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Low and high energy phenomenology of quark-lepton complementarity scenarios

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We conduct a detailed analysis of the phenomenology of two predictive seesaw scenarios leading to quark-lepton complementarity. In both cases we discuss the neutrino mixing observables and their correlations, neutrinoless double beta decay and lepton flavor violating decays such as $\mu \to e\gamma$. We also comment on leptogenesis. The first scenario is disfavored on the level of one to two standard deviations, in particular, due to its prediction for $|U_{e3}|$. There can be resonant leptogenesis with quasidegenerate heavy and light neutrinos, which would imply sizable cancellations in neutrinoless double beta decay. The decays $\mu \to e\gamma$ and $\tau \to \mu\gamma$ are typically observable unless the SUSY masses approach the TeV scale. In the second scenario leptogenesis is impossible. It is, however, in perfect agreement with all oscillation data. The prediction for $\mu \to e\gamma$ is in general too large, unless the SUSY masses are in the range of several TeV. In this case $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are unobservable.

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I. INTRODUCTION

The neutrino mass and mixing phenomena [1] have provided us with some exciting hints towards the structure of the underlying theory of flavor. In particular, based on observations implying that the Cabibbo-Kobayashi-Maskawa (CKM) and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrices are linked by a profound connection, an interesting class of models arises. The CKM matrix is to zeroth order the unit matrix plus a small correction, given by the sine of the Cabibbo angle, $\sin\theta_C = 0.23$. Hence, in the quark sector mixing is absent at zeroth order and the deviation from *no mixing* is small. To make a connection to the lepton sector, it was noted [2] that the deviation from *maximal mixing* is small. Indeed, using the bimaximal [3] mixing scenario as the zeroth order scheme and interpreting the observed deviation from maximal solar neutrino mixing as a small expansion parameter, one can write [2]

$$|U_{e2}| \equiv \sqrt{\tfrac{1}{2}}(1 - \lambda_\nu). \tag{1}$$

With current experimental information [4], we obtain $\lambda_\nu = 0.21^{+0.04,0.08,0.11}_{-0.03,0.07,0.11}$, where we have inserted the best-fit values and the 1, 2, and 3σ ranges of the relevant oscillation parameters. This number is remarkably similar to the Cabibbo angle [2]. In fact, the so-called quark-lepton complementarity (QLC) relation [5,6]

$$\theta_{12} + \theta_C = \frac{\pi}{4} \tag{2}$$

has been suggested and several situations in which it can be realized have been discussed [5–9]. In general, the PMNS matrix is given by $U_\ell^\dagger U_\nu$, where U_ν diagonalizes the neutrino mass matrix and U_ℓ originates from the charged lepton diagonalization. Apparently, deviations from maximal θ_{12} as implied by Eqs. (1) and (2) can be obtained if the neutrino mass matrix corresponds to bimaximal mixing and the charged lepton mass matrix is diagonalized by either the CKM or a CKM-like [10,11] matrix. The opposite case, namely, bimaximal mixing from the charged lepton sector and a CKM correction from the neutrinos, can also be realized, which indicates two possibilities for the approximate realization of Eq. (2).

In the present article we fully analyze the phenomenology of these two popular scenarios, proposed in [5,6], leading to an approximate realization of QLC within the seesaw mechanism [12]. The two scenarios show the feature that the matrix perturbing the bimaximal mixing scenario is exactly the CKM matrix and not just a CKM-like matrix, which minimizes the number of free parameters. We study the neutrino oscillation phenomenology, neutrinoless double beta decay and—in the context of the seesaw mechanism—lepton flavor violating decays such as $\mu \to e\gamma$. We present our results of the correlations between the observables in several plots. In contrast to many previous works, we include the full number of possible *CP* phases. This is a new approach particularly for the second scenario, where bimaximal mixing arises from the charged lepton sector. For both scenarios we comment on the prospects of leptogenesis. We begin in Sec. II with an introduction to the formalism required to study the observables. In Secs. III and IV we discuss the phenomenology of the two scenarios, before we conclude in Sec. V with a summary of our results.

II. FORMALISM

In this section we briefly introduce the required formalism to analyze the QLC scenarios. First, we discuss lepton and quark mixing before turning to lepton flavor violation, whose connection to low energy neutrino physics is im-

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plied by the seesaw mechanism. Conclusively, the principles of leptogenesis are outlined.

A. Neutrino masses, lepton and quark mixing

The two scenarios leading to QLC are set within the framework of the seesaw mechanism for neutrino mass generation [12]. In general, one has the Lagrangian

$$\mathcal{L} = \tfrac{1}{2}\bar{N}_R M_R N_R^c + \bar{\ell}_R m_\ell \ell_L + \bar{N}_R m_D \nu_L, \tag{3}$$

where N_R are the right-handed Majorana singlets, $\ell_{L,R}$ the left- and right-handed charged leptons, and ν_L the left-handed neutrinos. The mass matrix of the charged leptons is m_ℓ, m_D is the Dirac neutrino mass matrix, and M_R the heavy right-handed Majorana neutrino mass matrix. As $M_R \gg m_D$, Eq. (3) leads to an effective neutrino mass matrix at low energies, defined as

$$m_\nu = -m_D^T M_R^{-1} m_D = U_\nu^* m_\nu^{\text{diag}} U_\nu^\dagger, \tag{4}$$

where U_ν transforms m_ν to m_ν^{diag}, with the neutrino masses $m_{1,2,3}$ as diagonal entries. When diagonalizing the charged lepton mass matrix as $m_\ell = V_\ell m_\ell^{\text{diag}} U_\ell^\dagger$, we can rotate $\nu_L \to U_\nu^\dagger \nu_L$, $\ell_R \to V_\ell^\dagger \ell_R$, and $\ell_L \to U_\ell^\dagger \ell_L$. From the charged current term, which is proportional to $\bar{\ell}_L \gamma^\mu \nu_L$, we thus obtain the PMNS matrix

$$U = U_\ell^\dagger U_\nu, \tag{5}$$

which we parametrize as

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}), \tag{6}$$

where we have used the usual notations $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$. We have also introduced the Dirac CP-violating phase δ and the two Majorana CP-violating phases α and β [13]. The oscillation parameters can be expressed by two independent mass squared differences, $\Delta m^2_\odot = m_2^2 - m_1^2$ and $\Delta m^2_{\text{A}} = |m_3^2 - m_1^2|$, as well as three mixing angles, whose exact values are a matter of intense research projects [1]. Their current best-fit values and their 1, 2, and 3σ ranges are according to Ref. [4]:

$$\begin{aligned} \Delta m^2_\odot &= (7.9^{+0.3,0.6,1.0}_{-0.3,0.6,0.8}) \times 10^{-5}\ \text{eV}^2, \\ \sin^2\theta_{12} &= 0.31^{+0.02,0.06,0.09}_{-0.03,0.05,0.07}, \\ \Delta m^2_{\text{A}} &= (2.2^{+0.37,0.7,1.1}_{-0.27,0.5,0.8}) \times 10^{-3}\ \text{eV}^2, \\ \sin^2\theta_{23} &= 0.50^{+0.06,0.14,0.18}_{-0.05,0.12,0.16}, \\ \sin^2\theta_{13} &< 0.012\ (0.028, 0.046). \end{aligned} \tag{7}$$

The present best-fit value for $\sin^2\theta_{13}$ is 0 and there is no information on any of the phases.

Turning to the quark sector, the CKM matrix is [14]

$$V = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho + i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{8}$$

In analogy to the PMNS matrix it is a product of two unitary matrices, $V = V_{\text{up}}^\dagger V_{\text{down}}$, where V_{up} (V_{down}) is associated with the diagonalization of the up-(down-)quark mass matrix. As reported in [15] the best-fit values as well as the 1, 2, and 3σ ranges of the parameters λ, A, $\bar{\rho}$, $\bar{\eta}$ are

$$\begin{aligned} \lambda &= \sin\theta_C = 0.2272^{+0.0010,0.0020,0.0030}_{-0.0010,0.0020,0.0030}, \\ A &= 0.809^{+0.014,0.029,0.044}_{-0.014,0.028,0.042}, \\ \bar{\rho} &= 0.197^{+0.026,0.050,0.074}_{-0.030,0.087,0.133}, \\ \bar{\eta} &= 0.339^{+0.019,0.047,0.075}_{-0.018,0.037,0.057}, \end{aligned} \tag{9}$$

where $\bar{\rho} = \rho(1 - \lambda^2/2)$ and $\bar{\eta} = \eta(1 - \lambda^2/2)$. Effects caused by CP violation are always proportional to a Jarlskog invariant [16], defined as

$$\begin{aligned} J_{CP} &= -\text{Im}\{V_{ud}V_{cs}V_{us}^*V_{cd}^*\} \simeq A^2\lambda^6\bar{\eta} \\ &= (3.1^{+0.43,0.82,1.08}_{-0.37,0.74,0.96}) \times 10^{-5}. \end{aligned} \tag{10}$$

The leptonic analogue of Eq. (10) is

$$\begin{aligned} J_{CP}^{\text{lep}} &= \text{Im}\{U_{e1}U_{\mu 2}U_{e2}^*U_{\mu 1}^*\} \\ &= \tfrac{1}{8}\sin 2\theta_{12}\sin 2\theta_{23}\sin 2\theta_{13}\cos\theta_{13}\sin\delta, \end{aligned} \tag{11}$$

where we have also given the explicit form of J_{CP}^{lep} with the parametrization of Eq. (6). There are two additional invariants, S_1 and S_2 [17], related to the Majorana phases:

$$S_1 = \text{Im}\{U_{e1}U_{e3}^*\} \quad \text{and} \quad S_2 = \text{Im}\{U_{e2}U_{e3}^*\}, \tag{12}$$

which have no analogue in the quark sector.

B. Lepton flavor violation

The seesaw mechanism explains the smallness of neutrino masses, but due to the extreme heaviness of the right-handed Majorana neutrinos a direct test is not only challenging, but presumably impossible. Nonetheless a reconstruction of the seesaw parameter space is possible in supersymmetric (SUSY) scenarios. While being extremely suppressed when mediated by light neutrinos [18], lepton flavor violating (LFV) decays such as $\mu \to e\gamma$ depend in the context of SUSY seesaw on the very same parameters responsible for neutrino masses and can be observable in this case [19]. The size and relative magnitudes of the decays are known to be a useful tool to distinguish between different models. In this work we will focus on models where SUSY is broken by gravity mediation, so-called mSUGRA models. In this case there are four relevant parameters, which are defined at the GUT scale M_X, namely, the universal scalar mass m_0, the universal gaugino mass $m_{1/2}$, the universal trilinear coupling parameter A_0 and $\tan\beta$, which is the ratio of the vacuum expectation values of the uplike and downlike Higgs doublets. For the branching ratios of the decays $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ one can obtain in the leading-log approximation [19]

$$\mathrm{BR}(l_i \to l_j\gamma) \simeq \frac{\Gamma(l_i \to e\nu\bar{\nu})}{\Gamma_{\mathrm{total}}(l_i)} \frac{\alpha_{\mathrm{em}}^3}{G_F^2 m_S^8 v_u^4}\left(\frac{3m_0^2 + A_0^2}{8\pi^2}\right)^2 \times |(\tilde{m}_D^\dagger L \tilde{m}_D)_{ij}|^2 \tan^2\beta. \tag{13}$$

Here $v_u = v\sin\beta$ with $v = 174$ GeV, m_S represents a SUSY particle mass, and $L = \delta_{ij}\ln M_X/M_i$, with M_i the heavy Majorana masses and $M_X = 2 \times 10^{16}$ GeV. Note that the formulas relevant for lepton flavor violation and leptogenesis have to be evaluated in the basis in which the charged leptons and the heavy Majorana neutrinos are real and diagonal. In this very basis we have to replace

$$m_D \to \tilde{m}_D = V_R^T m_D U_\ell, \tag{14}$$

where V_R diagonalizes the heavy Majorana mass matrix via $M_R = V_R^* M_R^{\mathrm{diag}} V_R^\dagger$. The current limit on the branching ratio of $\mu \to e\gamma$ is 1.2×10^{-11} at 90% C.L. [20]. A future improvement of 2 orders of magnitude is expected [21]. In most parts of the relevant soft SUSY breaking parameter space, the expression

$$m_S^8 \simeq 0.5 m_0^2 m_{1/2}^2 (m_0^2 + 0.6 m_{1/2}^2)^2 \tag{15}$$

is an excellent approximation to the results obtained in a full renormalization group analysis [22]. In order to simplify comparisons of different scenarios, it can be convenient to use "benchmark values" of the SUSY parameters. We choose both pints and slopes of the Snowmass Points and Slopes (SPS) values [23] displayed in Table I.

In this context it might be worth commenting on renormalization aspects of the QLC relation (see also [6]). The running of the CKM parameters can always be neglected. However, the case of a large $\tan\beta \gtrsim 10$ in the minimal supersymmetric standard model (MSSM) can imprint sizable effects on the neutrino observables, if the neutrino masses are not normally ordered. In our analysis, this would affect only the SPS point 4, when the neutrinos have an inverted hierarchy or are quasidegenerate.

TABLE I. SPS Benchmark values for the mSUGRA parameters according to Ref. [23]. The values of m_0, $m_{1/2}$ and A_0 are in GeV. The slope for point 1a (2, 3) is $m_0 = -A_0 = 0.4 m_{1/2}$ ($m_0 = 2m_{1/2} + 850$ GeV, $m_0 = \frac{1}{4} m_{1/2} - 10$ GeV) with varying $m_{1/2}$.

Point	m_0	$m_{1/2}$	A_0	$\tan\beta$
1a	100	250	−100	10
1b	200	400	0	30
2	1450	300	0	10
3	90	400	0	10
4	400	300	0	50

It proves useful to consider also the "double" ratios,

$$R(21/31) \equiv \frac{\mathrm{BR}(\mu \to e + \gamma)}{\mathrm{BR}(\tau \to e + \gamma)} \simeq \frac{|(\tilde{m}_D^\dagger L \tilde{m}_D)_{21}|^2}{|(\tilde{m}_D^\dagger L \tilde{m}_D)_{31}|^2},$$
$$R(21/32) \equiv \frac{\mathrm{BR}(\mu \to e + \gamma)}{\mathrm{BR}(\tau \to \mu + \gamma)} \simeq \frac{|(\tilde{m}_D^\dagger L \tilde{m}_D)_{21}|^2}{|(\tilde{m}_D^\dagger L \tilde{m}_D)_{32}|^2}, \tag{16}$$

which are essentially independent of the SUSY parameters.

C. Leptogenesis

Since we will also comment on the possibility of leptogenesis in the QLC scenarios, we will summarize the key principles of this mechanism. An important challenge in modern cosmology is the explanation of the baryon asymmetry $\eta_B \simeq 6 \times 10^{-10}$ [24] of the Universe. One of the most popular mechanisms to create the baryon asymmetry is leptogenesis [25]. The heavy neutrinos, whose comparatively huge masses govern the smallness of the light neutrino masses, decay in the early Universe into Higgs bosons and leptons, thereby generating a lepton asymmetry, which in turn gets recycled into a baryon asymmetry via nonperturbative standard model processes. For recent reviews, see [26]. In principle, all three heavy neutrinos generate a decay asymmetry, which can be written as (summed over all flavors)

$$\varepsilon_i = \frac{1}{8\pi v_u^2} \frac{1}{(\tilde{m}_D \tilde{m}_D^\dagger)_{ii}} \sum_{j \neq i} \mathrm{Im}\{(\tilde{m}_D \tilde{m}_D^\dagger)_{ji}^2\} \times \sqrt{x_j}\left(\frac{2}{1 - x_j} - \ln\left(\frac{1 + x_j}{x_j}\right)\right),$$
$$\varepsilon_1 \simeq -\frac{3}{8\pi v_u^2} \frac{1}{(\tilde{m}_D \tilde{m}_D^\dagger)_{11}} \sum_{j=2,3} \mathrm{Im}\{(\tilde{m}_D \tilde{m}_D^\dagger)_{j1}^2\} \frac{M_1}{M_j}, \tag{17}$$

where $x_j = M_j^2/M_i^2$. This is the general form of ε_i and the limit for ε_1 in case of $M_3 \gg M_2 \gg M_1$. Note that the decay asymmetries depend on $\tilde{m}_D \tilde{m}_D^\dagger$, which has to be compared to the dependence on $\tilde{m}_D^\dagger \tilde{m}_D$ governing the LFV decays. In the case $M_3 \gg M_2 \gg M_1$ only ε_1 plays a role, and dedicated numerical studies [26,27] have shown that, in the case of the MSSM and a hierarchical spectrum of the heavy Majorana neutrino masses, successful thermal leptogenesis is only possible for

$$m_1 \lesssim 0.1 \text{ eV} \quad \text{and} \quad M_1 \gtrsim 10^9 \text{ GeV}. \tag{18}$$

However, it can occur in certain models that the lightest heavy neutrino mass is smaller than the limit of 10^9 GeV given above. We will encounter a scenario like this in the next section. There are three possible ways to resolve this problem:

(i) the decay of the second heaviest neutrino can in certain scenarios generate the baryon asymmetry. Flavor effects [28,29] are important in this respect;

(ii) if the heavy Majorana neutrinos are quasidegenerate in mass, the decay asymmetry can be resonantly enhanced, as has been analyzed in [30]. This requires some amount of tuning;

(iii) nonthermal leptogenesis, i.e., the production of heavy neutrinos via inflaton decay [31]. This possibility is a more model dependent case and complicates the situation, as the reheating temperature, the mass of the inflaton, and the corresponding branching ratios for its decay into the Majorana neutrinos need to be known.

Let us comment a bit on the first case: the expression for the decay asymmetry Eq. (17) has been obtained by summing over all flavors in which the heavy neutrino decays. Recently, however, is has been realized that flavor effects on leptogenesis can have a significant impact on the scenario [28,29]. The decay asymmetry for the decay of the heavy neutrino in a lepton of flavor $\alpha = e, \mu, \tau$ has to be evaluated individually and the wash-out or distribution for each flavor has to be followed individually by its own Boltzmann equation. However, the bound on the lightest heavy neutrino mass M_1 is essentially the same as in the ''summed over all flavors'' approach. In addition, the decay asymmetry in this approach can be enhanced by at most one order of magnitude. What will be interesting for our purpose is that if $M_1 \ll 10^9$ GeV the second heaviest neutrino with mass M_2 can in principle generate the baryon asymmetry [29], as long as the wash-out by the lightest heavy neutrino is low. We will discuss this in more detail in Sec. III C.

III. FIRST REALIZATION OF QLC

The first framework in which our analysis is set is the following:

(i) we assume the conventional seesaw mechanism to generate the neutrino mass matrix $m_\nu = -m_D^T M_R^{-1} m_D$. Diagonalization of m_ν is achieved via $m_\nu = U_\nu^* m_\nu^{\text{diag}} U_\nu^\dagger$ and U_ν produces exact bimaximal mixing;

(ii) the PMNS matrix is given by $U = U_\ell^\dagger U_\nu$, where U_ℓ corresponds to the CKM matrix V. This can be achieved in some $SU(5)$ models, in which $m_\ell = m_{\text{down}}^T$, where m_{down} is the down-quark mass matrix. Hence, $V_{\text{down}} = V$. Consequently, the up-quark mass matrix m_{up} is real and diagonal;

(iii) in some $SO(10)$ models it holds that $m_{\text{up}} = m_D$. It follows that the bimaximal structure of m_ν originates from M_R, which is diagonalized by $M_R = V_R^* M_R^{\text{diag}} V_R^\dagger$.

This scenario has been outlined already in [5,6]. Note that only $U_\ell = V$ is required for the low energy realization of QLC and that the relation $m_\ell = m_{\text{down}}^T$ will not be required to calculate the branching ratios of the LFV decays or the baryon asymmetry. It is known that $m_\ell = m_{\text{down}}^T$ is not realistic for the first and second fermion generation. More ''realistic'' scenarios have been analyzed in Refs. [8,32], in which the relation $m_\ell = m_{\text{down}}^T$ is modified by the Georgi-Jarlskog factor [33]. However, in this case the neutrinos cannot be diagonalized by a bimaximal mixing matrix, because a too large solar neutrino mixing angle would result. Consequently, the minimality of the scenarios is lost, and the QLC relation $\theta_{12} + \theta_C = \pi/4$ turns out to be just a numerical coincidence. Therefore, following most of the analyses in Refs. [5–7], we assume that there is a particular structure on the mass matrices in which mixing depends only weakly on the mass eigenvalues.

With the indicated set of properties, we can express Eq. (5) as

$$U = V^\dagger U_{\text{bimax}}, \tag{19}$$

with U_{bimax} corresponding to bimaximal mixing, which will be precisely defined in Eq. (21). Moreover, Eq. (14) changes to

$$\tilde{m}_D = V_R^T m_D V$$
$$\Rightarrow \begin{cases} \tilde{m}_D^\dagger \tilde{m}_D = V^\dagger \operatorname{diag}(m_u^2, m_c^2, m_t^2) V & \text{for LFV}, \\ \tilde{m}_D \tilde{m}_D^\dagger = V_R^T \operatorname{diag}(m_u^2, m_c^2, m_t^2) V_R^* & \text{for } \eta_B. \end{cases} \tag{20}$$

In the above equation we have given the two important matrices $\tilde{m}_D \tilde{m}_D^\dagger$ and $\tilde{m}_D^\dagger \tilde{m}_D$ describing leptogenesis and the branching ratios of the lepton flavor violating processes. Note, however, that for the latter we have for now neglected the logarithmic dependence on the heavy neutrino masses, cf. Eq. (13).

A. Low energy neutrino phenomenology

The matrix diagonalizing m_ν is called U_ν and corresponds to a bimaximal mixing matrix:

$$U_\nu = U_{\text{bimax}} = P_\nu \tilde{U}_{\text{bimax}} Q_\nu$$
$$= \operatorname{diag}(1, e^{i\phi}, e^{i\omega}) \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \operatorname{diag}(1, e^{i\sigma}, e^{i\tau}). \tag{21}$$

We have included two diagonal phase matrices P_ν and Q_ν. It has been shown in Ref. [11] that this is the most general form if all "unphysical" phases are rotated away. We have in total five phases, one phase in $U_\ell = V$ and four phases in U_ν. Note that Q_ν is "Majorana-like" [11], i.e., the phases σ and τ do not appear in neutrino oscillations, but contribute to the low energy Majorana phases. Multiplying the matrices of Eq. (8) and (21) yields for the oscillation parameters:

$$U = U_\ell^\dagger U_\nu = V^\dagger U_{\text{bimax}}$$
$$\Rightarrow \begin{cases} \sin^2\theta_{12} = \frac{1}{2} - \frac{\lambda}{\sqrt{2}}\cos\phi + \mathcal{O}(\lambda^3), \\ |U_{e3}| = \frac{\lambda}{\sqrt{2}} + \mathcal{O}(\lambda^3), \\ \sin^2\theta_{23} = \frac{1}{2} - (A\cos(\omega - \phi) + \frac{1}{4})\lambda^2 + \mathcal{O}(\lambda^4), \\ J_{CP}^{\text{lep}} = \frac{\lambda}{4\sqrt{2}}\sin\phi + \mathcal{O}(\lambda^3). \end{cases} \tag{22}$$

Apparently, Eq. (22) generates correlations between the observables. The solar neutrino mixing parameter depends on the CP phase ϕ, which originates from the neutrino sector and is to a very good approximation the phase governing leptonic CP violation in oscillation experiments. Note that, in order to have solar neutrino mixing of the observed magnitude, the phase has to be close to zero or 2π. Approximately, at 3σ it should be below $\pi/4$ or above $7\pi/4$. The smallest solar neutrino mixing angle is obtained for $\phi = 0$ and the prediction for $\sin^2\theta_{12}$ is

$$\sin^2\theta_{12} \gtrsim 0.334\ (0.333, 0.332, 0.331). \tag{23}$$

This value of $\sin^2\theta_{12} \gtrsim 0.33$ has to be compared to the experimental 1σ (2σ) limit of $\sin^2\theta_{12} \le 0.33$ (0.37), showing a small conflict. Note that, for the numerical values, as well as for the generation of the plots, which will be presented and discussed in the following, we did not use the approximate expressions in Eq. (22), but the exact formulas.[1] Besides the phases, we also vary the parameters of the CKM matrix in their 1, 2, and 3σ ranges (though, in particular, the error in λ is negligible), and also fix these parameters to their best-fit values. Even for the best-fit values of the CKM parameters, there results a range of values, which is caused by the presence of the unknown phases ϕ and ω. To a good approximation, $|U_{e3}|$ is the sine of the Cabibbo angle divided by $\sqrt{2}$, leading to a sharp prediction of $|U_{e3}|^2 = 0.0258$. Varying the phases and the CKM parameters, we find a range of

$$|U_{e3}| = 0.1607^{+0.0058,0.0069,0.0083,0.0096}_{-0.0059,0.0068,0.0080,0.0091}, \tag{24}$$

where we took the central value $\lambda/\sqrt{2} = 0.1607$. Recall that the 1σ (2σ) bound on $|U_{e3}|$ is 0.11 (0.17). Therefore, the prediction for $|U_{e3}|$ is incompatible with the current 1σ bound of $|U_{e3}|$ and even quite close to the 2σ limit. The experiments taking data in the next 5 to 10 years [34] will have to find a signal corresponding to nonvanishing $|U_{e3}|$ in order for this particular framework to survive. Leptonic CP violation is in leading order proportional to $\lambda\sin\phi$, which is five orders in units of λ larger than the J_{CP} of the quark sector. If the neutrino sector conserved CP, one would obtain $J_{CP}^{\text{lep}} = \frac{1}{8}A\eta\lambda^4$, which is still two orders of λ larger than the J_{CP} of the quark sector. If V was equal to the unit matrix, which corresponds to bimaximal mixing in the PMNS matrix, J_{CP}^{lep} would be zero. There is an interesting "sum-rule" between leptonic CP violation, solar neutrino mixing, and $|U_{e3}|$:

$$\sin^2\theta_{12} \simeq \frac{1}{2} - |U_{e3}|\cos\phi \simeq \frac{1}{2} \pm \sqrt{|U_{e3}|^2 - 16(J_{CP}^{\text{lep}})^2}. \tag{25}$$

Overall, the experimental result of $\sin^2\theta_{12} \simeq 0.31$ implies large $\cos\phi$, and therefore small $\sin\phi$, leading to small CP violating effects even though $|U_{e3}|$ is sizable. Atmospheric neutrino mixing stays very close to maximal and due to cancellations $\sin^2\theta_{23} = \frac{1}{2}$ can always occur. If $\cos(\omega - \phi) = 1$, then $\sin^2\theta_{23}$ takes its minimal value. We have seen above that the observed low value of the solar neutrino mixing angle requires $\phi \simeq 0$, so that $\omega \simeq 0$ is implied when θ_{23} is very close to maximal. The minimal and maximal values of $\sin^2\theta_{23}$ are given by

$$\sin^2\theta_{23} \ge 0.445\ (0.444, 0.443, 0.442) \quad \text{and}$$
$$\sin^2\theta_{23} \le 0.531\ (0.532, 0.533, 0.534). \tag{26}$$

Probing deviations from maximal mixing of order 10% could be possible in future experiments [34]. In Fig. 1 we show the correlations between the oscillation parameters which result from the relation $U = V^\dagger U_{\text{bimax}}$ in Eq. (22). We plot J_{CP}^{lep}, ϕ, and $\sin^2\theta_{23}$ against $\sin^2\theta_{12}$, as well as $\sin^2\theta_{23}$ against $|U_{e3}|$. We also indicate the current 1, 2, and 3σ ranges of the oscillation parameters. This shows again that solar neutrino mixing is predicted to be close to its 1σ bound and $|U_{e3}|$ even close to its 2σ bound.

Now we turn to the neutrino observables outside the oscillation framework and comment on the consequences for neutrinoless double beta decay. The two invariants related to the Majorana phases are

$$S_1 = \frac{\lambda}{2}\sin(\phi + \tau) + \frac{\lambda^2}{2\sqrt{2}}\sin\tau + \mathcal{O}(\lambda^3),$$
$$S_2 = \frac{\lambda}{2}\sin(\phi - \sigma + \tau) + \frac{\lambda^2}{2\sqrt{2}}\sin(\sigma - \tau) + \mathcal{O}(\lambda^3). \tag{27}$$

As expected, the two phases σ and τ in Q_ν only appear in these quantities. According to the parametrization of

[1]Note for instance that the next term in the expansion of $|U_{e3}|$ is of order $\lambda^3 \simeq 0.01$ and can contribute sizably.

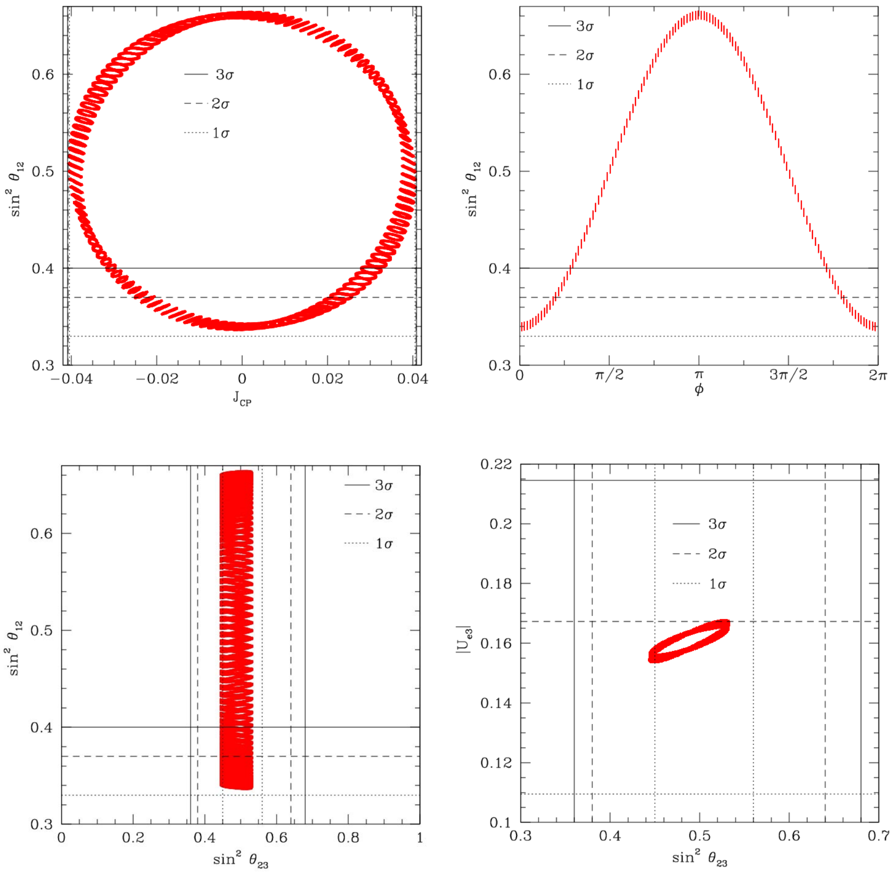

FIG. 1 (color online). First realization of QLC: neutrino observables resulting from Eq. (22) for the 3σ ranges of the CKM parameters. We also indicated the present 1, 2, and 3σ ranges of the oscillation parameters.

Eq. (6), we have $S_1 = -c_{12}c_{13}s_{13}s_\beta$ and $S_2 = s_{12}c_{13}s_{13}s_{\alpha-\beta}$. We can insert in Eq. (27) the expressions for the mixing angles from Eq. (22) to obtain in leading order $\sin\beta \simeq -\sin(\phi+\tau)$ and $\sin(\alpha-\beta) \simeq \sin(\phi - \sigma + \tau)$. Hence, the Majorana phase σ is related to the phase α in the parametrization of Eq. (6). It is interesting to study the form of the neutrino mass matrix, which is responsible for bimaximal mixing. It reads

$$m_\nu^{\rm bimax} = \begin{pmatrix} A & Be^{-i\phi} & -Be^{-i\omega} \\ \cdot & (D+\frac{A}{2})e^{-2i\phi} & (D-\frac{A}{2})e^{-i(\phi+\omega)} \\ \cdot & \cdot & (D+\frac{A}{2})e^{-2i\omega} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{-i\omega} \end{pmatrix} \begin{pmatrix} A & B & -B \\ \cdot & D+\frac{A}{2} & D-\frac{A}{2} \\ \cdot & \cdot & D+\frac{A}{2} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{-i\omega} \end{pmatrix}, \quad (28)$$

where

$$A = \frac{1}{2}(m_1 + m_2 e^{-2i\sigma}), \qquad B = \frac{1}{2\sqrt{2}}(m_2 e^{-2i\sigma} - m_1), \qquad D = \frac{m_3 e^{-2i\tau}}{2}. \quad (29)$$

The inner matrix in Eq. (28) is diagonalized by a real and bimaximal rotation and the masses are obtained as

$$m_1 = A - \sqrt{2}B, \qquad e^{-2i\sigma}m_2 = A + \sqrt{2}B, \qquad e^{-2i\tau}m_3 = 2D. \quad (30)$$

Up to now there has been no need to specify the neutrino mass ordering. This is however necessary in order to discuss neutrinoless double beta decay ($0\nu\beta\beta$) [35]. There are three extreme hierarchies often discussed; the normal hierarchy ($m_3 \simeq \sqrt{\Delta m_A^2} \gg m_2 \simeq \sqrt{\Delta m_\odot^2} \gg m_1$), the inverted hierarchy ($m_2 \simeq m_1 \simeq \sqrt{\Delta m_A^2} \gg m_3$), and the quasidegenerate case ($m_1 \simeq m_2 \simeq m_3 \gg \sqrt{\Delta m_A^2}$). The effective mass which can be measured in $0\nu\beta\beta$ experiments is the ee element of m_ν in the charged lepton basis. To first order in λ, one gets for a normal hierarchy that $\langle m\rangle \simeq \frac{1}{2}\sqrt{\Delta m_\odot^2}\lambda$. In case of an inverted hierarchy we have

$$\langle m\rangle \simeq \sqrt{\Delta m_{\rm A}^2}|c_\sigma + \sqrt{2}s_\sigma s_\phi \lambda|. \qquad (31)$$

The maximal (minimal) effective mass is obtained for $\sigma = 0$ ($\sigma = \pi/2$). On the other hand, we have $\langle m\rangle \simeq \sqrt{\Delta m_{\rm A}^2}\sqrt{1-\sin^2 2\theta_{12}\sin^2\alpha}$ in terms of the usual parametrization [35]. Therefore, as is also obvious from the discussion following Eq. (27), σ will be closely related to the Majorana phase α. Similar considerations apply to the quasidegenerate case.

B. Lepton flavor violation

Now we study the branching ratios of the LFV decays like $\mu \to e\gamma$ for this scenario. With our present assumptions we have that $m_D = m_{\rm up} = {\rm diag}(m_u, m_c, m_t)$. With this input and with Eq. (20), one easily obtains

$$|(\tilde{m}_D^\dagger \tilde{m}_D)_{21}|^2 \simeq A^4 m_t^4(\eta^2 - (1-\rho)^2)\lambda^{10} + \mathcal{O}(\lambda^{14}). \qquad (32)$$

Note that we have neglected the logarithmic dependence on M_i. The double ratios are[2]

$$\begin{aligned} R(21/31) &\simeq A^2\lambda^4 + \mathcal{O}(\lambda^8),\\ R(21/32) &\simeq A^2(\eta^2 - (1-\rho)^2)\lambda^6 + \mathcal{O}(\lambda^{10}). \end{aligned} \qquad (33)$$

The branching ratios behave according to

$$\mathrm{BR}\,(\mu \to e\gamma):\mathrm{BR}(\tau \to e\gamma):\mathrm{BR}(\tau \to \mu\gamma) \simeq \lambda^6:\lambda^2:1, \qquad (34)$$

which is in agreement with Ref. [32].

In order to conduct a more precise study of the rates of the LFV processes, we recall that there is some dependence on the heavy neutrino masses, as encoded in the matrix $L = \delta_{ij}\ln M_X/M_i$ in Eq. (13). Hence, we need to evaluate the values of the heavy Majorana neutrino masses, i.e., we need to invert the seesaw formula $m_\nu = -m_D^T M_R^{-1} m_D$ and diagonalize M_R [36–38]. The light neutrino mass matrix is displayed in Eq. (28). With $m_D = m_{\rm up} = {\rm diag}(m_u, m_c, m_t)$ the heavy neutrino mass matrix reads

$$-M_R = m_{\rm up} m_\nu^{-1} m_{\rm up} = P_\nu \begin{pmatrix} \tilde{A}m_u^2 & \tilde{B}m_u m_c & -\tilde{B}m_u m_t \\ \cdot & (\tilde{D}+\frac{\tilde{A}}{2})m_c^2 & (\tilde{D}-\frac{\tilde{A}}{2})m_c m_t \\ \cdot & \cdot & (\tilde{D}+\frac{\tilde{A}}{2})m_t^2 \end{pmatrix} P_\nu, \qquad (35)$$

where

$$\tilde{A} = \frac{1}{2m_1} + \frac{e^{2i\sigma}}{2m_2} = \frac{A}{A^2 - 2B^2}, \qquad \tilde{B} = \frac{e^{2i\sigma}}{2\sqrt{2}m_2} - \frac{1}{2\sqrt{2}m_1} = \frac{-B}{A^2-2B^2}, \qquad \tilde{D} = \frac{e^{2i\tau}}{2m_3} = \frac{1}{4D}.$$

A, B, and D are given in Eq. (29). The heavy Majorana mass matrix is related to the inverse of the light neutrino mass matrix and has for bimaximal mixing a very similar form. Because of the very hierarchical structure of M_R, and if none of the elements vanish, the eigenvalues are quite easy to obtain (see also [38]):

$$\begin{aligned} M_1 e^{i\phi_1} &\simeq \frac{m_u^2(\tilde{A}^2 - 2\tilde{B}^2)}{\tilde{A}} = \frac{2m_u^2}{m_1 + m_2 e^{-2i\sigma}},\\ M_2 e^{i\phi_2} &\simeq 2m_c^2 \frac{\tilde{A}\tilde{D}}{\tilde{D}+\tilde{A}/2} = 2e^{2i(\sigma+\tau)}m_c^2 \frac{m_1 + m_2 e^{-2i\sigma}}{m_2 m_3 + m_1 m_3 e^{2i\sigma} + 2m_1 m_2 e^{2i\tau}},\\ M_3 e^{i\phi_3} &\simeq m_t^2(\tilde{D} + \tilde{A}/2) = \frac{m_t^2}{4m_1 m_2 m_3}(2e^{2i\tau}m_1 m_2 + e^{2i\sigma}m_1 m_3 + m_2 m_3). \end{aligned} \qquad (36)$$

Here $M_{1,2,3}$ are real and positive, and $\phi_{1,2,3}$ denote the phases of the complex eigenvalues of the inner matrix in Eq. (35). We see that the values of the heavy Majorana masses depend on the phases σ and τ, which in turn are related to the low energy Majorana phases. Note that the requirement of M_3 from Eq. (36) being smaller than the Planck mass gives a lower bound on the smallest neutrino mass of

$$\begin{aligned} m_1 &\geq \frac{m_t^2}{4M_{\rm Pl}} \simeq 10^{-7}\ {\rm eV} \quad \text{and}\\ m_3 &\geq \frac{m_t^2}{2M_{\rm Pl}} \simeq 3\times 10^{-7}\ {\rm eV}, \end{aligned} \qquad (37)$$

for the normal and inverted hierarchy, respectively.

The matrix V_R is defined via $M_R = V_R^* M_R^{\rm diag} V_R^\dagger$, where $M_R^{\rm diag} = {\rm diag}(M_1, M_2, M_3)$ contains real and positive entries. We find

[2] The relative magnitude of the branching ratios has in this scenario been estimated in Ref. [32]. Here we take the dependence on M_i and m_i carefully into account and study in addition their absolute magnitude.

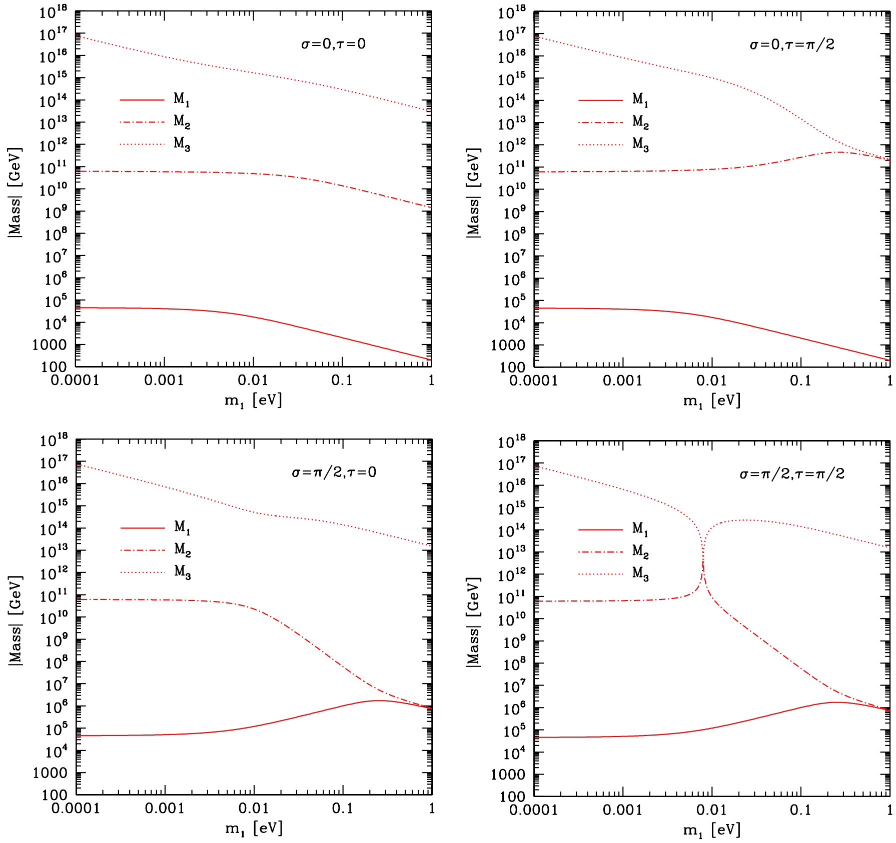

FIG. 2 (color online). First realization of QLC: the heavy neutrino masses resulting from the diagonalization of Eq. (35) as a function of the smallest neutrino mass for the normal mass ordering. We have chosen four different pairs of values for σ and τ, showing the possible degeneracy of the masses. See text for further discussion.

$$V_R = iP_\nu^* \tilde{V}_R P_\nu R_\nu, \qquad \text{where } \tilde{V}_R \simeq \begin{pmatrix} 1 & \frac{m_u}{m_c}\frac{\tilde{B}}{\tilde{A}} & -\frac{m_u}{m_t}\frac{2\tilde{B}(\tilde{A}^2-2\tilde{B}^2)}{\tilde{A}(\tilde{A}^2-2\tilde{B}^2+2\tilde{A}\tilde{D})-4\tilde{B}^2\tilde{D}} \\ -\frac{m_u}{m_c}\frac{\tilde{B}}{\tilde{A}} & 1 & -\frac{m_c}{m_t}\frac{\tilde{A}(\tilde{A}^2-2\tilde{B}^2-2\tilde{A}\tilde{D})+4\tilde{B}^2\tilde{D}}{\tilde{A}(\tilde{A}^2-2\tilde{B}^2+2\tilde{A}\tilde{D})-4\tilde{B}^2\tilde{D}} \\ \frac{m_u}{m_t}\frac{\tilde{B}}{\tilde{A}} & \frac{m_c}{m_t}\frac{\tilde{A}(\tilde{A}^2-2\tilde{B}^2-2\tilde{A}\tilde{D})+4\tilde{B}^2\tilde{D}}{\tilde{A}(\tilde{A}^2-2\tilde{B}^2+2\tilde{A}\tilde{D})-4\tilde{B}^2\tilde{D}} & 1 \end{pmatrix} \tag{38}$$

and $R_\nu = \mathrm{diag}(e^{-i\phi_1/2}, e^{-i\phi_2/2}, e^{-i\phi_3/2})$ contains the phases of the eigenvalues in Eq. (36). The above matrix is unitary to order m_u/m_c or m_c/m_t, which phenomenologically corresponds to an order of λ^4. The heavy neutrino masses are plotted in Fig. 2 as a function of the lightest neutrino mass in case of normal ordering. Figure 3 shows the same for inversely ordered light neutrinos. We have chosen four different pairs of values for σ and τ. For the plots we have fixed $\Delta m_\odot^2$ and Δm_A^2 to their best-fit values and have taken the quark masses as[3] $m_u = 0.45$ MeV, $m_c = 1.2$ GeV, and $m_t = 175$ GeV. The matrix M_R was diagonalized numerically. Equation (36) is nevertheless an excellent approximation if σ and τ are far away from $\pi/2$. Moreover, it holds that $V_R \simeq \mathbb{1}$ in this case. On the other hand, if $\sigma \simeq \pi/2$ it can occur that M_1 and M_2 are almost degenerate if m_1 takes a value around 0.5 eV. This happens if $\tilde{A} = 0$ or, strictly speaking, $\tilde{A} m_u^2 \ll \tilde{B} m_u m_c$ in which case Eqs. (36) and (38) are no longer valid [37,38], but M_1 and M_2 build a pseudo-Dirac pair with mass

$$M_1 \simeq M_2 \simeq \tilde{B} m_u m_c \simeq \frac{m_u m_c}{2\sqrt{2} m_1} \sim 10^6 \text{ GeV}. \tag{39}$$

Note that the indicated value of m_1 is in conflict with tight cosmological constraints [40]. There are similar situations for M_2 and M_3, which occur when $\tau \simeq \pi/2$. Neglecting these tuned cases, we plot the branching ratios in the case of $\tau = \sigma = 0$ for the normal ordering in Fig. 4 as a function of the smallest neutrino mass,[4] choosing the

[3]The values for the heavy neutrino masses are not much different when we take the quark masses [39] at a higher energy scale.

[4]Note that for inverse mass ordering the masses m_1 and m_2 are always rather close. As obvious from Eq. (36), this leads to slightly larger masses for the heavy neutrinos. This translates into branching ratios which for small m_3 are larger by a factor of roughly 3.

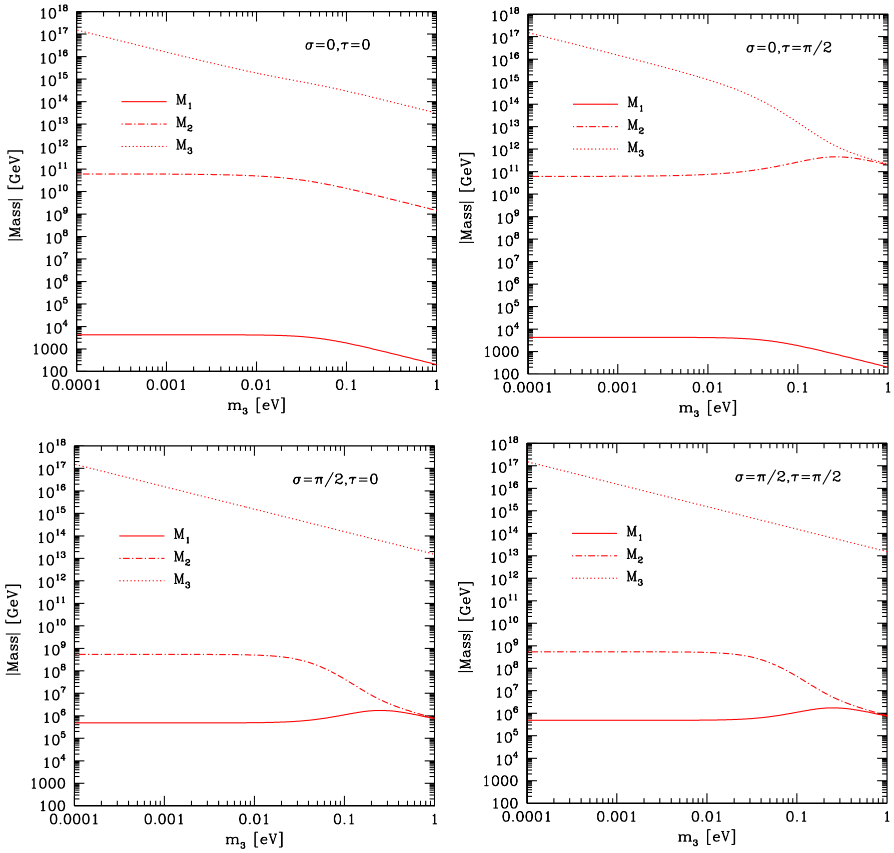

FIG. 3 (color online). Same as Fig. 2 for the inverted mass ordering.

SPS points 1a, 2, and 4. We do not use points 1b and 3, because the corresponding plots will be indistinguishable from the plots for points 1a and 2, respectively. The results are typical if both τ and σ are not close to $\pi/2$. In order to take renormalization aspects into account, we evaluated the branching ratios for quark masses at high scale [39]. For instance, we took $m_u = 0.7$ MeV, $m_c = 210$ MeV, and $m_t = 82.4$ GeV, which corresponds for $\tan\beta = 10$ to $m_u = 2.3$ MeV, $m_c = 677$ MeV, and $m_t = 181$ GeV at M_Z [39]. Because of the presence of the diagonal matrix $L = \delta_{ij} \ln M_X/M_i$ in the equation for the branching ratios, the possibility of cancellations arises, leading to a very small branching ratio. From Eq. (32) alone, such a cancellation is impossible. We have also indicated current and future sensitivities on the decays in Fig. 4. Typically, $\mu \to e\gamma$ can be observable for not too small neutrino masses, unless the SUSY masses approach the TeV scale. BR($\tau \to e\gamma$) is predicted to be very small, and observation of $\tau \to \mu\gamma$ requires rather large neutrino masses, small SUSY masses, or large $\tan\beta$. This is illustrated in Fig. 5, where we have plotted the branching ratios as a function of the SUSY parameter $m_{1/2}$ for the SPS slopes 1a and 2 from Table I. We have chosen two values for the neutrino masses (normal ordering), namely, 0.002 and 0.2 eV. The relative magnitude of the branching ratios, as estimated in Eq. (34), holds true for most of the parameter space.

C. Comments on leptogenesis

It is worth discussing leptogenesis in the scenario under study. As indicated in Sec. II C, the value of the baryon asymmetry crucially depends on the spectrum of the heavy Majorana neutrinos, which we have displayed in Figs. 2 and 3 for normally and inversely ordered light neutrino masses. It also depends on the matrix V_R, which in the case of σ far away from $\pi/2$ is given in Eq. (38). In this case the eigenvalues $M_{1,2,3}$ are strongly hierarchical. In general, M_1 does not exceed 10^6 GeV, as obvious from Eq. (36) and Figs. 2 and 3. According to Eq. (18), this is too small a value for successful thermal leptogenesis generated by this heavy neutrino. As pointed out in Sec. II C, it is in principle possible that the second heaviest neutrino generates the decay asymmetry. We will illustrate now that within the QLC scenario under study this is problematic. Taking advantage of the analysis in [29], we can estimate the resulting baryon asymmetry including flavor effects [28,29].[5] The decay asymmetry of the neutrino with mass

[5]For an analysis without flavor effects, see [41].

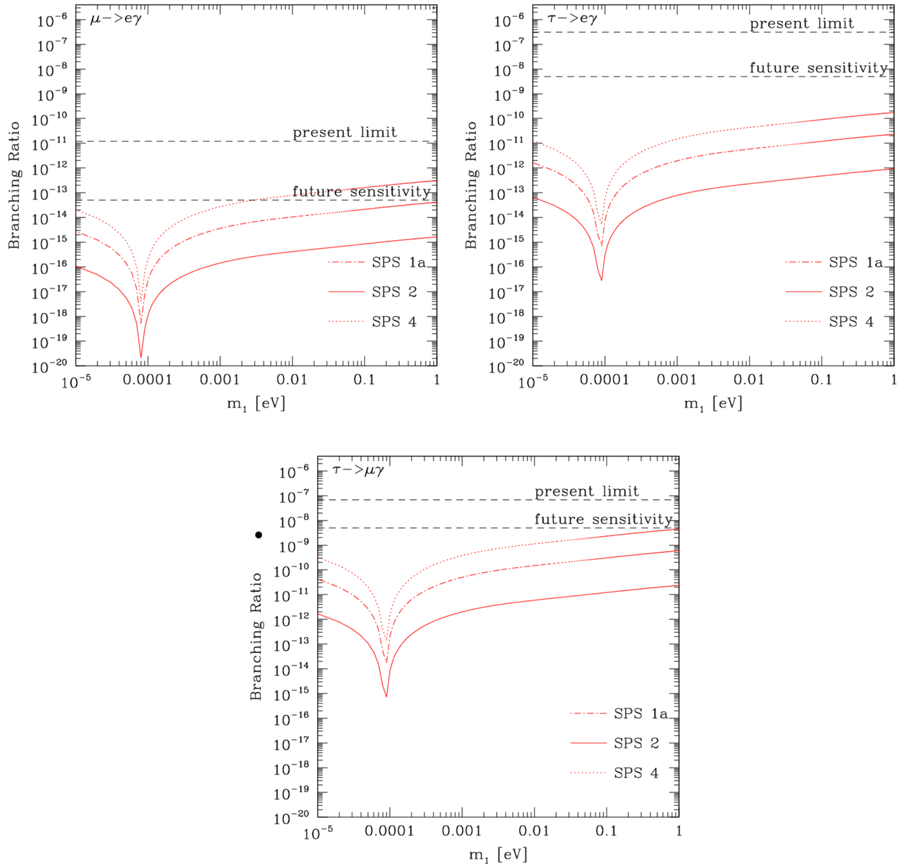

FIG. 4 (color online). First realization of QLC: the branching ratios for $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ against the smallest neutrino mass (normal ordering) for the SPS points 1a, 2, and 4, see Table I. Indicated are also the present and future experimental sensitivities.

M_2 in the flavor $\alpha = e, \mu, \tau$ reads [29,41]

$$\varepsilon_2^\alpha \simeq -\frac{1}{8\pi v_u^2}\frac{1}{(\tilde m_D \tilde m_D^\dagger)_{22}}\left[\frac{3}{2}\frac{M_2}{M_3}\,\mathrm{Im}\{(\tilde m_D)_{2\alpha}(\tilde m_D^\dagger)_{\alpha 3}(\tilde m_D\tilde m_D^\dagger)_{23}\} + \frac{M_1}{M_2}\left(\ln\frac{M_2}{M_1}-2\right)\mathrm{Im}\{(\tilde m_D)_{2\alpha}(\tilde m_D^\dagger)_{\alpha 1}(\tilde m_D\tilde m_D^\dagger)_{21}\}\right], \tag{40}$$

where $\tilde m_D$ is given in Eq. (20). In case of a normal hierarchy, we can neglect m_1 with respect to $m_2 \simeq \sqrt{\Delta m_\odot^2}$ and $m_3 \simeq \sqrt{\Delta m_{\rm A}^2}$ and find from Eq. (36) that

$$\begin{aligned} M_1 e^{i\phi_1} &\simeq \frac{2m_u^2}{\sqrt{\Delta m_\odot^2}} e^{2i\sigma}, \\ M_2 e^{i\phi_2} &\simeq \frac{2m_c^2}{\sqrt{\Delta m_{\rm A}^2}} e^{2i\tau}, \\ M_3 e^{i\phi_3} &\simeq \frac{m_t^2}{4m_1} \end{aligned} \tag{41}$$

which fixes $\phi_1 = 2\sigma$, $\phi_2 = 2\tau$, and $\phi_3 = 0$ in the phase matrix R_ν appearing in Eq. (38). The matrix $\tilde V_R$ in Eq. (38) simplifies considerably to

$$\tilde V_R \simeq \begin{pmatrix} 1 & -\frac{m_u}{\sqrt{2}m_c} & \sqrt{2}\frac{m_u}{m_t} \\ \frac{m_u}{\sqrt{2}m_c} & 1 & -\frac{m_c}{m_t} \\ -\frac{m_u}{\sqrt{2}m_t} & \frac{m_c}{m_t} & 1 \end{pmatrix}. \tag{42}$$

We can evaluate the decay asymmetries by making an expansion in terms of λ, for which we use that $m_c = c_c m_t \lambda^4$ and $m_u = c_u m_t \lambda^8$ with $c_{u,c} \simeq 1$. One finds that ε_2^τ is larger than ε_2^μ (ε_2^e) by two (four) orders in λ. The leading term in ε_2^τ comes from the contribution proportional to $M_2/M_3 \simeq 8c_c^2\lambda^8 m_1/m_3$ in Eq. (40). Thus, we obtain

$$\begin{aligned} \varepsilon_2^\tau &\simeq \frac{3c_c^2}{2\pi(1+c_c^2)}\frac{m_1}{\sqrt{\Delta m_{\rm A}^2}}\lambda^8 \sin 2(\omega - \phi + \tau) \\ &\simeq 5\times 10^{-9}\left(\frac{m_1}{10^{-4}\ \mathrm{eV}}\right)\sin 2(\omega - \phi + \tau). \end{aligned} \tag{43}$$

The second contribution in Eq. (40) proportional to M_1/M_2

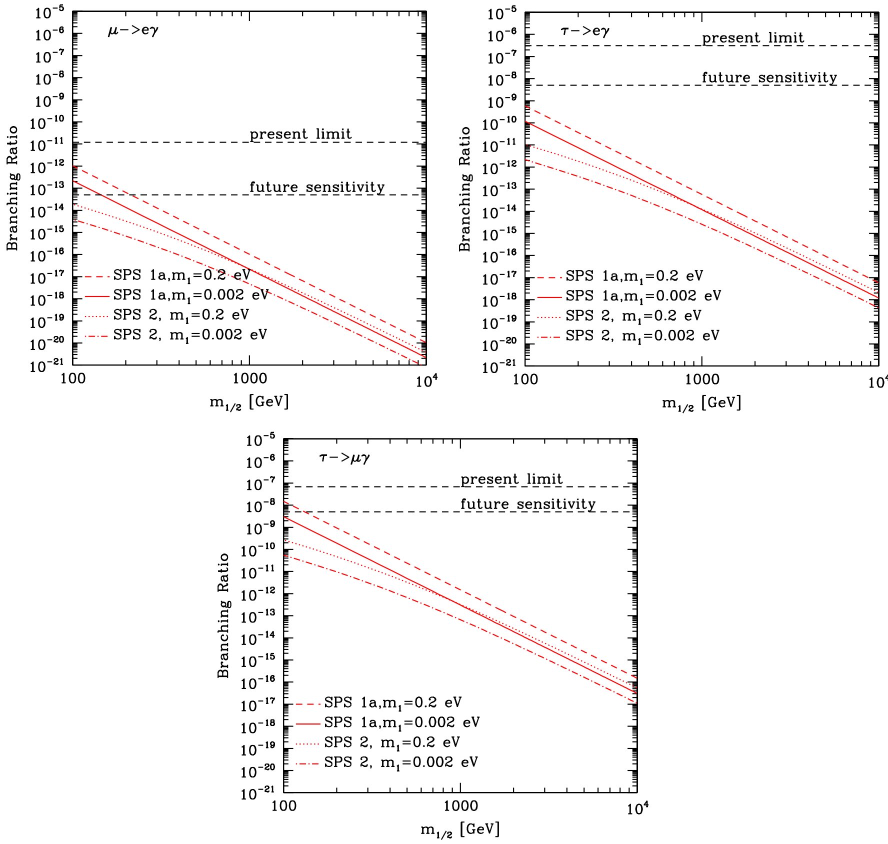

FIG. 5 (color online). First realization of QLC: the branching ratios for $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ against the SUSY parameter $m_{1/2}$ for the SPS slopes 1a and 2, see Table I. We have chosen two values for the neutrino masses (normal ordering), namely, 0.002 and 0.2 eV. Indicated are also the present and future experimental sensitivities.

is suppressed by $(1 - c_c^2)\lambda^{24} m_3/m_2$, which is always much smaller than $m_1/\sqrt{\Delta m_A^2}\lambda^8$ due to the lower limit on m_1 from Eq. (37). We can identify the leptogenesis phase $\omega - \phi + \tau$. This combination of phases is not directly measurable in low energy experiments, as is clear from the results in Sec. III A. Recall however that $\sin^2\theta_{23} - \frac{1}{2} \propto \lambda^2 \cos(\omega - \phi)$, $J_{CP} \propto \sin\phi$, and $\sin\beta \simeq -\sin(\phi + \tau)$, which *in principle* allows one to reconstruct the leptogenesis phase with low energy measurements. However, determining the Majorana phases in case of a normal hierarchy seems at present impossible. We still have to estimate the final baryon asymmetry from the decay asymmetry Eq. (43). The wash-out of ε_2^τ by the lightest neutrino is governed by $\tilde{m}_1^\tau/m^*$, where $m^* \simeq 10^{-3}$ eV and $\tilde{m}_i^\tau \simeq (\tilde{m}_D)_{i\tau}(\tilde{m}_D^\dagger)_{\tau i}/M_i$. In our case, $\tilde{m}_1^\tau/m^* \simeq \sqrt{\Delta m_\odot^2}/(4c_u^2 m^*) \simeq 2$, which confirms the result in Ref. [29], where it has been shown that the resulting wash-out is of order 0.2. Without flavor effects, the wash-out would be 2 orders of magnitude stronger [29], which clearly demonstrates their importance. However, there is very strong wash-out from interactions involving M_2: the efficiency is $m^*/\tilde{m}_2^\tau \simeq 2c_c^2 m^*/\sqrt{\Delta m_A^2} \simeq 1/25$ and the estimate for the total baryon asymmetry is [29]

$$\eta_B \simeq 6 \times 10^{-5} \epsilon_2^\tau \simeq 3 \times 10^{-13}\left(\frac{m_1}{10^{-4}\ \mathrm{eV}}\right)\sin 2(\omega - \phi + \tau), \tag{44}$$

which is much below[6] the observed value of 6×10^{-10}. Of course, these estimates will eventually have to be confirmed by a precise numerical analysis. Nevertheless, it serves to show that successful thermal leptogenesis with the second heaviest Majorana neutrino is quite problematic in the scenario.

We can perform similar estimates if the light neutrinos are governed by an inverted hierarchy. After some algebra in analogy to the normal hierarchical case treated above, we find that

$$\varepsilon_2^\tau \simeq \frac{3c_c^2}{16\pi(1 + c_c^2)} \frac{m_3}{\sqrt{\Delta m_A^2}} \lambda^8 \sin 2(\phi + \tau - \omega - \sigma/2) \simeq 7 \times 10^{-10}\left(\frac{m_3}{10^{-4}\ \mathrm{eV}}\right)\sin 2(\phi + \tau - \omega - \sigma/2), \tag{45}$$

[6]This is in agreement with the findings of Ref. [42].

which is always larger than $\varepsilon_2^{e,\mu}$. This expression is one order of magnitude smaller than the decay asymmetry for the normal hierarchy. It seems therefore that successful leptogenesis within the inverted hierarchy is even more difficult. A more precise statement would require solving the full system of Boltzmann equations. The leptogenesis phase is now $\phi + \tau - \omega - \sigma/2$ and this combination of phases can in principle be reconstructed using $\sin^2\theta_{23} - \frac{1}{2} \propto \lambda^2 \cos(\omega - \phi)$, $J_{CP} \propto \sin\phi$, $\sin\beta \simeq -\sin(\phi + \tau)$, and $\sin(\alpha - \beta) \simeq \sin(\phi - \sigma + \tau)$. However, determining both Majorana phases seems at present impossible.

There is another interesting situation in which successful leptogenesis can take place in this scenario, namely, resonant leptogenesis. This can occur if $\sigma \simeq \pi/2$, in which case two heavy neutrinos have quasidegenerate masses, see Eq. (39). In Ref. [38] a similar framework was considered, and the mass splitting required to generate an η_B of the observed size has been estimated. The result corresponds to $|1 - M_2/M_1| \simeq 10^{-5}$–$10^{-6}$, which is a rather fine-tuned situation. However, there are two rather interesting aspects to this case: as discussed in Sec. III A, the phase σ is related to the low energy Majorana phase α. If $\alpha = \pi/2$, it is known that for quasidegenerate neutrinos the stability with respect to radiative corrections is significantly improved [43]. Moreover, the resonant condition occurs if the smallest neutrino mass is approximately 0.5 eV, i.e., the light neutrinos are quasidegenerate. In this case, the effective mass for neutrinoless double beta decay reads

$$\langle m \rangle \simeq m_1(\sqrt{2}\lambda + \tfrac{1}{2}c_{\phi+2\tau}\lambda^2). \tag{46}$$

The maximum value of the effective mass for quasidegenerate neutrinos is roughly m_1 [35]. The suppression factor $\sqrt{2}\lambda$ is nothing but $\cos 2\theta_{12}$. Therefore there are sizable cancellations in the effective mass [44] when the resonance condition for the heavy neutrino masses is fulfilled. With $m_1 \simeq 0.5$ eV we can predict that $\langle m \rangle \simeq 0.16$ eV, a value which can be easily tested in running and up-coming experiments [45].

If $\tau \simeq \pi/2$, it is apparent from Figs. 2 and 3 that situations can occur in which M_2 and M_3 are quasidegenerate. Hence, their decay could create a resonantly enhanced decay asymmetry, but the lighter neutrino with mass M_1 should not wash out this asymmetry. Determining if this is indeed possible would again require a dedicated study and solution of the Boltzmann equations. Leaving the fine-tuned possibility of resonant leptogenesis aside, we can consider nonthermal leptogenesis. However, as also discussed in Ref. [38], the decay asymmetry ε_1 turns out to be too tiny: if we insert the phenomenological values $m_u/m_c \sim m_c/m_t \simeq \lambda^4$ in the exact equations and if we refrain from considering the possibility of resonant enhancements, ε_1 is of order $\lambda^{16} \simeq 10^{-11}$. In principle, the baryon asymmetry could be generated by the decays of the heavier neutrinos, i.e., by ε_2 and/or ε_3, which are indeed larger than ε_1. This possibility would indicate that the inflaton has a sizable branching ratio in the heavier neutrinos. However, this would also require that the lightest Majorana neutrino N_1 does not wash out the asymmetry generated by N_2 and N_3, making a detailed numerical analysis necessary.

IV. SECOND REALIZATION OF QLC

In this section we discuss another possible realization of QLC, which has also been outlined already in [5,6]:

(i) the conventional seesaw mechanism generates the neutrino mass matrix. Diagonalization of m_ν is achieved via $m_\nu = U_\nu^* m_\nu^{\rm diag} U_\nu^\dagger$ and U_ν is related to V (in the sense that $U_\nu^\dagger = P_\nu V Q_\nu$);

(ii) the matrix diagonalizing the charged leptons corresponds to bimaximal mixing: $U_\ell = U_{\rm bimax}^T$. This can be achieved when $V_{\rm up} = V^\dagger$, therefore $V_{\rm down} = \mathbb{1}$;

(iii) if indeed $V_{\rm up} = V^\dagger$, then $m_\nu = -m_D^T M_R^{-1} m_D$ is diagonalized by the CKM matrix. If M_R does not introduce additional rotations we can have the $SO(10)$-like relation $m_{\rm up} = m_D = V'_{\rm up} m_{\rm up}^{\rm diag} P_\nu V Q_\nu$. Here $V'_{\rm up}$ denotes in our convention the in-principle unknown right-handed rotation of m_D. The condition of M_R not introducing additional rotations means that $V_R = (V'_{\rm up})^*$, where $M_R = V_R^* M_R^{\rm diag} V_R^\dagger$.

Note that the equalities $U_\ell = U_{\rm bimax}^T$ and $V_{\rm down} = \mathbb{1}$ are consistent with the $SU(5)$-like relation $m_\ell = m_{\rm down}^T$. The same comments as in the first realization of QLC on whether the indicated scenario is realistic or not, would then apply here. If $m_\ell = m_{\rm down}^T$ was not assumed, the quark and lepton sector would not be related.

In the following, we will redo the calculations of the previous sections for all the observables with this second set of assumptions. First of all, we note that in the important basis in which the charged leptons and heavy neutrinos are real and diagonal the Dirac mass matrix reads

$$m_D \to \tilde{m}_D = V_R^T m_D U_\ell = m_{\rm up}^{\rm diag} P_\nu V Q_\nu U_{\rm bimax}^T \Rightarrow \begin{cases} \tilde{m}_D^\dagger \tilde{m}_D = U_{\rm bimax} Q_\nu^\dagger V^\dagger {\rm diag}(m_u^2, m_c^2, m_t^2) V Q_\nu U_{\rm bimax}^T & \text{for LFV,} \\ \tilde{m}_D \tilde{m}_D^\dagger = {\rm diag}(m_u^2, m_c^2, m_t^2) & \text{for } \eta_B. \end{cases} \tag{47}$$

The correspondence between the light and heavy Majorana neutrino masses is rather trivial:

$$M_1 = \frac{m_u^2}{m_1}, \qquad M_2 = \frac{m_c^2}{m_2}, \qquad M_3 = \frac{m_t^2}{m_3}. \tag{48}$$

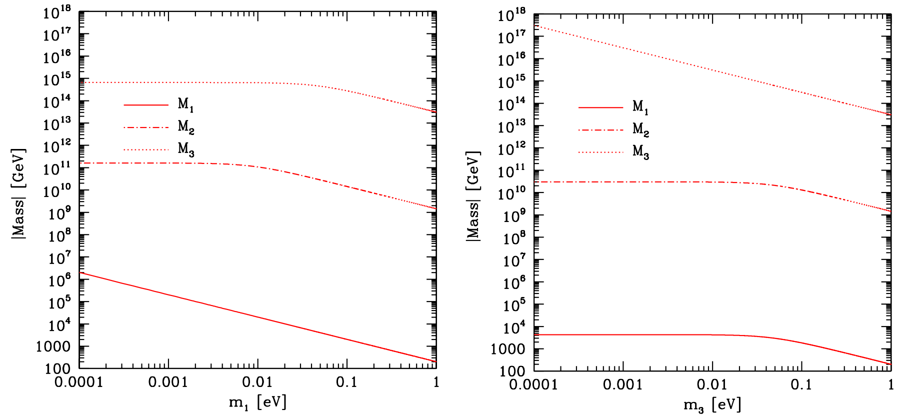

FIG. 6 (color online). Second realization of QLC: the heavy neutrino masses as a function of the smallest neutrino mass for the normal (left plot) and inverted (right plot) mass ordering.

In Fig. 6 we show the neutrino masses as a function of the smallest neutrino mass m_1 and m_3 for the normal and inverted ordering, respectively. Again, we have taken the best-fit points for $\Delta m^2_\odot$ and Δm^2_A and the quark masses are $m_u = 0.7$ MeV, $m_c = 210$ MeV, and $m_t = 82.4$ GeV. Note that in contrast to the first realization of QLC there is no possibility to enhance the neutrino masses, since they do not depend on phases. We can set a lower limit on m_1 or m_3 which stems from the requirement that M_1 or M_3 does not exceed the Planck mass:

$$m_1 \geq \frac{m_u^2}{M_{\rm Pl}} \simeq 4 \times 10^{-17}\ \text{eV} \quad \text{and} \qquad m_3 \geq \frac{m_t^2}{M_{\rm Pl}} \simeq 6 \times 10^{-7}\ \text{eV}. \tag{49}$$

This is for m_1 10 orders of magnitude smaller than the corresponding limit in the first realization of QLC, see Eq. (37).

Interestingly, there can be no leptogenesis in this scenario. First of all, M_1 is lighter than 10^7 GeV and this is—in analogy to the first realization of QLC—too small a value for successful leptogenesis. Can the decay of the second heaviest neutrino generate the baryon asymmetry? The answer is no, simply because $\tilde{m}_D \tilde{m}_D^\dagger$ is diagonal, as can be seen in Eq. (47). The decay asymmetries, both in the case when one sums over all flavors, Eq. (17), and the asymmetries for a given flavor, Eq. (40), are always proportional to off-diagonal entries of $\tilde{m}_D \tilde{m}_D^\dagger$ and therefore always vanish in this realization of QLC.

A. Low energy neutrino phenomenology

In our second case the PMNS matrix can be written as

$$U = U_\ell^\dagger U_\nu = R_{23} R_{12} (P_\nu V Q_\nu)^\dagger, \tag{50}$$

where R_{ij} is a rotation with $\pi/4$ around the (ij)-axis and P_ν and Q_ν are defined in Eq. (21). We remark that an analysis of this framework including all possible phases has not been performed before (see Refs. [5,6,9]). With our parametrization of the PMNS matrix, the two phases in P_ν are "Majorana-like" and do not show up in oscillations. All phases originate from the neutrino sector. The neutrino oscillation observables are

$$\begin{aligned} \sin^2\theta_{12} &= \frac{1}{2} - \lambda \cos\sigma + \mathcal{O}(\lambda^3), \\ |U_{e3}| &= \frac{A}{\sqrt{2}} \lambda^2 + \mathcal{O}(\lambda^3), \\ \sin^2\theta_{23} &= \frac{1}{2} - \sqrt{\frac{1}{2}} A \lambda^2 \cos(\tau - \sigma) + \mathcal{O}(\lambda^3), \\ J_{CP}^{\rm lep} &= \frac{\lambda^2}{4\sqrt{2}} \sin(\tau - \sigma) + \mathcal{O}(\lambda^3). \end{aligned} \tag{51}$$

The solar neutrino mixing parameter depends on the CP phase σ. Note that, in order to have solar neutrino mixing of the observed magnitude, the phase has to be close to zero or 2π, at 3σ typically below $\pi/4$ (or above $7\pi/4$). The prediction for $\sin^2\theta_{12}$ is[7]

$$\sin^2\theta_{12} \gtrsim 0.279\ (0.278, 0.277, 0.276). \tag{52}$$

These are lower values than in our first scenario. The numbers have to be compared to the 1σ (2σ) limit of $\sin^2\theta_{12} \leq 0.33$ (0.37). The parameter $|U_{e3}|$ has a "central value" of $A\lambda^2/\sqrt{2} \simeq 0.0295$. In the first scenario the prediction was $|U_{e3}|^2 = 0.0258$, which is by chance almost the same number. We find a range of

$$|U_{e3}| = 0.0295^{+0.0059,0.0070,0.0085,0.0099}_{-0.0058,0.0066,0.0076,0.0084}. \tag{53}$$

Recall that the 1σ (2σ) bound on $|U_{e3}|$ is 0.11 (0.17). Probing such small values of $|U_{e3}|$ is rather challenging

[7] Again, we do not use the approximate expressions in Eq. (51), but the exact equations. Besides the phases, we also vary the parameters of the CKM matrix in their 1, 2, and 3σ ranges, and also fix these parameters to their best-fit values.

and would require at least superbeams [34]. Because of cancellations $\sin^2\theta_{23} = \frac{1}{2}$ can always occur. In this case, $\cos(\tau - \sigma) = 0$ and $J_{CP}^{\rm lep}$ takes its maximal possible value. The minimal and maximal values of $\sin^2\theta_{23}$ are given by

$$\begin{aligned} &\sin^2\theta_{23} \geq 0.466\ (0.465, 0.463, 0.462) \quad \text{and} \\ &\sin^2\theta_{23} \leq 0.536\ (0.538, 0.539, 0.540), \end{aligned} \tag{54}$$

which is only a slightly larger range compared to the first scenario, and thus hard to probe experimentally. Leptonic CP violation is in leading order proportional to $\lambda^2 \sin(\tau - \sigma)$, which is four powers of λ larger than the J_{CP} of the quark sector. If the neutrino sector conserved CP, then $J_{CP}^{\rm lep}$ vanishes. Note that the phase combination $(\tau - \sigma)$ governs the magnitude of the atmospheric neutrino mixing. In the first scenario, $J_{CP}^{\rm lep}$ and the solar neutrino mixing were correlated in this way. In analogy to Eq. (25) we can write the sum-rule

$$\sin^2\theta_{23} \simeq \tfrac{1}{2} - |U_{e3}| \cos(\tau - \sigma) \simeq \tfrac{1}{2} \pm \sqrt{|U_{e3}| - 16 J_{CP}^2}. \tag{55}$$

In Fig. 7 we show the correlations between the oscillation parameters which result from the relation in Eq. (50). We plot $J_{CP}^{\rm lep}$ and $\sin^2\theta_{12}$ against $\sin^2\theta_{23}$, as well as σ and $|U_{e3}|$ against $\sin^2\theta_{12}$. We also indicate the current 1, 2, and 3σ ranges of the oscillation parameters, showing that the predictions of this scenario are perfectly compatible with all current data.

Turning aside again from the oscillation observables, the invariants for the Majorana phases are

$$\begin{aligned} S_1 &= -\frac{\lambda^2}{2} A \sin(\sigma + \omega) + \mathcal{O}(\lambda^3) \quad \text{and} \\ S_2 &= -\frac{\lambda^2}{2} A \sin(\omega - \phi) + \mathcal{O}(\lambda^3). \end{aligned} \tag{56}$$

In analogy to the discussion following Eq. (27), we can translate these formulas into expressions for the low energy Majorana phases α and β. This leads to $\sin\beta \simeq \sin(\sigma + \omega)$ and $\sin(\alpha - \beta) \simeq \sin(\phi - \omega)$ and indicates that α in the parametrization of Eq. (6) is related to $(\phi + \sigma)$. Indeed, a calculation of the effective mass in the case of an inverted hierarchy, where the Majorana phase α plays a crucial role [35], results in

$$\langle m \rangle \simeq \sqrt{\Delta m_{\rm A}^2} \left| c_{\phi+\sigma} + 2 \frac{s_\phi}{c_{\phi+\sigma}} \lambda^2 \right|. \tag{57}$$

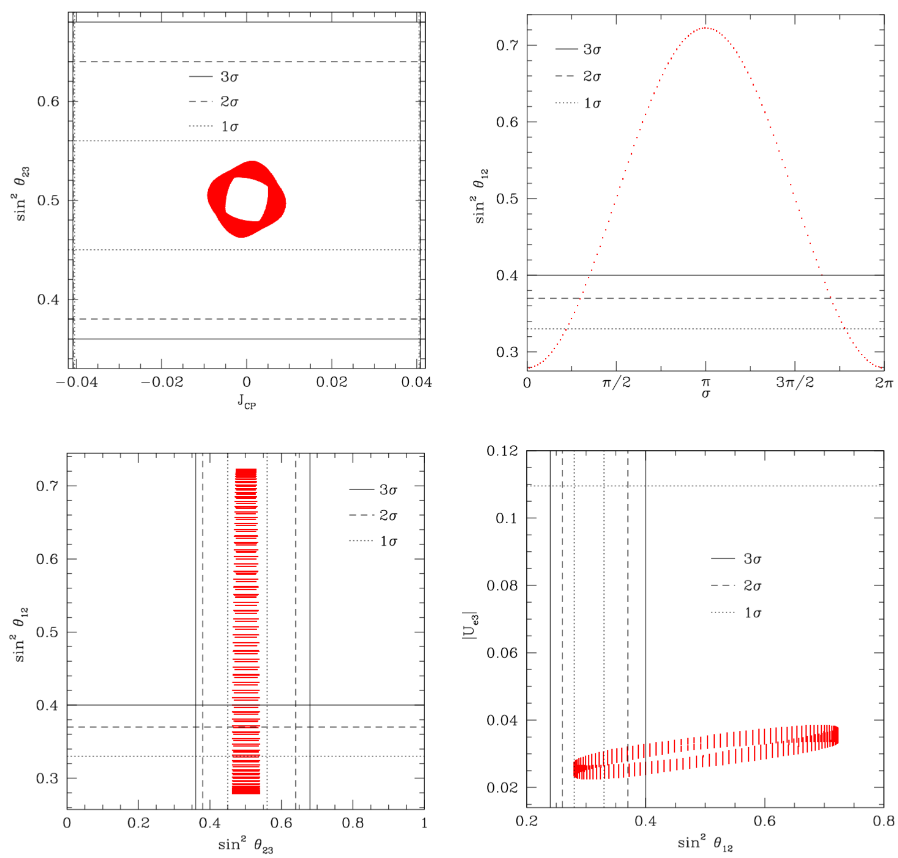

FIG. 7 (color online). Second realization of QLC: neutrino observables resulting from Eq. (51) for the 3σ ranges of the CKM parameters. We also indicated the current 1, 2, and 3σ ranges of the oscillation parameters.

Similar statements can be made for quasidegenerate neutrinos.

B. Lepton flavor violation

With the help of Eqs. (13) and (47), we can evaluate the branching ratios for LFV processes, ignoring for the moment the logarithmic dependence on the heavy neutrino masses. The decay $\mu \to e\gamma$ is found to be governed by

$$|(\tilde{m}_D^\dagger \tilde{m}_D)_{21}|^2 \simeq \frac{1}{4} A^2 m_t^4 \lambda^4 + \mathcal{O}(\lambda^5). \quad (58)$$

Comparing with Eq. (32), we see that in the second realization the branching ratio is larger than in the first realization by 6 inverse powers of λ, or $\lambda^{-6} \simeq 8820$, almost 4 orders of magnitude. For the double ratios of the branching ratios, we obtain

$$\begin{aligned} R(21/31) &\simeq 1 - 2\sqrt{2} A \cos(\sigma - \tau)\lambda^2 + \mathcal{O}(\lambda^3), \\ R(21/32) &\simeq A^2 \lambda^4 + \mathcal{O}(\lambda^5). \end{aligned} \quad (59)$$

There is a small dependence on the phase combination $(\sigma - \tau)$, which also governs leptonic CP violation in oscillation experiments and the magnitude of the atmospheric neutrino mixing angle. The branching ratios behave according to

$$\mathrm{BR}(\mu \to e\gamma):\mathrm{BR}(\tau \to e\gamma):\mathrm{BR}(\tau \to \mu\gamma) \simeq A^2\lambda^4 : A^2\lambda^4 : 1. \quad (60)$$

In Fig. 8 we show the branching ratios for $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ as a function of the smallest neutrino mass for a normal mass ordering, choosing the SPS points 1a, 2, and 4. The small dependence on the heavy neutrino masses is taken into account and plots for the inverted ordering look very similar. Note that from Fig. 8 it follows that the dependence on the neutrino masses is very small. The relative magnitude of the branching ratios, as estimated in Eq. (59), holds true to a very high accuracy. However, we immediately see that the prediction for $\mu \to e\gamma$ is at least one order of magnitude above the current limit. To obey the experimental limit on $\mathrm{BR}(\mu \to e\gamma)$, the SUSY masses should be in the several TeV range. This is illustrated in Fig. 9, where we have plotted the branching ratios as a function of the SUSY parameter $m_{1/2}$ for the SPS slopes 1a and 2 from Table I. We took the normal ordering of neutrino masses with a smallest mass $m_1 = 0.02$ eV. Once we have adjusted the SUSY parameters to have $\mathrm{BR}(\mu \to e\gamma)$ below its current limit, the other decays $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are too rare to be observed with presently planned experiments.

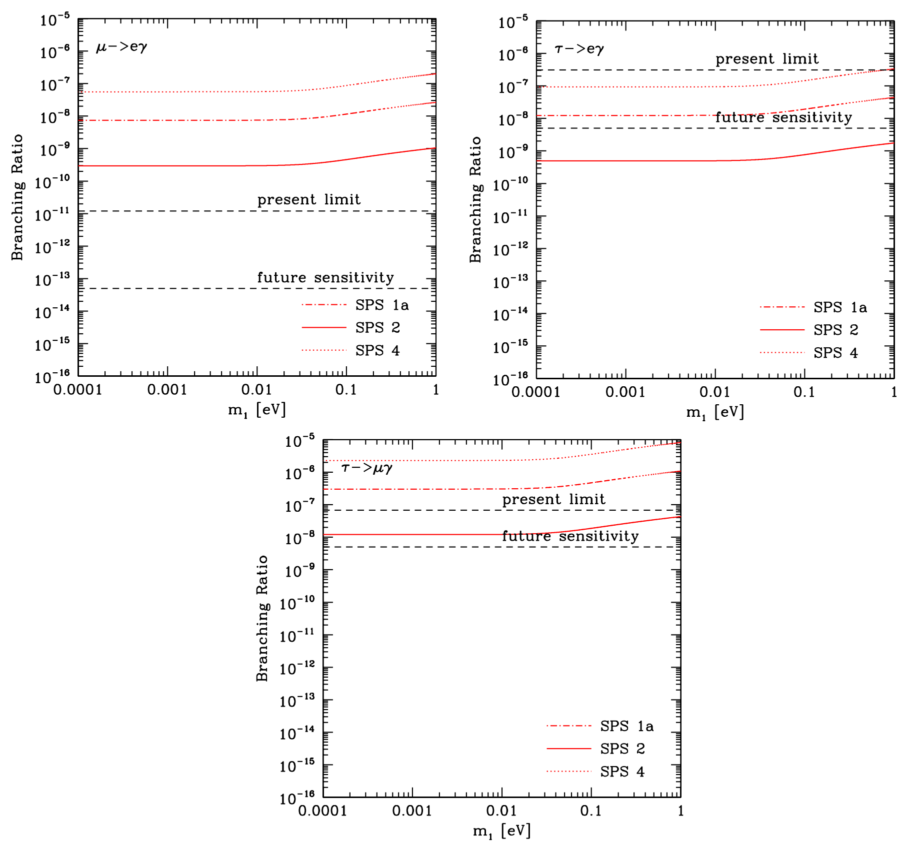

FIG. 8 (color online). Second realization of QLC: the branching ratios for $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ against the smallest neutrino mass (normal ordering) for the SPS points 1a, 2, and 4, see Table I. Indicated are also the present and future sensitivities.

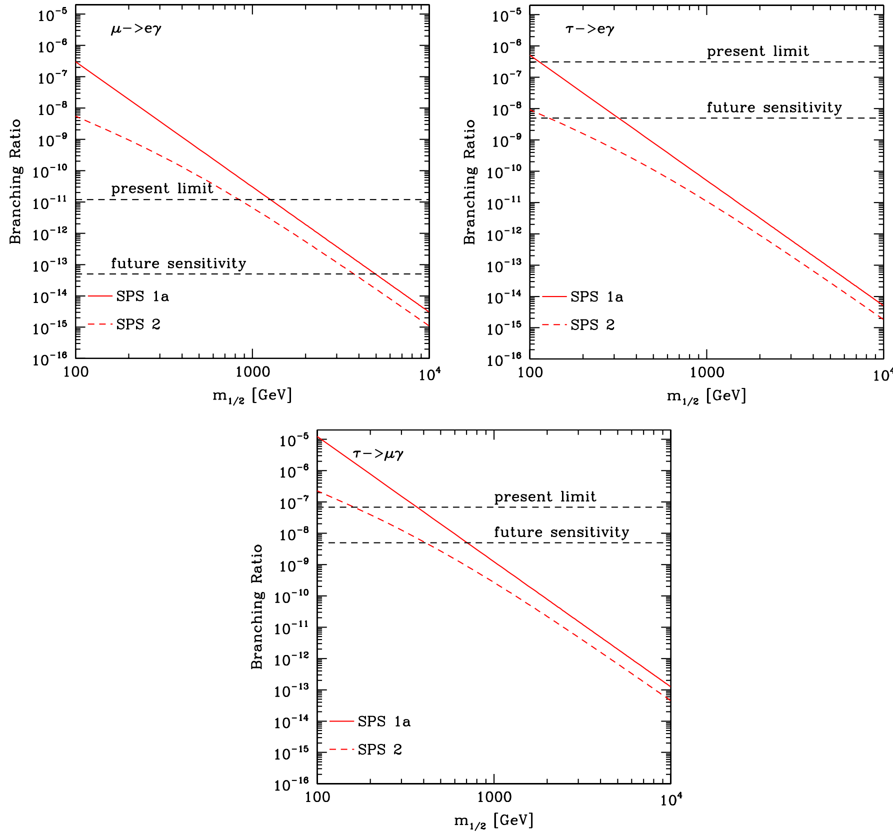

FIG. 9 (color online). Second realization of QLC: the branching ratios for $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ against the SUSY parameter $m_{1/2}$ for the SPS slopes 1a and 2, see Table I. We have chosen for the neutrino mass (normal ordering) 0.02 eV. Indicated are also the present and future sensitivities.

V. CONCLUSIONS AND SUMMARY

We have considered the phenomenology of two predictive seesaw scenarios leading approximately to quark-lepton complementarity. Both have in common that bimaximal mixing is corrected by the CKM matrix. We have studied the complete low energy phenomenology, including the neutrino oscillation parameters, where we have taken into account all possible phases, and neutrinoless double beta decay. Moreover, lepton flavor violating charged lepton decays have been studied and all results have been compared to presently available and expected future data. Finally, we have commented on the possibility of leptogenesis.[8]

In terms of the elements of the PMNS matrix U and the CKM matrix V, the QLC condition can be written as $|U_{e2}| + |V_{ud}| = 1/\sqrt{2}$. This defines the solar neutrino mixing parameter $\sin^2\theta_{12}$ to be $\sin^2(\frac{\pi}{4} - \lambda)$. Taking the best-fit, as well as the 1, 2, and 3σ values of λ from Eq. (9), we obtain

$$\sin^2\theta_{12} = 0.2805 \pm (0.0009, 0.0018, 0.0027). \tag{61}$$

A second QLC relation has also been suggested, namely $\theta_{23} + A\lambda^2 = \pi/4$, which is the analogue of Eq. (2) for the (23)-sector. This can also be written as $|U_{\mu 3}| + |V_{cb}| = 1/\sqrt{2}$ and its precise prediction is

$$\sin^2\theta_{23} = 0.4583^{+0.0011,0.0022,0.0032}_{-0.0011,0.0022,0.0034}. \tag{62}$$

We remark that in our scenario with all possible CP phases the above two relations correspond to at least one phase being zero.

The first scenario has bimaximal mixing from the neutrino sector and the matrix diagonalizing the charged leptons is the CKM matrix. The main results are:

(i) solar neutrino mixing is predicted close to its 1σ bound and $|U_{e3}|$ even close to its 2σ bound, see Fig. 1. The phase governing the magnitude of θ_{12} is the CP phase of neutrino oscillations and is implied to be small;

[8] As indicated at the beginning of Sec. II B, the decays $\mu \to e\gamma$, $\tau \to e\gamma$, and $\tau \to \mu\gamma$ are very strongly suppressed and cannot be observed if supersymmetry is not realized by nature. Hence, judging the validity of a given seesaw scenario based on its predictions for those decays can in this case not be done. Note that the predictions for leptogenesis do not depend on the presence of supersymmetry.

(ii) $|U_{e3}|$ is roughly 0.16, i.e., it should be observed soon;

(iii) the lowest value of $\sin^2\theta_{12}$ (corresponding to CP conservation) is roughly 0.33, which differs by about 15% from Eq. (61). For $\sin^2\theta_{23}$ the lowest value is 0.44, in moderate agreement with Eq. (62);

(iv) the decay $\mu \to e\gamma$ can be observable for not too small neutrino masses, unless the SUSY masses approach the TeV scale. BR$(\tau \to e\gamma)$ is predicted to be very small, and observation of $\tau \to \mu\gamma$ requires rather large neutrino masses, small SUSY masses, or large $\tan\beta$. The relative magnitude of the branching ratios is BR$(\mu \to e\gamma)$:BR$(\tau \to e\gamma)$:BR$(\tau \to \mu\gamma) \simeq \lambda^6$:$\lambda^2$:1;

(v) successful resonant leptogenesis depends on the low energy Majorana phases but is fine-tuned. One possibility occurs if $\sigma \simeq \pi/2$, leading to two quasidegenerate heavy neutrino masses. It also leads to quasidegenerate light neutrinos with mass around 0.5 eV and to sizable cancellations in neutrinoless double beta decay, corresponding to $\langle m \rangle \simeq 0.16$ eV. Leptogenesis via the decay of the second heaviest neutrino typically fails, even with the inclusion of flavor effects.

The second scenario has bimaximal mixing from the charged lepton sector and the matrix diagonalizing the neutrinos is the CKM matrix. The main results are:

(i) the neutrino oscillation parameters are perfectly compatible with all data, see Fig. 7. The phase governing the magnitude of θ_{23} is the CP phase of neutrino oscillations;

(ii) $|U_{e3}|$ is roughly 0.03, which is a rather small value setting a challenge for future experiments;

(iii) the lowest value of $\sin^2\theta_{12}$ (corresponding to CP conservation) is roughly 0.28, in perfect agreement with Eq. (61). For $\sin^2\theta_{23}$ the lowest value is 0.46 (but θ_{23} can be maximal), in perfect agreement with Eq. (62). If $\sin^2\theta_{23} = \frac{1}{2}$ then maximal CP violation is implied;

(iv) The branching ratio of $\mu \to e\gamma$ is larger than in the first scenario by six inverse powers of λ and therefore typically too large unless the SUSY masses are of several TeV scale. If they are so heavy that $\mu \to e\gamma$ is below its current limit, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are too small to be observed. The relative magnitude of the branching ratios is BR$(\mu \to e\gamma)$:BR$(\tau \to e\gamma)$:BR$(\tau \to \mu\gamma) \simeq A^2\lambda^4$:$A^2\lambda^4$:1;

(v) there can be no leptogenesis.

We conclude that both scenarios predict interesting and easily testable phenomenology. However, the first scenario is in slight disagreement with oscillation data and allows leptogenesis only for fine-tuned parameter values. In the second scenario, the predictions for LFV decays are in contradiction to experimental results unless the SUSY parameters are very large. Moreover, no leptogenesis is possible in this case.

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