenta $k_2$ ($\alpha = \widehat{\vec{p}_1, \vec{k}_2}$) in the center mass system of hadrons. This angle has a nontrivial relation with $\theta$ and $y$:

$$\alpha = \pi - \arccos\frac{\cos\theta - f}{\sqrt{1 + f^2 - 2f\cos\theta}} - \arcsin\frac{f\sin\theta}{\sqrt{1 + f^2 - 2f\cos\theta}}, \tag{46}$$

$$f = \frac{e^{2y} - 1}{e^{2y} + 1}.$$

Let us note here that the second standard CMS restriction $p_T(l) \geq 20$ GeV in the region of large $M$ considered here and at Eq. (44) is satisfied completely and automatically. It can be seen in Fig. 3 where all of the CMS experimental restrictions for $M = 1$ TeV have been shown.

In Fig. 4 the relative corrections are presented

$$\delta_M^C = \frac{\sigma_C(M)}{\sigma_0(M)}, \tag{47}$$

as functions of $M$ in the region 0.5 TeV $\leq M \leq$ 10 TeV with experimental restrictions of the CMS. Index "$C$" in all numerical analysis means

$$C = \text{ZZ, WW, HV, BSE, tot}; \qquad \text{tot} = \text{ZZ} + \text{WW} + \text{HV} + \text{BSE}. \tag{48}$$

It can be seen that the BSE, HV, and WW contributions are most significant, and the WW boxes give the dominant (negative) contribution. In Fig. 4 we can also see the significant contribution of zero power of Sudakov logs ($\sim l_{W,x}^0$); the difference "WW (DSL only)"—"WW (total)" is ~4% and depends weakly on $M$. Big contribution of $l_{W,x}^0$ justifies the detailed calculations which have been done in Sec. IV. The 4% difference is caused mainly by the $2\pi^2/3$ term in Eq. (31), the contribution of $D_2^{WW}$ (32) is far less than 1%. The total WRC effect is significant: in the region $M < 1.5$ TeV is positive, since the sum of positive HV and BSE contributions exceeds the negative WW boxes, but for $M > 2$ TeV the total correction is definitely negative and decreases with the increase of $M$. Thus, the $\delta_M^{\text{tot}}$ correction changes the dilepton mass distribution up to $\sim +3(-12)\%$ at $M = 1(5)$ TeV.

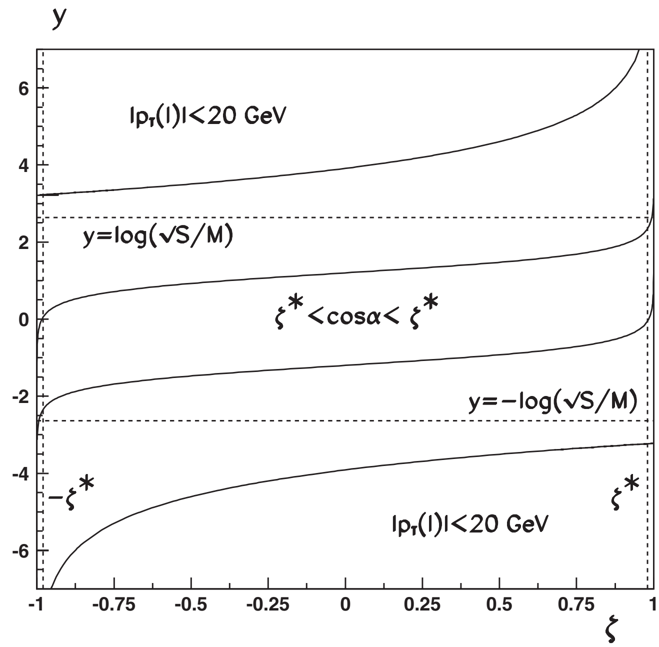


FIG. 3. The region of the cross section integration over $\zeta$ and $y$ taking into consideration the CMS experimental cuts at $\sqrt{S} =$ 14 TeV and $M = 1$ TeV.

Here we should note that the importance of the investigated problem demands the cross-checking of this part

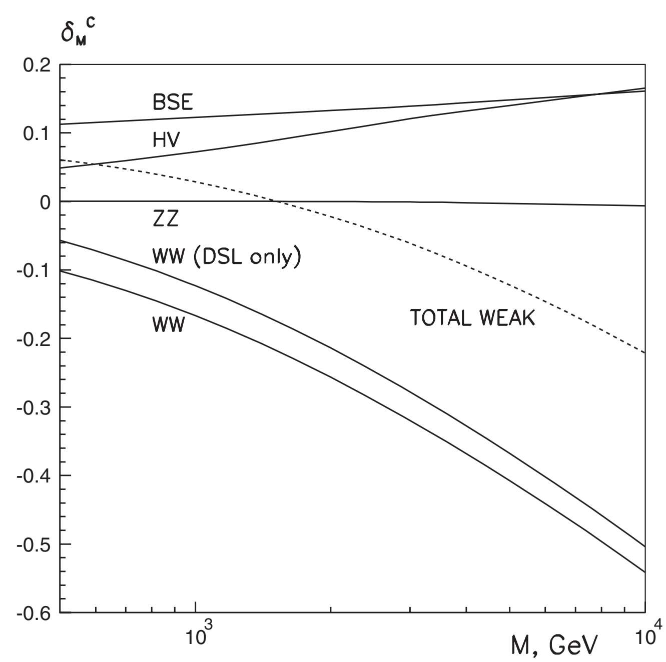


FIG. 4. The relative corrections $\delta_M^C$ corresponding to the ZZ, WW, HV, BSE contributions with experimental restrictions of the CMS as a functions of $M$. The dashed line corresponds to the sum of all the WRC contributions.

with the earlier obtained in Ref. [5]. The direct comparison between numerical results of the two works is not possible, since in Ref. [5] the effective Born approximation has been used to calculate the relative corrections. We do not use the effective Born approximation as the main subject of our investigation is the region of large invariant mass ($M \geq 1$ TeV) where this approach is illegal. Fortunately, the recent paper [10] provides the possibility to compare the results of Ref. [5] and presented by us in the region of high invariant dilepton mass (see the first picture in Fig. 7 of Ref. [10]). At first glance the results look absolutely different: for $M = 1(2)$ TeV the relative correction of Ref. [10] is $\sim -5\%(-11\%)$, whereas our result is $\sim +3\%(-2\%)$. However, the solid line in Fig. 7 of Ref. [10] corresponds to total one-loop result and to compare correctly we should add the photon corrections: soft and hard photon contributions. We have our own code for calculation of this part (FORTRAN program READY—"Radiative corrEctions to lArge invariant mass Drell-Yan process," which is supposed to be published in the near future, for more details see Ref. [22]). Main features of READY are the same as in Ref. [5]: using the soft–hard separator $\omega$ and independence of total result on it, lepton identification cuts reduce extremely the hard FSR contribution and so on. For the decision of quark mass singularity problem we used the procedure of linearization [19]. The soft part, which we should add to pure WRC for comparison with Ref. [10], is large and negative (for example, at $M = 1$ TeV, $\omega = 10$ GeV, and $l = e$ the FSR part of soft relative correction defined by formulas (72), (73) in Ref. [17] equals $-40.7\%$. The positive hard FSR part suppressed by lepton identification requirements will be far less, and the rest parts (ISR after quark mass removing, and the mass-independing interference) will be small as compared to the FSR. Hence, we are supposed to have a rather good (at least qualitative) agreement with hadron estimations of Ref. [10]; the relative correction will be negative and will decrease with $M$ in region $M \gtrsim 0.5$ TeV. Unfortunately the READY is not quite complete and here we can not present more exact numbers, but we hope future publications of the results of code READY will clarify this problem later.

In the end of this section we want to discuss the effect of the PDFs on the weak radiative corrections in this very high mass regime, and what uncertainty they introduce. Comparing the results for CTEQ6 set [21] and MRST 2004-QED set [23] of PDFs the essential effect of PDF choice for the cross sections has not been noticed, the relative error is on one per cent level and has no tendency to increase with the increase of $M$, at least at $M \sim 3.16$ TeV (unfortunately MRST 2004-QED set for $Q^2 > 10^7$ GeV, i.e. for $M > 3.16$ TeV, does not work, and it does not allow us to compare the result for both sets of PDF in the region of extremely high invariant masses).

## VI. CONCLUSIONS

The weak radiative corrections with the neutral current to the Drell-Yan process for large invariant mass of a dilepton pair have been studied. The compact asymptotic expressions have been obtained, which expand in the powers of the Sudakov electroweak logarithms. At the parton level the investigated radiative corrections have been compared with the existing results and a rather good coincidence at energy $\gtrsim 0.5$ TeV has been found. The numerical analysis in the high energy region has been performed with use of the standard CMS cuts. It has been found that the considered radiative corrections become very significant at high dilepton mass, and their large scale does not permit us to neglect this part of the radiative correction procedure in future experiments at LHC directed on the NP research in the Drell-Yan process. Some part of the corrections has become beyond the scope of the presented paper (the QCD corrections, the pure QED corrections, and the two-loop electroweak logarithms), and still requires a thorough analysis in order for us to completely solve the radiative corrections problem to the Drell-Yan process at extremely high energies.

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