

Quantum-classical interactions through the path integral

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(Received 2 October 2006; published 22 March 2007)

I consider the case of two interacting scalar fields, ϕ and ψ , and use the path integral formalism in order to treat the first classically and the second quantum-mechanically. I derive the Feynman rules and the resulting equation of motion for the classical field which should be an improvement of the usual semiclassical procedure. As an application I use this method in order to enforce Gauss's law as a classical equation in a non-Abelian gauge theory. I argue that the theory is renormalizable and equivalent to the usual Yang-Mills theory as far as the gauge field terms are concerned. There are additional terms in the effective action that depend on the Lagrange multiplier field λ that is used to enforce the constraint. These terms and their relation to the confining properties of the theory are discussed.

DOI: [10.1103/PhysRevD.75.065023](https://doi.org/10.1103/PhysRevD.75.065023)

PACS numbers: 11.15.Kc, 11.15.Bt

I. INTRODUCTION

Consider the case of two scalar fields, ϕ and ψ , interacting with the action

$$S(\phi, \psi) = \int d^4x L(\phi, \psi) \quad (1)$$

where

$$L = \frac{1}{2}(\partial\phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{g}{4!}\phi^4 + \frac{1}{2}(\partial\psi)^2 - \frac{1}{2}M^2\psi^2 - \frac{g'}{2}\phi^2\psi^2 - \frac{g''}{4!}\psi^4. \quad (2)$$

The classical equation for ϕ is obtained from

$$\frac{\delta S}{\delta\phi} = -(\square + m^2)\phi - \frac{g}{3!}\phi^3 - g'\phi\psi^2 = 0. \quad (3)$$

If one wishes to treat ψ quantum-mechanically the usual, semiclassical, procedure leads one to replace all terms containing ψ in (3) by their quantum expectation values. There are, however, several problems associated with this procedure, conceptual as well as technical: One may use a background field method in order to derive the first (one-loop) quantum corrections to the effective potential and associated mass terms. If, however, a higher order calculation is needed, one would have to modify the propagator using the one-loop result, and be careful in order to avoid double-counting for any relevant diagrams. The procedure depends on the model examined and on relative order-of-magnitude estimates. One would like to have a more self-consistent approach in order to incorporate quantum effects on classical fields and vice-versa.

Here I use the path integral formalism in order to describe this problem and I get the resulting Feynman rules that enable one to study the interactions between the classical and quantum fields. The result is what one would

expect: the classical field propagates only in tree diagrams, not in loops, and the quantum field propagates as usual, providing the quantum corrections. This leads to an effective equation of motion for the classical field that should be an improvement upon the semiclassical procedure. The Feynman rules presented here reorganize the entire perturbation series and enable one to treat these problems in a self-consistent manner that can easily be extended in other models.

The path integral approach has been used before in order to treat pure classical mechanics [1,2] and investigate problems of classical behavior in quantum field theory [3,4]. The formalism developed here has many similarities to these previous works. The main method is an extension of [1,2] to the case of interactions between classical and quantum fields (there are, however, some important differences even in the case of purely classical fields).

In Sec. II I develop the main formalism, derive the Feynman rules and get an effective action from which the equation of motion for the classical field can be obtained. An important advantage of this method is that it is a self-consistent procedure that can be used in order to calculate higher order effects.

In Sec. III I use this method in order to treat Gauss's law as a classical equation in a non-Abelian gauge theory. Because of the asymmetry of the Feynman rules described here, the effective action contains, besides the usual Yang-Mills terms, additional terms that depend on the Lagrange multiplier field that is used to enforce the classical constraint. These terms are of the Coleman-Weinberg type [5], and are reminiscent of phenomenological effective actions that have been used in order to describe confinement [6–8]. They have similar interpretation here, where one can also see the region of validity of perturbation theory.

Since the method presented here is new I include two sections of comments. In Sec. IV I discuss the applications to non-Abelian gauge theory and in Sec. V I make some general comments regarding this method.

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II. PATH INTEGRAL AND EFFECTIVE ACTION

In order to calculate

$$Z(J, J') = \int [d\phi][d\psi] \delta(\phi - \phi_{cl}) \times \exp\left(i \int L(\phi, \psi) + J\phi + J'\psi\right), \quad (4)$$

where ϕ_{cl} is the solution of (3), I use the Lagrange multiplier λ and ghost fields c, \bar{c} , similarly to the work in [1,2], and adding another source Λ , the path integral to be evaluated becomes

$$Z(J, J', \Lambda) = \int [d\phi][d\psi][d\lambda][dc][d\bar{c}] \times \exp\left(i \int \tilde{L} + J\phi + J'\psi + \Lambda\lambda\right) \quad (5)$$

where

$$\tilde{L} = L + \lambda \frac{\delta S}{\delta \phi} + \bar{c} \frac{\delta^2 S}{\delta \phi^2} c \quad (6)$$

is the modified Lagrangian with the corresponding modified action \tilde{S} . For the simplest case of two interacting scalar fields with (2) we get

$$\tilde{S} = S + \int_x \left(\lambda K \phi + \bar{c} K c - \frac{g}{3!} \lambda \phi^3 - g' \lambda \phi \psi^2 + \frac{g}{2} \bar{c} \phi^2 c + g' \bar{c} \psi^2 c \right), \quad (7)$$

where $K = -(\square + m^2)$. The propagators and the vertices can be deduced from here. For the λ and ϕ fields we get

$$\int [d\phi][d\lambda] e^{i \int (1/2) \phi K \phi + \lambda K \phi + J \phi + \Lambda \lambda} = N e^{-(i/2) \int (2JG\Lambda - \Lambda G \Lambda)}, \quad (8)$$

where N is a normalization factor, independent of the sources, and $G = 1/(k^2 - m^2 + i\epsilon)$, in momentum space, is the usual Feynman propagator. Accordingly there is no $\phi - \phi$ propagator, there are, however, a mixed $\lambda - \phi$ propagator equal to G , and a $\lambda - \lambda$ propagator equal to $-G$. The remaining $\psi - \psi$ and ghost propagators as well as the various vertices are as usual from (7).

One can now check: at one-loop order the loops with the $\lambda - \phi$ propagator cancel with the ghost loops and similar cancellations exist in higher loops, loops with the $\lambda - \lambda$ propagator do not appear because of the Feynman rules of the modified action (the vertices are at most linear in λ) with the final result that the ϕ field does not have quantum corrections but only propagates classically through tree diagrams. A typical line with the λ and ϕ fields is either the sum of two $\lambda - \phi$ and one $\lambda - \lambda$ propagator, or a single $\lambda - \phi$ propagator, in both cases equal to G . The ψ field, of course, propagates also in loops as a genuine

quantum field and gives the quantum corrections to the classical field ϕ .

I should note here that the propagator G that we get for the classical field is the Feynman propagator, and not the retarded one that is usually employed in classical mechanics. There are two reasons for that: first, the boundary conditions used in the path integral (the field goes to zero at infinity) are different than the ones usually employed in classical mechanics (the field configuration is given at an initial time). One can check with a more careful evaluation of (8) that we get, indeed, the Feynman prescription. This is true even if we have only the classical field in our theory. One can presumably use the path integral formalism with different boundary conditions in order to attack purely classical problems. Then the retarded propagator would probably emerge, as in [2]. A second, physical reason that is relevant here, is that we want to study interactions between the classical and quantum fields. The possibility of particle creation and annihilation is essential for both the classical and quantum fields. Our classical field, therefore, admits both particles and antiparticles propagating classically.

Renormalization of the theory proceeds as usual. All divergent terms come from loops of the quantum field ψ and depend on the quantum-classical coupling g' or the quantum coupling g'' , not on g . The classical parameters g and m get also renormalized. However, pairs of terms like $g\lambda\phi^3$ and $g\phi^4$ in the modified action \tilde{S} have the same divergencies associated with, accordingly the renormalization procedure does not affect the classical nature of the field ϕ .

I will now proceed to show how the classical equation for ϕ gets modified in the presence of quantum interactions. If we use the generating functional $Z(J, J', \Lambda)$ to construct $W(J, J', \Lambda)$, the generating functional for connected diagrams, and from that $\Gamma(\phi, \psi, \lambda)$, the effective action, we see that the appropriate equation is

$$\left(\frac{\delta \Gamma}{\delta \lambda}\right)_{\lambda=0} = 0. \quad (9)$$

This can be verified if we consider the theory with only the classical field and use the properties of the Legendre transformation in order to express the classical equation $\delta S / \delta \phi = -\Lambda = 0$ in terms of the effective action. So the equation of motion for the classical field ϕ can be taken from the tadpole one-particle irreducible graphs with one external field λ that contain ψ loops and lines of the λ and ϕ fields, but not loops of the classical field. For the case of the two interacting scalar fields of (2) we get at one-loop order

$$-(\square + m^2)\phi = \frac{\partial V_{\text{eff}}(\phi, \psi)}{\partial \phi} \quad (10)$$

where

$$V_{\text{eff}} = \frac{g}{4!} \phi^4 + \frac{g'}{2} \phi^2 \psi^2 + \frac{i}{2} \int \frac{d^4 k}{(2\pi)^4} \times \ln \left(\frac{(k^2 - m_\phi^2)(k^2 - M_\psi^2) + (g' \phi \psi)^2}{(k^2 - m_\phi^2)} \right) \quad (11)$$

is the usual one-loop effective potential of the original theory without the ϕ loop ($m_\phi^2 = m^2 + \frac{1}{2} g \phi^2 + g' \psi^2$, $M_\psi^2 = M^2 + g' \phi^2 + \frac{1}{2} g'' \psi^2$). In fact (11) also contains the loop with only the ψ field. Since this is, however, ϕ -independent it does not contribute to (10). One can check that when the quantum-classical coupling g' is zero this reduces to the ordinary equation for a classical Klein-Gordon field. Regularization and renormalization are performed as usual, as was discussed above.

This result can, of course, be also obtained using a background field perturbation theory. However, if one wishes to study higher order corrections, perturbation theory has to be reorganized at any order so as to incorporate the previous order result and avoid any multiple-counting. This problem does not appear here since we have a well-defined effective action, that depends on the auxiliary field λ , with fixed Feynman rules from the beginning.

The resulting equation for the classical field is written as an equation of motion from an effective action that contains what one would expect—arbitrary loops of the quantum field but not loops of the classical field. It depends on the vacuum expectation value of the quantum field ψ , enabling one to also study problems of symmetry breaking. Even for $\psi = 0$, of course, it provides the quantum corrections to the classical equations of motion. What is more important, this method allows one to include higher order corrections self-consistently through the effective action formalism.

The complete effective action also describes the quantum properties of the field ψ . One can then determine the effects of the classical field on the quantum field via its effective potential or other terms. There are also higher order terms that involve powers or derivatives of the auxiliary field λ . Their relevance, if any, to the combined dynamics of the system is not clear from this work. In the simple example described here, since there is no symmetry breaking, we have to set $\lambda = 0$ anyway in order to derive the effective equation of motion. In cases with symmetry breaking, however, these terms may turn out to be important. One interesting case will be described in the next section in the context of the non-Abelian gauge theory, where they may be relevant to the confining properties of the theory.

III. AN APPLICATION IN NON-ABELIAN GAUGE THEORY

The formalism of the preceding section applies strictly in the case of two interacting fields. Here, however, I will show how it can be used in the case of the non-Abelian

gauge theory, in order to treat Gauss's law as a classical equation. The strategy will be the following: I will choose first a noncovariant gauge in which pure Yang-Mills theory is well-defined and renormalizable, and use the Feynman rules derived here in order to map a sector of the theory onto the usual Yang-Mills theory, and identify the remaining (or missing) terms as additional contributions to the effective action, that depend on λ .

For the non-Abelian gauge theory with action S and Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} \quad (12)$$

where $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$, I will use Lagrange multipliers λ^a and ghost fields \bar{c}^a , c^a , and get the modified action

$$\tilde{S} = S + \int \lambda^a \frac{\delta S}{\delta A_0^a} + \bar{c}^a \frac{\delta^2 S}{\delta A_0^a \delta A_0^b} c^b \quad (13)$$

in order to treat Gauss's law

$$\frac{\delta S}{\delta A_0} = -D \cdot \vec{E} = 0 \quad (14)$$

classically (S and L in this section will denote the Yang-Mills values). In the modified action we add a source term $\Lambda^a \lambda^a$ together with the source terms $J_\mu^a A^{a\mu}$, as before, in order to derive the Feynman rules. We also have to add a gauge-fixing term, so I will choose a noncovariant gauge that is well-defined and does not depend on A_0 , namely, the axial gauge-fixing term

$$L_{\text{ax}} = -\frac{(n \cdot A)^2}{2\xi} \quad (15)$$

with a purely spatial four-vector $n^\mu = (0, \vec{n})$.

The usual Feynman rules for Yang-Mills theory with action S in the axial gauge involve the gauge field propagator

$$G_{\mu\nu}^{ab} = \frac{-\delta^{ab}}{k^2} \left(g_{\mu\nu} - \frac{k_\mu n_\nu + k_\nu n_\mu}{k \cdot n} + k_\mu k_\nu \frac{n^2 + \xi k^2}{(k \cdot n)^2} \right) \quad (16)$$

and the usual QCD vertices [9]. The new Feynman rules that we get for the modified action, using (8), involve an $A_0 - A_0$ propagator

$$\tilde{G}_{00} = G_{00} - G_c \quad (17)$$

(I will not denote the color indices where obvious) and $A_0 - \lambda$ as well as $\lambda - \lambda$ propagators

$$G_{0\lambda} = -G_{\lambda\lambda} = G_c \quad (18)$$

where $G_c = 1/\vec{k}^2$ is the Coulomb propagator. The remaining propagators G_{0i} and G_{ij} are the same as the usual Yang-Mills theory. That is, the effect of the classical constraint of Gauss's law has been to split the Coulomb interaction from

the propagator and treat it classically. We also have the usual vertices of QCD, and an additional set of vertices: for every QCD vertex that contains A_0 we have a vertex where an A_0 leg is replaced by λ . There is also the ghost sector which is similar to the previous section, and will be described shortly. This ghost sector is not related to the usual Fadeev-Popov ghosts, which I will assume that decouple [9]. We can now discuss renormalizability and relation to the usual Yang-Mills theory.

As far as the gauge field terms A^2 , A^3 and A^4 terms are concerned we get the same terms as usual Yang-Mills theory: what used to be a G_{00} propagator can now be formed as either the sum of \tilde{G}_{00} , two $G_{0\lambda}$ and one $G_{\lambda\lambda}$, or \tilde{G}_{00} and one $G_{0\lambda}$, in both cases equal to the usual G_{00} . This can be verified for every diagram. There is one difference: The ghost loop cancels an instantaneous Coulomb loop from every diagram in the usual Yang-Mills theory that contains a closed A_0 loop. I will assume that regularization can be performed so that these loops vanish, that is, relations like

$$\int_k \frac{1}{k^2(\vec{k} + \vec{p})^2} = 0 \quad (19)$$

hold, together with usual axial gauge integrals like

$$\int_k \frac{1}{(n \cdot k)^2} = 0. \quad (20)$$

This assumption is supported by results of split dimensional regularization [10]. In fact, even without this assumption, it is possible that the theory will be renormalizable, since it is the same closed loop that is missing from every would-be Yang-Mills diagram, the verification of this, however, would be highly noncovariant. In any case, with the previous assumption, the theory is renormalizable and equivalent to the usual Yang-Mills theory as far as the gauge fields are concerned. The divergencies of the remaining terms $\lambda \delta S / \delta A_0$ in the modified action are the same as their counterparts in S , as in the previous section, so these are renormalizable too. We now turn to the discussion of the λ terms.

Consider first the one-loop terms with external legs λ with zero momentum, for what would be an effective potential term $U(\lambda)$. All the propagators can run inside the loop with one exception: The $A_0 - \lambda$ and $\lambda - \lambda$ propagators cannot appear because the vertices are linear in λ . An example of such a diagram is shown in Fig. 1. So the Coulomb interaction is missing. Had these terms been there we would have the full covariant propagator $G_{\mu\nu}$ in the loop. All these diagrams, therefore, would add up to zero (their value would be the same as an effective potential term for A_0 in the usual Yang-Mills theory which does not exist because of gauge invariance). Accordingly the sum of the diagrams that contain a Coulomb interaction between two external legs λ has to be subtracted. The relevant vertices ($\lambda - A_0 - A_i$) are the same as the QCD

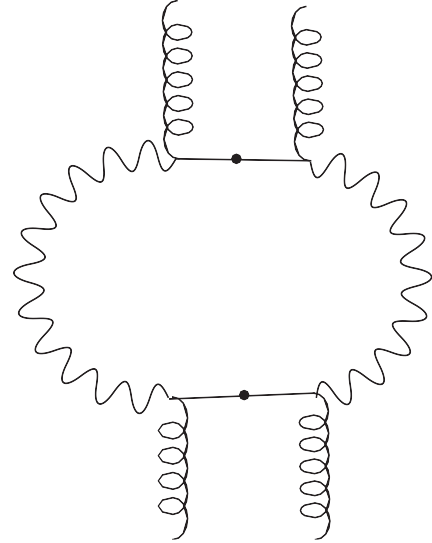


FIG. 1. An example of a diagram used for the calculation of $U(\lambda)$ and the derivation of Eqs. (21) and (22). The wiggly lines denote A_i, A_j fields. The curly lines denote the λ field and the solid lines denote A_0 . The solid lines with a dot denote the modified $A_0 - A_0$ propagator, \tilde{G}_{00} , derived in the text, that does not contain the Coulomb interaction; the $\lambda - A_0$ and $\lambda - \lambda$ lines that carry the Coulomb propagator cannot be also added in its place, because the vertices are only linear in λ .

vertex ($A_0 - A_0 - A_i$) and the missing terms with a Coulomb interaction running between two external legs λ correspond to an effective interaction term

$$\delta L = m^2 A_i \frac{k_i k_j}{k^2} A_j \quad (21)$$

(in momentum space, with k the momentum running in the loop) with

$$m^2 = g^2 C \lambda^2 \quad (22)$$

where $f^{acd} f^{bcd} = C \delta^{ab}$, $\lambda^2 = \lambda^a \lambda^a$. One can add this effective interaction term in the usual Yang-Mills action in order to derive the terms in the effective action that depend on λ . It corresponds to missing terms so the pieces calculated have to be subtracted. The calculation is highly noncovariant, however, once we have identified the effect of the missing diagrams as an effective interaction term to be added to the usual Yang-Mills action, there is no reason to continue in the axial gauge in order to complete the calculation [11]. It is more convenient to choose a Feynman gauge-fixing term, $(\partial_\mu A_\mu)^2/2$, in which case the effective propagators become

$$D_{00} = \frac{-1}{k^2} \quad (23)$$

$$D_{ij} = \frac{1}{k^2} \left(\delta_{ij} + \frac{m^2 k_i k_j}{(k^2 - m^2) k^2} \right). \quad (24)$$

The usual Fadeev-Popov ghosts do not contribute and the calculation of the relevant terms gives the final form of the effective action:

$$\Gamma = \int_x -Z(\lambda)\lambda D \cdot \vec{E} - \frac{1}{4}Z(\lambda)F^2 + U(\lambda) \quad (25)$$

with

$$U(\lambda) = c_1 g^4 \lambda^4 \left(\ln \frac{\lambda^2}{\mu^2} - \frac{1}{2} \right) \quad (26)$$

$$Z(\lambda) = 1 + c_2 g^2 \ln \frac{\lambda^2}{\mu^2} \quad (27)$$

where $c_1 = C^2/64\pi^2$, $c_2 = C/6\pi^2$. We have an effective action of the Coleman-Weinberg form [5] with a few differences:

First, the potential appears with the opposite sign, since it corresponds to missing diagrams. The counterterms that were needed for its renormalization were chosen so that $U''(0) = 0$ and $U'(\mu) = 0$. The condition $U'(0) = 0$ does not fix the counterterms, so we have to pick the scale μ (primes denote derivatives with respect to λ).

The factors Z in the first two terms of the effective action (25) can be calculated in the same manner, in terms of missing diagrams. They are expected to be the same by individual diagram inspection. A first calculation gives the result presented above. $Z(\lambda)$ was calculated from the λ -dependent, \vec{p}^2 coefficient, of the $A_0 - A_0$ wavefunction renormalization diagrams with external momentum p (subtracted from the tree level term). Other terms ($A_i - A_j$ for example) should give the same value for Z . This, however, will have to be verified, because of the asymmetry of the Feynman rules described here. In any case, for the preliminary analysis presented below, the main fact that we need is that renormalization conditions can be chosen so that $Z(\mu) = 1$.

Although there is only one coupling constant in the theory the one-loop terms are reliable when $g^2 \ln(\lambda/\mu)$ is small, since the tree value for U is 0 and for Z is 1.

Another possibility that was not encountered in the simple model of the previous section, is the generation of $(\nabla\lambda)^2$ terms, because of the asymmetry of the Feynman rules described here. The tree level value of this term should be set to zero by counterterms, it is a possible fine-tuning problem of the method. Generation of these terms at higher order via the Coleman-Weinberg mechanism does not affect the analysis presented below.

At $\lambda \approx \mu$ we have $Z \approx 1$ and ordinary perturbative Yang-Mills theory. As a first approximation to the effective action we take the Abelian, electric parts, for $Z \approx 1$

$$\Gamma_0 = \int_x \lambda \nabla^2 A_0 + \frac{1}{2} A_0 \nabla^2 A_0 + U(\lambda). \quad (28)$$

The equations

$$\frac{\delta \Gamma_0}{\delta \lambda} = 0 \quad (29)$$

$$\frac{\delta \Gamma_0}{\delta A_0} = 0 \quad (30)$$

have the simultaneous solution

$$A_0 = -\lambda_B \quad (31)$$

with λ_B the solution of

$$\nabla^2 \lambda = \frac{\partial U}{\partial \lambda}. \quad (32)$$

Note, first, that, although similar in form, the two Eqs. (29) and (30) are quite different conceptually: The first equation is an expression of the classical nature of Gauss's law and should be satisfied identically to all orders. The second equation is a usual semiclassical equation, only to be satisfied approximately.

Equation (32) has a soliton (bounce, bubble) solution $\lambda_B(r)$, spherically symmetric in the three-dimensional radius r , similar to the bounce solutions that are associated with tunneling at zero and finite temperature [12,13], although here it is not related to either tunneling or finite temperature. $\lambda_B(r)$ is of order μ for sufficiently small r , and goes rapidly to zero for r larger than the radius R_B of the bounce (which is of order $\frac{1}{g^2\mu}$). It corresponds to a confining potential for the electric field (31), reminiscent of a bag model, such that deep inside the bounce we have ordinary perturbative Yang-Mills theory with a zero electric field, and a strong electric field appearing as we get closer to the bounce radius (in fact, the bounce solution is not of the thin wall type, it resembles more the three-dimensional thick wall bubbles of [13], and has to be determined numerically).

As we approach the radius of the bubble, however, when λ goes to zero, the approximation $Z \approx 1$ does not hold; in fact, when λ is nonperturbatively small Z goes to zero and the full nonperturbative features of QCD become important [6,7]. Our perturbative parameter is the same as that for Coleman-Weinberg models, namely $g^2 \ln(\lambda/\mu)$, and only when this is small can the higher loop effects be neglected. It is, of course, possible to improve on these results with the use of the renormalization group, and even when this is not the case, the effective action presented here may be quite useful phenomenologically.

Another important fact that should be mentioned for the effective action (25) proposed here, as well as for the "dynamically induced" mass term (22), is that they are gauge invariant, provided the auxiliary field λ is gauge covariant (under a gauge transformation V we have $\lambda \rightarrow V\lambda V^{-1}$). The consequences of this gauge invariance, and its possible associated BRS symmetries, are not obvious because of the peculiarities of the Feynman rules described here; this is, however, another indication in support of the arguments presented above, namely, that the theory is renormalizable and compatible with ordinary, perturbative Yang-Mills theory.

IV. COMMENTS

The upshot of this work, as far as the non-Abelian gauge theory is concerned is the following: there is a sector of the theory, namely, the Coulomb interaction, that is purely classical in nature, by virtue of Gauss's law; this cannot be expressed self-consistently in perturbation theory unless one employs a skewed set of Feynman rules of the type described here. This has the effect of generating additional terms in the effective action that reveal both the appearance of the confining properties of the theory and the limits of validity of perturbation theory.

The appearance of the inverted effective potential term $U(\lambda)$ that was described in the previous section does not indicate an energetic instability of the theory, it is, however, related to the instability of the perturbative vacuum. In fact, there is no direct Hamiltonian interpretation for the effective action presented here, λ remains an auxiliary Lagrange multiplier that has to be eliminated via $\delta\Gamma/\delta\lambda = 0$. The perturbative vacuum $\lambda = \mu$, however, cannot exist for all space, since it has infinite action. A finite action soliton solution that was presented in the previous section shows signs of confining behavior and, at the same time, gives clues about the region of validity of perturbation theory.

It is not clear whether this method hints on fundamental results (problems rather) of the usual quantization procedure of non-Abelian gauge theories, or is purely of a heuristic value, I would like, however, to make some comments in support of the former: First of all, something that is obvious, but should be, nevertheless, mentioned, is that this method does not change at all the rules for the Abelian gauge theory. The Coulomb interaction splits again as presented here. Since, however, photons do not have any self-interactions, the result is equivalent to the usual Feynman rules for Abelian gauge theory (the inclusion of fermions is also straightforward). The usual Fadeev-Popov procedure (Abelian or non-Abelian) also is not related and does not change by the method presented here. However, in the process of quantization of the non-Abelian gauge theory, while changing from the Hamiltonian formalism with noncovariant gauge fixing to the Lagrangian formalism with covariant gauge fixing there are many manipulations of constraints such as Gauss's law. The treatment of these constraints is not completely justified from the present point of view. In many cases the actual, physical, fields are used as Lagrange multipliers, and a relation such as (8) does not appear. It is possible that some important piece of information of the theory is lost in the process.

V. DISCUSSION

In this work I developed a formalism in order to treat interactions between quantum and classical fields through the path integral. It seems that one is able to describe quantum-classical interactions in field theory self-

consistently with this method. The path integral formalism is particularly simple and can hopefully be generalized in other cases involving such interactions.

Some further applications of this work, besides the conceptual problem of quantum-classical interactions, would be in cases of a nonrenormalizable classical interaction, such as gravity. Note that, in the example described in Sec. II, there is nothing that demands the classical field, ϕ , to have a renormalizable quantum equivalent. Even a nonrenormalizable self-interaction of the classical field does not generate higher order terms since there are no classical loops. There may be of course quantum-classical interaction terms that generate higher order terms, so the problem of renormalizability then depends on the specifics of the model. Even so, this method may be useful in treating the stress-tensor renormalization equations

$$G_{\mu\nu} = -8\pi G\langle T_{\mu\nu} \rangle \quad (33)$$

in a self-consistent way.

I have also given an example of an application of this work in the case of the non-Abelian gauge field theory by using this method in order to treat Gauss's law as a classical equation. This is different in spirit than the previous discussion, it shows, however, another application of this method in this case, where it can be useful for studying the infrared properties of the theory. The treatment of the Coulomb interaction as purely classical in nature has resulted in the generation of purely quantum terms of the Coleman-Weinberg type that are related to the confinement mechanism.

Even though the treatment of the non-Abelian gauge theory presented here has a different motivation than previous works, there are several common features with various other approaches to confinement [6–8,14–18]. Namely we find: the appearance of a negative metric propagator (the $\lambda - \lambda$ line which is crucial in describing the classical nature); the emergence of a “dynamical” mass term (22) and an effective potential generated through radiative corrections; an effective action reminiscent of older phenomenological actions that were used to describe confinement (although with important differences described before).

This method can probably be useful in various other problems in theories that have infrared singularities: one can, presumably, use this method in order to study the infrared modes of these theories classically, and include the quantum corrections from higher momentum modes self-consistently. The problem here, of course, is the use of an arbitrary cut-off scale, and care should be taken in order to derive cut-off independent results.

ACKNOWLEDGMENTS

This work was done in the National Technical University of Athens. I am grateful to the people of the Physics Department for their hospitality.

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