

$\mathcal{N} = 1$ supersymmetric quantum chromodynamics: How confined non-Abelian monopoles emerge from quark condensationA. Gorsky,^{1,2} M. Shifman,² and A. Yung^{1,2,3}¹*Institute of Theoretical and Experimental Physics, Moscow 117259, Russia*²*William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*³*Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia*

(Received 11 January 2007; published 30 March 2007)

We consider $\mathcal{N} = 1$ supersymmetric QCD with the gauge group $U(N)$ and $N_f = N$ quark flavors. To get rid of flat directions we add a meson superfield. The theory has no adjoint fields and, therefore, no 't Hooft-Polyakov monopoles in the quasiclassical limit. We observe a non-Abelian Meissner effect: condensation of color charges (squarks) gives rise to *confined monopoles*. The very fact of their existence in $\mathcal{N} = 1$ supersymmetric QCD without adjoint scalars was not known previously. Our analysis is analytic and is based on the fact that the $\mathcal{N} = 1$ theory under consideration can be obtained starting from $\mathcal{N} = 2$ SQCD in which the 't Hooft-Polyakov monopoles do exist, through a certain limiting procedure allowing us to track the status of these monopoles at various stages. Monopoles are confined by Bogomol'nyi-Prasad-Sommerfield (BPS) non-Abelian strings (flux tubes). Dynamics of string orientational zero modes are described by a supersymmetric $CP(N - 1)$ sigma model on the string world sheet. If a dual of $\mathcal{N} = 1$ SQCD with the gauge group $U(N)$ and $N_f = N$ quark flavors could be identified, in this dual theory our demonstration would be equivalent to the proof of the non-Abelian dual Meissner effect.

DOI: [10.1103/PhysRevD.75.065032](https://doi.org/10.1103/PhysRevD.75.065032)

PACS numbers: 11.30.Pb, 12.38.Aw

I. INTRODUCTION

Seiberg and Witten demonstrated [1] that the dual Meissner effect takes place in $\mathcal{N} = 2$ Yang-Mills theories. Shortly after [1], Hanany, Strassler, and Zaffaroni discussed [2] formation and structure of the chromoelectric flux tubes in the Seiberg-Witten solution. Their analysis showed that details of the Seiberg-Witten confinement are quite different from those we expect in QCD-like theories. The confining strings in the Seiberg-Witten solution are, in fact, Abelian strings of the Abrikosov-Nielsen-Olesen type [3]. The “hadronic” spectrum in the Seiberg-Witten model is much richer than that in QCD (for a review, see e.g. [4].) The discovery of non-Abelian strings [5,6] and non-Abelian confined monopoles [7,8] was a significant step towards QCD. They were originally found in $\mathcal{N} = 2$ models which are quite distant relatives of QCD. To get closer to QCD one needs to have less supersymmetry. Another conspicuous feature of $\mathcal{N} = 2$ Yang-Mills theories which drastically distinguishes them from QCD-like theories is the presence of scalar and spinor fields in the adjoint representation.

To advance along these lines it is highly desirable to break $\mathcal{N} = 2$ down to $\mathcal{N} = 1$ and get rid of the adjoint superfield by making it very heavy, without destroying non-Abelian strings and confined monopoles. A partial success in this direction was reported in Ref. [9]. Adding a mass term to the adjoint superfield of the type $\delta\mathcal{W} = \mu\mathcal{A}^2$ breaks $\mathcal{N} = 2$. As long as the mass parameter μ is kept finite, the non-Abelian string in this $\mathcal{N} = 1$ model is well defined and supports confined monopoles. However, at $\mu \rightarrow \infty$, as the adjoint superfield becomes heavy and we approach the limit of $\mathcal{N} = 1$ SQCD, an infrared problem

develops. This is due to the fact that in $\mathcal{N} = 1$ SQCD defined in a standard way the vacuum manifold is not an isolated point; rather, there exists a flat direction (a Higgs branch). On the Higgs branch there are no finite-size Bogomol'nyi-Prasad-Sommerfield (BPS) strings [10]. Thus one arrives at a dilemma: either one has to abandon the attempt to decouple¹ the adjoint superfield, or, if this decoupling is performed, confining non-Abelian strings cease to exist [9].

In this paper we report that a relatively insignificant modification of the benchmark $\mathcal{N} = 2$ model solves the problem. All we have to do is to add a neutral meson superfield M coupled to the quark superfields through a superpotential term. Acting together with the mass term of the adjoint superfield, it breaks $\mathcal{N} = 2$ down to $\mathcal{N} = 1$. The limit $\mu \rightarrow \infty$ in which the adjoint superfield completely decouples, becomes well defined. No flat directions emerge. The limiting theory is $\mathcal{N} = 1$ SQCD supplemented by the meson superfield. We show that it supports non-Abelian strings. The junctions of these strings present confined monopoles, or, it is better to say, what becomes of the monopoles in the theory where there are no adjoint scalar fields. There is a continuous path following which one can trace the evolution in its entirety: from the

¹Below we use the word decouple in two opposite meanings: first, if a field becomes very heavy and disappears from the physical spectrum, so that it can be integrated out; second, if all coupling constants of a certain field vanish so that it becomes sterile. With regards to the adjoint fields decoupling, it means making them very heavy. With regards to the meson superfield M decoupling, it means sterility. Each time it is perfectly clear from the context what is meant. We hope this will cause no confusion.

't Hooft-Polyakov monopoles which do not exist without the adjoint scalars to the confined monopoles in the adjoint-free environment. As far as we know, this is the first demonstration (in a fully controllable weak coupling regime) of the Meissner effect in $\mathcal{N} = 1$ theories without adjoint superfields. If a dual of $\mathcal{N} = 1$ SQCD with the additional meson superfield could be found, in this dual theory our demonstration would be equivalent to the proof of the non-Abelian dual Meissner effect.

The organization of the paper is as follows. In Sec. III we review the benchmarks $\mathcal{N} = 2$ super-Yang-Mills theory with the gauge group $U(N)$ and $N_f = N$ quark flavors. We introduce the Fayet-Iliopoulos (FI) term [11] in the $U(1)$ subgroup, crucial for the string construction, and a meson superfield M , coupled to the quark superfields through a cubic superpotential. We add the mass terms to the adjoint superfields. The latter two terms in the superpotential break $\mathcal{N} = 2$. In Sec. III we discuss the spectrum of elementary excitations, in particular, in the limit $\mu \rightarrow \infty$. We show that the limiting theory is essentially $\mathcal{N} = 1$ SQCD. The only distinction is the meson superfield which survives in the limit $\mu \rightarrow \infty$. The vacuum of this theory, which will be referred to as the M model, is isolated (i.e. there are no flat directions). As usual, we construct non-Abelian strings and determine the world sheet theory. This is the contents of Sec. IV. One of the crucial points of our analysis is determination of the fermion zero modes. To count these modes we engineer an appropriate index theorem (Sec. V). This theorem applies to the two-dimensional Dirac operator which we encounter in the string analysis. In Sec. we derive the index $\nu = 4N$. We observe four supertranslational zero modes and $4(N - 1)$ superorientational modes. In Sec. VI we discuss how the monopoles evolve when we vary the adjustable parameters of the M model: from the 't Hooft-Polyakov limit to the limit of confined monopoles in a highly quantum regime in $\mathcal{N} = 1$ SQCD. In Sec. VII the same issue is discussed from the brane perspective. Section VIII summarizes our findings. Finally, in the appendix we present explicit expressions for the fermion zero modes in the case of two flavors.

II. FROM $\mathcal{N} = 2$ SQCD TO $\mathcal{N} = 1$

To begin with, let us briefly review $\mathcal{N} = 2$ supersymmetric QCD. The gauge symmetry of our benchmark model is $SU(N) \times U(1)$. It has $N_f = N$ matter hypermultiplets. The field content of this model is as follows. The $\mathcal{N} = 2$ vector multiplet consists of the $U(1)$ gauge fields A_μ and $SU(N)$ gauge field A_μ^a , (here $a = 1, \dots, N^2 - 1$), their Weyl fermion superpartners $(\lambda_\alpha^1, \lambda_\alpha^2)$ and $(\lambda_\alpha^{1a}, \lambda_\alpha^{2a})$, and complex scalar fields a and a^a , the latter in the adjoint of $SU(N)$. The spinorial index of λ 's runs over $\alpha = 1, 2$. In this sector the global $SU(2)_R$ symmetry inherent to the $\mathcal{N} = 2$ model at hand manifests itself through rotations $\lambda^1 \leftrightarrow \lambda^2$.

The quark multiplets of the $SU(N) \times U(1)$ theory consist of the complex scalar fields q^{kA} and \tilde{q}_{Ak} (squarks) and the Weyl fermions ψ^{kA} and $\tilde{\psi}_{Ak}$, all in the fundamental representation of the $SU(N)$ gauge group. Here $k = 1, \dots, N$ is the color index while A is the flavor index, $A = 1, \dots, N$. Note that the scalars q^{kA} and \tilde{q}^{kA} form a doublet under the action of the global $SU(2)_R$ group.

In addition, we introduce the Fayet-Iliopoulos D term for the $U(1)$ gauge field which triggers the squark condensation.

The undeformed $\mathcal{N} = 2$ theory we start from has a superpotential,

$$\mathcal{W}_{\mathcal{N}=2} = \sqrt{2} \text{Tr} \left\{ \frac{1}{2} \tilde{Q} \mathcal{A} Q + \tilde{Q} \mathcal{A}^a T^a Q \right\} + \text{Tr} m \tilde{Q} Q, \quad (2.1)$$

where \mathcal{A}^a and \mathcal{A} are chiral superfields, the $\mathcal{N} = 2$ superpartners of the gauge bosons of $SU(N)$ and $U(1)$, respectively, while T^a are generators of $SU(N)$ normalized by the condition

$$\text{Tr} T^a T^b = \frac{1}{2} \delta^{ab}.$$

Moreover, m is the quark mass matrix, a numerical $N \times N$ matrix m_A^B (to be elevated to a superfield matrix later on). We write the quark superfields Q^{kA} as $N \times N$ matrices in color and flavor indices. The trace in (2.1) runs over the appropriate indices.

Now we deform this theory in two ways each of which breaks $\mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$. First, we add superpotential mass terms for the adjoint chiral superfields from the $U(1)$ and $SU(N)$ sectors, respectively,

$$\delta \mathcal{W} = \sqrt{\frac{N}{2}} \frac{\mu_1}{2} \mathcal{A}^2 + \frac{\mu_2}{2} (\mathcal{A}^a)^2, \quad (2.2)$$

where μ_1 and μ_2 are mass parameters. Clearly, the mass term (2.2) splits the gauge $\mathcal{N} = 2$ supermultiplets, breaking $\mathcal{N} = 2$ supersymmetry down to $\mathcal{N} = 1$. As will be discussed later in detail, in the large- μ limit the adjoint multiplets decouple and then we recover $\mathcal{N} = 1$ SQCD with $N_f = N$ flavors. This theory has a Higgs branch (see, for example, [12]). The presence of quark massless states in the bulk associated with this Higgs branch obscure physics of non-Abelian strings in this theory [9]. In particular, the strings become infinitely thick.

Can one avoid this shortcoming? The answer is yes. To this end we introduce another $\mathcal{N} = 2$ breaking deformation. Namely, we uplift the quark mass matrix m_A^B to the superfield status,

$$m_A^B \rightarrow M_A^B,$$

where M represents N^2 chiral superfields of the mesonic type (they are color singlets). With this uplifting we have to add a kinetic term for M_A^B ,

$$\delta S_{M\text{kin}} = \int d^4x d^2\theta d^2\bar{\theta} \frac{2}{h} \text{Tr} \bar{M} M, \quad (2.3)$$

where h is a new coupling constant (it supplements the set of the gauge couplings). At $h = 0$ the matrix field M becomes sterile, it is frozen and in essence returns to the status of a constant numerical matrix as in Ref. [9]. The theory acquires flat directions (a moduli space). With non-vanishing h these flat directions are lifted and M is determined by the minimum of the scalar potential, see below.

The elevation of the quark mass matrix to superfield is a crucial step which allows us to lift the Higgs branch which would develop in this theory in the large μ limit if M were a constant matrix.

The bosonic part of our $SU(N) \times U(1)$ theory has the form

$$S = \int d^4x \left[\frac{1}{4g_2^2} (F_{\mu\nu}^a)^2 + \frac{1}{4g_1^2} (F_{\mu\nu})^2 + \frac{1}{g_2^2} |D_\mu a^a|^2 + \frac{1}{g_1^2} |\partial_\mu a|^2 + \text{Tr} |\nabla_\mu q|^2 + \text{Tr} |\nabla_\mu \tilde{q}|^2 + \frac{1}{h} |\partial_\mu M^0|^2 + \frac{1}{h} |\partial_\mu M^a|^2 + V(q, \tilde{q}, a^a, a, M^0, M^a) \right]. \quad (2.4)$$

where f^{abc} stands for the $SU(N)$ structure constants. The first and second terms here represent D terms, the next two terms are $F_{\mathcal{A}}$ terms, while the term in the curly brackets represents the squark F terms. Two last terms are F terms of the M field. In Eq. (2.7) we also introduced the FI D term for the $U(1)$ field, with the FI parameter ξ . Note that the FI term does not break $\mathcal{N} = 2$ supersymmetry [2,13]. The three parameters which do break $\mathcal{N} = 2$ down to $\mathcal{N} = 1$ are μ_1 , μ_2 , and h .

The FI term triggers the spontaneous breaking of the gauge symmetry. The vacuum expectation values (VEV's) of the squark fields can be chosen as

$$\langle q^{kA} \rangle = \sqrt{\xi} \begin{pmatrix} 1 & 0 & \dots \\ \dots & \dots & \dots \\ \dots & 0 & 1 \end{pmatrix}, \quad \langle \tilde{q}^{kA} \rangle = 0, \quad (2.8)$$

$k = 1, \dots, N, \quad A = 1, \dots, N,$

up to gauge rotations. The VEV's of the adjoint fields vanish,

$$\langle a^a \rangle = 0, \quad \langle a \rangle = 0, \quad (2.9)$$

and so do those of the M fields,

Here D_μ is the covariant derivative in the adjoint representation of $SU(2)$, while

$$\nabla_\mu = \partial_\mu - \frac{i}{2} A_\mu - i A_\mu^a T^a. \quad (2.5)$$

Moreover, the matrix M_B^A can be always decomposed as

$$M_B^A = \frac{1}{2} \delta_B^A M^0 + (T^a)_B^A M^a. \quad (2.6)$$

We use this decomposition in Eq. (2.4). The coupling constants g_1 and g_2 correspond to the $U(1)$ and $SU(N)$ sectors, respectively. With our conventions the $U(1)$ charges of the fundamental matter fields are $\pm 1/2$.

The potential $V(q^A, \tilde{q}_A, a^a, a, M^0, M^a)$ in the Lagrangian (2.4) is a sum of various D and F terms,

$$V(q^A, \tilde{q}_A, a^a, a, M^0, M^a) = \frac{g_2^2}{2} \left(\frac{1}{g_2^2} f^{abc} \tilde{a}^b a^c + \text{Tr} \tilde{q} T^a q - \text{Tr} \tilde{q} T^a \tilde{q} \right)^2 + \frac{g_1^2}{8} (\text{Tr} \tilde{q} q - \text{Tr} \tilde{q} \tilde{q} - N \xi)^2 + \frac{g_2^2}{2} |2 \text{Tr} \tilde{q} T^a q + \sqrt{2} \mu_2 a^a|^2 + \frac{g_1^2}{2} |\text{Tr} \tilde{q} q + \sqrt{N} \mu_1 a|^2 + \frac{1}{2} \text{Tr} \left\{ \left| (a + 2T^a a^a) q + \frac{1}{\sqrt{2}} q (M^0 + 2T^a M^a) \right|^2 + \left| (a + 2T^a a^a) \tilde{q} + \frac{1}{\sqrt{2}} \tilde{q} (M^0 + 2T^a M^a) \right|^2 \right\} + \frac{h}{4} |\text{Tr} \tilde{q} q|^2 + h |\text{Tr} q T^a \tilde{q}|^2, \quad (2.7)$$

$$\langle M^a \rangle = 0, \quad \langle M^0 \rangle = 0. \quad (2.10)$$

The color-flavor locked form of the quark VEV's in Eq. (2.8) and the absence of VEV's of the adjoint scalar a^a and the meson scalar M^a in Eqs. (2.9) and (2.10) result in the fact that, while the theory is fully Higgsed, a diagonal $SU(N)_{C+F}$ symmetry survives as a global symmetry. Namely, the global rotation

$$q \rightarrow U q U^{-1}, \quad a^a T^a \rightarrow U a^a T^a U^{-1}, \quad (2.11)$$

$$M \rightarrow U^{-1} M U,$$

where U is a matrix from $SU(N)$ is not broken by the VEV's (2.8), (2.9), and (2.10). This is a particular case of the Bardakçi-Halpern mechanism [14]. The presence of this symmetry leads to the emergence of orientational zero modes [6] of the Z_N strings in the model (2.4).

Note that the vacuum expectation values (2.8), (2.9), and (2.10) do not depend on the supersymmetry breaking parameters μ_1 and μ_2 . This is because our choice of parameters in (2.4) ensures vanishing of the adjoint VEV's, see (2.9). In particular, we have the same pattern of symmetry

breaking all the way up to very large values μ_1 and μ_2 , where the adjoint fields decouple.

With N matter hypermultiplets, the $SU(N)$ part of the gauge group is asymptotically free, implying generation of a dynamical scale Λ . In the ultraviolet (UV) we start with a small g_2^2 , and let the theory evolve in the infrared. If the descent to Λ were uninterrupted, the gauge coupling g_2^2 would explode at this scale. Moreover, strong coupling effects in the $SU(N)$ subsector at the scale Λ would break the $SU(N)$ subgroup through the Seiberg-Witten mechanism [1]. Since we want to stay at weak coupling, we assume that $\sqrt{\xi} \gg \Lambda$, so that the $SU(N)$ coupling running is frozen by the squark condensation at a small value, namely,

$$\frac{8\pi^2}{Ng_2^2} = \ln \frac{\sqrt{\xi}}{\Lambda} + \dots \gg 1. \quad (2.12)$$

Now let us discuss the elementary excitation spectrum in the theory (2.4). Since both $U(1)$ and $SU(N)$ gauge groups are broken by the squark condensation, all gauge bosons become massive. From (2.4) we get for the $U(1)$ gauge boson mass (we refer to it as photon)

$$m_{\text{ph}} = g_1 \sqrt{\frac{N}{2}} \xi, \quad (2.13)$$

while $(N^2 - 1)$ gauge bosons of the $SU(N)$ group acquire a common mass

$$m_W = g_2 \sqrt{\xi}. \quad (2.14)$$

This is typical of the Bardakçı-Halpern mechanism. To get the masses of the scalar bosons we expand the potential (2.7) near the vacuum (2.8), (2.9), and (2.10) and diagonalize the corresponding mass matrix. The N^2 components of the $2N^2$ -component² scalar q^{kA} are eaten by the Higgs mechanism for $U(1)$ and $SU(N)$ gauge groups. Another N^2 components are split as follows: one component acquires the mass (2.13). It becomes a scalar component of a massive $\mathcal{N} = 1$ vector $U(1)$ gauge multiplet. The remaining $N^2 - 1$ components acquire masses (2.14) and become scalar superpartners of the $SU(N)$ gauge boson in the $\mathcal{N} = 1$ massive gauge supermultiplet.

Moreover, $6N^2$ real scalar components of the fields \tilde{q}_{Ak} , a^a , M^a , and M^0 produce the following states: six states have masses determined by the roots of the cubic equation

$$\lambda_i^3 - \lambda_i^2(2 + \omega_i^2 + 2\gamma_i) + \lambda_i(1 + 2\gamma_i + \gamma_i^2 + 2\gamma_i\omega_i) - \gamma_i^2\omega_i^2 = 0, \quad (2.15)$$

for $i = 1$. Namely, these states form degenerate pairs with the masses

²We mean here *real* components.

$$m_{U(1)} = g_1 \sqrt{\frac{N}{2}} \xi \lambda_1. \quad (2.16)$$

Each root of Eq. (2.15) for $i = 1$ determines masses of two degenerate states.

Above we introduced $\mathcal{N} = 2$ supersymmetry breaking parameters ω and γ associated with the $U(1)$ and $SU(N)$ gauge groups, respectively,

$$\omega_1 = \frac{g_1 \mu_1}{\sqrt{\xi}}, \quad \omega_2 = \frac{g_2 \mu_2}{\sqrt{\xi}}, \quad (2.17)$$

and

$$\gamma_1 = \frac{h}{2g_1^2}, \quad \gamma_2 = \frac{h}{2g_2^2}. \quad (2.18)$$

Now we are left with $6(N^2 - 1)$ states. They acquire masses

$$m_{SU(N)} = g_2 \sqrt{\xi \lambda_2}, \quad (2.19)$$

where each root of Eq. (2.15) for $i = 2$ determines masses of $2(N^2 - 1)$ degenerate states.

When the supersymmetry breaking parameters ω_i and γ_i vanish, two mass eigenvalues (2.16) coincide with the $U(1)$ gauge boson mass (2.13). The corresponding states form the bosonic part of the $\mathcal{N} = 2$ long massive $U(1)$ vector supermultiplet [13]. The one remaining eigenvalue in (2.16) becomes zero. It corresponds to the massless field M^0 which decouples (becomes sterile) in this limit. With nonvanishing values of ω_1 and γ_1 this supermultiplet splits into the massive $\mathcal{N} = 1$ vector multiplet, with mass (2.13), plus three chiral multiplets with masses given by Eq. (2.16).

The same happens with the states with masses (2.19). If ω 's and γ 's vanish they combine into the bosonic parts of $(N^2 - 1)$ $\mathcal{N} = 2$ massive vector supermultiplets, with mass (2.14), plus the massless field M^a . If ω 's and γ 's do not vanish, these multiplets split into $(N^2 - 1)$ $\mathcal{N} = 1$ vector multiplets (for the $SU(N)$ group), with mass (2.14), and $3(N^2 - 1)$ chiral multiplets, with masses (2.19).

III. $\mathcal{N} = 1$ SQCD WITH THE MESONIC M FIELD

Now let us take a closer look at the spectrum obtained above, assuming the limit of very large $\mathcal{N} = 2$ supersymmetry breaking parameters ω_i ,

$$\omega_i \gg 1.$$

In this limit the largest masses $m_{U(1)}$ and $m_{SU(N)}$ become

$$m_{U(1)}^{(\text{largest})} = m_{U(1)} \omega_1 = \sqrt{\frac{N}{2}} g_1^2 \mu_1, \quad (3.1)$$

$$m_{SU(N)}^{(\text{largest})} = m_{SU(2)} \omega_2 = g_2^2 \mu_2.$$

Clearly, in the limit $\mu_i \rightarrow \infty$ these are the masses of the

heavy adjoint scalars a and a^a . At $\omega_i \gg 1$ these fields leave the physical spectrum; they can be integrated out.

The low-energy bulk theory in this limit contains massive gauge $\mathcal{N} = 1$ multiplets and chiral multiplets with two lower masses $m_{U(1)}$ and two lower masses $m_{SU(N)}$. Equation (2.15) gives for these masses

$$m_{U(1)}^{(1)} = \sqrt{\frac{hN\xi}{4}} \left\{ 1 + \frac{1}{2\omega_1} \sqrt{\gamma_1(\gamma_1 + 1)} + \dots \right\}, \quad (3.2)$$

$$m_{U(1)}^{(2)} = \sqrt{\frac{hN\xi}{4}} \left\{ 1 - \frac{1}{2\omega_1} \sqrt{\gamma_1(\gamma_1 + 1)} + \dots \right\},$$

for the U(1) sector and

$$m_{SU(N)}^{(1)} = \sqrt{\frac{h\xi}{2}} \left\{ 1 + \frac{1}{2\omega_2} \sqrt{\gamma_2(\gamma_2 + 1)} + \dots \right\}, \quad (3.3)$$

$$m_{SU(N)}^{(2)} = \sqrt{\frac{h\xi}{2}} \left\{ 1 - \frac{1}{2\omega_2} \sqrt{\gamma_2(\gamma_2 + 1)} + \dots \right\},$$

for the SU(N) sector.

It is worth emphasizing again that there are no massless states in the bulk theory. As we have already mentioned, at $h = 0$ the theory (2.4) develops a Higgs branch in the large- μ limit (see, for example, [9]). If $h \neq 0$, M becomes a fully dynamical field, and the Higgs branch is lifted, as follows from Eqs. (3.2) and (3.3).

At large μ one can readily integrate out the adjoint fields \mathcal{A}^a and $\bar{\mathcal{A}}$. Instead of the superpotential terms (2.1) and (2.2) we get

$$\mathcal{W} = -\frac{(\text{Tr}\tilde{Q}Q)^2}{4\mu_1} - \frac{(\text{Tr}\tilde{Q}T^aQ)^2}{\mu_2} + \text{Tr}M\tilde{Q}Q. \quad (3.4)$$

At $\mu_{1,2} \rightarrow \infty$ the first two terms disappear, we are left with $\mathcal{W} = \text{Tr}M\tilde{Q}Q$, and our model (2.4) reduces to $\mathcal{N} = 1$ SQCD with the extra mesonic M field. The bosonic part of the action takes the form

$$S = \int d^4x \left[\frac{1}{4g_2^2} (F_{\mu\nu}^a)^2 + \frac{1}{4g_1^2} (F_{\mu\nu})^2 + \text{Tr}|\nabla_\mu q|^2 \right. \\ \left. + \text{Tr}|\nabla_\mu \tilde{q}|^2 + \frac{1}{h} |\partial_\mu M^0|^2 + \frac{1}{h} |\partial_\mu M^a|^2 + \frac{g_2^2}{2} \right. \\ \left. \times (\text{Tr}\tilde{q}T^a q - \text{Tr}\tilde{q}T^a \tilde{q})^2 + \frac{g_1^2}{8} (\text{Tr}\tilde{q}q - \text{Tr}\tilde{q}\tilde{q} - N\xi)^2 \right. \\ \left. + \text{Tr}|qM|^2 + \text{Tr}|\tilde{q}M|^2 + \frac{h}{4} |\text{Tr}\tilde{q}q|^2 + h |\text{Tr}qT^a \tilde{q}|^2 \right]. \quad (3.5)$$

The vacuum of this theory is given by Eqs. (2.8) and (2.10). The mass spectrum of elementary excitations over this vacuum consists of the $\mathcal{N} = 1$ gauge multiplets for the U(1) and SU(N) sectors with masses given by Eqs. (2.13) and (2.14), and the chiral multiplets of the U(1) and SU(N) sectors with masses given by the leading

terms in Eqs. (3.2) and (3.3). The scale of the theory (3.5) is determined by the scale of the theory (2.4) in the $\mathcal{N} = 2$ limit by the relation

$$\Lambda_{\mathcal{N}=1}^{2N} = \mu_2^N \Lambda^N. \quad (3.6)$$

In order to keep the theory (3.5) at weak coupling we assume that

$$g_2\sqrt{\xi} \gg \Lambda_{\mathcal{N}=1}. \quad (3.7)$$

Our $\mathcal{N} = 1$ SQCD with the M field, the M model, belongs to the class of theories introduced by Seiberg [15] to give a dual description of conventional $\mathcal{N} = 1$ SQCD with the SU(N_c) gauge group and N_f flavors of fundamental matter, where

$$N_c = N_f - N$$

(for reviews see Refs. [12,16]). There are significant distinctions, however.

Let us point out the main differences of the M model (3.5) from those introduced [15] by Seiberg:

- (i) Our theory has the U(N) gauge group rather than SU(N);
- (ii) Our theory has the FI D term instead of a linear in M superpotential in Seiberg's models;
- (iii) We consider the case $N_f = N$ which would correspond to Seiberg's $N_c = 0$. Our theory (3.5) is asymptotically free while Seiberg's dual theories give the most reliable description of the original $\mathcal{N} = 1$ SQCD in the range $N_f < 3/2N_c$ which corresponds to $N_f > 3N$. In this range the theory (3.5) is not asymptotically free.

In addition, it is worth noting that at $N_f > N$ the vacuum (2.8) and (2.10) becomes metastable: supersymmetry is broken [17]. The $N_c = N_f - N$ supersymmetry-preserving vacua have vanishing VEV's of the quark fields and nonvanishing VEV of the M field.³ The latter vacua are associated with the gluino condensation in pure SU(N) theory, $\langle \lambda\lambda \rangle \neq 0$, arising upon decoupling of N_f flavors [12]. In the case $N_f = N$ considered here the vacuum (2.8) and (2.10) preserves supersymmetry. Thus, despite a conceptual similarity between Seiberg's models and ours, dynamical details are radically different.

To conclude this section let us mention that if a theory dual to the one in (3.5) were known our results would imply a non-Abelian confinement of quarks in the former theory. We will qualitatively discuss this issue in Sec. VIII.

IV. NON-ABELIAN STRINGS

Non-Abelian strings were shown to emerge at weak coupling in $\mathcal{N} = 2$ supersymmetric gauge theories [5–8]. The main feature of the non-Abelian strings is the

³This is correct for the version of the theory with ξ -parameter introduced via superpotential.

presence of orientational zero modes associated with rotations of their color flux in the non-Abelian gauge group. This feature makes such strings genuinely non-Abelian.

As long as the solution for the non-Abelian string suggested and discussed in [6,7] for $\mathcal{N} = 2$ SQCD does not depend on the adjoint fields it can be generalized to $\mathcal{N} = 1$ SQCD upon introducing the mass term (2.2) for the adjoint fields and then taking the limit $\mu_{1,2} \rightarrow \infty$. This was done in Ref. [9]. However, as we have already explained above, $\mathcal{N} = 1$ SQCD has the Higgs branch which obscures physics of the non-Abelian strings. The string becomes infinitely thick in the limit $\mu_i \rightarrow \infty$ due to the presence of massless fields in the bulk.

In particular, in [9] it turned out impossible to follow the fate of the confined monopoles (present in $\mathcal{N} = 2$ SQCD) all the way down to $\mathcal{N} = 1$ SQCD which one recovers in the limit $\mu_{1,2} = \infty$. Below we will show that this obstacle does not arise in the model (2.4). The reason is that $\mathcal{N} = 1$ SQCD with the mesonic field M has no massless states in the bulk in the limit $\mu_i \rightarrow \infty$, as was demonstrated in Sec. III.

Let us generalize the string solutions found in [6,7] to the model (2.4). In addition to the conventional Abrikosov-Nielsen-Olesen (ANO) strings [3] this model supports Z_N strings. These arise due to a nontrivial homotopy group

$$\pi_1(\text{SU}(N) \times \text{U}(1)/Z_N) \neq 0. \quad (4.1)$$

It is easy to see that this nontrivial topology amounts to winding of just one element of the diagonal matrix (2.8) at infinity. Such strings can be called elementary; their tension is $1/N$ of that of the ANO string. The ANO string can be viewed as a bound state of N elementary strings.

More concretely, the Z_N string solution (a progenitor of the non-Abelian string) can be written [6] as follows:

$$q = \begin{pmatrix} \phi_2(r) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & \phi_2(r) & 0 \\ 0 & 0 & \dots & e^{i\alpha} \phi_1(r) \end{pmatrix}, \quad \tilde{q} = 0,$$

$$A_i^{\text{SU}(N)} = \frac{1}{N} \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} \quad (4.2)$$

$$\times (\partial_i \alpha)[-1 + f_{\text{NA}}(r)],$$

$$A_i^{\text{U}(1)} = \frac{I}{2} A_i = \frac{I}{N} (\partial_i \alpha)[1 - f(r)],$$

$$a = a^a = M^0 = M^a = 0,$$

where $i = 1, 2$ labels coordinates in the plane orthogonal to the string axis, r and α are the polar coordinates in this plane, and I is the unit $N \times N$ matrix. The profile functions $\phi_1(r)$ and $\phi_2(r)$ determine the profiles of the scalar fields, while $f_{\text{NA}}(r)$ and $f(r)$ determine the $\text{SU}(N)$ and $\text{U}(1)$ fields

of the string solution, respectively. These functions satisfy the following rather obvious boundary conditions:

$$\phi_1(0) = 0, \quad f_{\text{NA}}(0) = 1, \quad f(0) = 1, \quad (4.3)$$

at $r = 0$, and

$$\begin{aligned} \phi_1(\infty) &= \sqrt{\xi}, & \phi_2(\infty) &= \sqrt{\xi}, \\ f_{\text{NA}}(\infty) &= 0, & f(\infty) &= 0 \end{aligned} \quad (4.4)$$

at $r = \infty$.

As long as our *ansatz* (4.2) does not involve the fields \tilde{q} , a , and M the classical string solution does not depend on $\mathcal{N} = 2$ SUSY breaking parameters. The classical solution is the same as that found [6] in the $\mathcal{N} = 2$ SQCD limit. In particular, the profile functions satisfy the following first-order equations:

$$\begin{aligned} r \frac{d}{dr} \phi_1(r) - \frac{1}{N} (f(r) + (N-1)f_{\text{NA}}(r)) \phi_1(r) &= 0, \\ r \frac{d}{dr} \phi_2(r) - \frac{1}{N} (f(r) - f_{\text{NA}}(r)) \phi_2(r) &= 0, \\ -\frac{1}{r} \frac{d}{dr} f(r) + \frac{g_1^2 N}{4} [(\phi_1(r))^2 + (N-1)(\phi_2(r))^2 - N\xi] &= 0, \\ -\frac{1}{r} \frac{d}{dr} f_{\text{NA}}(r) + \frac{g_2^2}{2} [(\phi_1(r))^2 - (\phi_2(r))^2] &= 0. \end{aligned} \quad (4.5)$$

Numerical solutions of the Bogomol'nyi equations (4.5) for $N = 2$ (Z_2 strings) were found in Ref. [6].

The string (4.8) is 1/2-BPS saturated. It automatically preserves two supercharges out of four present in the bulk theory. The tension of this elementary string is

$$T_1 = 2\pi\xi, \quad (4.6)$$

to be compared with the ANO string tension

$$T_{\text{ANO}} = 2N\pi\xi \quad (4.7)$$

in our normalization.

The elementary strings are *bona fide* non-Abelian. This means that, besides trivial translational moduli, they acquire moduli corresponding to spontaneous breaking of a non-Abelian symmetry. Indeed, while the ‘‘flat’’ vacuum (2.8), (2.9), and (2.10) is $\text{SU}(N)_{C+F}$ symmetric, the solution (4.2) breaks this symmetry down to $\text{U}(1) \times \text{SU}(N-1)$.

To obtain the non-Abelian string solution from the Z_N string (4.2) we apply the diagonal color-flavor rotation (2.11) which preserves the vacuum. To this end it is convenient to pass to the singular gauge where the scalar fields have no winding at infinity, while the string flux comes from the vicinity of the origin. In this gauge we have (for details see the review paper [18])

$$\begin{aligned}
 q &= \frac{1}{N}[(N-1)\phi_2 + \phi_1] + (\phi_1 - \phi_2)\left(n \cdot n^* - \frac{1}{N}\right), \\
 A_i^{\text{SU}(N)} &= \left(n \cdot n^* - \frac{1}{N}\right)\varepsilon_{ij} \frac{x_i}{r^2} f_{\text{NA}}(r), \\
 A_i^{\text{U}(1)} &= \frac{1}{N}\varepsilon_{ij} \frac{x_i}{r^2} f(r), \\
 \tilde{q} &= 0, \\
 a &= a^a = M^0 = M^a = 0,
 \end{aligned} \tag{4.8}$$

where we parametrize the matrices U of $\text{SU}(N)_{C+F}$ rotations as follows:

$$\begin{aligned}
 &\frac{1}{N} \left\{ U \begin{pmatrix} 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots \\ 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -(N-1) \end{pmatrix} U^{-1} \right\}_p \\
 &= -n^l n_p^* + \frac{1}{N} \delta_p^l.
 \end{aligned} \tag{4.9}$$

Here n^l is a complex vector in the fundamental representation of $\text{SU}(N)$, and

$$n_i^* n^i = 1, \tag{4.10}$$

($l, p = 1, \dots, N$ are color indices). In Eq. (4.8) for brevity we suppress all $\text{SU}(N)$ indices. At $n = \{0, \dots, 1\}$ we get the field configuration quoted in Eq. (4.2).

The vector n^l parametrizes orientational zero modes of the string associated with flux rotations in $\text{SU}(N)$. The presence of these modes makes the string genuinely non-Abelian.

To derive an effective world sheet theory for the orientational collective coordinates n^l of the non-Abelian string we follow Refs. [6,7,19], see also the review [18]. From the string solution (4.2) it is quite clear that not each element of the matrix U will give rise to a modulus. The $\text{SU}(N-1) \times \text{U}(1)$ subgroup remains unbroken by the string solution under consideration; therefore the moduli space is

$$\frac{\text{SU}(N)}{\text{SU}(N-1) \times \text{U}(1)} \sim \text{CP}(N-1). \tag{4.11}$$

Assume that the orientational collective coordinates n^l are slowly varying functions of the string world sheet coordinates $x_k, k = 0, 3$. Then the moduli n^l become fields of a $(1+1)$ -dimensional sigma model on the world sheet. Since the vector n^l parametrizes the string zero modes, there is no potential term in this sigma model.

To obtain the kinetic term we substitute our solution, which depends on the moduli n^l , in the action (2.4), assuming that the fields acquire a dependence on the coordinates x_k via $n^l(x_k)$. Then we arrive at the $\text{CP}(N-1)$ sigma model (for details see the review paper [18]),

$$S_{\text{CP}(N-1)}^{(1+1)} = 2\beta \int dt dz \{(\partial_k n^* \partial_k n) + (n^* \partial_k n)^2\}, \tag{4.12}$$

where the coupling constant β is given by a normalizing integral defined in terms of the string profile functions. Using the first-order equations for the string profile functions (4.5) one can see that this integral reduces to a total derivative and is given by the flux of the string determined by $f_{\text{NA}}(0) = 1$, namely

$$\beta = \frac{2\pi}{g_2^2}. \tag{4.13}$$

The two-dimensional coupling constant is determined by the four-dimensional non-Abelian coupling.

The relation between the four-dimensional and two-dimensional coupling constants (4.13) is obtained at the classical level. In quantum theory both couplings run. So we have to specify a scale at which the relation (4.13) takes place. The two-dimensional $\text{CP}(N-1)$ model is an effective low-energy theory good for the description of internal string dynamics at low energies, much lower than the inverse thickness of the string which, in turn, is given by $g_2\sqrt{\xi}$. Thus, $g_2\sqrt{\xi}$ plays the role of a physical ultraviolet cutoff in (4.12). This is the scale at which Eq. (4.13) holds. Below this scale, the coupling β runs according to its two-dimensional renormalization-group flow.

The sigma model (4.12) is asymptotically free [20]; at large distances (low energies) it gets into the strong coupling regime. The running coupling constant as a function of the energy scale E at one loop is given by

$$4\pi\beta = N \ln\left(\frac{E}{\Lambda_{\text{CP}(N-1)}}\right) + \dots, \tag{4.14}$$

where $\Lambda_{\text{CP}(N-1)}$ is the dynamical scale of the $\text{CP}(N-1)$ model. As was mentioned above, the UV cutoff of the sigma model at hand is determined by $g_2\sqrt{\xi}$. Hence,

$$\Lambda_{\text{CP}(N-1)}^N = g_2^N \xi^{N/2} e^{-(8\pi^2/g_2^2)}. \tag{4.15}$$

Note that in the bulk theory, due to the VEV's of the squark fields, the coupling constant is frozen at $g_2\sqrt{\xi}$. There are no logarithms in the bulk theory below this scale. Below $g_2\sqrt{\xi}$ the logarithms of the world sheet theory take over.

In the limit of large μ_2 we are interested in here,

$$\mu_2 \gg g_2\sqrt{\xi},$$

the coupling constant g_2 of the bulk theory is determined by the scale $\Lambda_{\mathcal{N}=1}$ of the $\mathcal{N} = 1$ SQCD (3.5) with the M field included, as shown in Eq. (3.6). In this limit Eq. (4.15) gives

$$\Lambda_{\text{CP}(N-1)} = \frac{\Lambda_{\mathcal{N}=1}^2}{g_2\sqrt{\xi}}, \tag{4.16}$$

where we take into account that the first coefficient of the β function in $\mathcal{N} = 1$ SQCD is $2N$.

To conclude this section let us note a somewhat related development: *non-BPS non-Abelian strings* were recently

considered in metastable vacua of a dual description of $\mathcal{N} = 1$ SQCD at $N_f > N$ in Ref. [21].

V. FERMIONIC SECTOR OF THE WORLD SHEET THEORY

In this section we discuss the fermionic sector of the low-energy effective theory on the world sheet of the non-Abelian string in $\mathcal{N} = 1$ SQCD with the M field, as well as supersymmetry of the world sheet theory. First we note that our string is 1/2 BPS saturated. Therefore in the $\mathcal{N} = 2$ limit (when $\mathcal{N} = 2$ breaking parameters μ_i and h vanish) four supercharges out of eight present in the bulk theory are automatically preserved on the string world sheet. They become supercharges in the $CP(N-1)$ model (4.12).

For simplicity in this section we will discuss the case $N = 2$ limiting ourselves to the $CP(1)$ model. Generalization to arbitrary N is straightforward. The action of the (2, 2) supersymmetric $CP(1)$ model is

$$S_{CP(1)}^{(1+1)} = \beta \int dt dz \left\{ \frac{1}{2} (\partial_k S^a)^2 + \frac{1}{2} \chi_1^a i(\partial_0 - i\partial_3) \chi_1^a + \frac{1}{2} \chi_2^a i(\partial_0 + i\partial_3) \chi_2^a - \frac{1}{2} (\chi_1^a \chi_2^a)^2 \right\}, \quad (5.1)$$

where we used the fact that $CP(1)$ is equivalent to the $O(3)$ sigma model defined in terms of a unit real vector S^a ,

$$S^a = n_l^* \tau^a n^l, \quad (S^a)^2 = 1. \quad (5.2)$$

This model has two real bosonic degrees of freedom. Two real fermion fields χ_1^a and χ_2^a are subject to constrains

$$\chi_1^a S^a = 0, \quad \chi_2^a S^a = 0. \quad (5.3)$$

Altogether we have four real fermion fields in the model (5.1).

Now we break $\mathcal{N} = 2$ supersymmetry of the bulk model by switching on parameters μ_i and h . The 1/2 ‘‘BPS-ness’’ of the string solution requires only two supercharges. However, as we will show below, the number of the fermion zero modes on the string does not change. This number is fixed by the index theorem. Thus, the number of (classically) massless fermion fields in the world sheet $CP(N-1)$ model does not change. It was shown in [9]

that the (2, 2) supersymmetric sigma model with the $CP(N-1)$ target space does not admit (0, 2) supersymmetric deformations. Therefore, it was concluded in [9] that the world sheet theory has ‘‘accidental’’ SUSY enhancement. A similar phenomenon was found earlier in [22] for domain walls.

On the other hand in the recent publication [23] it was suggested that superorientational zero modes can mix with supertranslational modes. It was shown that the sigma model with the $C \times CP(N-1)$ target space does admit (0, 2) supersymmetric deformations. It is not clear at the moment if this mixing really occurs in the effective theory on the string. If it occurs then the emerging (0, 2) supersymmetric $C \times CP(N-1)$ model has a μ -deformed four-fermion interaction

$$S_{CP(1)}^{(1+1)} = \beta \int dt dz \left\{ \frac{1}{2} (\partial_k S^a)^2 + \frac{1}{2} \chi_1^a i(\partial_0 - i\partial_3) \chi_1^a + \frac{1}{2} \chi_2^a i(\partial_0 + i\partial_3) \chi_2^a - \frac{1}{2} \frac{1}{1 + c|\mu_2|^2/(g_2^2 \xi)} (\chi_1^a \chi_2^a)^2 \right\}, \quad (5.4)$$

where c is an unknown coefficient. Also the first constraint in (5.3) is replaced with $\chi_1^a S^a = c/2(\mu_2 \zeta_1 + \bar{\mu}_2 \bar{\zeta}_1)$, where ζ_1 is the right-moving two-dimensional fermion field associated with the supertranslational zero modes. If this conjecture [23] is correct the four-fermion term disappears in the large μ limit. To find out which scenario is correct one has to calculate the coefficient in front of the four-fermion term in (5.4). We left this for future work.

In any case, the world sheet supersymmetric model has N vacua which are interpreted as N elementary strings of the bulk theory. This number is protected by Witten’s index and survives $\mathcal{N} = 2$ breaking deformations. We will use this result in the next section. The kinks which interpolate between these vacua are confined monopoles. Below we will show that the occurrence of four (4(N-1) in the general case) superorientational fermion zero modes on the non-Abelian strings follows from an index theorem. In the appendix we present explicit solutions for these modes for the case $N = 2$.

A. Index theorem

The fermionic part of the action of the model (3.5) is

$$S_{\text{ferm}} = \int d^4 x \left\{ \frac{i}{g_2^2} \bar{\lambda}^a \bar{\psi} \lambda^a + \frac{i}{g_1^2} \bar{\lambda} \bar{\psi} \lambda + \text{Tr}[\bar{\psi} i \not{\partial} \psi] + \text{Tr}[\bar{\psi} i \not{\partial} \bar{\psi}] + \frac{2i}{h} \text{Tr}[\bar{\zeta} \not{\partial} \zeta] + \frac{i}{\sqrt{2}} \text{Tr}[\bar{q}(\lambda \psi) - (\bar{\psi} \lambda) \bar{q}] + \frac{i}{\sqrt{2}} \text{Tr}[\bar{q} 2T^a (\lambda^a \psi) - (\bar{\psi} \lambda^a) 2T^a \bar{q}] + (\bar{\psi} \bar{\lambda}^a) 2T^a q - \bar{q} 2T^a (\bar{\lambda}^a \bar{\psi})] + i \text{Tr}[\bar{q}(\psi \zeta) + (\bar{\psi} q \zeta) + (\bar{\psi} \bar{q} \bar{\zeta}) + \bar{q}(\bar{\psi} \bar{\zeta})] + i \text{Tr}[\bar{\psi} \psi M + \bar{\psi} \bar{\psi} \bar{M}] \right\}, \quad (5.5)$$

where the matrix color-flavor notation is used for matter fermions $(\psi^\alpha)^{kA}$ and $(\tilde{\psi}^\alpha)_{Ak}$ and the traces are performed over the color-flavor indices. Contraction of the spinor indices is assumed inside all parentheses, for instance, $(\lambda\psi) \equiv \lambda_\alpha\psi^\alpha$. Moreover, ζ denotes the fermion component of the matrix M superfield,

$$\zeta_B^A = \frac{1}{2}\delta_B^A\zeta^0 + (T^a)_B^A\zeta^a. \quad (5.6)$$

In order to find the number of the fermion zero modes in the background of the non-Abelian string solution (4.8) we have to carry out the following program. Since our string solution depends only on two coordinates x_i ($i = 1, 2$), we can reduce our theory to two dimensions. Given the theory defined on the (x_1, x_2) plane we have to identify an axial current and derive the anomalous divergence for this current. In two dimensions the axial current anomaly takes the form

$$\partial_i j_{i5} \sim F^*, \quad (5.7)$$

where $F^* = (1/2)\epsilon_{ij}F_{ij}$ is the dual U(1) field strength in two dimensions.

Then the integral over the left-hand side over the (x_1, x_2) plane gives us the index of the 2D Dirac operator ν coinciding with the number of the 2D left-handed minus 2D right-handed zero modes of this operator in the given background field. The integral over the right-hand side is proportional to the string flux. This will fix the number of the chiral fermion zero modes⁴ of the string with the given flux. Note that the reduction of the theory to two dimensions is an important step in this program. The anomaly relation in four dimensions involves the instanton charge F^*F rather than the string flux and is therefore useless for our purposes.

The reduction of $\mathcal{N} = 1$ gauge theories to two dimensions is discussed in detail in [24] and here we will be brief. Following [24] we use the rules

$$\begin{aligned} \psi^\alpha &\rightarrow (\psi^-, \psi^+), & \tilde{\psi}^\alpha &\rightarrow (\tilde{\psi}^-, \tilde{\psi}^+), \\ \lambda^\alpha &\rightarrow (\lambda^-, \lambda^+), & \zeta^\alpha &\rightarrow (\zeta^-, \zeta^+). \end{aligned} \quad (5.8)$$

With these rules the Yukawa interactions in (5.5) take the form

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= i\sqrt{2}\text{Tr}[-\bar{q}(\hat{\lambda}_-\psi_+ - \hat{\lambda}_+\psi_-) \\ &\quad + (\tilde{\psi}_-\hat{\lambda}_+ - \tilde{\psi}_+\hat{\lambda}_-)\bar{q}] + \text{c.c.}] \\ &\quad - i\text{Tr}[\bar{q}(\psi_-\zeta_+ - \psi_+\zeta_-) \\ &\quad + (\tilde{\psi}_-q\zeta_+ - \tilde{\psi}_+q\zeta_-) + \text{c.c.}], \end{aligned} \quad (5.9)$$

where the color matrix $\hat{\lambda} = (1/2)\lambda + T^a\lambda^a$.

It is easy to see that $\mathcal{L}_{\text{Yukawa}}$ is classically invariant under the chiral $U(1)_R$ transformations with the $U(1)_R$ charges presented in Table I. The axial current associated with this

TABLE I. The $U(1)_R$ and $U(1)_{\tilde{R}}$ charges of fields of the two-dimensional reduction of the theory.

Field	ψ_+	ψ_-	$\tilde{\psi}_+$	$\tilde{\psi}_-$	λ_+	λ_-	ζ_+	ζ_-	q	\tilde{q}
$U(1)_R$ charge	-1	1	-1	1	-1	1	-1	1	0	0
$U(1)_{\tilde{R}}$ charge	-1	1	1	-1	-1	1	1	-1	0	0

$U(1)_R$ is not anomalous [24]. This is easy to understand. In two dimensions the chiral anomaly comes from the diagram shown in Fig. 1. The $U(1)_R$ chiral charges of the fields ψ and $\tilde{\psi}$ are the same while their electric charges are opposite. This leads to cancellation of their contributions to this diagram.

It turns out that for the particular string solution we are interested in the classical two-dimensional action has more symmetries than generically, for a general background. To see this, please note that the field \tilde{q} vanishes on the string solution (4.8). Then the Yukawa interactions (5.9) reduce to

$$\begin{aligned} i\sqrt{2}\text{Tr}[-\bar{q}(\hat{\lambda}_-\psi_+ - \hat{\lambda}_+\psi_-)] - i\text{Tr}[\tilde{\psi}_-q\zeta_+ - \tilde{\psi}_+q\zeta_-] \\ + \text{c.c.} \end{aligned} \quad (5.10)$$

The fermion ψ interacts only with λ 's while the fermion $\tilde{\psi}$ interacts only with ζ . Note also that the interaction in the last line in (5.5) is absent because $M = 0$ on the string solution. This property allows us to introduce another chiral symmetry in the theory, the one which is relevant for the string solution. We will refer to this extra chiral symmetry as $U(1)_{\tilde{R}}$.

The $U(1)_{\tilde{R}}$ charges of our set of fields are also shown in Table I. Note that ψ and $\tilde{\psi}$ have the opposite charges under this symmetry. The corresponding current then has the form

$$\tilde{j}_{i5} = \left(\begin{array}{l} \bar{\psi}_-\psi_- - \bar{\psi}_+\psi_+ - \bar{\tilde{\psi}}_-\tilde{\psi}_- + \bar{\tilde{\psi}}_+\tilde{\psi}_+ + \dots \\ -i\bar{\psi}_-\psi_- - i\bar{\tilde{\psi}}_+\psi_+ + i\bar{\tilde{\psi}}_-\tilde{\psi}_- + i\bar{\psi}_+\tilde{\psi}_+ + \dots \end{array} \right), \quad (5.11)$$

where the ellipses stand for terms associated with the λ and ζ fields which do not contribute to the anomaly relation.

Clearly, in quantum theory this symmetry is anomalous. Now the contributions of the fermions ψ and $\tilde{\psi}$ double in the diagram in Fig. 1 rather than cancel. It is not difficult to find the coefficient in the anomaly formula

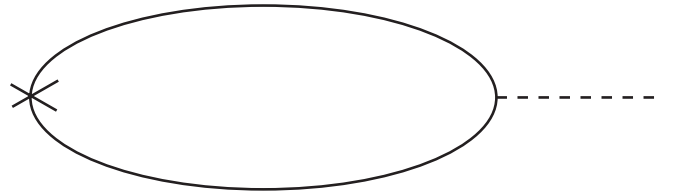


FIG. 1. Diagram for the chiral anomaly in two dimensions. The solid lines denote fermions $\psi, \tilde{\psi}$, the dashed line denotes the photon, while the cross denotes insertion of the axial current.

⁴Chirality is understood as the two-dimensional chirality.

$$\partial_i \tilde{j}_{i5} = \frac{N^2}{\pi} F^*, \quad (5.12)$$

which can be normalized, e.g. from [25]. The factor N^2 appears due to the presence of $2N^2$ fermions ψ^{kA} and $\tilde{\psi}_{Ak}$.

Now, taking into account that the flux of the Z_N string under consideration is

$$\int d^2x F^* = \frac{4\pi}{N}, \quad (5.13)$$

(see the expression for the U(1) gauge field for the solution (4.2) or (4.8)) we conclude that the total number of the fermion zero modes in the string background

$$\nu = 4N. \quad (5.14)$$

This number can be decomposed as

$$\nu = 4N = 4(N - 1) + 4, \quad (5.15)$$

where 4 is the number of the supertranslational modes while $4(N - 1)$ is the number of the superorientational modes. Four supertranslational modes are associated with four fermion fields in the two-dimensional effective theory on the string world sheet, which are superpartners of the bosonic translational moduli x_0 and y_0 . Furthermore, $4(N - 1)$ corresponds to $4(N - 1)$ fermion fields in the $\mathcal{N} = 2$ CP(N - 1) model on the string world sheet (4.12). CP(N - 1) describes dynamics of the orientational moduli of the string. For $N = 2$ the latter number ($4(N - 1) = 4$) counts four fermion fields χ_1^a, χ_2^a in the model (5.1) or (5.4).

We explicitly determine four superorientational fermion zero modes for the case $N = 2$ in the appendix. Note that the fermion zero modes of the string in $\mathcal{N} = 1$ SQCD with the M field are perfectly normalizable provided we keep the coupling constant h nonvanishing. Instead, in conventional $\mathcal{N} = 1$ SQCD without the M field the second pair of the fermion zero modes (proportional to χ_1^a) become non-normalizable [9]. This is related to the presence of the Higgs branch and massless bulk states in conventional $\mathcal{N} = 1$ SQCD. As was already mentioned more than once, in the M model, Eq. (3.5), we have no massless states in the bulk.

Note that in both translational and orientational sectors the number of the fermion zero modes is twice larger than the one dictated by $1/2$ ‘‘BPS-ness.’’

VI. EVOLUTION OF THE MONOPOLES

Since the supersymmetric CP(N - 1) model is an effective low-energy theory describing world sheet physics of the non-Abelian string, all consequences of this model ensue, in particular, N degenerate vacua and kinks which interpolate between them—the same kinks that we had discovered in $\mathcal{N} = 2$ SQCD [7] and interpreted as (confined) non-Abelian monopoles, the descendants of the ’t Hooft-Polyakov monopole [26].

Let us briefly review the reason for this interpretation [7,8,27] and discuss what happens with these monopoles as we deform our theory and eventually end up with the M model. It is convenient to split this deformation into several distinct stages. We will describe what happens to the monopoles as one passes from one stage to another.

A qualitative evolution of the monopoles under consideration as a function of the relevant parameters is presented in Fig. 2.

- (i) We start from $\mathcal{N} = 2$ SQCD turning off the $\mathcal{N} = 2$ breaking parameters h and μ ’s as well as the FI parameter in the theory (2.4), i.e. we start from the Coulomb branch of the theory,

$$\mu_1 = \mu_2 = 0, \quad h = 0, \quad \xi = 0, \quad M \neq 0. \quad (6.1)$$

As was explained in Sec. II, the field M is frozen in this limit and can take arbitrary values (the notorious flat direction). The matrix M_B^A plays the role of fixed mass parameter matrix for the quark fields. First we consider the diagonal matrix M , with distinct diagonal entries,

$$M_B^A = \text{diag}\{M_1, \dots, M_N\}. \quad (6.2)$$

Shifting the field a one can always make $\sum_A M_A = 0$ in the limit $\mu_1 = 0$, therefore $M^0 = 0$. If all M_A ’s are different the gauge group SU(N) is broken down to $U(1)^{(N-1)}$ by a VEV of the SU(N) adjoint scalar

$$\langle a_i^k \rangle = -\frac{1}{\sqrt{2}} \delta_i^k M_i. \quad (6.3)$$

Thus, there are ’t Hooft-Polyakov monopoles embedded in the broken gauge SU(N). Classically, on the Coulomb branch the masses of $(N - 1)$ elementary monopoles are proportional to

$$|(M_A - M_{A+1})|/\xi^2.$$

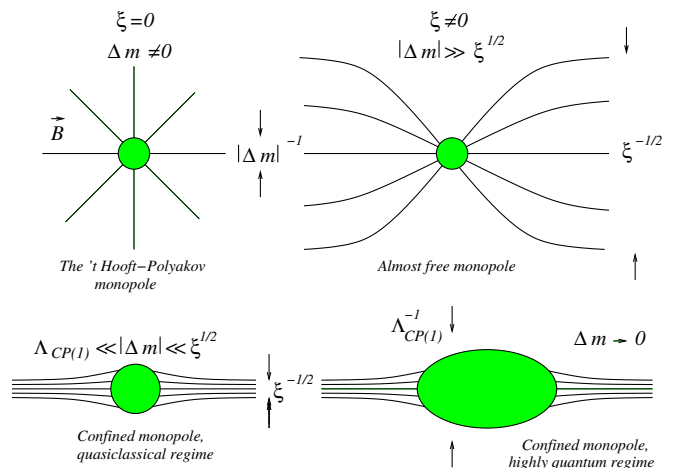


FIG. 2 (color online). Various regimes for the monopoles and flux tubes in the simplest case of two flavors.

This is shown in the upper left corner of Fig. 2 for the case

$$N = 2, \quad \Delta m \equiv M_1 - M_2.$$

In the limit $(M_A - M_{A+1}) \rightarrow 0$ the monopoles tend to become massless, formally, in the classical approximation. Simultaneously their size becomes infinite [28]. The mass and size are stabilized by confinement effects which are highly quantum. The confinement of monopoles occurs in the Higgs phase, at $\xi \neq 0$.

- (ii) Now we introduce the FI parameter ξ which triggers the squark condensation. The theory is in the Higgs phase. We still keep $\mathcal{N} = 2$ breaking parameters h and μ 's vanishing,

$$\begin{aligned} \mu_1 = \mu_2 = 0, \quad h = 0, \\ \xi \neq 0, \quad M \neq 0. \end{aligned} \quad (6.4)$$

If we allow ξ to be nonvanishing but small,

$$|M_A| \gg \sqrt{\xi}, \quad (6.5)$$

then the effect which comes into play first is the spontaneous breaking of the gauge $SU(N)$ by the condensation of the adjoint scalars. The next gauge symmetry breaking, due to $\xi \neq 0$, which leads to complete Higgsing of the model and the string formation (confinement of monopoles) is much weaker. Thus, we deal here with the formation of ‘‘almost’’ ’t Hooft-Polyakov monopoles, with a typical size $\sim |(M_A - M_{A+1})|^{-1}$. Only at much larger distances, $\sim \xi^{-1/2}$, the chromoelectric charge condensation enters the game, and forces the magnetic flux, rather than spreading evenly *à la* Coulomb, to form flux tubes (the upper right corner of Fig. 2).

Let us verify that the confined monopole is a junction of two strings. At $M_A \neq 0$ the global $SU(N)_{C+F}$ group is broken by condensation of the adjoint scalars (6.3), and non-Abelian strings become Abelian Z_N strings. Their orientational moduli space is lifted [7,8]. Consider the junction of two Z_N strings (4.8), namely the A th string with

$$n^l = \delta_A^l \quad (6.6)$$

and the ‘‘neighboring’’ $(A + 1)$ th string with

$$n^l = \delta_{A+1}^l, \quad (6.7)$$

(cf. solution (4.2) which is written for $A + 1 = N$). The flux of the junction is given by the difference of the fluxes of these two strings. Using (4.8) we get that the flux of the junction is

$$4\pi \times \text{diag}_2^l \{ \dots, 0, 1, -1, 0, \dots \} \quad (6.8)$$

with the nonvanishing entries located at the posi-

tions A and $(A + 1)$. These are exactly the fluxes of $N - 1$ distinct ’t Hooft-Polyakov monopoles occurring in the $SU(N)$ gauge theory provided that $SU(N)$ is spontaneously broken down to $U(1)^{N-1}$. We see that in the quasiclassical limit of large $|M_A|$ the Abelian monopoles and the junctions of the Abelian Z_N strings are in one-to-one correspondence.

At large M_A the monopoles, albeit confined, are weakly confined. Now, if we further reduce $|M_A|$,

$$\Lambda_{CP(N-1)} \ll |M_A| \ll \sqrt{\xi}, \quad (6.9)$$

the size of the monopole $\sim |(M_A - M_{A+1})|^{-1}$ becomes larger than the transverse size of the attached strings. The monopole gets squeezed in earnest by the strings—it becomes a *bona fide* confined monopole (the lower left corner of Fig. 2). At nonzero M_A the effective $CP(N - 1)$ model on the string world sheet becomes massive with the potential determined by so-called twisted-mass terms [7,8,27]. Two Z_N strings correspond to two neighboring vacua of the $CP(N - 1)$ model. The monopole (aka the string junction of two Z_N strings) is interpreted as a kink interpolating between these two vacua.

In [7] the first-order equations for the 1/4 BPS string junction of two Z_2 strings were explicitly solved in the case $N = 2$, and the solution shown to correspond to the kink solution of the two-dimensional $CP(1)$ model. Moreover, it was shown that the mass of the monopole matches the mass of the $CP(1)$ -model kink both in the quasiclassical ($\Delta m \gg \Lambda_{CP(1)}$) and quantum ($\Delta m \ll \Lambda_{CP(1)}$) limits.

- (iii) Now let us switch off the mass differences M_A still keeping the $\mathcal{N} = 2$ breaking parameters vanishing,

$$\begin{aligned} \mu_1 = \mu_2 = 0, \quad h = 0, \\ \xi \neq 0, \quad M = 0. \end{aligned} \quad (6.10)$$

The values of the twisted mass in the $CP(N - 1)$ model equal M_A while the size of the twisted-mass sigma-model kink/confined monopole is of the order of $\sim |(M_A - M_{A+1})|^{-1}$.

As we further diminish M_A approaching $\Lambda_{CP(N-1)}$ and then getting below $\Lambda_{CP(N-1)}$, the size of the monopole grows, and, classically, it would explode. This is where quantum effects in the world sheet theory take over. It is natural to refer to this domain of parameters as the ‘‘regime of highly quantum dynamics.’’ While the thickness of the string (in the transverse direction) is $\sim \xi^{-1/2}$, the z -direction size of the kink representing the confined monopole in the highly quantum regime is much larger, $\sim \Lambda_{CP(N-1)}^{-1}$, see the lower right corner in Fig. 2.

In this regime the confined monopoles become truly non-Abelian. They no longer carry average magnetic flux since

$$\langle n^l \rangle = 0, \quad (6.11)$$

in the strong coupling limit of the $CP(N-1)$ model [29]. The kink/monopole belongs to the fundamental representation of the $SU(N)_{C+F}$ group [29,30].

Let us stress that in the limit $M_A = 0$ the global group $SU(N)_{C+F}$ is restored in the bulk and both strings and confined monopoles become non-Abelian. One might argue that this restoration could happen only at the classical level. One could suspect that in quantum theory a ‘‘dynamical Abelianization’’ (i.e. a cascade breaking of the gauge symmetry $U(N) \rightarrow U(1)^N \rightarrow$ discrete group) might occur. This could have happened if the adjoint VEV’s that are classically vanished at $M = 0$ (see (2.9)) could have developed dynamically in quantum theory. At $M_A \neq 0$ the global $SU(N)_{C+F}$ group is explicitly broken down to $U(1)^{N-1}$ by quark masses. At $M_A = 0$ this group is classically restored. If it could still be dynamically broken this would mean a spontaneous symmetry breaking.

Let us show that this does not happen in the theory at hand. First of all, if a global symmetry is not spontaneously broken at the tree level then it cannot be broken by quantum effects at weak coupling in ‘‘isolated’’ vacua. Second, if the global group $SU(N)_{C+F}$ were broken spontaneously at $M_A = 0$ this would ensure the presence of massless Goldstone bosons. However, we know that there are no massless states in the spectrum of the bulk theory, see Secs. II and III.

Finally, the breaking of $SU(N)_{C+F}$ in the $M_A = 0$ limit would mean that the twisted masses of the world sheet $CP(N-1)$ model would not be given by M_A ; instead they would be shifted, $m_A^{(tw)} = M_a + c_A \Lambda_{CP(N-1)}$, where c_A are some coefficients. In [7,8] it was shown that the BPS spectrum of the $CP(N-1)$ model on the string should coincide with the BPS spectrum of the four-dimensional bulk theory on the Coulomb branch because the central charges which determine masses of the BPS states cannot depend on the nonholomorphic parameter ξ . The BPS spectrum of the $CP(N-1)$ model is determined by $m_A^{(tw)}$ while the BPS spectrum of the bulk theory on the Coulomb branch is determined by M_A . In [31] it was shown that the BPS spectrum of both theories coincide at $m_A^{(tw)} = M_A$. Thus, we conclude that $c_A = 0$ and the twisted masses go to zero in the $M_A = 0$ limit. Again we conclude that the global $SU(N)_{C+F}$ group is not broken in the bulk and both strings and confined

- monopoles become non-Abelian at $M_A = 0$.
 (iv) Thus, at zero M_A we still have confined ‘‘monopoles’’ (interpreted as kinks) stabilized by quantum effects in the world sheet $CP(N-1)$ model. Now we can finally switch on the $\mathcal{N} = 2$ breaking parameters μ_i and h ,

$$\mu_i \neq 0, \quad h \neq 0, \quad \xi \neq 0, \quad M = 0. \quad (6.12)$$

Note that the last equality here is automatically satisfied in the vacuum, see Eq. (2.10).

As we discussed in Secs. IV and V the effective world sheet description of the non-Abelian string is still given by the supersymmetric $CP(N-1)$ model. This model obviously still has N vacua which should be interpreted as N elementary non-Abelian strings in the quantum regime, and kinks can interpolate between these vacua. These kinks should still be interpreted as non-Abelian confined monopoles/string junctions.

Note that although the adjoint fields are still present in the theory (2.4) their VEV’s vanish (see (2.9)) and the monopoles cannot be seen in the semiclassical approximation. They are seen as the $CP(N-1)$ model kinks. Their mass and inverse size is determined by $\Lambda_{CP(N-1)}$ which in the limit of large μ_i is given by Eq. (4.16).

- (v) Now, at the last stage, we take the limit of large masses of the adjoint fields in order to eliminate them from the physical spectrum altogether,

$$\mu_i \rightarrow \infty, \quad h \neq 0, \quad \xi \neq 0, \quad M = 0. \quad (6.13)$$

The theory flows to $\mathcal{N} = 1$ SQCD extended by the M field.

In this limit we get a remarkable result: although the adjoint fields are eliminated from our theory and the monopoles cannot be seen in any semiclassical description, our analysis shows that confined non-Abelian monopoles still exist in the theory (3.5). They are seen as $CP(N-1)$ -model kinks in the effective world sheet theory on the non-Abelian string.

VII. A BRANE PERSPECTIVE

Let us elucidate how some important features of the consideration above are seen in the brane picture. To this end we will rely on Type IIA string approach to our M model. Consider the brane picture for $\mathcal{N} = 2$ and $\mathcal{N} = 1$ SQCD (see Ref. [32] for a review). We will limit ourselves to the special case of equal numbers of colors and flavors relevant to the present work.

The $\mathcal{N} = 2$ theory involves N D4 branes extended in the directions of the (0, 1, 2, 3, 6) coordinates, two NS5 branes with coordinates along (0, 1, 2, 3, 4, 5), localized at $x_6 = 0$ and $x_6 = 1/g^2$ and $N_f = N$ D6 branes with the world volume along (0, 1, 2, 3, 7, 8, 9). The D4 branes are stretched between NS5 branes along x_6 , while the coordinates of D6 branes in x_6 are arbitrary. The NS5 branes can be split in the x_7 direction which corresponds to the introduction of the Fayet-Iliopoulos term in the U(1) factor of U(N), namely,

$$\delta x_7 = \xi.$$

The Higgs branch in this theory occurs when the D6 branes touch the D4 branes. After this, the D4 branes can split in pieces which can be moved in the (7, 8, 9) directions. The coordinates of these pieces in the (7, 8, 9) directions, along with the Wilson line of A_6 , yield coordinates on the Higgs branch of the moduli space. Fluctuations of the ends of the D4 branes in the (4, 5) plane provide the coordinates on the Coulomb branch of the moduli space.

To break $\mathcal{N} = 2$ SUSY down to $\mathcal{N} = 1$ we rotate one of the NS5 branes. The angle of rotation corresponds to the mass of the adjoint scalar in the superpotential (2.2). The fact that this superpotential does not vanish removes the Coulomb branch of the moduli space. The positions of the D4 branes in the (4, 5) plane are now fixed.

Now, let us switch on the meson field M . It turns out that it emerges as a particular limiting brane configuration in the setup described above, without any additional branes. Consider the situation when the x_6 coordinates of all D6 branes are the same. First, in this limit the open strings connecting the pairs of the D6 branes yield a massless field which is in the adjoint representation with respect to the flavor group U(N). In the field-theory language this is nothing but our M field. Taking into account the standard three-string vertex we immediately derive the superpotential $\mathcal{W}_M = \text{Tr} M \tilde{Q} Q$. On the other hand, since all D6 branes have the same x_6 coordinate, it is impossible to split the pieces of the D4 branes—such a splitting would require different values of x_6 for the pair of the D6 branes. Thus, the Higgs branch disappears. We see that in the brane language the introduction of the M field is in one-to-one correspondence with the disappearance of the Higgs branch.

Consider now the evolution of the monopoles discussed in Sec. VI within the framework of the brane picture. In the $\mathcal{N} = 2$ theory in the regime (6.1) the monopole is represented by a D2 brane stretched between two NS5 branes in the x_6 direction and two D4 branes located at $x_{4A} = M_A$ and $x_{4(A+1)} = M_{(A+1)}$, which yields the correct monopole mass

$$\frac{|(M_A - M_{A+1})|}{g_2^2}.$$

Switching on the Fayet-Iliopoulos parameter ξ in the regime (6.4) corresponds to a displacement of one of the NS5 branes in the x_7 direction. Since the D4 branes split in two pieces at the common x_6 coordinate where the D6 branes are located, and each piece is attached to the NS5 brane, a squark condensate develops. It is proportional to $\sqrt{\xi}$. This regime supports quasiclassical non-Abelian strings which have a transparent geometrical interpretation [5]. The non-Abelian strings are identified with the D2 brane parallel to the D6 branes stretched between two NS5 branes along the x_7 coordinate. Geometrically, the string tension equals δx_7 , in full agreement with the field-theory result.

The D2 brane representing the monopole in the Higgs phase is located as follows. It extends along two coordinates, x_6 and x_4 . Along the x_6 coordinate the D2 brane is stretched between the common position of the D6 branes and the NS5 brane. In the x_4 direction it is stretched between two D4 branes.

From this picture one immediately recognizes the monopole to be a junction of two non-Abelian strings since it is stretched between two different non-Abelian strings in the x_4 direction. If one switches off the Fayet-Iliopoulos term then the monopole in the Higgs phase geometrically smoothly transforms into the 't Hooft-Polyakov monopole.

This picture implies that in the semiclassical regime of large M_A the monopole mass is the same as the mass of the 't Hooft-Polyakov monopole. With M_A decreasing we eventually find ourselves in the purely quantum regime described by lifting type IIA string to M theory and, hence, lifting the D2 brane to M2 brane. In M theory the monopole in the Higgs phase can be easily described by the M2 brane wrapping the appropriate circle on the Riemann surface, using its identification with the kink in the $CP(N - 1)$ model [31].

Finally in the regime (6.12) we rotate one of the NS5 branes which results in vanishing vacuum expectation values of the adjoint scalars. However, the M2 brane representing the non-Abelian string is still clearly identified. The monopoles are the M2 branes wrapped around the Riemann surface responsible for this regime upon rotation of the branes.

Let us emphasize that the monopoles in all regimes are represented by the M2 branes, and their evolution from the Coulomb branch to the Higgs one corresponds just to different placement of one and the same brane in a certain brane background.

Note that the brane picture suggests the possibility of a more general situation, when only k of the D6 branes have the same x_6 coordinates. Then, the massless meson field M belongs to the U(k) subgroup of the flavor group. In particular, one can consider the case $N_f > N$, introduce a meson field of some rank, and perform the standard Seiberg duality transformation by exchanging two NS5 branes.

VIII. DISCUSSION AND CONCLUSIONS

Let us summarize our findings. Deformation of $\mathcal{N} = 2$ SQCD leads us to the M model, $\mathcal{N} = 1$ SQCD supplemented by the M field, see (3.5). We observe confined non-Abelian monopoles in this model which has no monopoles whatsoever in the semiclassical limit. Why are we sure that the objects we observe are “non-Abelian monopoles?” We know this because we can start from the $\mathcal{N} = 2$ theory on the Coulomb branch where the standard ’t Hooft-Polyakov monopoles are abundant, and trace their evolution stage by stage, as one varies the adjustable parameters to eventually arrive at $\mathcal{N} = 1$ SQCD.

This is the main result of the present paper. As was mentioned above the confined monopoles are in the highly quantum regime so they do not carry average magnetic flux (see Eq. (6.11)). They are genuinely non-Abelian. Moreover, they acquire global flavor quantum numbers. In fact, they belong to the fundamental representation of the global $SU(N)_{C+F}$ group (see Refs. [29,30] where this phenomenon is discussed in the context of the $CP(N-1)$ -model kinks).

In particular, the monopole-antimonopole “meson” formed by the string configuration shown in Fig. 3 belongs to the adjoint representation of the global “flavor” group $SU(N)_{C+F}$, in accordance with our expectations. Similar there are “baryons” built of N monopoles connected by strings to each other to form a close necklace configuration.

We believe that the emergence of these non-Abelian monopoles can shed light on mysterious objects introduced by Seiberg: “dual magnetic” quarks which play an important role in the description of $\mathcal{N} = 1$ SQCD at strong coupling [12,15].

It is curious to note that monopolelike configurations apparently occur in lattice QCD. In particular, in the recent publications [33] the occurrence of the monopolelike configurations is traced back to the color-octet operator $\tilde{q}T^a q$. We would like to stress that the non-Abelian monopoles observed here are totally different. In the limit $\mu \rightarrow \infty$ all traces of “Abelization” (i.e. cascade breaking of the gauge symmetry $U(N) \rightarrow U(1)N \rightarrow$ discrete subgroup) typical of the $\mathcal{N} = 2$ limit are erased. In fact, it is clear from (2.8) that $\langle \tilde{q}T^a q \rangle = 0$ in the M -model vacuum and cannot be used to construct monopoles. Our monopoles are not

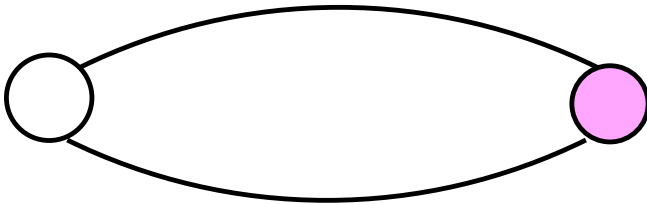


FIG. 3 (color online). Monopole and antimonopole bound by strings in a meson. Open and closed circles denote monopole and antimonopole, respectively.

seen classically. The confined non-Abelian monopoles emerge as $CP(N-1)$ -model kinks living on the string, deep in the quantum regime.

Now, let our imagination run away with the hypothetical dual of the M model. In this model it is not chromomagnetic but rather chromoelectric flux tubes that will form (upon “monopole” condensation) in a highly quantum regime. The number of degenerate chromoelectric flux tubes must grow with N . Quarks are confined; inside mesons a quark and its antipartner must be attached to a pair of strings, in contradistinction with QCD where the confining bond between quark and antiquark is built from a single string. It is thus clear that even if a dual to the M model is found, it is not yet quite QCD. However, it is pretty close.

ACKNOWLEDGMENTS

We are grateful to Arkady Vainshtein for useful discussions, to Maxim Chernodub, and especially to David Tong for helpful communications. This work was supported in part by DOE Grant No. DE-FG02-94ER408. A. G. was funded in part by FTPI, University of Minnesota, Grant No. CRDF-RUP2-261-MO-04 and RFBR Grant No. 06-02-17382. The work of A. Y. was supported by FTPI, University of Minnesota, by INTAS Grant No. 05-100008-7865, by RFBR Grant No. 06-02-16364a, and by Russian State Grant for Scientific School RSGSS-11242003.2.

APPENDIX: SUPERORIENTATIONAL ZERO MODES

In this appendix we find explicit expressions for four superorientational fermion zero modes of the non-Abelian string in the theory (3.5) with $N = 2$. Half-criticality of the string in question ensures that two supercharges are preserved in the world sheet theory. Following the general method of [7,9] we generate two superorientational fermion zero modes applying SUSY transformations to our string solution (4.8). Essentially repeating the calculation made in [9] we get

$$\begin{aligned} \bar{\psi}_{Ak2} &= \left(\frac{\tau^a}{2}\right)_{Ak} \frac{1}{2\phi_2} (\phi_1^2 - \phi_2^2) [\chi_2^a + i\varepsilon^{abc} S^b \chi_2^c], \\ \bar{\psi}_{Aki} &= 0, \\ \lambda^{a1} &= \frac{i}{\sqrt{2}} \frac{x_1 - ix_2}{r^2} f_{NA} \frac{\phi_1}{\phi_2} [\chi_2^a + i\varepsilon^{abc} S^b \chi_2^c], \\ \lambda^{a2} &= 0. \end{aligned} \tag{A1}$$

We see that supersymmetry generates for us only two fermion superorientational modes out of four predicted by the index theorem. They are parametrized by the two-dimensional fermion field χ_2^a . This was expected, of course. The modes proportional to χ_1^a do not appear.

The nonzero fermion fields here have the $U(1)_{\bar{R}}$ charge +1 while the fields which are zero have charge -1 . Clearly we need to find two more zero modes of charge +1. We do it by explicitly solving the Dirac equations. From the fermion part of the action of the model (5.5) we get the relevant Dirac equations

$$\begin{aligned} \frac{i}{g_2^2} \bar{\psi} \lambda^a + \frac{i}{\sqrt{2}} \text{Tr} \bar{\psi} \tau^a q = 0, & \quad \frac{i}{h} \bar{\psi} \zeta^a + \frac{i}{2} \text{Tr} \bar{q} \bar{\psi} \tau^a = 0, \\ i \not{D} \bar{\psi} - \frac{i}{\sqrt{2}} \lambda^a (\tau^a \bar{q}) = 0, & \quad i \not{D} \bar{\psi} + \frac{i}{2} \zeta^a (q \tau^a) = 0. \end{aligned} \quad (\text{A2})$$

After some algebra one can check that Eqs. (A1) do satisfy the first and the third of the Dirac equations (A2) provided the first-order equations for the string profile functions (4.5) are satisfied.

Now let us find two additional fermion zero modes by solving the second and the fourth of the Dirac equations (A2). The fields with the $U(1)_{\bar{R}}$ chiral charge -1 vanish,

$$\bar{\psi}_2^{kA} = 0, \quad \zeta^{a1} = 0. \quad (\text{A3})$$

In order to find the fields with the $U(1)_{\bar{R}}$ chiral charge +1 we use the following *ansatz*⁵ (cf. Ref. [9]),

$$\begin{aligned} \zeta^{a2} &= \zeta(r) [\chi_1^a + i \varepsilon^{abc} S^b \chi_1^c], \\ \bar{\psi}_1^{kA} &= \frac{x_1 - i x_2}{r} \psi(r) \left(\frac{\tau^a}{2} \right)^{kA} [\chi_1^a + i \varepsilon^{abc} S^b \chi_1^c]. \end{aligned} \quad (\text{A4})$$

Here we introduce two profile functions $\zeta(r)$ and $\psi(r)$ parametrizing the fermion fields ζ^2 and $\bar{\psi}_1$.

Substituting (A4) into the Dirac equations (A2) we get the following equations for fermion profile functions:

$$\begin{aligned} \frac{d}{dr} \psi + \frac{1}{r} \psi - \frac{1}{2r} (f + f_{\text{NA}}) \psi + i \phi_1 \zeta = 0, \\ -\frac{d}{dr} \zeta + i \frac{h}{2} \phi_1 \psi = 0. \end{aligned} \quad (\text{A5})$$

Below we present the solution to these equations in the limit

$$h \ll g_1^2 \sim g_2^2. \quad (\text{A6})$$

This latter assumption is not a matter of principle, rather it is just a technical point. It allows us to find an approximate analytic solution to Eqs. (A5). If the condition (A6) is met the mass of the fermions $\bar{\psi}$ and ζ ,

$$m_0 = \sqrt{\frac{h}{2}} \xi \quad (\text{A7})$$

(see Eqs. (3.2) and (3.3)), becomes much smaller than the masses of the gauge bosons (see Eqs. (2.13) and (2.14); note that the fermions $\bar{\psi}$ and ζ are superpartners of \bar{q} and M and have the same mass). Thus, the fields $\bar{\psi}$ and ζ develop long range tails with the exponential falloff determined by the small masses (A7). This allows us to solve Eqs. (A5) analytically treating separately the domains of large and small r .

In the large r domain, at $r \gg m_W$, we can drop the terms in (A5) containing f and f_{NA} and use the first equation to express ψ in terms of ζ . We then get

$$\psi = -\frac{2i}{h\sqrt{\xi}} \frac{d}{dr} \zeta. \quad (\text{A8})$$

Substituting this into the second equation in (A5) we obtain

$$\frac{d^2}{dr^2} \zeta + \frac{1}{r} \frac{d}{dr} \zeta - m_0^2 \zeta = 0. \quad (\text{A9})$$

This is a well-known equation for a free field with mass m_0 in the radial coordinates. Its solution is well known too,

$$\zeta = -\frac{ih}{2} \sqrt{\xi} K_0(m_0 r), \quad (\text{A10})$$

where $K_0(x)$ is the imaginary-argument Bessel function, and we fix a certain convenient normalization (in fact, the normalization constant of the profile functions is included in χ_1^a). At infinity $K_0(x)$ falls off exponentially,

$$K_0(x) \sim \frac{e^{-x}}{\sqrt{x}}, \quad (\text{A11})$$

while at $x \rightarrow 0$ it has a logarithmic behavior,

$$K_0(x) \sim \ln \frac{1}{x}. \quad (\text{A12})$$

Taking into account Eq. (A8) we get the solutions for the fermion profile functions at $r \gg 1/m_W$,

$$\zeta = -\frac{ih}{2} \sqrt{\xi} K_0(m_0 r), \quad \psi = -\frac{d}{dr} K_0(m_0 r). \quad (\text{A13})$$

In particular, at $r \ll 1/m_0$ we have

$$\zeta \sim -\frac{ih}{2} \sqrt{\xi} \ln \frac{1}{m_0 r}, \quad \psi \sim \frac{1}{r}. \quad (\text{A14})$$

In the intermediate domain $r \leq 1/m_W$ we neglect the small mass terms in (A5). We then arrive at

⁵One can show that profile functions in front of all other possible structures have singular behavior either at $r = 0$ or at $r = \infty$.

$$\frac{d}{dr}\zeta = 0, \quad \frac{d}{dr}\psi + \frac{1}{r}\psi - \frac{1}{2r}(f + f_{\text{NA}})\psi = 0. \quad (\text{A15})$$

The first equation here shows that $\zeta = \text{const}$, while the second one is identical to the equation for the string profile function ϕ_1 , see Eq. (4.5). This gives the fermion profile functions at intermediate r ,

$$\zeta = -\frac{ih}{2}\sqrt{\xi}\ln\frac{m_W}{m_0}, \quad \psi_- = \frac{1}{r\sqrt{\xi}}\phi_1, \quad (\text{A16})$$

where we fixed the normalization constants matching these solutions with the ones in the large- r region, see (A14).

Equations (A13) and (A16) present our final result for the fermion profile functions. They determine two extra fermion superorientational zero modes proportional to χ_1^a via Eq. (A4).

Now if we substitute the fermion zero modes (A1) and (A4) in the action (5.5) we get the effective $\mathcal{N} = 2$ CP(1) model (5.1) on the string world sheet,⁶ cf. Ref. [9].

⁶In doing so one has to redefine the normalization of the fields χ_1^a .

-
- [1] N. Seiberg and E. Witten, Nucl. Phys. **B426**, 19 (1994); **B430**, 485(E) (1994); **B431**, 484 (1994).
- [2] A. Hanany, M.J. Strassler, and A. Zaffaroni, Nucl. Phys. **B513**, 87 (1998).
- [3] A. Abrikosov, Sov. Phys. JETP **32**, 1442 (1957); in *Solitons and Particles*, edited by C. Rebbi and G. Soliani (World Scientific, Singapore, 1984), p. 356; H. Nielsen and P. Olesen, Nucl. Phys. **B61**, 45 (1973); in *Solitons and Particles*, edited by C. Rebbi and G. Soliani (World Scientific, Singapore, 1984), p. 365.
- [4] M.J. Strassler, *At the Frontier of Particle Physics/ Handbook of QCD*, edited by M. Shifman (World Scientific, Singapore, 2001), Vol. III, p. 1859.
- [5] A. Hanany and D. Tong, J. High Energy Phys. 07 (2003) 037.
- [6] R. Auzzi, S. Bolognesi, J. Evslin, K. Konishi, and A. Yung, Nucl. Phys. **B673**, 187 (2003).
- [7] M. Shifman and A. Yung, Phys. Rev. D **70**, 045004 (2004).
- [8] A. Hanany and D. Tong, J. High Energy Phys. 04 (2004) 066.
- [9] M. Shifman and A. Yung, Phys. Rev. D **72**, 085017 (2005).
- [10] A. A. Penin, V.A. Rubakov, P.G. Tinyakov, and S.V. Troitsky, Phys. Lett. B **389**, 13 (1996).
- [11] P. Fayet and J. Iliopoulos, Phys. Lett. B **51**, 461 (1974).
- [12] K.A. Intriligator and N. Seiberg, Nucl. Phys. B, Proc. Suppl. **45BC**, 1 (1996).
- [13] A.I. Vainshtein and A. Yung, Nucl. Phys. **B614**, 3 (2001).
- [14] K. Bardakçı and M.B. Halpern, Phys. Rev. D **6**, 696 (1972).
- [15] N. Seiberg, Nucl. Phys. **B435**, 129 (1995).
- [16] M. Shifman, Prog. Part. Nucl. Phys. **39**, 1 (1997).
- [17] K. Intriligator, N. Seiberg, and D. Shih, J. High Energy Phys. 04 (2006) 021.
- [18] M. Shifman and A. Yung (unpublished).
- [19] A. Gorsky, M. Shifman, and A. Yung, Phys. Rev. D **71**, 045010 (2005).
- [20] A.M. Polyakov, Phys. Lett. B **59**, 79 (1975).
- [21] M. Eto, K. Hashimoto, and S. Terashima, hep-th/0610042.
- [22] A. Ritz, M. Shifman, and A. Vainshtein, Phys. Rev. D **70**, 095003 (2004); A. Ritz, *Continuous Advances in QCD 2004*, edited by T. Gherghetta (World Scientific, Singapore, 2004), p. 428.
- [23] M. Edalati and D. Tong, hep-th/0703045.
- [24] E. Witten, Nucl. Phys. **B403**, 159 (1993).
- [25] M. A. Shifman, Phys. Rep. **209**, 341 (1991).
- [26] G. 't Hooft, Nucl. Phys. **B79**, 276 (1974); A.M. Polyakov, Pis'ma Zh. Eksp. Teor. Fiz. **20**, 430 (1974) [JETP Lett. **20**, 194 (1974)].
- [27] D. Tong, Phys. Rev. D **69**, 065003 (2004).
- [28] E.J. Weinberg, Nucl. Phys. **B167**, 500 (1980); **B203**, 445 (1982).
- [29] E. Witten, Nucl. Phys. **B149**, 285 (1979).
- [30] K. Hori and C. Vafa, hep-th/0002222.
- [31] N. Dorey, J. High Energy Phys. 11 (1998) 005.
- [32] A. Giveon and D. Kutasov, Rev. Mod. Phys. **71**, 983 (1999).
- [33] M.N. Chernodub, Phys. Rev. Lett. **95**, 252002 (2005); M.N. Chernodub and S.M. Morozov, Phys. Rev. D **74**, 054506 (2006).