Renormalization effects on neutrino masses and mixing in a string-inspired $\mathbf{SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_X}$ model

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We discuss renormalization effects on neutrino masses and mixing angles in a supersymmetric string-inspired $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_X$ model, with matter in fundamental and antisymmetric tensor representations and singlet Higgs fields charged under the anomalous $U(1)_X$ family symmetry. The quark, lepton and neutrino Yukawa matrices are distinguished by different Clebsch-Gordan coefficients. The presence of a second $U(1)_X$ breaking singlet with fractional charge allows a more realistic, hierarchical light neutrino mass spectrum with bi-large mixing. By numerical investigation we find a region in the model parameter space where the neutrino mass-squared differences and mixing angles at low energy are consistent with experimental data.

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The experimentally measured values of gauge coupling constants α_3, α_{em}, and the weak mixing angle $\sin^2\theta_W$ are correctly predicted in the minimal supersymmetric standard model (MSSM), assuming a unification scale of the order 10^{16} GeV. Moreover, the existing data from neutrino oscillation experiments provide an important clue to physics beyond the successful standard model (SM) and MSSM.

Experimental data on atmospheric and solar neutrino oscillations [1] imply tiny but nonzero neutrino mass-squared differences $\Delta m^2_{\nu_{ij}}$. The negligible size of the neutrino masses, as compared to those of quarks and charged leptons, might suggest that a theory beyond MSSM should incorporate the right-handed neutrinos and an appropriate (seesaw) mechanism to suppress adequately the neutrino masses. Moreover, the observed large neutrino mixing angles $\theta_{\nu_{12}}$, $\theta_{\nu_{23}}$ present challenges for additional symmetries and a unified framework in which neutrinos and quarks form part of same multiplet. Examples of higher symmetries including the SM gauge group and incorporating the right-handed neutrino in the fermion spectrum, are the partially unified Pati-Salam model, based on $SU(4)\times SU(2)_L\times SU(2)_R$ [2], and the fully unified SO(10). When embedded into perturbative string or D-brane models, these may be extended by additional Abelian or discrete fermion family symmetries. Semirealistic models with these characteristics have been discussed in the literature [3–5].

Recently some models have been proposed to explain the presence of large mixing angles in the neutrino sector, in contrast to the smaller quark mixings. For example, the mixing angle $\theta_{\nu_{12}}$ and the Cabibbo mixing θ_C could satisfy the so called quark-lepton complementarity (QLC) relation $\theta_{\nu_{12}}+\theta_C\approx\frac{\pi}{4}$ [6]. It has been shown [7] that this relation can be reproduced if some symmetry exists among quarks and leptons. Attempts to realize QLC in the context of models unifying quarks and leptons such as Pati-Salam have been made [8].

As another possibility, the symmetry $L_e-L_\mu-L_\tau$ implies an inverted neutrino mass hierarchy and bimaximal mixing $\theta_{\nu_{12}}=\theta_{\nu_{23}}=\frac{\pi}{4}$, with $\theta_{\nu_{13}}=0$ [9]. This symmetry alone does not give a consistent description of current experimental data, but additional corrections and renormalization effects have still to be taken into account. It has been shown [10] in the context of MSSM extended by a spontaneously broken $U(1)_X$ factor, that the neutrino sector respects an $L_e-L_\mu-L_\tau$ symmetry. Small corrections from other singlet vacuum expectation values (v.e.v.'s), which are usually present in a string spectrum, can lead to a soft breaking of this symmetry and describe accurately the experimental neutrino data.

Another important issue is the renormalization group (RG) flow of the neutrino parameters from the high energy scale where the neutrino mass matrices are formed, down to their low-energy measured values. One can attempt to determine the neutrino mass matrices from experimental data directly at the weak scale. However, the Yukawa couplings and other relevant parameters are not known at the unification scale. A knowledge of these quantities at the unification mass could provide a clue for the structure of the unified or partially unified theory and the exact (family) symmetries determining the neutrino mass matrices at the grand unified theory (GUT) scale. Attempts to determine the neutrino mass parameters in a top-bottom or bottom-up approach have recently discussed in the literature [11–13].

In this paper we investigate further the neutrino mass spectrum of a model with gauge symmetry $SU(4)\times SU(2)_L\times SU(2)_R\times U(1)_X$ based on the 4-2-2 models [3,14–16], whose implications for quark and lepton masses were recently investigated in Ref. [17]. These models present several attractive features. First, only "small" Higgs representations are needed and these commonly arise in string models. Second, third generation fermion Yukawa couplings are unified [18] up to possible small corrections, fixing the value of $\tan\beta$ (the ratio of the up-type to down-type Higgs v.e.v.'s) to be large, of order 50. Unification of gauge couplings is allowed and, if one assumes the model embedded in supersymmetric string, might be predicted [19], even though the four-dimensional gauge group has separate factors. Furthermore, the doublet-triplet splitting problem is absent, since the colored triplets can be given mass by coupling to antisymmetric tensor representations which also arise from explicit string constructions [3,14].

In string derived models a large number of standard model singlet fields carrying quantum numbers only under $U(1)_X$ appear in the spectrum of the effective field theory. D- and F-flatness conditions require some of them to obtain nonzero v.e.v.'s of the order of the $U(1)_X$ breaking scale. In the present model, in order to describe accurately the low-energy neutrino data we introduce two such singlets charged under $U(1)_X$ [20]. The previous model [17] with one singlet could achieve a normal hierarchy of light neutrino masses with bi-large mixing. However, after study of the renormalization group (RG) evolution and unification it was found that the scale of light neutrino masses was too large to be compatible with observation, due to strong running in the large tan beta regime. Conversely, if we imposed the correct scale of light neutrino masses, then some heavy right-handed neutrinos (RHN) would have masses above the unification scale, which is incompatible with our effective field theory approach.

Thus three *a priori* independent expansion parameters arise from the superpotential, two coming from the singlets and one from the Higgses which receive v.e.v.'s at the $SU(4) \times SU(2)_R$ breaking scale. In general, a nonrenormalizable operator may contain several products of the $SU(4) \times SU(2)_R$ breaking Higgses, thus different contractions of gauge group indices are possible leading to different contributions depending on the Clebsch factors. We use a minimal set of nontrivial Clebsch factors to construct the Dirac mass matrices. Right-handed ($SU(2)_L$ singlet) neutrinos acquire Majorana masses through nonrenormalizable couplings to the $U(1)_X$-charged singlets and to Higgses, while light neutrinos will obtain masses via the seesaw mechanism.

The renormalization group equations (RGEs) for the neutrino masses and mixing angles above, between and below the seesaw scales are solved numerically, for several sets of order 1 parameters which specify the heavy RHN matrix. In each case the results at low energy are consistent with current experimental data, given a normal hierarchy of light neutrino masses, and provide further predictions for the 1–3 neutrino mixing angle and for neutrinoless double beta decay.

I. DESCRIPTION OF THE MODEL

In this section we present salient features of the string-inspired Pati-Salam model extended by a $U(1)_X$ family symmetry, the total gauge group being $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_X$. The field content includes three copies of $(4, 2, 1) + (\bar{4}, 1, \bar{2})$ representations to accommodate the three fermion generations $F_i + \bar{F}_i$ $(i = 1, 2, 3)$,

$$F_i = \begin{pmatrix} u_i & u_i & u_i & \nu_i \\ d_i & d_i & d_i & e_i \end{pmatrix}_{\alpha_i},$$

$$\bar{F}_i = \begin{pmatrix} u_i^c & u_i^c & u_i^c & \nu_i^c \\ d_i^c & d_i^c & d_i & e_i^c \end{pmatrix}_{\bar{\alpha}_i},$$

where the subscripts α_i, $\bar{\alpha}_i$ indicate the $U(1)_X$ charge. In order to break the Pati-Salam symmetry down to SM gauge group, Higgs fields $H = (4, 1, 2)$ and $\bar{H} = (\bar{4}, 1, \bar{2})$ are introduced

$$H = \begin{pmatrix} u_H & u_H & u_H & \nu_H \\ d_H & d_H & d_H & e_H \end{pmatrix}_x,$$

$$\bar{H} = \begin{pmatrix} u_{\bar{H}}^c & u_{\bar{H}}^c & u_{\bar{H}}^c & \nu_{\bar{H}}^c \\ d_{\bar{H}}^c & d_{\bar{H}}^c & d_{\bar{H}} & e_{\bar{H}}^c \end{pmatrix}_{\bar{x}},$$

which acquire v.e.v.'s of the order M_G along their neutral components

$$\langle H \rangle = \langle \nu_H \rangle = M_G, \qquad \langle \bar{H} \rangle = \langle \nu_{\bar{H}} \rangle = M_G. \tag{1}$$

The Higgs sector also includes the $h = (1, \bar{2}, 2)$ field which after the breaking of the PS symmetry is decomposed to the two Higgs superfields of the MSSM. Further, two $D = (6, 1, 1)$ scalar fields are introduced to give mass to color triplet components of H and $\bar{H}$ via the terms HHD and $\bar{H}\bar{H}D$ [14].

Finally, we introduce two scalar singlet fields ϕ, χ, charged under $U(1)_X$ whose v.e.v.'s will play a crucial role in the fermion mass matrices through nonrenormalizable terms of the superpotential. In the stable SUSY vacuum the two singlets obtain v.e.v.'s to satisfy the D-flatness condition including the anomalous Fayet-Iliopoulos term [21]. The anomalous D-flatness conditions allow solutions where the v.e.v.'s of the conjugate fields $\bar{\phi}$ and $\bar{\chi}$ are zero and we will restrict our analysis to this case. Note that in

TABLE I. Field content and $U(1)_X$ charges.

	$SU(4)$	$SU(2)_L$	$SU(2)_R$	$U(1)_X$
F_i	4	2	1	α_i
$\bar{F}_i$	$\bar{4}$	1	$\bar{2}$	$\bar{\alpha}_i$
H	4	1	2	x
$\bar{H}$	$\bar{4}$	1	$\bar{2}$	$\bar{x}$
ϕ	1	1	1	z
χ	1	1	1	z'
h	1	$\bar{2}$	2	$-\alpha_3 - \bar{\alpha}_3$
D_1	6	1	1	$-2x$
D_2	6	1	1	$-2\bar{x}$

general a string model may have more than two singlets and more than one set of Higgses H_i, $\bar{H}_i$, with different $U(1)_X$ charge. All such fields may in principle also obtain v.e.v.'s, however we find that two of them are sufficient to give a set of mass matrices in accordance with all experimental data. Hence we consider any additional singlet v.e.v.'s to be significantly smaller.

The Higgses H_i, $\bar{H}_i$ may obtain masses through $H\bar{H}\phi$, $H\bar{H}\chi$ and $H\bar{H}\phi\chi$ couplings. However, in order to break the Pati-Salam group while preserving SUSY we require that one H-$\bar{H}$ pair be massless at this level. This "symmetry-breaking" Higgs pair could be a linear combination of fields with different $U(1)_X$ charges, which would in general complicate the expressions for fermion masses. The chiral spectrum is summarized in Table I. We choose the charge of the Higgs field h to be $-\alpha_3 - \bar{\alpha}_3$ so that the 3rd generation coupling $F_3\bar{F}_3h$ is allowed at tree level.

We now turn to the terms in the superpotential which can give rise to fermion masses. Dirac type mass terms arise after electroweak symmetry-breaking from couplings of the form

$$\mathcal{W}_D = y_0^{33}F_3\bar{F}_3h + F_i\bar{F}_jh\Bigg(\sum_{m>0} y_m^{ij}\left(\frac{\phi}{M_U}\right)^m + \sum_{m'>0} y_{m'}^{ij}\left(\frac{\chi}{M_U}\right)^{m'} + \sum_{n>0} y'^{ij}{}_n\left(\frac{H\bar{H}}{M_U^2}\right)^n + \sum_{k,\ell>0} y_{k,\ell}^{ij}\left(\frac{\phi}{M_U}\right)^k\left(\frac{\chi}{M_U}\right)^\ell + \sum_{p,q>0} y_{p,q}^{ij}\left(\frac{H\bar{H}}{M_U^2}\right)^p\left(\frac{\phi}{M_U}\right)^q + \sum_{r,s>0} y_{r,s}^{ij}\left(\frac{H\bar{H}}{M_U^2}\right)^r\left(\frac{\chi}{M_U}\right)^s + \cdots\Bigg). \tag{2}$$

Apart from the heaviest generation, all masses arise at nonrenormalizable level, suppressed by powers of the fundamental scale or unification scale M_U. The couplings $y_{m,m'}^{ij}$, $y'^{ij}{}_n$ etc. are nonvanishing and generically of order 1 whenever the $U(1)_X$ charge of the corresponding operator vanishes, thus

$$\alpha_i - \alpha_3 + \bar{\alpha}_j - \bar{\alpha}_3 = \{-mz, -m'z', -n(x+\bar{x}), -kz - \ell z', -p(x+\bar{x}) - qz, -r(x+\bar{x}) - sz'\}.$$

Other higher-dimension operators may arise by multiplying any term by factors such as $(H\bar{H})^\ell\phi^s/M_U^{2\ell+s}$ where $\ell(x+\bar{x}) + sz = 0$. Such terms are negligible unless the leading term vanishes.

Neutrinos may in addition receive also Majorana type masses. These arise from the operators

$$W_M = \frac{\bar{F}_i\bar{F}_jHH}{M_U}\Bigg(\mu_0^{ij} + \sum_{t>0}\mu_t^{ij}\left(\frac{\phi}{M_U}\right)^t + \sum_{t'>0}\mu_{t'}^{ij}\left(\frac{\chi}{M_U}\right)^{t'} + \sum_{w>0}\mu'^{ij}_w\left(\frac{H\bar{H}}{M_U^2}\right)^w + \sum_{k',\ell'>0}\mu_{k',\ell'}^{ij}\left(\frac{\phi}{M_U}\right)^{k'}\left(\frac{\chi}{M_U}\right)^{\ell'} + \sum_{p',q'>0}\mu_{p',q'}^{ij}\left(\frac{H\bar{H}}{M_U^2}\right)^{p'}\left(\frac{\phi}{M_U}\right)^{q'} + \sum_{r,s>0}\mu_{r',s'}^{ij}\left(\frac{H\bar{H}}{M_U^2}\right)^{r'}\left(\frac{\chi}{M_U}\right)^{s'} + \cdots\Bigg). \tag{3}$$

Couplings of this type are nonvanishing whenever the following conditions are satisfied:

$$\bar{\alpha}_i + \bar{\alpha}_j + 2x = \{-tz, -t'z', -w(x+\bar{x}), -k'z - \ell'z' - p'(x+\bar{x}) - q'z, -r'(x+\bar{x}) - s'z'\}.$$

II. FERMION MASS MATRICES

A. General structure

As can be seen from the superpotential Yukawa couplings (2) and (3), three different expansion parameters appear in the construction of the fermion mass matrices. These are

$$\epsilon \equiv \frac{\langle\phi\rangle}{M_U}, \qquad \epsilon' \equiv \frac{M_G^2}{M_U^2}, \qquad \epsilon'' \equiv \frac{\langle\chi\rangle}{M_U}, \tag{4}$$

given $\langle H\bar{H}\rangle = M_G^2$. Note that, for nonrenormalizable Dirac mass terms involving several products of $H\bar{H}/M_U^2$, the gauge group indices may be contracted in different ways [16]. This can lead to different contributions to the up, down quarks and charged leptons, depending on the Clebsch factors $C_{n(u,d,e,\nu)}^{ij}$ multiplying the effective Yukawa couplings. Also, although the Clebsch coefficient for a particular operator O_n may vanish at order n, the coefficient for the operator $O_{(n+p);q}$ containing p additional factors $(H\bar{H})$ and q factors of ϕ and/or χ is generically nonzero.

In our analysis we wish to estimate the effects of the second singlet (χ) contributions on the neutrino sector as compared to the analysis presented in Ref. [17] without affecting essentially the results in the quark sector. In order to obtain a set of fermion mass matrices with the minimum number of new operators, we assume fractional $U(1)_X$ charges for H, $\bar{H}$, and χ fields, while the combination $H\bar{H}$ and the singlet ϕ are assumed to have integer charges. Thus α_i, $\bar{\alpha}_i$, $x+\bar{x}$, and z are integers, while z', x and $\bar{x}$ are fractional. As a result, the Dirac mass terms involving v.e.v.'s of χ are expected to be subleading compared to other terms. Suppressing higher-order terms involving products of ϵ, ϵ', and ϵ'', the Dirac mass terms at the unification scale are

$$m_{ij} \approx \delta_{i3}\delta_{j3}m_3 + (\epsilon^m + (\epsilon'')^{m'} + C_{ij}(\epsilon')^n)v_{u,d}, \qquad (5)$$

where $m_3 \equiv v_{u,d}y_0^{33}$, with v_u and v_d being the up-type and down-type Higgs v.e.v.'s, respectively, and we omit the order-one Yukawa coefficients y_m^{ij} etc. for simplicity.

The Majorana mass terms are proportional to the combination HH [see Eq. (3)] which has fractional $U(1)_X$ charge. Thus, terms proportional to χ/M_U become now important for the structure of the mass matrix. The general form of the Majorana mass matrix is then

$$M_N \approx M_R(\mu_t^{ij}\epsilon^t + \mu_{t'}^{ij}(\epsilon'')^{t'} + \mu_w'^{ij}(\epsilon')^w + \mu_{k',l'}^{ij}\epsilon^{k'}(\epsilon'')^{l'} + \mu_{p',q'}^{ij}(\epsilon')^{p'}\epsilon^{q'} + \mu_{r',s'}^{ij}(\epsilon')^{r'}(\epsilon'')^{s'}),$$

where we define $M_R \equiv M_G^2/M_U \equiv \epsilon' M_U$.

B. Choice of $U(1)_X$ charges

Before we proceed to a specific, viable set of mass matrices, we first make use of the observation [17] that the form of the fermion mass terms above is invariant under the shifts

$$\alpha_i \to \alpha_i + \zeta, \qquad \bar{\alpha}_i \to \bar{\alpha}_i + \bar{\zeta}, \qquad x \to x - \bar{\zeta}, \qquad \bar{x} \to \bar{x} + \bar{\zeta}, \qquad (6)$$

so that we are free to assign $\alpha_3 = \bar{\alpha}_3 = 0$. We further fix $x + \bar{x} = 1$ and $z = -1$; we will choose the values of x and z' to be fractional such that the v.e.v. of χ only affects the overall scale of neutrino masses, as explained below. The resulting $U(1)_X$ charges are presented in Table II.

The charge entries of the common Dirac mass matrix for quarks, charged leptons and neutrinos are then

$$Q_X[M_D] = \begin{pmatrix} -6 & -3 & -4 \\ -5 & -2 & -3 \\ -2 & 1 & 0 \end{pmatrix}, \qquad (7)$$

and the charge matrix for heavy neutrino Majorana masses is

$$Q_X[M_N] = 2x + \begin{pmatrix} -4 & -1 & -2 \\ -1 & 2 & 1 \\ -2 & 1 & 0 \end{pmatrix}. \qquad (8)$$

Now, we relate ϵ, ϵ', ϵ'' with a single expansion parameter η, assuming the relations

$$\epsilon' = \sqrt{\eta}, \qquad \epsilon = b_1\sqrt{\eta}, \qquad \epsilon'' \equiv b_2\eta, \qquad (9)$$

where b_1, b_2 are numerical coefficients of order one. Then

TABLE II. Specific choice of $U(1)_X$ charges.

Field	F_1	F_2	F_3	$\bar{F}_1$	$\bar{F}_2$	$\bar{F}_3$	h	H	$\bar{H}$	ϕ	χ
$U(1)_X$	-4	-3	0	-2	1	0	0	x	$1-x$	-1	z'

the effective Yukawa couplings for quarks and leptons may include terms

$$Y_f^{ij} = b_1^m\eta^{m/2} + b_1^{1+m}\eta^{1+m/2} + C_f^{ij}\eta^{n/2} + b_1\eta^{1+n/2} + \cdots \qquad (10)$$

with $f = u, d, e, \nu$, up to order 1 coefficients y_f^{ij}. Which of these terms survives, depends on the sign of the charge of the corresponding operator. For a negative charge entry, the first two terms are not allowed and only the third and fourth contribute. Further, if a particular C_f^{ij} coefficient is zero, then we consider only the fourth term.

Therefore, we need to specify the Clebsch-Gordan coefficients C_f^{ij} for the terms involving powers of $\langle H\bar{H}\rangle/M_U^2$. These coefficients could be found if the fundamental theory was completely specified at the unification or string scale. In the absence of a specific string model, here we present a minimal number of operators which lead to a simple and viable set of mass matrices. Up to possible complex phases, we choose $C_d^{12} = C_d^{22} = \frac{1}{3}$, $C_u^{23} = 3$ and $C_u^{11} = C_u^{12} = C_u^{21} = C_u^{22} = C_u^{31} = C_\nu^{22} = C_\nu^{31} = 0$ with all others being equal to unity. The effective Yukawa matrices at the GUT scale obtained under the above assumptions are

$$Y_u = \begin{pmatrix} b_1\eta^4 & b_1\eta^{5/2} & \eta^2 \\ b_1\eta^{7/2} & b_1\eta^2 & 3\eta^{3/2} \\ b_1\eta^2 & b_1\eta^{1/2} & 1 \end{pmatrix},$$
$$Y_d = \begin{pmatrix} \eta^3 & \frac{\eta^{3/2}}{3} & \eta^2 \\ \eta^{5/2} & \frac{\eta}{3} & \eta^{3/2} \\ \eta & b_1\eta^{1/2} & 1 \end{pmatrix},$$
$$Y_e = \begin{pmatrix} \eta^3 & \eta^{3/2} & \eta^2 \\ \eta^{5/2} & \eta & \eta^{3/2} \\ \eta & b_1\eta^{1/2} & 1 \end{pmatrix},$$
$$Y_\nu = \begin{pmatrix} \eta^3 & \eta^{3/2} & \eta^2 \\ \eta^{5/2} & b_1\eta^2 & \eta^{3/2} \\ b_1\eta^2 & b_1\eta^{1/2} & 1 \end{pmatrix}, \qquad (11)$$

where we suppress order-one coefficients. The quark sector as well as the neutrino sector were studied in Ref. [17]. However, full renormalization group effects were not calculated for the neutrino sector and as it turns out one singlet is inadequate to accommodate the low-energy data. Consequently, we introduced the second singlet χ, with fractional charge, whose v.e.v. affects only the overall scale of neutrino masses.

The desired matrix for the right-handed Majorana neutrinos may result from more than one choice of charge for the H field and the χ singlet field. These can be seen in Table III.

TABLE III. Possible choices for the $U(1)_X$ charges of the H and χ fields.

$Q_X[H] = x$	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{5}{3}$	$-\frac{6}{5}$	$-\frac{7}{8}$	$-\frac{1}{4}$	$-\frac{3}{4}$
$Q_X[\chi] = z'$	$-\frac{5}{3}$	$-\frac{1}{3}$	$\frac{1}{3}$	$-\frac{3}{5}$	$-\frac{5}{4}$	$-\frac{5}{2}$	$-\frac{3}{2}$

We choose the H charge to be $x = -\frac{6}{5}$ so that $2x$ is noninteger, and set the χ singlet charge to $-\frac{3}{5}$. The analysis for the quarks and charged leptons remains the as in Ref. [17] since operators with nonzero powers χ^r do not exist for powers $r < 5$ and are negligible compared to the leading terms.

With these assignments, the charge entries of the heavy Majorana matrix Eq. (8) are

$$Q_X[M_N] = \begin{pmatrix} -\frac{32}{5} & -\frac{17}{5} & -\frac{22}{5} \\ -\frac{17}{5} & -\frac{2}{5} & -\frac{7}{5} \\ -\frac{22}{5} & -\frac{7}{5} & -\frac{12}{5} \end{pmatrix}. \tag{12}$$

Because of the fractional $U(1)_X$ charges contributions from ϕ or $H\bar{H}$ alone vanish. However, we also have the singlets $H\bar{H}\chi/M_U^3$ with v.e.v. $b_2\eta^{3/2}$ and $\phi\chi/M_U^2$ with a v.e.v. $b_1b_2\eta^2$, while for some entries one may have to consider higher-order terms since the leading order will be vanishing. In Table IV we explicitly write the operator for every entry of M_N. The Majorana right-handed neutrino mass matrix is then

$$M_N = \begin{pmatrix} \mu_{11}\eta^{9/2} & \mu_{12}\eta^3 & \mu_{13}\eta^{7/2} \\ \mu_{12}\eta^3 & \mu_{22}\eta^{3/2} & \mu_{23}\eta^2 \\ \mu_{13}\eta^{7/2} & \mu_{23}\eta^2 & \eta^{5/2} \end{pmatrix} b_2 M_R, \tag{13}$$

with $M_R = \epsilon' M_U = \sqrt{\eta} M_U$.

Having defined the Dirac and heavy Majorana mass matrices for the neutrinos, it is straightforward to obtain the light Majorana mass matrix from the seesaw formula

$$m_\nu = -m_{D\nu} M_N^{-1} m_{D\nu}^T \tag{14}$$

at the GUT scale.

TABLE IV. Operators producing the Majorana right-handed neutrino matrix M_N.

M_N entry	Operator	v.e.v.
M_N^{11}	$(\frac{H\bar{H}}{M_U^2})^7 \frac{\chi}{M_U}$	$b_2\eta^{9/2}$
M_N^{12}	$(\frac{H\bar{H}}{M_U^2})^4 \frac{\chi}{M_U}$	$b_2\eta^3$
M_N^{13}	$(\frac{H\bar{H}}{M_U^2})^5 \frac{\chi}{M_U}$	$b_2\eta^{7/2}$
M_N^{22}	$(\frac{H\bar{H}}{M_U^2}) \frac{\chi}{M_U}$	$b_2\eta^{3/2}$
M_N^{23}	$(\frac{H\bar{H}}{M_U^2})^2 \frac{\chi}{M_U}$	$b_2\eta^2$
M_N^{33}	$(\frac{H\bar{H}}{M_U^2})^3 \frac{\chi}{M_U}$	$b_2\eta^{5/2}$

C. Setting the expansion parameters

Given the fermion mass textures in terms of the $U(1)_X$ charges and expansion parameters, we need now to determine the values of the latter in order to obtain consistency with the low-energy experimentally known quantities (masses and mixing angles). Note that the coefficient b_2 defined in Eq. (9) will determine the overall neutrino mass scale through Eq. (13).

Consistency with the measured values of quark masses and mixings fixes the value of $\eta \approx 5 \times 10^{-2}$: for example, the angle θ_{12} of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix is given by $\sqrt{\eta} \approx 0.22$ up to small corrections [17]. Hence the ratio of the SU(4) breaking scale M_G to the fundamental scale M_U is also fixed through $\frac{M_G^2}{M_U^2} = \sqrt{\eta} \approx 0.22$: the Pati-Salam group is unbroken over only a small range of energy. We perform a renormalization group analysis in order to check the consistency of this prediction with the low-energy values of the gauge couplings α_s, α_{em}, and the weak mixing angle $\sin^2\theta_W$ [22]

$$\sin^2\theta_W = 0.23120, \qquad \alpha_3 = 0.118 \pm 0.003,$$
$$a_{em} = \frac{1}{127.906}.$$

If the underlying model at M_U has a single unified gauge coupling, then M_G is fixed to be just below the unification scale according to the analysis of gauge coupling unification in the MSSM. Because of this fact, the low-energy measured range for α_3 affects the unification of the gauge couplings. Thus, we add the following extra states:

$$h_L = (1, 2, 1), \qquad h_R = (1, 1, 2),$$

which are usually present in a string spectrum [3]. It turns

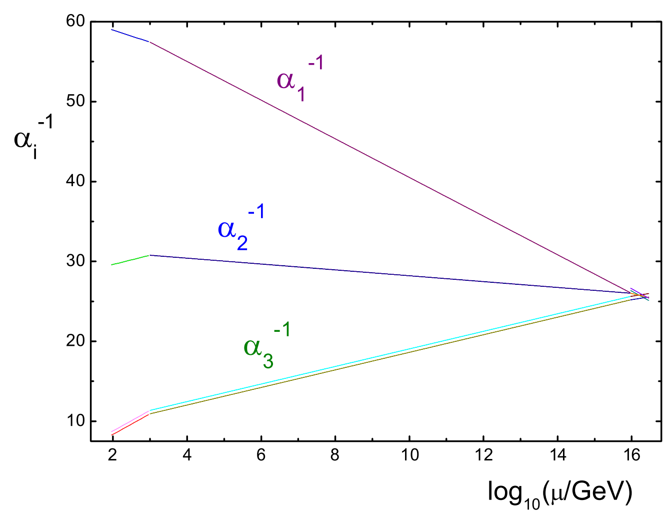

FIG. 1 (color online). Evolution of the gauge couplings. The two lines for α_3 indicate the range of initial conditions at M_Z.

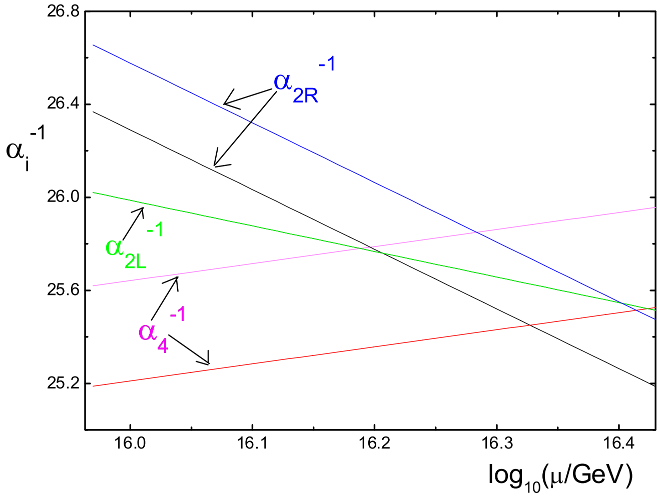

FIG. 2 (color online). Close-up of the gauge couplings in the Pati-Salam energy region.

out that we need 4 of each of these extra states for $M_G = 9.32 \times 10^{15}$ GeV to be consistent with the value of ϵ' deduced from quark mass matrices.

In Fig. 1 we plot the evolution of the gauge couplings from M_Z to M_U. In Fig. 2 we show in more detail the evolution of the gauge couplings in the Pati-Salam energy region. The two bands for the α_4 and α_{2R} couplings are due to strong coupling uncertainty at M_Z. For $\alpha_3(M_Z) = 0.1176$, as can be seen from Fig. 3, we obtain $M_U = 1.96 \times 10^{16}$ GeV.

The remaining parameters to be determined are b_1, b_2, and μ_{ij} of the right-handed neutrino mass matrix. We find that M_N is proportional to b_2, $M_N \approx b_2 M_R M'_N$, thus b_2 is related to the scale of the light matrix m_ν. Also, the choice $b_1 \approx 1.1$ leads to agreement with the data, while implying $\epsilon = 1.1\sqrt{\eta} \approx 0.25$.

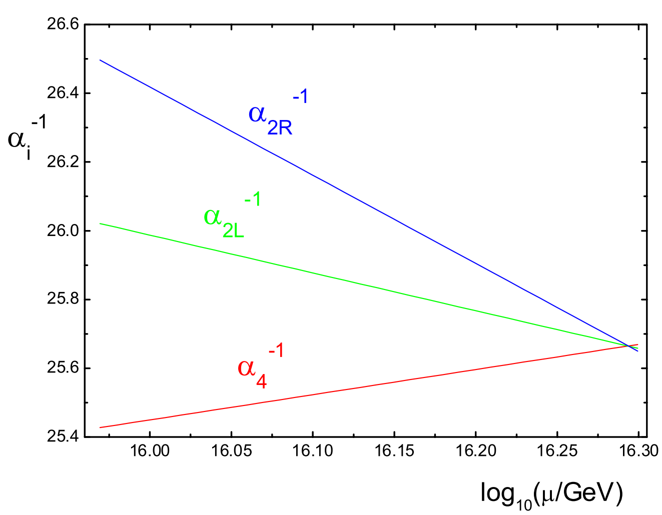

FIG. 3 (color online). The unification point of Pati-Salam gauge couplings for $\alpha_3(M_Z) = 0.1176$.

III. RELATION OF THEORY TO NEUTRINO OBSERVABLES

A. Running of neutrino masses and mixing angles

One of the problems one encounters when searching for a specific mass matrix for the light neutrinos via the seesaw formula is the effects induced by the renormalization group equations. The low-energy neutrino data could be considerably different from the results at the seesaw scale. The running of neutrino masses and mixing angles has been extensively discussed for energies below the seesaw scales [23–25] as well as above [11,12,26]. The running of the effective neutrino mass matrix m_ν above and between the seesaw scales is split into two terms

$$m_\nu = -\frac{v^2}{4}(\overset{(n)}{\kappa} + 2\overset{(n)}{Y}_\nu \overset{(n)}{M}{}^{-1} \overset{(n)}{Y}{}^T_\nu), \quad (15)$$

where κ is related to the coefficient of the effective 5 dimensional operator $LLh_u h_u$, (n) labels the effective field theories with M_n right-handed neutrino integrated out $(M_n \geq M_{n-1} \geq M_{n-2}, \ldots)$ and $\overset{(n)}{Y}_\nu$ are the neutrino couplings at an energy scale M between two RH neutrino masses $M_n \geq M \geq M_{n-1}$, while $\overset{(n)}{Y}_\nu = 0$ below the lightest RH neutrino mass. These effective parameters govern the evolution below the highest seesaw scale and obey the differential equation [23–25]

$$16\pi^2 \frac{d\overset{(n)}{X}}{dt} = (Y_e Y_e^\dagger + \overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger)^T \overset{(n)}{X} + \overset{(n)}{X}(Y_e Y_e^\dagger + \overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger)^T + (2\,\mathrm{Tr}(\overset{(n)}{Y}_\nu \overset{(n)}{Y}{}_\nu^\dagger + 3 Y_u Y_u^\dagger) - 6/5 g_1^2 - 6 g_2^2)\overset{(n)}{X}, \quad (16)$$

where $X = \kappa$, $Y_\nu M^{-1} Y_\nu^T$. The RGEs have been solved both numerically and also analytically [11,12,25]. Numerically, below the lightest heavy RH neutrino mass large renormalization effects can be experienced only in the case of degenerate light neutrino masses for very large $\tan\beta$ [25,27]. Above this mass things are more complicated due to the nontrivial dependence of the heavy neutrino mass couplings, unless M_{ν^c} is diagonal. For the leptonic mixing angles, in the case of normal hierarchy relevant to our model, one expects negligible effects for the solar mixing angle while θ_{13} and θ_{23} are expected to run faster [13].

On the other hand, studying the analytical expressions obtained after approximation, exactly the opposite behavior is predicted and the solar mixing angle receives larger renormalization effects than θ_{13} or θ_{23}. However, possible cancellations may occur and enhancement or suppression factors may appear, thus the numerical solutions may differ considerably from these estimates.

In our string-inspired model the Dirac and heavy Majorana mass matrices at the unification scale are parametrized in terms of order 1 superpotential coefficients

$\mu_{ij}(M_U)$ whose exact numerical values are not known. The flavour structure at the unification scale might also be different from that at the electroweak scale M_Z. Thus, even if the Yukawa parameters are determined at M_Z, to understand the structure of the theory at M_U, and consequently any possible family symmetry, we would certainly need the parameter values at M_U.

In this section we study the renormalization group flow of the neutrino mass matrices "top down" from the Pati-Salam scale M_G to the weak scale. We choose sets of values of the undetermined order 1 coefficients at the high scale and run the renormalization group equations down to M_Z where we calculate $\Delta m^2_{\nu_{ij}}$ and $\theta_{\nu_{ij}}$ and compare them with the experimental values. Study of a bottom-up approach has been performed [13] and we will compare our results to this work. The renormalization group analyses of the neutrino parameters, successively integrating out the right-handed neutrinos, is performed using the software packages REAP/MPT [11].

(i) We generate appropriate numerical values for the coefficients μ_{11}, μ_{12}, μ_{13}, μ_{22}, μ_{23}, so that after the evolution of m_ν to low energy we obtain values in agreement with the experimental data. The coefficient μ_{33} is set to unity (which can always be done by adjusting the value of b_2). Experimentally acceptable solutions can be seen in Table V. In Table VI we present the resulting values of θ_{ij} and Δm^2_{ij} at the scale M_Z. The mass-squared differences lie in the ranges $\Delta m^2_{\text{atm}} = [1.33\text{–}3.39] \times 10^{-3}\ \text{eV}^2$, $\Delta m^2_{\text{sol}} = [7.24\text{–}8.85] \times 10^{-5}\ \text{eV}^2$. These are consistent with the experimental data $\Delta m^2_{\text{atm,exp}} = [1.3\text{–}3.4] \times 10^{-3}\ \text{eV}^2$ and $\Delta m^2_{\text{sol,exp}} = [7.1\text{–}8.9] \times 10^{-5}\ \text{eV}^2$. The mixing angles are also found in the allowed ranges $\theta_{12} = [29.4\text{–}37.6]$, $\theta_{23} = [36.9\text{–}51.0]$, and $\theta_{13} = [0.86\text{–}12.50]$.

(ii) In Fig. 4 we plot the running of the three light neutrino Majorana masses ($m_1 < m_2 < m_3$) in the energy range $M_G - M_Z$. The initial (GUT) neutrino eigenmasses are all larger than their low-energy values. Significant running is observed mainly for the heaviest eigenmass m_{ν_3}. For experimentally acceptable mass-squared differences $\Delta m^2_{\nu_{ij}}$ at M_Z, in all cases their corresponding values at the GUT scale lie out of the acceptable range. In this scenario with hierarchical light neutrino masses, we find that large renormalization effects occur above the heavy neutrino threshold since the Yukawa couplings Y_ν are large and the second term in Eq. (15) dominates. Also, since $m_{\nu_1} < \sqrt{\Delta m^2_{\text{sol}}}$, the solar angle turns out to be more stable compared to the running of the θ_{23}, as can be seen in Fig. 5. 'The running of light neutrino mass-squared differences is displayed in Fig. 6. These results are in agreement with the findings of Ref. [13].

(iii) In Fig. 7 we plot the distribution Δm^2_{atm} versus Δm^2_{sol} at the two scales M_G (Table VII) and M_Z for the ten models of Table V. We find that the hierarchy of the neutrino masses at the Pati-Salam breaking scale tends to be greater than that at low

TABLE V. Numerical values of parameters μ_{11}, μ_{12}, μ_{13}, μ_{22}, μ_{23} at M_G.

Solution	μ_{11}	μ_{12}	μ_{13}	μ_{22}	μ_{23}
1	0.105 35	0.109 72	0.860 12	0.104 91	0.910 14
2	0.119 39	0.109 54	0.809 12	0.106 83	0.938 32
3	0.103 92	0.117 87	0.977 96	0.105 12	0.987 49
4	0.091 43	0.109 62	0.876 16	0.100 63	0.937 98
5	0.126 97	0.127 45	0.998 60	0.116 52	0.999 80
6	0.109 20	0.096 38	1.009 75	0.102 38	0.935 61
7	0.101 24	0.116 82	0.985 68	0.106 88	0.991 56
8	0.123 58	0.095 14	0.995 80	0.104 34	0.956 46
9	0.130 06	0.119 73	1.022 35	0.103 78	0.894 60
10	0.126 65	0.121 37	1.000 29	0.106 95	0.915 78

TABLE VI. Values of the physical parameters Δm^2_{atm}, Δm^2_{sol}, θ_{12}, θ_{13}, θ_{23} at M_Z (mass units eV^2).

Solution	$\Delta m^2_{\text{atm}} \cdot 10^3$	$\Delta m^2_{\text{sol}} \cdot 10^5$	θ_{12}	θ_{13}	θ_{23}
1	2.7149	7.9621	29.442	3.9859	44.114
2	2.3145	7.9514	34.289	12.507	51.047
3	1.8978	8.6141	30.560	0.8656	46.230
4	3.0062	8.3217	34.347	1.8512	44.333
5	3.3905	7.2468	30.245	2.9355	36.900
6	3.2459	7.5351	34.296	1.3701	46.947
7	2.0171	7.9464	34.432	1.0086	50.279
8	1.3321	7.9060	37.646	6.1490	43.067
9	2.4867	8.8561	29.592	5.6007	42.970
10	2.1652	7.8869	29.189	3.1512	37.220

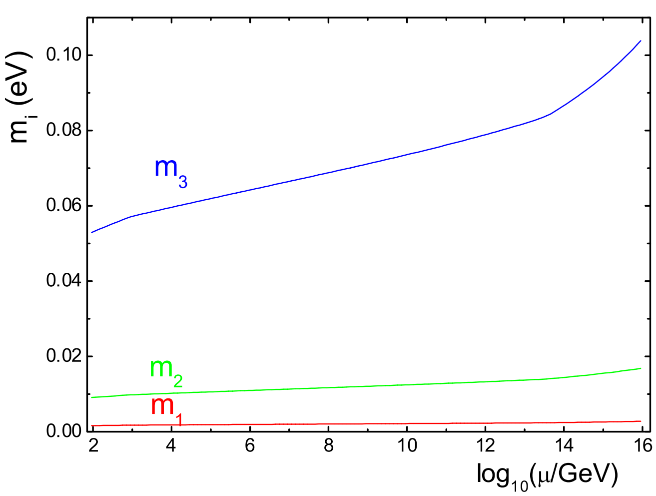

FIG. 4 (color online). The running of the light neutrino masses.

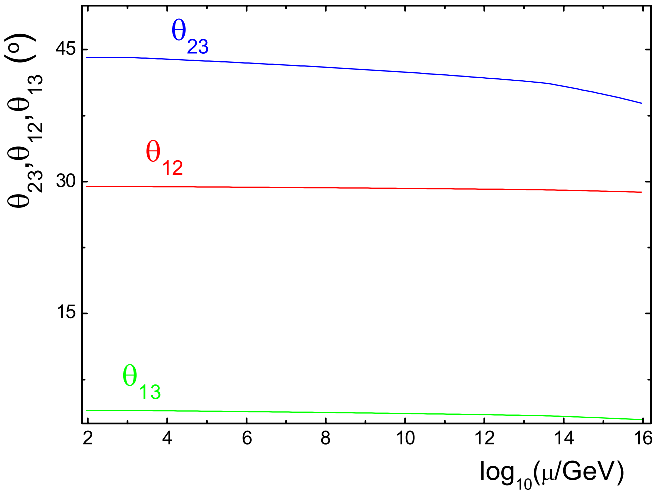

FIG. 5 (color online). The evolution of the mixing angles.

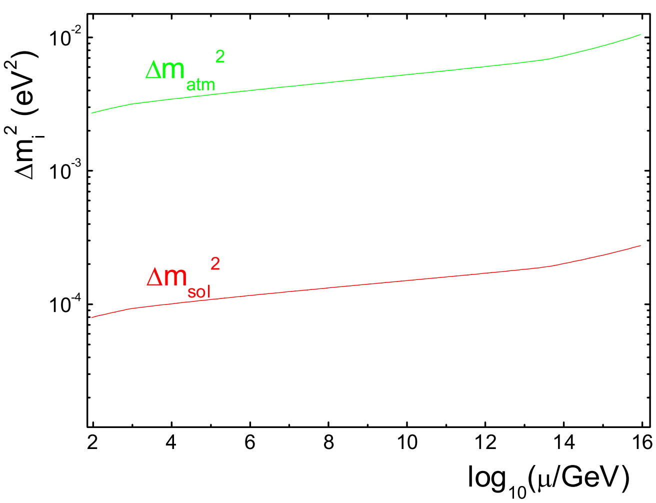

FIG. 6 (color online). Running of $\Delta m^2_{\mathrm{sol}}$ and $\Delta m^2_{\mathrm{atm}}$.

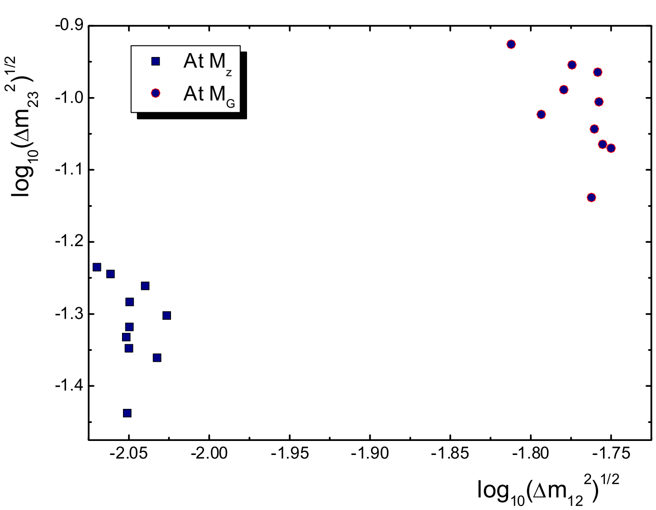

FIG. 7 (color online). Δm^2_{12} and Δm^2_{23} at M_Z and at M_G.

TABLE VII. Values of the physical parameters $\Delta m^2_{\mathrm{atm}}$ and $\Delta m^2_{\mathrm{sol}}$ at M_G; the effective neutrino mass $\langle m\rangle$ related to $\beta\beta_{0\nu}$ decay; and the parameter b_2 which determines the scale of the light matrix m_ν.

Solution	$\Delta m^2_{\mathrm{atm}}(M_G)\cdot 10^3$	$\Delta m^2_{\mathrm{sol}}(M_G)\cdot 10^5$	$\langle m\rangle$	b_2
1	10.5294	27.6096	0.003 61	3.41
2	8.1987	30.1333	0.006 33	2.88
3	7.2533	31.6108	0.006 07	1.50
4	11.7749	30.4352	0.007 53	1.28
5	14.093	23.7416	0.003 46	2.85
6	12.347	28.2642	0.008 22	1.26
7	7.4256	30.855	0.008 21	1.23
8	5.2910	29.8775	0.008 52	1.17
9	9.746 56	30.5418	0.003 79	3.64
10	8.996 95	25.8838	0.003 27	3.42

energies. Several models predict $\Delta m^2_{23}/\Delta m^2_{21}$ out of the experimental range at M_G, although after the running at M_Z they are consistent with the data.

(iv) Finally, we check the predictions of our model for the effective neutrino mass parameter relevant for $\beta\beta_{0\nu}$ decay. This parameter can be written in terms of the observable quantities as

$$|\langle m\rangle| = |(m_1\cos^2\theta_\odot + e^{i\alpha}\sqrt{\Delta m_{\mathrm{sol}}}\sin^2\theta_\odot)\cos^2\theta_{13} + \sqrt{\Delta m_{\mathrm{atm}}}\sin^2\theta_{13}e^{i\beta}|. \tag{17}$$

In the last column of Table VII the $\beta\beta_{0\nu}$-decay predictions are presented for solutions 1–10. Many current experiments attempt to measure this quantity [28]; the best current limit on the effective mass is given by the Heidelberg-Moscow Collaboration [29]

$$\langle m\rangle \leq 0.35z\ \mathrm{eV}, \tag{18}$$

where the parameter $z = \mathcal{O}(1)$ allows for uncertainty arising from nuclear matrix elements.

In a recent analysis of neutrinoless double beta decay [30] the allowable range of the effective mass parameter was given for specific scenarios. In the case of the normal hierarchy the bounds are

$$0 < \langle m\rangle < 0.007\ \mathrm{eV} \tag{19}$$

thus our results are in the experimentally acceptable region.

B. Number of free parameters and predictivity

We now discuss the number of continuously or discretely adjustable parameters involved in fitting the observed values of neutrino masses and mixings, taking into account also the quark and charged lepton masses and quark mixings and the preservation of gauge unification. The number of parameters in the theory is *a priori* large, consisting of the U(1) charges, the expansion parameters arising from the singlet v.e.v. and $SU(4) \times SU(2)_R$ breaking Higgses, the underlying y_{ij} and μ_{ij} parameters, and the Clebsch factors for (only) those operators containing one or more powers of $H\bar{H}$.

In order to restrict our search and provide some nontrivial relations between observables, we chose not to consider all of these as independently adjustable, and to restrict the values of others to respect some notion of "naturalness." For the theory to provide a meaningful explanation of the hierarchy and mixing of fermion masses, numbers much smaller or larger than unity should not be put in by hand, but arise from some power of a v.e.v. breaking an underlying symmetry of the model. Thus, all the coefficients of the operators producing fermion mass terms in Eqs. (2) and (3) are numbers of order 1. We do not rule out the possibility that some string model might give rise to underlying superpotential coefficients much larger or smaller than 1, but this would not lead to an explanation of the observed hierarchy of fermion masses within our framework.

To further remove free parameters, the magnitudes of the y_{ij} parameters can be set to unity while still fitting the observed fermion masses. (We did not include complex phases.) We also imposed that the Clebsch factors should be equal to unity except when this was incompatible with observations, in which case we restricted their values to zero or some simple rational number. Thus the 9 quark and charged lepton masses and 3 quark mixing angles are exchanged for the four discrete charges of $F_{1,2}$ and $\bar{F}_{1,2}$, the two expansion parameters η and b_1 and a small set of discretely chosen Clebsch factors not equal to 1.

Thus the basic hierarchy of (both heavy and light) neutrino masses is already fixed, up to the discrete choice of x and z' charges and the parameter b_2, which mainly influence the overall scale of neutrino masses without affecting the structure of the mass matrix. As shown in Table III and the accompanying discussion there is a degeneracy since different choices for the charges can produce the same effective mass matrices. Now we introduce the μ_{ij} parameters which multiply the entries of the RHN mass matrix: clearly if the choice of these were totally unrestricted, there would be no prediction for the light neutrino mass matrix. We apply the restriction that their sizes should not differ by more than an order of magnitude. In practice we may set $\mu_{33} = 1$ and impose that the magnitude of μ_{ij} must be inside the range $[0.1, 1]$.

With this restriction we find it is impossible to achieve a solution with inverted hierarchy or quasidegenerate light neutrino masses (after extensive searches, and using freedom to adjust Clebsch coefficients in the neutrino Dirac matrix). In order to explain maximal mixing, some entries in the light neutrino mass matrix must be very close in magnitude, which together with the hierarchy of powers of η in the Dirac and Majorana mass matrices, restricts the possibilities considerably. However, we cannot give an algebraic proof that other possibilities beyond normal hierarchy are impossible, because the RG equations are not analytically solved in the regime we are considering.

We considered the possibility of reducing the number of continuously adjustable parameters by setting $b_1 = 1$ exactly (thus $\epsilon = \sqrt{\eta}$) but this would require some μ_{ij} values to be outside the range $[0.1, 1]$ hence it should be rejected by our "naturalness" criterion.

IV. CONCLUSIONS

In this work, we studied the running of neutrino masses and mixing angles in a supersymmetric string-inspired $SU(4) \times SU(2)_L \times SU(2)_R \times U(1)_X$ model. An accurate description of the low-energy neutrino data forced us to introduce two singlets charged under the $U(1)_X$, leading to two expansion parameters. The mass matrices are then constructed in terms of three expansion parameters

$$\epsilon \equiv \frac{\langle \phi \rangle}{M_U}, \qquad \epsilon' \equiv \frac{\langle H\bar{H} \rangle}{M_U^2}, \qquad \epsilon'' \equiv \frac{\langle \chi \rangle}{M_U}, \tag{20}$$

where ϕ and χ are singlets and H, $\bar{H}$ the $SU(4) \times SU(2)_R$-breaking Higgses. The model is simplified by the fractional $U(1)_X$ charges of H and χ, which ensure that the parameter ϵ'' only appears as a prefactor to the heavy Majorana neutrino masses.

The expansion parameter arising from the Higgs v.e.v.'s defines the ratio of the SU(4) breaking scale M_G to the unification scale M_U: we performed a renormalization group analysis of gauge couplings under this constraint and found successful unification with the addition of extra states usually present in a string spectrum.

Assuming that only the third generation of quarks and charged leptons acquire masses at tree level and under a specific choice of $U(1)_X$ charges as well as Clebsch factors, we examined the implications for the light neutrino masses resulting from the seesaw formula. We found that the light neutrino mass spectrum must be hierarchical, given the requirement from naturalness that underlying superpotential couplings are of order 1. Also the mass hierarchy of light neutrinos tends to be larger at the GUT scale than at M_Z due the renormalization group running. The solar mixing angle θ_{12} is stable under RG evolution while larger renormalization effects are found for the atmospheric mixing angle θ_{23} and θ_{13}, always with their values at M_Z in agreement with experiment.

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