Baryon octet magnetic moments to all orders in flavor breaking: An application to the problem of the strangeness in the nucleon

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Using the general QCD parametrization (GP) we display the magnetic moments of the octet baryons including all flavor breaking terms to any order. The hierarchy of the GP parameters allows to estimate a parameter g_0 related to the quark loops contribution of the proton magnetic moment; its magnitude is predicted to be inside a comparatively small interval including the value given recently by Leinweber *et al.* from a lattice QCD calculation.

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I. INTRODUCTION

The $s\bar{s}$ contribution $\mu_p^s = -\frac{1}{3}G_M^s(0)$ to the proton magnetic moment (compare, for the experimental situation, Refs. [1,2]) has been the object of many QCD calculations leading to a variety of results. A list of references is found in Table 3 of the review of Beck-McKeown [3] and also in recent papers by Leinweber *et al.* [4]. Although, as we shall see, our results are compatible with the most recent ones of Leinweber *et al.* [4], it is known that QCD (either in the chiral or in the lattice approaches) has produced in the course of the years a variety of predictions for $G_M^s(0)$, differing in magnitude by a factor larger than 10 (and also differing in sign).

To treat this problem we use here the general QCD parametrization (GP) [5–8] of the magnetic moments of the baryons, including (for the first time) terms of all orders in flavor breaking. Exploiting a dynamical property of the GP, its "hierarchy" of the parameters [9], we can estimate the order of magnitude of a parameter g_0 related to a part of the s-quark loops contribution of the proton magnetic moment (g_0 is $-3O_N$ in the notation of Ref. [4]). The value of g_0 is predicted inside a rather narrow interval that includes the value obtained by Leinweber *et al.* [4] by a lattice QCD calculation.[1]

II. THE GENERAL QCD PARAMETRIZATION OF THE MAGNETIC MOMENTS

Consider the operator

$$\mathbf{M} = (1/2)\int d^3\boldsymbol{r}(\boldsymbol{r}\times \boldsymbol{j}(\boldsymbol{r})), \tag{1}$$

where the current

$$j_\mu(x) = e\bar{q}(x)Q\gamma_\mu q(x) \tag{2}$$

is expressed in terms of the quark (u, d, s) fields $q(x)$ and of a charge-diagonal flavor matrix Q. The GP parametrizes (compare [5,7]—see, in particular, the footnote 14 in [7]) the expectation value of $\mathbf{M}$ (Eq. (1)) on the exact eigenstate $|\psi_B\rangle$ of the QCD Hamiltonian for the baryon B at rest as:

$$\begin{aligned}\langle\psi_B|\mathbf{M}|\psi_B\rangle &= \langle\Phi_B|V^\dagger \mathbf{M} V|\Phi_B\rangle \\ &= \langle W_B|\sum_\nu g_\nu \mathbf{G}_\nu(\boldsymbol{\sigma}, f)|W_B\rangle \\ &\equiv \langle W_B|\mathbf{G}(\boldsymbol{\sigma}, f)|W_B\rangle. \end{aligned}\tag{3}$$

The symbols in the above equations were used repeatedly in previous work [5–8], but, for convenience, we recall part of them again: Thus $|\Phi_B\rangle$ is an *auxiliary* three quarks-no gluon state—having simple properties (e.g. $L = 0$); V is a QCD unitary transformation leading from the auxiliary state $|\Phi_B\rangle$ to the *exact* QCD eigenstate $|\psi_B\rangle$ ($|\psi_B\rangle = V|\Phi_B\rangle$). In Eq. (3) the space coordinates are eliminated in the second step, as described in [5]; the $\mathbf{G}_\nu$'s are operators, linear in the matrix Q, depending only on the spin $\boldsymbol{\sigma}$ and flavor f of three quarks; $|W_B\rangle$ are the usual spin (1/2)-flavor octet states (with spin up); the g_ν's are parameters independent of Q, the same for the magnetic moments ($Q = Q^\gamma = \mathrm{Diag}[2/3, -1/3, -1/3]$) and the Z-moments ($Q = Q^Z \propto \mathrm{Diag}[1-(8/3)\sin^2\theta_W, -1+(4/3)\sin^2\theta_W, -1+(4/3)\sin^2\theta_W]$).

In the limit of exact $SU(3)$ ($m_u = m_d = m_s$) $\mathbf{G}(\boldsymbol{\sigma}, f)$ is the sum of only three possible operators. Writing: $\mathbf{G}_1 = \sum_i Q_i\boldsymbol{\sigma}_i$, $\mathbf{G}_3 = \sum_{i\neq k} Q_i\boldsymbol{\sigma}_k$, it is:

$$\mathbf{G}(\boldsymbol{\sigma}, f) = g_1\mathbf{G}_1 + g_3\mathbf{G}_3 + \hat{g}_0\,\mathrm{Tr}[Q]\sum_i \boldsymbol{\sigma}_i. \tag{4}$$

In $\mathbf{G}_1$, $\mathbf{G}_3$ the index i on Q_i arises from the γ (or Z) being attached to a quark line with that index. The meaning of a quark line and its index in the exact state $|\psi_B\rangle$ is illustrated in Fig. 1 of Ref. [9].[2]

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[1]Recently [10], applying the GP to this problem, we related the key formula of Beck-McKeown [3] for $\mu_p^s \equiv -(1/3)G_M^s$, to the quark loops corresponding to the Z-trace terms. Note: In our notation the negative charge of s is not included in the definition of $G_M^s(0)$.

[2]We recall that in the figure just cited the boxes describe the effect of the transformation V in Eq. (3) on the three quarks in $|\Phi_B\rangle$. A quark line may zigzag inside the box as much as one likes, due to emission and reabsorption of gluons; also each box contains quark loops and gluon lines connected in all possible ways consistent with the prescriptions of QCD.

The *Trace* terms (e.g. the last term in Eq. (4) and others to appear in the following sections) can be due only to quark loops; each loop is connected to the other quark lines by gluons; because the quark loops associated to *Trace* terms including a charge must have on their contour an incoming γ (or Z), the number of gluons connecting these loops to the other quark lines must be at least three, due to color conservation and the Furry theorem (see, e.g., Sec. 4 of [11]). From Eq. (4), the contribution to the proton magnetic moment from the s-quarks would be $(-1/3)\hat{g}_0$, but, in the electromagnetic case, this s-quark contribution is cancelled by those of the u and d quark loops, due to

$$\mathrm{Tr}[Q^\gamma] = 0. \tag{5}$$

The symmetry breaking due to the heavier mass of s (we work here assuming charge symmetry $m_u = m_d \equiv m$) is represented in the GP by terms containing an s projector $P^s = \mathrm{Diag}[0, 0, 1]$ or the product of two such projectors with different indices (products of more than two P^s with different indices do not contribute to matrix elements with octet states). Below we list all possible P^s structures.

III. FLAVOR BREAKING IN THE GP

A. Terms with one P^s

The terms with one P^s to be considered in this subsection were often indicated in the past as first order flavor breaking; but it was *underlined* since the beginning [5] that, because $P_i^s \equiv (P_i^s)^n$ for any n, such terms contained also contributions of higher order in flavor breaking ($\Delta m/m_s \equiv (m_s - m)/m_s$).

There are five operators with one indexed P^s (see [5,7]): They are: $\mathbf{G}_2 = \sum_i Q_i P_i^s \boldsymbol{\sigma}_i$; $\mathbf{G}_4 = \sum_{i\neq k} Q_i P_i^s \boldsymbol{\sigma}_k$; $\mathbf{G}_5 = \sum_{i\neq k} Q_k P_i^s \boldsymbol{\sigma}_i$; $\mathbf{G}_6 = \sum_{i\neq k} Q_i P_k^s \boldsymbol{\sigma}_i$; $\mathbf{G}_7 = \sum_{i\neq j\neq k} Q_i P_j^s \boldsymbol{\sigma}_k$. Of course they act only on the strange baryons. There is also a *Trace* term containing one P^s (that acts, clearly, also on p and n):

$$\mathbf{G}_0 = g_0 \,\mathrm{Tr}[QP^s] \sum_i \boldsymbol{\sigma}_i. \tag{6}$$

The magnetic moments μ_B for the octet baryons B are:

$$\mu_B = \langle \psi_B | (\mathbf{M})_z | \psi_B \rangle = \langle W_B | \sum_{\nu=0}^{7} g_\nu (\mathbf{G}_\nu)_z | W_B \rangle. \tag{7}$$

In Eqs. (6) and (7) g_0 is associated to the flavor breaking s-quark contribution to the proton moment, so that

$$G_M^s(0) = \hat{g}_0 + g_0 \tag{8}$$

(multiplied by $-1/3$) is the whole s-contribution to μ_p that one is measuring in the γ-Z interference experiments [1,2].

At this stage one has 8 parameters and 8 data for the octet (7 magnetic moments plus the $\Sigma\Lambda$ transition moment) but the expectation values of the $\mathbf{G}_\nu$'s ($\nu = 0, \ldots, 7$) in the $|W_B\rangle$'s are not independent (see the identity in Eq. (8) of [8]). Thus [10] it is impossible to derive the strangeness content of the nucleon only from the octet-baryons magnetic moments without exploiting some additional property of QCD.

We now list the flavor breaking structures with two (or more) P^s.

B. Terms with two (or more) P^s

To proceed we consider the following flavor breaking structures with two P^s (they act only on Ξ^0 and Ξ^-):

$$\begin{aligned}
\mathbf{G}_{7b} &= \sum_{i\neq k} Q_i P_i^s P_k^s \boldsymbol{\sigma}_i,\\
\mathbf{G}_{7c} &= \sum_{i\neq k} Q_i P_i^s P_k^s \boldsymbol{\sigma}_k,\\
\mathbf{G}_8 &= \sum_{i\neq j\neq k (j>k)} Q_i P_j^s P_k^s \boldsymbol{\sigma}_i,\\
\mathbf{G}_9 &= \sum_{i\neq j\neq k} Q_i P_k^s P_j^s \boldsymbol{\sigma}_k,\\
\mathbf{G}_{10} &= \sum_{i\neq j\neq k} Q_i P_i^s P_k^s \boldsymbol{\sigma}_j.
\end{aligned} \tag{9}$$

We have also four *Trace* structures of higher order:

$$\begin{aligned}
\mathbf{G}_0^a &= \mathrm{Tr}[QP^s] \sum_i P_i^s \boldsymbol{\sigma}_i,\\
\mathbf{G}_0^b &= \mathrm{Tr}[QP^s] \sum_{i\neq k} P_i^s \boldsymbol{\sigma}_k,\\
\mathbf{G}_0^c &= \mathrm{Tr}[QP^s] \sum_{i\neq k} P_i^s P_k^s \boldsymbol{\sigma}_k,\\
\mathbf{G}_0^d &= \mathrm{Tr}[QP^s] \sum_{i\neq j\neq k (i>j)} P_i^s P_j^s \boldsymbol{\sigma}_k.
\end{aligned} \tag{10}$$

Using Eq. (3) with $Q = Q^\gamma$, one gets the magnetic moments as linear combinations of the parameters g_ν's. Because the nine parameters multiplying the structures in Eqs. (9) and (10) intervene only in a few combinations,[3] it is convenient to define:

$$\begin{aligned}
g_\Xi &= \tfrac{2}{9}(-2g_{7b} - 2g_{7c} - g_8 + 4g_9 + g_{10}),\\
g_0^\Lambda &= g_0 + g_0^a, \qquad g_0^\Sigma = g_0 - g_0^a/3 + 4g_0^b/3,\\
g_0^\Xi &= g_0 + 4g_0^a/3 + 2g_0^b/3 + 4g_0^c/3 - g_0^d/3.
\end{aligned} \tag{11}$$

We get finally (compare Eq. (12)) the magnetic moments to all orders in flavor breaking (in Eq. (12) the baryon symbol indicates the magnetic moment). The contributions of the $1P^s$ terms (see, e.g. [8]) are obtained by omitting in (12) g_8, g_9 and the terms with g_Ξ, g_0^Λ, g_0^Σ, g_0^Ξ defined in Eq. (11)

[3] On using $Q_i P_i^s = -(1/3) P_i^s$ and $Tr(QP^s) = -(1/3)$ many operators in (9) and (10) reduce to the same, or to some already considered (e.g. $\mathbf{G}_0^a \equiv \mathbf{G}_2$). One may ask why these structures are listed separately; the reason is that they correspond to different QCD diagrams; only displaying them separately, the order of magnitude of their parameters can be estimated by the hierarchy—see Sec. IV.

above:

$$\begin{aligned}
p &= g_1 - g_0/3,\\
n &= -(2/3)(g_1 - g_3) - g_0/3,\\
\Lambda &= -(1/3)(g_1 - g_3 + g_2 - g_5) - g_0^{\Lambda}/3,\\
\Sigma^+ &= g_1 + (1/9)(g_2 - 4g_4 - 4g_5 + 8g_6 + 8g_7)\\
&\quad - g_0^{\Sigma}/3,\\
\Sigma^- &= -(1/3)(g_1 + 2g_3) + (1/9)(g_2 - 4g_4 + 2g_5\\
&\quad - 4g_6 - 4g_7) - g_0^{\Sigma}/3,\\
\Xi^0 &= -(2/3)(g_1 - g_3) + (2/9)(-2g_2 - g_4 + 2g_5\\
&\quad - 4g_6 + 5g_7) + g_{\Xi} - g_0^{\Xi}/3,\\
\Xi^- &= -(1/3)(g_1 + 2g_3) + (2/9)(-2g_2 - g_4 - 4g_5\\
&\quad - g_6 - g_7 + \tfrac{3}{2}g_8 - 6g_9) + g_{\Xi} - g_0^{\Xi}/3,\\
\mu(\Sigma\Lambda) &= -\tfrac{1}{\sqrt{3}}(g_1 - g_3 + g_6 - g_7). \qquad (12)
\end{aligned}$$

From Eqs. (11) and (12) one gets Eq. (13), also QCD exact to all orders in flavor breaking. Note that $\mu(\Sigma\Lambda)$ can have an imaginary part [5]; in principle this might contribute, via other terms, to the $\Sigma^0 \to \gamma\Lambda$ decay *rate*. But, because all *open* $\Sigma^0 \to \Lambda$ channels are γ channels, such terms would imply additional γ's and are therefore negligible,

$$\sqrt{3}\mu(\Sigma\Lambda) = n - (3/2)\Lambda - (\Sigma^+ + \Sigma^-)/4 + \Xi^0 - g_{\Xi} + (4/9)g_0^c - (1/9)g_0^d. \qquad (13)$$

Equation (13) (barring the higher order parameters g_{Ξ}, g_0^c, g_0^d) is the Okubo relation [12].

From the previous formulas one can calculate the contribution of each specific flavor $f = (u, d, s)$ to the baryon magnetic moments and compare the resulting equations with Eqs. (4) and (5) of Ref. [4], obtained exactly from QCD by Leinweber *et al.* using only u, d charge symmetry.[4] The contribution of the flavor f to the magnetic moment of B (except for the loops and to be still multiplied by the charges) is indicated as f^B similarly to Ref. [4]. In the GP treatment u^p, u^n, etc. are calculated as follows: Replace the matrix Q in Eq. (2) with P^f. We then obtain for u^p, u^n, etc. the results[5]:

$$\begin{aligned}
u^p &= (4/3)g_1 + (2/3)g_3,\\
u^n &= (-1/3)g_1 + (4/3)g_3,\\
u^{\Sigma^+} &= (4/3)g_1 + (2/3)g_3 + (-2/3)g_5 + (4/3)g_6\\
&\quad + (4/3)g_7,\\
u^{\Xi^0} &= (-1/3)g_1 + (4/3)g_3 + (4/3)g_5 + (-2/3)g_6\\
&\quad + (4/3)g_7 + (-1/3)g_8 + (4/3)g_9. \qquad (14)
\end{aligned}$$

We easily find using Eqs. (12) and (14) the relations:

$$u^{\Sigma^+} = \Sigma^+ - \Sigma^-; \qquad u^{\Xi^0} = \Xi^0 - \Xi^-, \qquad (15)$$

$$u^p = 2p + n + g_0; \qquad u^n = p + 2n + g_0 \qquad (16)$$

that are exact consequences of QCD (plus u, d charge symmetry) since the GP displays and takes into account all the operators $\mathbf{G}_\nu$ compatible with QCD. Because of the entirely different derivation, it is expected and satisfactory that Eqs. (15) and (16) lead to the same equations and therefore to the same constraint (Fig. 2 of [4]) between the ratios u^p/u^{Σ} and u^n/u^{Ξ} as those of [4]. This is seen by inserting in Eqs. (16) the expression (17) for g_0. To summarize: Because Eqs. (15) and (16) follow exactly from QCD plus charge symmetry, the general parametrization developed above—that includes exactly *all possible terms consistent with QCD*, plus charge symmetry—must lead necessarily to the above equations. Note, by the way, that some papers (e.g. [14]) misinterpreted the GP as some model "a little more sophisticated than the simplest constituent quark model."

The connection of our g_0, $\hat{g}_0$ (Eqs. (4), (6), and (8)) with the symbols in [4] is:

$$g_0 = -3O_N = -G_M^s(0)(1 - {}^lR_d^s)/{}^lR_d^s, \qquad (17)$$

$${}^lR_d^s = 1 + g_0/\hat{g}_0. \qquad (18)$$

IV. THE HIERARCHY OF THE COEFFICIENTS IN THE GENERAL QCD PARAMETRIZATION

Everything, so far, is an exact consequence of QCD (plus u, d charge symmetry). Now we turn to the determination of g_0. As already noted [10] its value cannot be determined from the magnetic moments only. One has to exploit a dynamical property of QCD, "the hierarchy of the parameters," as anticipated at the end of Sec. I. The hierarchy is an empirical property, noted long ago [9] and used in past work (e.g. [13,15]); below we recall only its main points. Although the hierarchy was discussed in detail in the references cited above, for convenience we will recall briefly (at the end of this section) how it works in the first case to which it was applied (the extension of the Gell Mann-Okubo mass formula to second order flavor breaking).

[4]Charge symmetry for u and d has always been part of our general QCD parametrization, although, of course, in some cases—e.g. for the explanation of the extraordinary level of precision of the Coleman-Glashow electromagnetic baryon mass formula [13]—its applicability was examined in detail (see Sec. 4 of [13]).

[5]To obtain (referring to the u quarks) the full contribution μ_B^u to the magnetic moment of baryon B, one must add to u^B the loop contributions—for the proton simply $\hat{g}_0$—and multiply the result by the u-charge Q_u (for p and n see Eqs. (20) and (21) of Ref. [10])

It is a fact that the coefficients g_ν of the various structures (the "parameters") are seen to decrease with increasing complexity of the structure. This was first seen in the GP of the **8** + **10** baryon masses, but applies generally. It appears from the previous analysis that each baryon property (e.g. the magnetic moments) is QCD parametrized as a sum of structures each being a sum of terms having a maximum of three different indices (as an example, see the structure $\mathbf{G}_8 = \sum_{i\neq j\neq k(j>k)} Q_i P_j^s P_k^s \boldsymbol{\sigma}_i$); these indices are appended to a quark charge, or to a P^s or to a $\boldsymbol{\sigma}$ in the term under consideration. If the rules of Ref. [9] are adopted to normalize univocally the sums, the coefficients of the various structures—that is the parameters g_ν—decrease in a well-defined way with the number of different indices present in the terms being summed and, of course, also with the number of P^s (flavor breaking) factors in the structure. Each factor P_i^s produces a reduction[6] $\approx 1/3$ and each pair of indices produces a reduction—as discussed in [7]—by a factor from 0.2 to 0.37 (see the footnote 16 of [7]). This "pair of indices" reduction is due to the exchange of a gluon between two quark lines and applies to baryons or mesons composed of quarks u, d, s.[7] In the calculations performed so far (e.g. [13,15,17,19]) this hierarchy of the coefficients was used to show why certain formulas (the Gell Mann-Okubo baryon mass formula, or the Coleman-Glashow or the Gal-Scheck formulas) work so well. Observe that to do this—that is to show that the terms neglected were indeed negligible—any choice of the "pair of indices" reduction factor given above (from 0.2 to 0.37) was equivalent; we selected 0.3 or 0.33 in the above evaluations for convenience (the same factor as that due to P^s); however, since below we are going to use the hierarchy to estimate the order of magnitude of g_0, we will consider the whole interval 0.2 to 0.37 mentioned above.

To exemplify the above presentation we will summarize the main points of the first application of the GP [15] to the baryon masses. In the general parametrization the masses of the eight lowest octet + decuplet baryons can be expressed in terms of 8 parameters (the electromagnetic mass differences are neglected). The known values of the masses allow to calculate all the above parameters; one can check that the parameters multiplying the two more complicated structures (sums over 3 indices—containing 2 or 3 P^s) are negligible. Neglecting them one finds a new relation between the masses which generalizes the Gell Mann-Okubo formula including to a very good approximation the contribution of the $2P^s$ terms.

[6]Of course the reduction, for the magnetic moments, is relative to $g_1 \approx 2.8$.

[7]Incidentally these reduction factors explain why the naive NRQM, based essentially on terms with few indices, works reasonably; to understand this fact on the basis of QCD was precisely the aim of the GP (see also [16]).

V. ESTIMATING g_0

For the baryon octet magnetic moments at the $1P^s$ level it has been seen repeatedly that the hierarchy for the g_ν ($\nu = 1\ldots7$) applies fairly well except for the ratio p/n. It is well known that for the ratio p/n the hierarchy does not work; indeed the extraordinary smallness of $(p/n + 3/2)$ is accidental; therefore one cannot determine the order of magnitude of g_0 from the p, n system alone. (For this compare [9] and [8]; see also [18]).[8]

Leaving aside g_3, there is however a way to estimate the order of magnitude of g_0 from Eqs. (12); indeed from them one derives the relation (19) (again, of course, an exact QCD equation):

$$g_0 = -\tfrac{5}{4}p - \tfrac{7}{4}n + \tfrac{\sqrt{3}}{2}\mu(\Sigma\Lambda) + \tfrac{1}{2}(\Sigma^+ - \Sigma^-) + \tfrac{1}{4}(\Xi^0 - \Xi^-) - \tfrac{3}{2}g_7 + \tfrac{1}{12}g_8 - \tfrac{1}{3}g_9. \quad (19)$$

Inserting experimental values in Eq. (19) one obtains (in nuclear magnetons):

$$g_0 = 0.13 \pm 0.07 - \tfrac{3}{2}g_7 + \tfrac{1}{12}g_8 - \tfrac{1}{3}g_9, \quad (20)$$

where the error comes essentially from $|\mu(\Sigma\Lambda)| = 1.61 \pm 0.08$ (the sign of $\mu(\Sigma\Lambda)$ is negative, as can be seen from Eq. (13)).[9]

In (20) g_8 and g_9 are parameters associated to a structure with three indices plus two P^s operators; they are both expected to be of order 0.03. Thus $|(g_8/12) - (g_9/3)| \leq 0.01$. So the contributions of g_8 and g_9 can only affect negligibly the already large uncertainty in the first term of Eq. (20). Instead, the parameter g_7 is associated to a structure with three indices plus one P^s; its contribution cannot be neglected. Depending on the choice of the gluon exchange reduction factors in the hierarchy (from 0.2 to 0.37), one expects

$$0.036 \leq |g_7| \leq 0.13. \quad (21)$$

We thus obtain from Eq. (20), if g_7 is negative: $0.18 \pm 0.07 \leq g_0 \leq 0.33 \pm 0.07$ and, if g_7 is positive: $-0.07 \pm 0.07 \leq g_0 \leq +0.08 \pm 0.07$. Altogether:

$$-0.07 \pm 0.07 \leq g_0 \leq 0.33 \pm 0.07. \quad (22)$$

This interval includes $g_0 = 0.28 \pm 0.07$, the value that can be extracted from Fig. 2 and Eqs. (11) of Ref. [4]. As a matter of fact the environment dependence of the u-quark

[8]Actually the $M1$ transition $\Delta \to p + \gamma$ allows to understand, via the hierarchy applied to the whole octet-decuplet system, the smallness of g_3; compare Secs. V and VI—see Eq. (22)—in [8].

[9]Neglecting g_8 and g_9, one gets from (20) $\tilde{g}_7 \equiv g_7 + (2/3)g_0 \simeq 0.09 \pm 0.04$ instead of $\tilde{g}_7 \simeq 0.16$, the value given in [8]. This is because, to determine g_7, we now used $\mu(\Sigma\Lambda) = -1.61 \pm 0.08$ deduced from its experimental value $|\mu(\Sigma\Lambda)| = 1.61 \pm 0.08$ (instead in [5,7] the $1P^s$ calculation was used giving $\mu(\Sigma\Lambda) = -1.48 \pm 0.04\mu_N$). A new measurement of $\mu(\Sigma\Lambda)$ would be of interest; the present value is the result of a Primakoff effect determination [20] dating back to 1986.

magnetic moment as reflected by u^n/u^{Ξ^0}, discussed in [4] and rederived above using the GP with charge symmetry—Eqs. (15) and (16)—suggests to stay in the region of positive g_0, so that the interval to be considered reduces to:

$$0 \leq g_0 \leq 0.33 \pm 0.07. \tag{23}$$

Because $u^n/u^{\Xi^0} = (g_0 + 2n + p)/(\Xi^0 - \Xi^-)$ its value is $\simeq 1.2$ for $g_0 = 0.33$ and $\simeq 1.7$ for $g_0 = 0$. As to $u^p/u^{\Sigma^+} = (g_0 + 2p + n)/(\Sigma^+ - \Sigma^-)$ its values are $\simeq 1.1$ for $g_0 = 0.33$ and $\simeq 1.0$ for $g_0 = 0$.

VI. CONCLUSION

The main point of this paper has been to show that in $G_M^s(0) = g_0 + \hat{g}_0$ one can evaluate g_0 by performing a complete general QCD parametrization of the baryon octet magnetic moments and exploiting the GP "hierarchy of parameters." The point of interest is that by the simple general QCD parametrization one obtains a value (compare Eq. (23)) compatible with the value (0.28 ± 0.07) obtained in Ref. [4].

Of course $G_M^s(0)$ is the sum of g_0 and $\hat{g}_0$ (Eq. (8)), where $\hat{g}_0$ is related to ${}^lR_d^s$ of Ref. [4] by Eq. (18) [${}^lR_d^s = 1 + g_0/\hat{g}_0$]. Even if ${}^lR_d^s$ had a value 2 times larger than that 0.139 ± 0.042 of Ref. [4]—as it seems possible—the value of μ_s^p would stay in the region of very small values (that is at the limit of the present experimental possibilities) recently indicated by HAPPEX (compare the HAPPEX JLAB report 2006 in Ref. [2]) and its experimental determination (especially at $Q^2 = 0$) would remain extremely difficult.

Note added.—Recently the results of the HAPPEX collaboration have been published [21] confirming the above indications. While of course we refer to [21] for any detail, we note only that the authors state that the experiments will continue at values of Q^2 higher than the present one $\approx 0.1\ \text{GeV}^2$. At such low values of Q^2 the results (in the words of the authors) "leave little room for observable nucleon strangeness dynamics."

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