Bulk viscosity of spin-one color superconductors with two quark flavors

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We consider the contribution of the Urca-type processes to the bulk viscosity of several spin-one color-superconducting phases of dense two-flavor quark matter. In the so-called transverse phases which are suggested to be energetically favorable at asymptotic densities, the presence of ungapped quasiparticle modes prevents that spin-one color superconductivity has a large effect on the bulk viscosity. When all modes are gapped, as for one particular color-spin-locked phase, the effect on the viscosity can be quite large, which may have important phenomenological implications.

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I. INTRODUCTION

Dissipative processes play an important role in the evolution of neutron stars. These processes are governed by transport coefficients, such as the heat and electrical conductivities, the neutrino diffusion coefficient, as well as the shear and bulk viscosities. The conductivities are important for stellar cooling as well as for the magnetic field decay. Shear viscosity dampens differential rotation in a star and, thus, leads to a uniform rigid-body rotation. Bulk viscosity, on the other hand, dampens density oscillations inside the star. Both differential rotation and oscillations could be excited in newly formed (hot) neutron stars, or could develop in old (cold) stars due to external perturbations, e.g., such as matter accreted from a companion star.

It is interesting to note that, in the absence of viscosity, all rotating stars would be unstable. The reason is that such stars spontaneously develop instabilities as a result of the emission of gravitational waves [1–5] (for reviews on this topic see, e.g., Refs. [6,7]). The so-called r-mode (or rotation-dominated) instabilities might be the most important ones. They can develop at a relatively low angular velocity [8], and therefore may be relevant for a large number of compact stars.

The main theoretical uncertainty in predicting whether the r-mode instabilities develop in a star lies in the poor understanding of the viscosity of dense baryon matter, as well as in the limited knowledge of the stellar composition. There is hope, however, that a systematic approach, based on a broad understanding of various properties of dense baryonic matter, can eventually result in a clear picture regarding the neutron star composition.

The viscosity of nuclear and mixed phases of dense baryonic matter has been calculated under various conditions and assumptions over the last three decades [9–18]. The bulk viscosity of normal conducting strange quark matter was also calculated [19,20]. The latter might be relevant if the baryon density in the central regions of neutron stars is so high that matter becomes deconfined.

The physical conditions in the interior of such stars are quite unique: this is the only place in the Universe where a deconfined state of cold and dense baryonic matter can naturally exist. This possibility has attracted a lot of attention since the notion of quarks was introduced [21–25].

If deconfined quark matter does exist inside stars, it is most likely color-superconducting. (For reviews on color superconductivity, see Refs. [26–32].) It is therefore of great interest to study various transport properties of color-superconducting phases of quark matter. First attempts have already been made to estimate the heat and electrical conductivity [33,34], as well as the bulk and shear viscosities [35,36] in the color-flavor-locked (CFL) phase of quark matter. Also, in the case of the two-flavor color-superconducting (2SC) phase, one can argue that most transport coefficients are dominated by the two ungapped (blue) quasiparticles [32]. (For a recent detailed study of the bulk viscosity in the 2SC phase, see Ref. [37].)

In this paper, we calculate the bulk viscosity of the four most popular spin-one color-superconducting phases of two-flavor (nonstrange) quark matter: the color-spin-locked (CSL), planar, polar, and the *A* phase [38–44]. One of these is likely to be the ground state of dense baryon matter if the spin-zero Cooper pairing of quarks is prevented by the constraints of charge neutrality and β-equilibrium. Moreover, cooling calculations for neutron stars favor small gaps of the order of 1 MeV [45] which is the typical size of the gap in spin-one color superconductors [38–44]. The absence of strange quarks in the system may be natural if the medium-modified constituent value of the strange quark mass is larger than the corresponding value of the chemical potential. The generalization of this study to the case of spin-one color-superconducting strange quark matter will be reported elsewhere [46].

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As is well known, in fully gapped spin-zero color-superconducting phases, the thermal densities of the quark- and hole-type quasiparticles are suppressed exponentially by the energy gap, $n_{qp} \propto \exp(-\phi_0/T)$ where ϕ_0 and T are the values of the spin-zero gap and the temperature, respectively. This then translates into an exponential suppression of the quasiparticle contributions to the transport coefficients. In Ref. [35] such an argument was used in order to get a simple estimate of the viscosity in the CFL phase. It should be noted, however, that many transport properties are not dominated by the quasiparticles when there exist Nambu-Goldstone excitations in the low-energy spectrum, as is the case in the CFL phase [33,34,36]. The situation is expected to be different also in the case of spin-one phases which, in general, are not isotropic, and whose gap functions may have nodes for some directions of momenta.

The effect of nonisotropic gaps and various topologies of nodes on the neutrino emission and the cooling rate of spin-one color-superconducting phases were recently discussed in detail [47,48]. Following a similar approach, in this work we study the bulk viscosity.

Since the order parameter in spin-one color superconductors breaks rotational invariance, dissipative hydrodynamics is more complicated than for isotropic media. For instance, we expect that the viscosity becomes a tensor [49]. We shall avoid these complications by making the implicit assumption that the hydrodynamic equations are averaged over solid angle. In this way, only one (angular-averaged) bulk viscosity coefficient, ζ, will appear in the hydrodynamic equations and will have to be extracted from the (angular-averaged) neutrino-emission rate.

In addition, one of the spin-one superconducting phases studied here, namely, the CSL phase, breaks baryon number, i.e., it is also a superfluid. Fortunately, it is not an anisotropic superfluid because the order parameter does not break rotational invariance. (Here we assume that the magnetic field of the star is not strong enough to align the spins of the Cooper pairs.) Nevertheless, isotropic superfluids still have a rather complicated hydrodynamic behavior, involving three (instead of only one) bulk viscosity coefficients [50,51]. However, it is not completely unrealistic to assume that the *relative* velocity between normal and superfluid components is negligible compared to the *absolute* velocity of the normal component. In this case, only one coefficient contributes to energy dissipation. In this sense, our treatment is completely analogous to that of Ref. [16], the difference being that here we consider quark matter instead of nucleonic matter. Let us finally note that the dissipative hydrodynamics of *anisotropic* superfluids is even more complicated, see for instance the case of superfluid He-3 [49,52].

In the CSL phase the breaking of baryon number gives rise to a phonon as the corresponding Goldstone excitation. We neglect the contribution of the Goldstone mode to the bulk viscosity coefficient for the following reason. First, the effective theory for phonons is approximately scale-invariant at very low energies. For scale-invariant superfluids, however, two of the three bulk viscosity coefficients have been shown to vanish [51]. The remaining coefficient may be nonzero, but corresponds to dissipation due to relative motion of superfluid and normal component, which we have already assumed to vanish.

The remainder of this paper is organized as follows. In the next section, we introduce the formalism for calculating the bulk viscosity in nonstrange quark matter. In Sec. III we calculate the bulk viscosity in the normal phase of two-flavor quark matter. Then, in Sec. IV we present our results for the bulk viscosity in spin-one color-superconducting phases. The discussion of the results is given in Sec. V.

II. BULK VISCOSITY

As mentioned in the introduction, the bulk viscosity is responsible for the damping of density oscillations in compact stars. The characteristic frequencies of interest (e.g., set by the r-mode instabilities) are comparable to the rotational frequencies of stars, i.e., $\omega \lesssim 10^4 \text{s}^{-1}$ [6,7]. These frequencies are many orders of magnitude smaller than the typical rates of strong interactions, and therefore quark matter cannot be driven substantially out of equilibrium with respect to strong processes. This is the reason why the bulk viscosity is dominated by the much slower, flavor-changing weak processes [19,20]. In the case of nonstrange quark matter studied here, the relevant processes are electron capture by u quarks and β decay of d quarks, see Fig. 1.

Let us assume that small oscillations of the quark matter density are described by $\delta n = \delta n_0 \text{Re}(e^{i\omega t})$ where δn_0 is the magnitude of the density variations. For such a periodic process, the bulk viscosity ζ is defined as the coefficient in the expression for the energy-density dissipation averaged over one period, $\tau = 2\pi/\omega$,

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = -\frac{\zeta}{\tau} \int_0^\tau dt (\nabla \cdot \vec{v})^2, \qquad (1)$$

where $\vec{v}$ is the hydrodynamic velocity associated with the
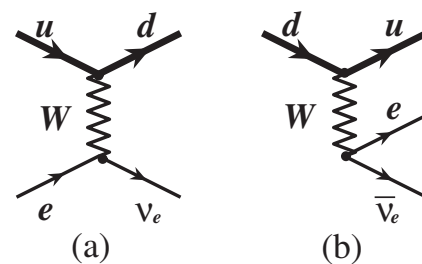

FIG. 1. Diagrammatic representation of the weak processes that contribute to the bulk viscosity of nonstrange quark matter in stellar cores.

density oscillations. By making use of the continuity equation, $\dot{n} + n\nabla \cdot \vec{v} = 0$, we derive

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = -\frac{\zeta \omega^2}{2}\left(\frac{\delta n_0}{n}\right)^2. \qquad (2)$$

In order to solve for ζ, the dissipated energy on the left-hand side has to be calculated explicitly. This can be done as follows.

The density oscillations drive quark matter slightly out of β equilibrium, but not out of thermal equilibrium which is restored almost without delay by strong processes. The corresponding instantaneous quasiequilibrium state can be unambiguously characterized by the total baryon number density n and the lepton fraction X_e,

$$n = \tfrac{1}{3}(n_u + n_d), \qquad (3a)$$

$$X_e = \frac{n_e}{n}, \qquad (3b)$$

where n_u and n_d are the number densities of up and down quarks, while n_e is the number density of electrons. (In the case of strange quark matter, one should also add the strangeness fraction $X_s = n_s/n$ where n_s is the number density of strange quarks [46].) Charge neutrality requires

$$\tfrac{2}{3}n_u - \tfrac{1}{3}n_d - n_e = 0. \qquad (4)$$

Using this constraint together with the definitions in Eq. (3), one can express the number densities and, in fact, all thermodynamic quantities of quark matter in terms of n and X_e. For the number densities, for example, one finds

$$n_e = X_e n, \qquad (5a)$$

$$n_u = (1 + X_e)n, \qquad (5b)$$

$$n_d = (2 - X_e)n. \qquad (5c)$$

These number densities can also be expressed in terms of the corresponding chemical potentials, $n_i = n_i(\mu_i)$. In β equilibrium, the three chemical potentials are related as follows: $\mu_d = \mu_u + \mu_e$. In pulsating matter, on the other hand, the instantaneous departure from equilibrium is described by the small parameter

$$\delta\mu \equiv \mu_d - \mu_u - \mu_e = \delta\mu_d - \delta\mu_u - \delta\mu_e, \quad (6)$$

where $\delta\mu_i$ denotes the deviation of chemical potential μ_i from its value in β equilibrium. The quantity $\delta\mu$ can be conveniently expressed in terms of the variations of the two independent variables δn and δX_e,

$$\delta\mu = C\frac{\delta n}{n} + B\delta X_e, \qquad (7)$$

where, as follows from the definition, the coefficient functions C and B are given by

$$C = n_d \frac{\partial \mu_d}{\partial n_d} - n_u \frac{\partial \mu_u}{\partial n_u} - n_e \frac{\partial \mu_e}{\partial n_e}, \qquad (8a)$$

$$B = -n\left(\frac{\partial \mu_d}{\partial n_d} + \frac{\partial \mu_u}{\partial n_u} + \frac{\partial \mu_e}{\partial n_e}\right). \qquad (8b)$$

When $\delta\mu$ is nonzero the two Urca processes, shown diagrammatically in Fig. 1, have slightly different rates. To leading order in $\delta\mu$, we could write

$$\Gamma_\nu - \Gamma_{\bar{\nu}} = -\lambda\delta\mu. \qquad (9)$$

(Note that our λ is defined so that it is non-negative.) The net effect of having different rates for the two processes is a change of the electron fraction in the system:

$$n\frac{d(\delta X_e)}{dt} = \lambda\delta\mu, \qquad (10)$$

This has the tendency to restore the equilibrium value of X_e. Since the rate is finite, however, the weak processes always lag behind the density oscillations. In order to see this explicitly, we substitute $\delta\mu$ from Eq. (7) into Eq. (10) and get the equation for δX_e in a closed form,

$$n\frac{d(\delta X_e)}{dt} = \lambda\left(C\frac{\delta n}{n} + B\delta X_e\right). \qquad (11)$$

The periodic solution to this equation can be found most easily by making use of complex variables. Denoting $\delta X_e \equiv \text{Re}(\delta X_{e,0} e^{i\omega t})$, we derive the following result:

$$\delta X_{e,0} = \frac{\delta n_0}{n}\frac{C}{i\alpha - B}, \qquad (12)$$

where, by definition, $\alpha \equiv n\omega/\lambda$. In the last equation, the lagging of the weak processes is indicated by a nonvanishing imaginary part of $\delta X_{e,0}$. Such an imaginary part controls the phase shift of the δX_e oscillations with respect to the oscillations of density.

As we show in a moment, the same phase shift also leads to a nonvanishing dissipation of the energy density,

$$\langle \dot{\mathcal{E}}_{\text{diss}} \rangle = \frac{n}{\tau}\int_0^\tau P\dot{V}dt \qquad (13)$$

where $V \equiv 1/n$ is the specific volume.

The pressure oscillations around the equilibrium value are driven by the oscillations of its two independent variables, i.e., the quark number density and the lepton fraction,

$$\delta P = \frac{\partial P}{\partial n}\delta n - nC\delta X_e, \qquad (14)$$

where C is the same as in Eq. (8a). In the derivation we took into account that $n_i = \partial P/\partial\mu_i$ and that the total pressure is given by the sum of the partial contributions of the quarks and electrons, $P = \sum_i P_i(\mu_i)$.

After taking into account the relation (14) together with the solution for $\delta X_{e,0}$ in Eq. (12), the expression (13) becomes

$$\langle\dot{\mathcal{E}}_{\text{diss}}\rangle = \frac{C}{2}\omega\delta n_0\,\text{Im}(\delta X_{e,0})$$

$$= -\frac{1}{2}\left(\frac{\delta n_0}{n}\right)^2\frac{\lambda\omega^2 C^2}{\omega^2 + (\lambda B/n)^2}. \tag{15}$$

By comparing this with the definition in Eq. (2), we finally derive an explicit expression for the bulk viscosity,

$$\zeta = \frac{\lambda C^2}{\omega^2 + (\lambda B/n)^2}. \tag{16}$$

This expression shows that the viscosity is maximum in the limit of zero frequency, $\zeta_{\text{max}} = \zeta_{\omega=0}$, and that it falls off as $1/\omega^2$ at high frequencies, $\omega \gg \omega_0 \equiv \lambda B/n$. It should be also noted that $\omega_0 \sim \lambda$, and that the maximum viscosity is inversely proportional to the rate, i.e., $\zeta_{\text{max}} \sim 1/\lambda$. Because the rates of the weak processes in a dense medium usually have a power-law (or even an exponential) dependence on the temperature, the bulk viscosity is a very sensitive function of the temperature, too.

III. BULK VISCOSITY IN THE NORMAL PHASE

In order to calculate the bulk viscosity in the normal phase of two-flavor quark matter, we need to determine the corresponding thermodynamic coefficients B and C [see Eq. (8)] and calculate the difference of the rates of the two Urca processes shown in Fig. 1.

By making use of the following relations valid for non-interacting quark matter

$$n_{u,d} = \frac{1}{\pi^2}(\mu_{u,d}^2 - m_{u,d}^2)^{3/2}, \tag{17a}$$

$$n_e = \frac{1}{3\pi^2}\mu_e^3, \tag{17b}$$

we derive

$$C \simeq \frac{m_u^2}{3\mu_u} - \frac{m_d^2}{3\mu_d}, \tag{18a}$$

$$B \simeq \frac{\pi^2}{3}n\left(\frac{1}{\mu_u^2} + \frac{1}{\mu_d^2} + \frac{3}{\mu_e^2}\right). \tag{18b}$$

Here we made use of the equilibrium relation satisfied by the chemical potentials, $\mu_d = \mu_u + \mu_e$, and neglected higher-order mass corrections in the expression for B. In the temperature regime of interest, $T \lesssim m_{u,d}$, we also do not need to take into account any corrections due to a nonzero temperature.

It should be noted that the coefficient C, and therefore the bulk viscosity which is proportional to C^2, vanishes in the case of massless quarks. Moreover, this statement remains true even if the following (non-)Fermi liquid correction due to strong forces are taken into account [53–56],

$$C' = \frac{4\alpha_s}{3\pi}\left[\frac{m_d^2}{\mu_d}\left(\ln\frac{2\mu_d}{m_d} - \frac{2}{3}\right) - \frac{m_u^2}{\mu_u}\left(\ln\frac{2\mu_u}{m_u} - \frac{2}{3}\right)\right]. \tag{19}$$

(For a recent discussion of (non-)Fermi liquid corrections see, for example, Ref. [56].) Because of the large logarithms on the right-hand side of Eq. (19), this correction can become even larger than the leading-order result for C in Eq. (18a). Our estimates show that, in two-flavor quark matter, taking C' into account may increase the value of the viscosity by approximately an order of magnitude. Therefore, in all numerical estimates below we add the contributions from Eqs. (18a) and (19).

Now let us turn to the calculation of λ defined by Eq. (9). Following the original approach of Iwamoto [57], we get the rate for β decay in the following form:

$$\Gamma_{\bar{\nu}}(\delta\mu) = 6\int\frac{d^3p_d d^3p_u d^3p_{\bar{\nu}} d^3p_e}{(2\pi)^8 E_d E_u E_{\bar{\nu}} E_e}|M|^2\delta^4(P_d - P_u$$
$$- P_e - P_{\bar{\nu}})f(E_d - \mu_d)[1 - f(E_u - \mu_u)]$$
$$\times [1 - f(E_e - \mu_e)]. \tag{20}$$

Here, P_i and p_i are the 4- and 3-momenta of the ith particle, respectively, and $f(E) \equiv 1/(e^{E/T} + 1)$ is the Fermi distribution function. The scattering amplitude squared is given by [57]

$$|M|^2 = 64G_F^2\cos^2\theta_C(P_d \cdot P_{\bar{\nu}})(P_u \cdot P_e)$$
$$\approx \frac{2^8\alpha_s}{3\pi}G_F^2\cos^2\theta_C E_u E_d E_{\bar{\nu}} E_e(1 - \cos\theta_{d\bar{\nu}}). \tag{21}$$

After substituting this approximate form of $|M|^2$, all angular integrals in Eq. (20) can be done exactly. Then, using the dimensionless variables $x_i = (E_i - \mu_i)/T$ (note that $\mu_{\bar{\nu}} = 0$), we obtain

$$\Gamma_{\bar{\nu}}(\xi) = \frac{4\alpha_s}{\pi^6}G_F^2\cos^2\theta_C\mu_d\mu_u\mu_e T^5\int_0^\infty dx_{\bar{\nu}}x_{\bar{\nu}}^2 J(x_{\bar{\nu}} - \xi), \tag{22}$$

where $\xi \equiv \delta\mu/T$, and

$$J(x) = \left[\prod_{j=1}^3\int_{-\infty}^\infty dx_j f(x_j)\right]\delta(x_1 + x_2 + x_3 - x)$$
$$= \frac{\pi^2 + x^2}{2(1 + e^x)}. \tag{23}$$

By noting that $\Gamma_{\bar{\nu}}(\xi) = \Gamma_\nu(-\xi)$, we finally derive the expression

$$\lambda = \frac{17}{15\pi^2}G_F^2\cos^2\theta_C\alpha_s\mu_d\mu_u\mu_e T^4$$
$$\simeq \frac{17}{15}G_F^2\cos^2\theta_C\alpha_s(6X_e)^{1/3}nT^4[1 + \mathcal{O}(X_e)]. \tag{24}$$

This result, together with the explicit form for the coefficient functions C, C', and B in Eqs. (18) and (19), is sufficient to calculate the bulk viscosity of the normal phase of dense quark matter. The numerical results are presented in Fig. 2. In the calculation, we used the following representative values of the parameters:
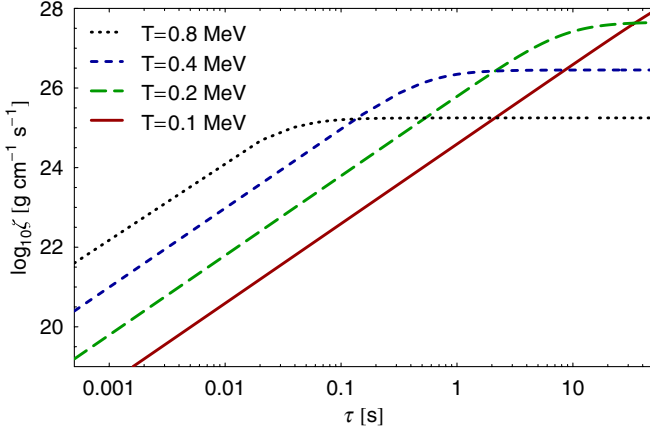
FIG. 2 (color online). The bulk viscosity for the normal phase of two-flavor quark matter as a function of the period of the density oscillations.

$$\mu_d = 500 \text{ MeV}, \qquad m_d = 9 \text{ MeV}, \qquad (25a)$$

$$\mu_u = 400 \text{ MeV}, \qquad m_u = 5 \text{ MeV}, \qquad (25b)$$

$$\mu_e = 100 \text{ MeV}, \qquad \alpha_s = 1. \qquad (25c)$$

In order to understand the numerical results better, we calculate the maximum value of the bulk viscosity, $\zeta_{\max} \equiv \zeta(\omega = 0)$, and the characteristic frequency $\omega_0 = \lambda B/n$, which separates the low- and high-frequency regimes,

$$\zeta_{\max} \approx 7.26 \times 10^{24} \left(\frac{T}{1 \text{ MeV}}\right)^{-4} \text{g cm}^{-1} \text{ s}^{-1}, \qquad (26)$$

$$\omega_0 \approx 460 \left(\frac{T}{1 \text{ MeV}}\right)^4 \text{ s}^{-1}. \qquad (27)$$

The knowledge of these two quantities is sufficient for determining the bulk viscosity at all frequencies,

$$\zeta = \frac{\zeta_{\max}}{1 + (\omega/\omega_0)^2}. \qquad (28)$$

Taking into account the relations in Eqs. (26) and (27), the temperature dependence of the results in Fig. 2 becomes clear. In accordance with the first relation, the maximum value of the viscosity (approached at low frequencies or large τ) becomes larger with decreasing temperature. Also, in agreement with Eq. (27), the location of the "shoulder" (occurring at $\tau_0 \equiv 2\pi/\omega_0$) shifts to smaller periods (higher frequencies) when the temperature gets higher.

IV. BULK VISCOSITY IN SPIN-ONE COLOR-SUPERCONDUCTING PHASES

In this section, we calculate the bulk viscosity of several spin-one color-superconducting phases that have been proposed and studied in detail in Refs. [39,40,42–44]. Following Ref. [47], we focus on the so-called transverse phases in which only quarks of opposite chiralities pair.

Theoretical studies at asymptotically large densities suggest that such phases are preferred [40,44].

In β equilibrium, the rates of the Urca processes were calculated in Ref. [47] in four different spin-one phases. Following the same method, here we generalize the calculation to the quasiequilibrium state, characterized by a small but nonvanishing value of $\delta\mu = \mu_d - \mu_u - \mu_e$.

Starting from equations analogous to Eq. (36) and (37) in Ref. [47], we derive

$$\Gamma_\nu - \Gamma_{\bar\nu} = \frac{2}{3\pi^6} \alpha_s G^2 \mu_e \mu_u \mu_d T^5 \sum_r \int_{-1}^1 d\Xi$$
$$\times \int x_\nu^2 dx_\nu [F^{rr}_{\varphi_u \varphi_d}(\Xi, x_\nu + \xi)$$
$$- F^{rr}_{\varphi_u \varphi_d}(\Xi, x_\nu - \xi)], \qquad (29)$$

where $\Xi \equiv \cos\theta_u = \cos\theta_d$ is the angle between the three-momentum of the u/d quark and the z-axis. The functions $F^{rr}_{\varphi_u \varphi_d}(\Xi, x)$ are the same as in Ref. [47]. Their explicit form is given by

$$F^{rr}_{\varphi_u \varphi_d}(\Xi, x) = \omega_{rr}(\Xi) \sum_{e_1, e_2 = \pm} \int_0^\infty \int_0^\infty dx_d dx_u$$
$$\times (e^{-e_1 \sqrt{x_u^2 + \lambda_{\Xi,r} \varphi_u^2}} + 1)^{-1} (e^{e_2 \sqrt{x_d^2 + \lambda_{\Xi,r} \varphi_d^2}} + 1)^{-1}$$
$$\times (e^{x_\nu + e_1 \sqrt{x_u^2 + \lambda_{\Xi,r} \varphi_u^2} - e_2 \sqrt{x_d^2 + \lambda_{\Xi,r} \varphi_d^2}} + 1)^{-1}, \qquad (30)$$

where, by definition, $\xi = \delta\mu/T$ and $\varphi_i = \phi_i/T$. For an explicit form of the functions $\lambda_{\Xi,r}$ and ω_{rr}, we refer the reader to Refs. [44,47].

By expanding the rate difference (29) in powers of ξ and extracting the coefficient of the linear term, we derive an expression for λ in the following general form:

$$\lambda(\varphi_d, \varphi_u) = \lambda(0)[\tfrac{1}{3} + \tfrac{2}{3} H(\varphi_d, \varphi_u)], \qquad (31)$$

where $\lambda(0)$ is the same as in the normal phase of quark matter, see Eq. (24), and $H(\varphi_d, \varphi_u)$ is a reduction factor whose explicit form depends on the choice of a specific spin-one color-superconducting phase. Note that, as in the case of the neutrino-emission rates [47,48], λ consists of two qualitatively different contributions. The first one is given by the term $1/3$ in the square brackets of Eq. (31). It originates from the ungapped modes that are present in all considered spin-one phases. The second contribution is given by the term proportional to $H(\varphi_u, \varphi_d)$. This one originates from the gapped modes. An explicit form of the function $H(\varphi_u, \varphi_d)$ in the case of the CSL, planar, and polar phases is given by

$$H(\varphi_d, \varphi_u) = \frac{60}{17\pi^4} \int_{-1}^1 d\Xi \int_0^\infty dx_\nu x_\nu F^{11}_{\varphi_u \varphi_d}(\Xi, x_\nu), \qquad (32)$$

and, in the case of the A phase, it reads

$$H(\varphi_d, \varphi_u) = \frac{60}{17\pi^4} \int_{-1}^{1} d\Xi \int_{0}^{\infty} dx_\nu x_\nu \times [F_{\varphi_u\varphi_d}^{11}(\Xi, x_\nu) - F_{\varphi_u\varphi_d}^{22}(\Xi, x_\nu)]. \tag{33}$$

In the derivation we integrated by parts so that the results are given in terms of the function $F_{\varphi_u\varphi_d}^{rr}(\Xi, x)$. The reduction factors can be easily calculated numerically for each phase. The results are shown in Fig. 3 for the case $\varphi_u = \varphi_d \equiv \varphi$. In the limit of large φ analytical expressions for the suppression functions for all four spin-one color-superconducting phases are given in Appendix A.

For many applications, it is of interest to know the temperature dependence of the rates. Therefore, in Fig. 4, we also show the temperature dependence of the suppression factor $H(T)$. In the calculation we used the following temperature dependence of the gap parameter,

$$\phi(T) = \phi_0 \sqrt{1 - \left(\frac{T}{T_c}\right)^2}, \tag{34}$$

with ϕ_0 being the value of the gap parameter at $T = 0$, and T_c being the value of the critical temperature. Note that the ratio T_c/ϕ_0 depends on the choice of the phase [44]. The approximate values of this ratio are 0.8 (CSL), 0.66 (planar), 0.49 (polar), and 0.81 (A phase).

With the expression for λ at hand, we are now in the position to calculate the bulk viscosity of the spin-one color-superconducting phases. Before we do this, it might be appropriate to mention that the coefficient functions B and C do not change much due to superconductivity. Corrections to B are of order $(\phi_i/\mu_i)^2$ and, thus, are strongly suppressed. The function C, on the other hand, gets corrections of order ϕ_i^2/μ_i which are negligible only if $\phi_i^2 \ll m_i^2$. We assume that this is indeed the case. Notably, the spin-one gap corrections to C should be comparable to the corrections due to a nonzero temperature that we neglected in our calculations.

It should be emphasized that the bulk viscosity in the color-superconducting phases has the same general struc-
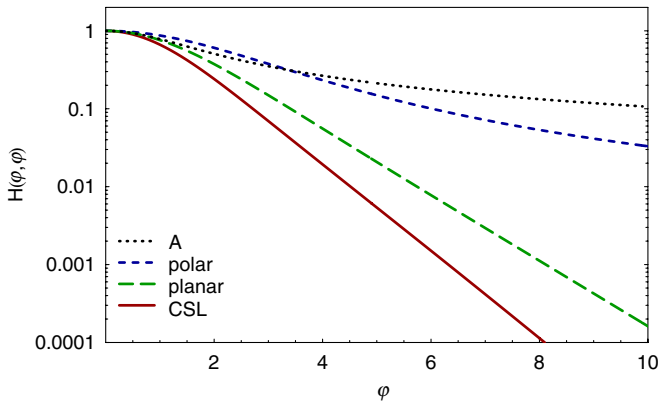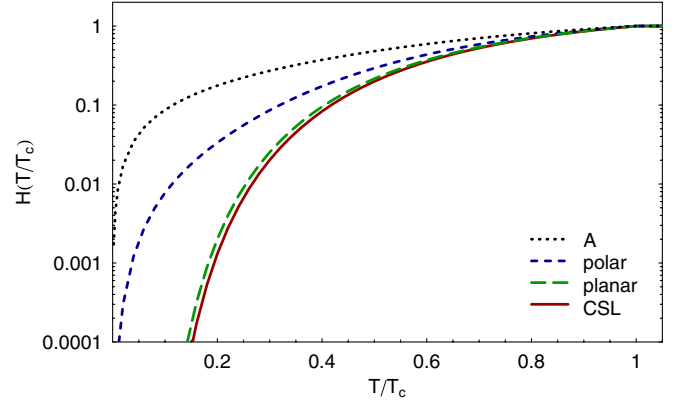

FIG. 4 (color online). The reduction factor as a function of the temperature for the CSL, planar, polar, and A phases.

ture as in the normal phase, see Eq. (28), but the quantities $\zeta_{\max}$ and ω_0 should be redefined to take into account the rate suppression factors:

$$\zeta_{\max}^{sp1} = \frac{\zeta_{\max}}{h_{sp1}}, \tag{35}$$

$$\omega_0^{sp1} = h_{sp1}\omega_0, \tag{36}$$

where $\zeta_{\max}$ and ω_0 are given in Eqs. (26) and (27), respectively, and

$$h_{sp1} = \tfrac{1}{3} + \tfrac{2}{3}H(T/T_c). \tag{37}$$

We see that the effect of the suppression of λ due to Cooper pairing manifests itself in a nontrivial way in the expression for the bulk viscosity, i.e.,

$$\zeta_{sp1} = \frac{\zeta_{\max}h_{sp1}}{h_{sp1}^2 + (\omega/\omega_0)^2}. \tag{38}$$

From the analysis of this representation, we find that the suppression of the rates tends to decrease the viscosity at high frequencies ($\omega > h_{sp1}\omega_0$) and to increase it at low frequencies ($\omega < h_{sp1}\omega_0$). One should note, though, that the relevant range of low frequencies would shrink a lot if the suppression happened to be strong.

The representation in Eq. (38) shows that the effect of color superconductivity cannot be very large. Even if the suppression of the weak rates due to gapped modes is maximal, i.e., $H(T/T_c) = 0$, the results for $\zeta_{\max}$ and ω_0 could not change by more than a factor of 3 compared to the normal phase of matter. Of course, this is the consequence of having ungapped quasiparticle modes in the energy spectra.

The numerical results for the bulk viscosity in the normal phase and spin-one color-superconducting phases with different values of the critical temperature are shown in Fig. 5. For each of the three choices of T_c, we show a shaded area (color online: blue, green and red for $T_c = 0.25, 1,$ and 4 MeV, respectively), that is bounded by the

FIG. 3 (color online). The reduction factor as a function of $\varphi \equiv \phi/T$ for the CSL, planar, polar, and A phases.
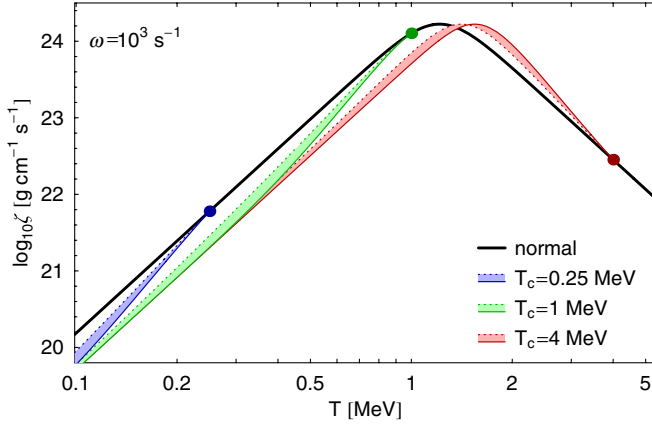
FIG. 5 (color online). The bulk viscosity as a function of the temperature for spin-one color-superconducting quark matter. The oscillation frequency is $\omega = 10^3 \text{s}^{-1}$.

values of the bulk viscosity in the CSL phase (solid boundary) and in the *A* phase (dotted boundary). In the calculation, we used the set of parameters given in Eq. (25).

For comparison, in Fig. 6 we also show the results for the bulk viscosity in a toy model in which all quasiparticles modes are gapped. In this case, the effect is indeed very dramatic. The largest deviations from the result of the normal phase correspond to the CSL phase (thin solid lines), and the smallest to the *A* phase (dotted lines). The viscosity in the planar phase (long-dashed lines) is numerically close to that in the CSL phase, and the viscosity in the polar phase (short-dashed lines) is typically somewhere in between.

At this point it is natural to ask ourselves if there exists the possibility that all quasiparticles in a spin-one color superconductor are gapped. The answer is affirmative. This is the case, e.g., in the version of the CSL phase proposed in Ref. [58], when the quarks are massive. In our notation,
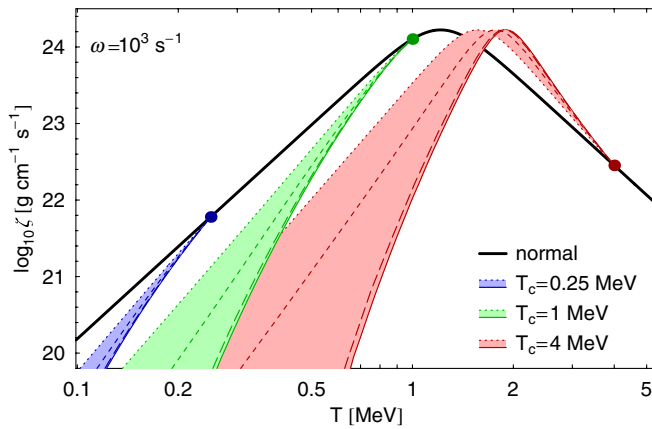

FIG. 6 (color online). The bulk viscosity as a function of the temperature for a toy model of a spin-one color superconductor in which all quasiparticle modes are gapped. The oscillation frequency is $\omega = 10^3 \text{s}^{-1}$.

the three quasiparticle gaps are given by

$$\phi_{i,1} = \phi_{i,2} \simeq \phi_i, \qquad \phi_{i,3} \simeq \frac{m_i \phi_i}{\sqrt{2}\mu_i}, \qquad (i = u, d). \tag{39}$$

The qualitative change in the low-energy spectrum comes from a different choice of the gap matrix in the CSL phase. (For a discussion of the specific differences, see Sec. VII in Ref. [47].) In fact, it is likely that the ansatz that produces the fully gapped phase is energetically favored [59]. The most important effect of the additional nonzero gap is the appearance of another suppression factor multiplying the first term in the expression for λ, see Eq. (31). Therefore, the modified expression for the CSL phase reads

$$\lambda(\varphi_d, \varphi_u) = \lambda(0)\left[\frac{1}{3}H\left(\frac{m_d\varphi_d}{\sqrt{2}\mu_d}, \frac{m_u\varphi_u}{\sqrt{2}\mu_u}\right) + \frac{2}{3}H(\varphi_d, \varphi_u)\right]. \tag{40}$$

Note that the additional suppression factor is given in terms of the same function $H(\varphi_d, \varphi_u)$ which was calculated numerically for the transverse CSL phase, see Figs. 3 and 4. If the up and down quark masses are rather small, the effect from the additional suppression cannot be easily seen before the temperature becomes much lower than the critical value, i.e., $T \ll T_c$. The situation changes dramatically, however, if the relevant constituent values of the quark masses happen to be considerably larger than the current masses of quarks. As suggested by the analysis in Ref. [58], this is indeed possible. Then, the bulk viscosity could be affected almost as much as in the toy model in Fig. 6.

V. DISCUSSION

In this paper we have calculated the bulk viscosity for the normal phase as well as for four spin-one color-superconducting phases of two-flavor dense quark matter. The main contributions come from the Urca processes shown diagrammatically in Fig. 1. Note that the results for the normal phase are also relevant for the 2SC phase. Indeed, after taking into account that there are two (blue) ungapped modes of quasiparticles in the low-energy spectrum of the 2SC phase, the low-temperature bulk viscosity is approximately given by the same expression (28), provided the following redefinitions are made: $\zeta_{\text{max}}^{2SC} = 3\zeta_{\text{max}}$ and $\omega_0^{2SC} = \omega_0/3$, where the normal-phase quantities are given in Eqs. (26) and (27), respectively. The redefinitions account for the decrease of the weak rates by a factor of 3 at $T \ll \Delta_0$ where Δ_0 is the value of the 2SC gap.

The microscopic calculations of the bulk viscosity in the spin-one color-superconducting phases suggests that quasiparticles with different types of gapless nodes (e.g., points or lines at the Fermi sphere) could potentially play a very important role. In the case of the transverse phases, however, the presence of a single ungapped quasiparticle

mode washes out essentially all information about spin-one Cooper pairing, see Fig. 5. The presence of nonzero quark masses may provide a gap for such a mode and the situation changes. In this paper, we briefly discussed such a possibility in connection with the CSL phase of Ref. [58]. The results are shown in Fig. 6.

In agreement with the general expectation, we find that the bulk viscosity often tends to decrease when there is Cooper pairing of quarks whose main effect is to suppress the rates of the weak processes. In some cases (e.g., at sufficiently low frequencies and/or at temperatures close to the critical value) the behavior may reverse because of the nontrivial dependence of the bulk viscosity on the suppression factor, see Eq. (38). Such an increase of the viscosity in the color-superconducting CSL phase is seen, for example, in a range of temperatures below T_c in Figs. 5 and 6 in the case when $T_c = 4$ MeV.

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APPENDIX: ASYMPTOTIC FORM OF THE SUPPRESSION FUNCTION $H(\varphi, \varphi)$ AT $\varphi \to \infty$

In this appendix, we calculate the asymptotic form of the suppression function $H(\varphi, \varphi)$ at $\varphi \to \infty$. This is relevant for the bulk viscosity of spin-one color superconductors in the low-temperature regime.

Let us start from the CSL and planar phases whose gap functions have no nodes in momentum space. To this end, it is useful to compute the asymptotic behavior of the following integral:

$$J(\varphi, \varphi) \equiv \sum_{e_1, e_2 = \pm} \int_0^\infty \int_0^\infty \int_0^\infty dx_u dx_d dx_\nu x_\nu$$
$$\times (e^{-e_1\sqrt{x_d^2 + \varphi^2}} + 1)^{-1}(e^{e_2\sqrt{x_u^2 + \varphi^2}} + 1)^{-1}$$
$$\times (e^{x_\nu + e_1\sqrt{x_u^2 + \varphi^2} - e_2\sqrt{x_d^2 + \varphi^2}} + 1)^{-1}. \quad (A1)$$

By neglecting the terms of order $\mathcal{O}(e^{-2\varphi})$, we arrive at the following leading-order asymptotic behavior:

$$J(\varphi, \varphi) \simeq 4\varphi e^{-\varphi} \int_0^\infty \int_0^\infty \int_0^\infty \frac{dy_u dy_d dx_\nu x_\nu}{e^{y_u^2} + e^{x_\nu + y_d^2}}$$
$$\approx 5.047 \varphi e^{-\varphi}. \quad (A2)$$

Note that in the derivation we changed the integration variables, $x_i = y_i\sqrt{2\varphi}$ for $i = u, d$, and calculated the remaining integral numerically.

By making use of the result for $J(\varphi, \varphi)$, we derive the large φ asymptotic behavior for the suppression function $H(\varphi, \varphi)$ in the CSL phase:

$$H^{\text{CSL}}(\varphi, \varphi) = \frac{240}{17\pi^4} J(\sqrt{2}\varphi, \sqrt{2}\varphi) \simeq 1.034 \varphi e^{-\sqrt{2}\varphi}. \quad (A3)$$

Similarly, after taking into account the angular dependence of the gap in the planar phase, we derive

$$H^{\text{planar}}(\varphi, \varphi) = \frac{120}{17\pi^4} \int_{-1}^1 d\Xi J(\sqrt{1 + \Xi^2}\varphi, \sqrt{1 + \Xi^2}\varphi)$$
$$\simeq 0.917\sqrt{\varphi} e^{-\varphi}. \quad (A4)$$

The derivation in the polar and A phases is slightly more complicated because the corresponding gap functions have zeros for some directions in momentum space. By approximating the angular integrals in the same way as in Ref. [47] (see Appendix E there), we arrive at the following asymptotic behaviors:

$$H^{\text{polar}}(\varphi, \varphi) \simeq \frac{\pi}{\varphi^2}, \quad (A5)$$

$$H^{\text{A}}(\varphi, \varphi) \simeq \frac{1}{\varphi}. \quad (A6)$$

In connection to these last two results, we should mention that while the parametric dependence on φ is easy to extract analytically, it is much harder to determine the overall coefficients in their power-law asymptotic behavior. In our derivation, therefore, we combined the analytical derivation with the numerical calculations.

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