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General class of braneworld wormholes

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The brane cosmology scenario is based on the idea that the Universe is a 3-brane embedded in a five-dimensional bulk. In this work, a general class of braneworld wormholes is explored with $R \neq 0$, where R is the four dimensional Ricci scalar, and specific solutions are further analyzed. A fundamental ingredient of traversable wormholes is the violation of the null energy condition (NEC). However, it is the effective total stress-energy tensor that violates the latter, and in this work, the stress-energy tensor confined on the brane, threading the wormhole, is imposed to satisfy the NEC. It is also shown that in addition to the local high-energy bulk effects, nonlocal corrections from the Weyl curvature in the bulk may induce a NEC violating signature on the brane. Thus, braneworld gravity seems to provide a natural scenario for the existence of traversable wormholes.

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I. INTRODUCTION

An important and intriguing challenge in wormhole physics is the quest to find a realistic matter source that will support these exotic spacetimes. Traversable wormholes are supported by a null energy condition violating stress-energy tensor, denoted as *exotic matter* [1]. Several candidates have been proposed in the literature, such as scalar fields [2]; wormhole solutions in semiclassical gravity (see Ref. [3] and references therein); solutions in Brans-Dicke theory [4]; solutions in higher dimensions, for instance in Einstein-Gauss-Bonnet theory [5], wormholes on the brane [6–8], etc. (see Ref. [9] for more details and references). More recently, it has been argued that traversable wormholes may be supported by several equations of state responsible for the late time accelerated expansion of the Universe, namely, phantom energy [10], the generalized Chaplygin gas [11], and the van der Waals quintessence fluid [12]. Despite the fact that, in a cosmological context, these equations of state represent homogeneous fluids, inhomogeneities may arise through gravitational instabilities. Therefore, it seems that traversable wormholes may possibly originate from density fluctuations in the cosmological background. In fact, it has been shown recently that quantum fluctuations may self sustain phantom wormholes with an equation of state varying with the radial coordinate [3].

Moving on to braneworld cosmology, the latter is an interesting scenario based on the idea that the Universe is a 3-brane embedded in a five-dimensional bulk. In the context of braneworld wormholes, a class of static and spherically symmetric solutions, with $R = 0$, where R is the four dimensional Ricci scalar, was considered in Ref. [7]. The authors also consider a vacuum brane, so that the wormhole is supported by bulk Weyl effects. In this work, we generalize to nonvacuum branes with nonexotic matter, and with $R \neq 0$. In fact, in addition to wormholes, several static and spherically symmetric spacetimes on the brane have been analyzed to some extent in the literature, for instance, stars on the brane [13,14], black holes [15,16] and constraints from solar system experiments were also found [16]. In this work, in addition to analyzing the wormhole case of $R \neq 0$, we impose that the stress-energy tensor confined on the brane, threading the wormhole satisfies the NEC. We also show that the local high-energy bulk effects and nonlocal corrections from the Weyl curvature in the bulk could leave a NEC violating signature on the brane. This argument is supported by the fact that negative energy densities are induced by gravitational waves or black strings in the bulk [17]. Thus, it seems that braneworld gravity provides a natural scenario for the existence of traversable wormholes.

In this work, we shall follow the formalism outlined in Ref. [18]. The five-dimensional Einstein equation in the bulk takes the form

$$ {}^{(5)}G_{AB} = -\Lambda_5 {}^{(5)}g_{AB} + k_5^2 {}^{(5)}T_{AB}. \tag{1} $$

If the bulk is empty, i.e., ${}^{(5)}T_{AB} = 0$, the induced field equations on the brane [18], are given by

$$ G_{\mu\nu} = -\Lambda g_{\mu\nu} + k^2 T_{\mu\nu} + \frac{6k^2}{\lambda} S_{\mu\nu} - \mathcal{E}_{\mu\nu}, \tag{2} $$

with

$$ k^2 = \frac{\lambda k_5^2}{6}, \qquad \Lambda = \frac{1}{2}(\Lambda_5 + k^2\lambda), \tag{3} $$

where k^2 and k_5^2 are the gravitational coupling constants, Λ and Λ_5 the cosmological constants on the brane and in the bulk, respectively; λ is the tension on the brane.

$T_{\mu\nu}$ is the stress-energy tensor confined on the brane, so $T_{AB}n^B = 0$, where n^A is the unit normal to the brane. The first correction term relative to Einstein's general relativity

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is the inclusion of a quadratic term $S_{\mu\nu}$ in the stress-energy tensor, arising from the extrinsic curvature terms in the projected Einstein tensor, and is given by

$$S_{\mu\nu} = \tfrac{1}{12}TT_{\mu\nu} - \tfrac{1}{4}T_{\mu\alpha}T^{\alpha}{}_{\nu} + \tfrac{1}{8}g_{\mu\nu}[T_{\alpha\beta}T^{\alpha\beta} - \tfrac{1}{3}T^2], \quad (4)$$

with $T = T^{\mu}{}_{\mu}$.

The second correction term, $\mathcal{E}_{\mu\nu}$, is the projection of the 5-dimensional Weyl tensor, ${}^{(5)}C_{ABCD}$, onto the brane, and is defined as $\mathcal{E}_{\mu\nu} = \delta^A_\mu \delta^{C(5)}_\nu C_{ABCD} n^B n^D$, and encompasses nonlocal bulk effects. The only general known property of this nonlocal term is that it is traceless, i.e., $\mathcal{E}^{\mu}{}_{\mu} = 0$.

Taking into account, the traceless property of the projected 5-dimensional Weyl tensor, then Eq. (2) implies

$$R = 4\Lambda - k^2 T - \frac{3k^2}{2\lambda}\left(T_{\alpha\beta}T^{\alpha\beta} - \frac{1}{3}T^2\right). \quad (5)$$

This paper is outlined in the following manner: In Sec. II, we outline the effective field equations governing braneworld wormholes, and provide two general strategies for finding specific solutions. In Sec. III, we further explore specific solutions, and finally, in Sec. IV, we conclude.

II. EFFECTIVE FIELD EQUATION ON THE BRANE

Consider a static and spherically symmetric wormhole metric given in the following form

$$ds^2 = -e^{2\Phi(r)}dt^2 + \frac{dr^2}{1 - b(r)/r} + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (6)$$

where $\Phi(r)$ and $b(r)$ are arbitrary functions of the radial coordinate, r, denoted as the redshift function, and the form function, respectively [1]. The radial coordinate has a range that increases from a minimum value at r_0, corresponding to the wormhole throat, and extends to infinity.

To be a wormhole solution, several properties need to be imposed [1], namely: The throat is located at $r = r_0$ and $b(r_0) = r_0$. A flaring out condition of the throat is imposed, i.e., $(b - b'r)/b^2 > 0$, which reduces to $b'(r_0) < 1$ at the throat. The condition $1 - b/r \geq 0$ is also imposed. To be traversable, one must demand that there are no horizons present, which are identified as the surfaces with $e^{2\Phi} \to 0$, so that $\Phi(r)$ must be finite everywhere.

Note that the field equation on the brane can take the form

$$G_{\mu\nu} = 8\pi T^{\rm eff}_{\mu\nu}, \quad (7)$$

with the total effective stress-energy tensor, $T^{\rm eff}_{\mu\nu}$, being given by

$$T^{\rm eff}_{\mu\nu} = T_{\mu\nu} - \frac{1}{8\pi}\mathcal{E}_{\mu\nu} + \frac{6}{\lambda}S_{\mu\nu}, \quad (8)$$

with $k^2 = 8\pi$. We have considered, for simplicity, that the cosmological constant on the brane is zero. Note that the quadratic term, i.e., $S_{\mu\nu} \sim (T_{\mu\nu})^2$, is the high-energy correction term. From the following approximations $|S_{\mu\nu}/\lambda|/|T_{\mu\nu}| \sim |T_{\mu\nu}|/\lambda \sim \rho/\lambda$, one readily verifies that $S_{\mu\nu}$ is dominant for $\rho \gg \lambda$, and negligible in the regime $\rho \ll \lambda$, where $\lambda > (1\ \mathrm{Tev})^4$ [18].

In general relativity, the flaring out condition implies that the wormhole should be threaded with matter violating the null energy condition (NEC). Note that the NEC is given by $T_{\mu\nu}k^\mu k^\nu \geq 0$, where k^μ is *any* null vector. Now, an important feature of wormholes on the brane, governed by the induced field equations on the brane, Eq. (2), is that it is the total effective stress-energy tensor that should violate the NEC. In particular, the stress-energy tensor confined on the brane, $T_{\mu\nu}$, could perfectly satisfy the NEC. Therefore, the NEC violation arises from a combination of the local bulk effects, through the quadratic term in the stress-energy tensor, $S_{\mu\nu}$, and the nonlocal effects from the bulk, $\mathcal{E}_{\mu\nu}$.

The analysis is simplified working in an orthonormal reference frame, so that the Einstein tensor components, for the metric (6), are given by the following relationships

$$G_{\hat{t}\hat{t}} = \frac{b'}{r^2}, \quad (9)$$

$$G_{\hat{r}\hat{r}} = 2\left(1 - \tfrac{b}{r}\right)\tfrac{\Phi'}{r} - \tfrac{b}{r^3}, \quad (10)$$

$$G_{\hat{\theta}\hat{\theta}} = G_{\hat{\phi}\hat{\phi}} = \left(1 - \frac{b}{r}\right)\left[\Phi'' + (\Phi')^2 + \frac{\Phi'}{r} - \frac{b'r - b}{2r^2(r-b)} - \frac{b'r - b}{2r(r-b)}\Phi'\right], \quad (11)$$

where the prime denotes a derivative with respect to the radial coordinate, r.

We shall consider an isotropic fluid confined on the brane, where the stress-energy tensor components, in the orthonormal reference frame, are given by $T_{\hat{\mu}\hat{\nu}} = \mathrm{diag}(\rho, p, p, p)$, where $\rho(r)$ is the energy density, $p(r)$ is the isotropic pressure. Taking into account the static and spherically symmetric nature of the problem, the projected Weyl tensor has the form $\mathcal{E}_{\hat{\mu}\hat{\nu}} = \mathrm{diag}[\epsilon(r), \sigma_r(r), \sigma_t(r), \sigma_t(r)]$. The quadratic correction term components, $S_{\hat{\mu}\hat{\nu}}$, which are the local effects of the bulk arising from the brane extrinsic curvature, included for self-completeness, are provided by

$$S_{\hat{t}\hat{t}} = \tfrac{1}{12}\rho^2, \quad (12)$$

$$S_{\hat{r}\hat{r}} = S_{\hat{\theta}\hat{\theta}} = S_{\hat{\phi}\hat{\phi}} = \tfrac{1}{12}\rho(\rho + 2p). \quad (13)$$

Thus, the effective stress-energy tensor components, Eq. (8), take the following form

$$\rho^{\rm eff} = \rho\left(1 + \frac{\rho}{2\lambda}\right) - \frac{\epsilon}{8\pi}, \quad (14)$$

$$p_r^{\rm eff} = p\left(1+\frac{\rho}{\lambda}\right) + \frac{\rho^2}{2\lambda} - \frac{\sigma_r}{8\pi}, \tag{15}$$

$$p_t^{\rm eff} = p\left(1+\frac{\rho}{\lambda}\right) + \frac{\rho^2}{2\lambda} - \frac{\sigma_t}{8\pi}. \tag{16}$$

It is interesting that the nonlocal bulk effects can contribute with an effective anisotropic fluid, even in the presence of an isotropic fluid on the brane.

In the analysis outlined below, the Ricci scalar will play an important role, and is given by

$$R = -2\left(1-\frac{b}{r}\right)\left[\Phi'' + (\Phi')^2 - \frac{b'}{r(r-b)} - \frac{b'r+3b-4r}{2r(r-b)}\Phi'\right]. \tag{17}$$

Evaluated at the throat, r_0, this reduces to

$$R|_{r_0} = \frac{2b_0'}{r_0^2} + \frac{(b_0'-1)\Phi_0'}{r_0}. \tag{18}$$

The Ricci scalar, from Eq. (5), may also be given in terms of the energy density and the isotropic pressure confined on the brane, and assumes the form

$$R = k^2(\rho - 3p) - \frac{3k^2}{2\lambda}\left[\rho^2 + 3p^2 - \frac{1}{3}(\rho-3p)^2\right]. \tag{19}$$

The imposition of the flaring out condition implies that the effective total stress-energy tensor violates the NEC, i.e., $T^{\rm eff}_{\mu\nu}k^\mu k^\nu < 0$. Now, considering a radial null vector, $k^{\hat{\mu}} = (1, 1, 0, 0)$, the latter inequality takes the form $\rho^{\rm eff} + p_r^{\rm eff} < 0$. From the following relationship

$$\rho^{\rm eff} + p_r^{\rm eff} = \rho + p - \frac{1}{8\pi}(\epsilon + \sigma_r) + \frac{1}{\lambda}\rho(\rho+p), \tag{20}$$

the NEC violation, $\rho^{\rm eff} + p_r^{\rm eff} < 0$, provides the following generic restriction

$$8\pi(\rho+p)(1+\tfrac{\rho}{\lambda}) < \epsilon + \sigma_r, \tag{21}$$

in order to obtain wormhole solutions. In particular, considering the low energy regime, $\rho \ll \lambda$, one may neglect the quadratic term components, and the inequality (21) reduces to $8\pi(\rho+p) < \epsilon + \sigma_r$. If the Weyl components are zero, then one recovers the usual general relativistic NEC violation. For high energies, $\rho \gg \lambda$, the quadratic term dominates, and the inequality (21) takes the form $8\pi\rho(\rho+p)/\lambda < \epsilon + \sigma_r$. Thus, in addition to nonlocal corrections from the Weyl curvature in the bulk (as in Ref. [7]), local high-energy bulk effects imprints a NEC violating signature on the brane. It is also possible to consider a zero Weyl curvature term, and generalize the stress-energy tensor to incorporate an anisotropic pressure contribution on the brane. This latter consideration would generalize standard general relativistic wormholes with the inclusion of a high-energy contribution. It seems that braneworld gravity provides a natural scenario for the existence of traversable wormholes. In summary, by choosing $T_{\mu\nu}$, this fixes $S_{\mu\nu}$. Therefore, one needs to find $\mathcal{E}_{\mu\nu}$ which produces a wormhole violating the NEC at all energies. However, the question of what 5-dimensional geometry produces this $\mathcal{E}_{\mu\nu}$ is much more difficult to answer, and shall not be explored here.

Now, one may adopt several strategies to find solutions of wormholes on the brane. For instance, specifying the functions $b(r)$ and $\Phi(r)$, the Ricci scalar R is determined through Eq. (17). Then considering an equation of state such that $p = p(\rho)$, the Ricci scalar through Eq. (19) would be given as a function of ρ. Thus, from Eqs. (17) and (19), one would then completely determine $\rho = \rho(r)$, and consequently $p = p(\rho)$. Finally, through the effective field equations induced on the brane, Eqs. (14)–(16), the projected 5-dimensional Weyl tensor components are determined.

One may also consider an analogous strategy as the one obtained in Ref. [14]. If one specifies the source, then $R(r)$ is determined through Eq. (19). Now, integrating Eq. (17) provides the following relationship for the form function

$$b(r) = e^{-\Gamma(r,r_0)}\left\{r_0 + \int_{r_0}^r \frac{\bar{r}e^{\Gamma(\bar{r},r_0)}[2\Phi''\bar{r} + 2(\Phi')^2\bar{r} + 4\Phi' + R(\bar{r})\bar{r}]}{2+\Phi'\bar{r}}d\bar{r}\right\}, \tag{22}$$

where $\Gamma(r, r_0)$ is defined as

$$\Gamma(r, r_0) = \int_{r_0}^r \frac{2\Phi''\bar{r} + 2(\Phi')^2\bar{r} + 3\Phi'}{2+\Phi'\bar{r}}d\bar{r}. \tag{23}$$

As the bulk is considered empty, ${}^{(5)}T_{AB} = 0$, the brane stress-energy tensor satisfies the usual conservation equation, $T^{\hat{\mu}\hat{\nu}}{}_{;\hat{\nu}} = 0$, which reflects that the interaction between the bulk and the brane is purely gravitational, i.e., there is no exchange of stress-energy between the two [18]. The conservation equation then provides the following relationship

$$p' = -(\rho+p)\Phi'. \tag{24}$$

Integrating, the pressure provides

$$p(r) = e^{-\Phi(r)}\left[-\int_{r_0}^r \Phi'(\bar{r})\rho(\bar{r})e^{\Phi(\bar{r})}d\bar{r} + C\right], \tag{25}$$

where C is an integrating constant. The latter may be defined, for instance, evaluating the pressure at the throat, so that $C = p(r_0)e^{\Phi(r_0)}$.

The algorithm runs as follows: providing Φ and one of the following functions ρ and p, one determines the second through Eq. (24). In particular, providing ρ and Φ, and specifying $p(r_0)$, then p is determined through Eq. (25), which yields the full stress-energy tensor $T_{\mu\nu}$. The Ricci scalar R is known through Eq. (19), which in turn is used to find the form function, Eq. (22), thus fixing the intrinsic geometry on the brane. Finally, the components of the anisotropic contribution of the projected Weyl tensor are computed through Eqs. (14)–(16).

III. SPECIFIC SOLUTIONS

A. Dust

The specific case of dust shall be explored in some detail as an illustrative example. For instance, consider dust with a positive energy density threading the wormhole. The effective stress-energy tensor components reduce to

$$\rho^{\rm eff}(r) = \rho\left(1+\frac{\rho}{2\lambda}\right) - \frac{1}{8\pi}\epsilon(r), \tag{26}$$

$$p_r^{\rm eff}(r) = \frac{1}{2\lambda}\rho^2 - \frac{1}{8\pi}\sigma_r(r), \tag{27}$$

$$p_t^{\rm eff}(r) = \frac{1}{2\lambda}\rho^2 - \frac{1}{8\pi}\sigma_t(r), \tag{28}$$

The NEC violation provides the following restriction

$$8\pi\rho(1+\tfrac{\rho}{\lambda}) < \epsilon + \sigma_r, \tag{29}$$

and the Ricci scalar reduces to

$$R = 8\pi\rho(1-\tfrac{\rho}{\lambda}). \tag{30}$$

Now, specifying the functions $b(r)$ and $\Phi(r)$, then the Ricci scalar as a function of the r-coordinate is determined and one deduces the energy density, which is given by

$$\rho = \frac{\lambda}{2}\left(1 \pm \sqrt{1-\frac{R}{2\pi\lambda}}\right). \tag{31}$$

For this particular case, note that the generic NEC violation, inequality (29), evaluated at the throat, and taking into account Eq. (18), takes the following form

$$(\epsilon+\sigma_r)|_{r_0} > -\frac{2b_0'}{r_0^2} - \frac{(b_0'-1)\Phi_0'}{r_0} + 8\pi\lambda\left(1 \pm \sqrt{1-\frac{b_0'}{\pi\lambda r_0^2} - \frac{(b_0'-1)\Phi_0'}{2\pi\lambda r_0}}\right), \tag{32}$$

in terms of the metric coefficients.

Considering a constant redshift function, for simplicity, we have $R = 2b'/r^2$, and Eq. (31) takes the form

$$\rho = \frac{\lambda}{2}\left(1 \pm \sqrt{1-\frac{b'}{\pi\lambda r^2}}\right). \tag{33}$$

One may impose the negative sign, and assuming that $b'/r^2 \to 0$ at spatial infinity, to allow $\rho \to 0$. However, we shall also analyze the positive sign, which provides interesting results. If we impose that $\rho \geq 0$, and considering the negative sign, it is a simple matter to prove that $b' \geq 0$. The condition $b' < \pi\lambda r^2$ is also imposed. The projected Weyl tensor components provide the following relationships

$$\epsilon(r) = -\frac{2b'}{r^2} + 6\pi\lambda\left(1 \pm \sqrt{1-\frac{b'}{\pi\lambda r^2}}\right), \tag{34}$$

$$\sigma_r(r) = \frac{b-b'r}{r^3} + 2\pi\lambda\left(1 \pm \sqrt{1-\frac{b'}{\pi\lambda r^2}}\right), \tag{35}$$

$$\sigma_t(r) = -\frac{b'r+b}{2r^3} + 2\pi\lambda\left(1 \pm \sqrt{1-\frac{b'}{\pi\lambda r^2}}\right). \tag{36}$$

Note that the traceless nature of the Weyl term is obeyed, $\mathcal{E}^\mu{}_\mu = -\epsilon + \sigma_r + 2\sigma_t = 0$, as expected.

One may now consider specific cases for the form function. For instance, consider the "spatial Schwarzschild" solution, with $b(r) = r_0$. The negative sign in Eq. (33) provides the vacuum $\rho = 0$, where $\mathcal{E}_{\hat\mu\hat\nu} = \mathrm{diag}(0, \sigma_r, -\sigma_r, -\sigma_r)$ with $\sigma_r = r_0/r^3$. This is a case analyzed in Ref. [7]. Now, considering the positive sign, so that $\rho = \lambda$, and the projected Weyl tensor components reduce to

$$\epsilon(r) = 12\pi\lambda, \qquad \sigma_r(r) = 4\pi\lambda + \tfrac{r_0}{r^3}, \tag{37}$$

$$\sigma_t(r) = 4\pi\lambda - \tfrac{r_0}{r^3}. \tag{38}$$

Note that this corresponds to a nonasymptotically flat solution, which, in this context, does not have a correspondence in general relativity.

B. Linear equation of state

As we are imposing an isotropic pressure NEC nonviolating distribution of matter on the brane, one may generalize the latter dust solution. As an example, consider the following linear equation of state, $p = \omega\rho$. In addition to this, we shall assume that the energy conditions are satisfied. In particular, the weak energy condition (WEC), which states that for a diagonal stress-energy tensor, we have $\rho \geq 0$ and $\rho + p \geq 0$. The strong energy condition (SEC) states that $\rho + p \geq 0$ and $\rho + 3p \geq 0$. The latter conditions then impose the following inequalities: $1+\omega \geq 0$ and $1 + 3\omega \geq 0$.

Now, Eq. (19) provides the following relationship

$$R = 8\pi\alpha\rho - 8\pi\frac{\beta}{2\lambda}\rho^2, \tag{39}$$

where

$$\alpha = 1 - 3\omega, \tag{40}$$

$$\beta = \tfrac{2}{3}(1 + 3\omega). \tag{41}$$

From the imposition of the SEC, considered above, we verify that $\beta \geq 0$. Specifying the functions $b(r)$ and $\Phi(r)$, and using Eq. (17), where the Ricci scalar as a function of the r-coordinate is considered, $R = R(r)$, the energy density $\rho(r)$ is finally given by

$$\rho = \frac{\lambda}{3\beta}\left(\alpha \pm \sqrt{\alpha^2 - \frac{3\beta R}{4\pi\lambda}}\right). \tag{42}$$

Note that a generic restriction is imposed, i.e., $\alpha^2 \geq \frac{3\beta R}{4\pi\lambda}$. Now, as the energy density is positive, $\rho \geq 0$, one needs to analyze two cases: (i) For the positive sign, if $\alpha \geq 0$, then the imposition of the above generic condition suffices; if $\alpha \leq 0$, then one needs to impose $\frac{3\beta R}{4\pi\lambda} \leq 0$. (ii) For the negative sign, the case of $\alpha < 0$ is ruled out; if $\alpha \geq 0$, then the additional restriction $\frac{3\beta R}{4\pi\lambda} \geq 0$ is imposed. Finally, the components of $\mathcal{E}_{\mu\nu}$ are provided by Eqs. (14)–(16).

In this context, one may also write down a total effective equation of state, $\omega^{\rm eff} = p_r^{\rm eff}/\rho^{\rm eff}$, using the linear equation of state outlined above. Thus, $\omega^{\rm eff}$ is given by

$$\omega^{\rm eff} = \frac{\omega(1 + \frac{\rho}{\lambda}) + \frac{\rho}{2\lambda} - k^2\sigma_r/\rho}{1 + \frac{\rho}{2\lambda} - k^2\epsilon/\rho}, \tag{43}$$

and in order to violate the NEC, one needs to impose $\omega^{\rm eff} < -1$ at the throat, consequently mimicking a traversable wormhole supported by phantom energy [10].

C. Asymptotically flat spacetime

In this section, we shall consider the second strategy following the analysis of Eq. (22). For simplicity, a constant redshift function, $\Phi' = 0$, is imposed, so that Eq. (22) reduces to

$$b(r) = r_0 + \frac{1}{2}\int_{r_0}^{r} R(\bar{r})\bar{r}^2 d\bar{r} \tag{44}$$

Consider once again, the dust solution with a specific choice for the energy density, given by the following relationship

$$\rho = \frac{\lambda}{2}\left(1 - \sqrt{1 - \frac{\gamma^2 r_0^2}{\pi\lambda r^4}}\right). \tag{45}$$

We shall only consider the negative sign, to allow $\rho \to 0$ at spatial infinity. Note that the wormhole throat radius obeys the inequality $r_0^2 > \gamma^2/(\pi\lambda)$. Now, from Eq. (19), the Ricci scalar is given by

$$R(r) = \frac{2\gamma^2}{r_0^2}\left(\frac{r_0}{r}\right)^4. \tag{46}$$

Substituting this function into Eq. (44), we deduce the following form function

$$b(r) = r_0 + \gamma^2 r_0(1 - \tfrac{r_0}{r}), \tag{47}$$

with $0 < \gamma^2 < 1$, so that one obtains an asymptotically flat wormhole solution with $b'(r) = \gamma^2 r_0^2/r^2$, and at the throat we have $b(r_0) = r_0$ and $b'(r_0) = \gamma^2 < 1$.

The projected Weyl tensor components are provided by Eqs. (34)–(36), by substituting the functions $b(r)$ and $b'(r)$, and are given by the following relationships

$$\epsilon(r) = -\frac{2\gamma^2 r_0^2}{r^4} + 6\pi\lambda\left(1 - \sqrt{1 - \frac{\gamma^2 r_0^2}{\pi\lambda r^4}}\right), \tag{48}$$

$$\sigma_r(r) = \frac{r_0}{r^3}\left[1 + \gamma^2\left(1 - \frac{2r_0}{r}\right)\right] + 2\pi\lambda\left(1 - \sqrt{1 - \frac{\gamma^2 r_0^2}{\pi\lambda r^4}}\right), \tag{49}$$

$$\sigma_t(r) = -\frac{r_0(1 + \gamma^2)}{2r^3} + 2\pi\lambda\left(1 - \sqrt{1 - \frac{\gamma^2 r_0^2}{\pi\lambda r^4}}\right), \tag{50}$$

which tend to zero as $r \to \infty$.

IV. CONCLUSION

In this work, we have adopted the viewpoint of a braneworld observer, by considering a general class of wormholes with $R \neq 0$, where R is the four dimensional Ricci scalar. In addition, two strategies were outlined in order to construct general solutions and specific cases were further explored. First, the specific case of dust, with a positive energy density was considered, and several general physical properties and characteristics were analyzed. Second, a linear equation of state of the stress-energy tensor components on the brane were analyzed, and finally, an asymptotically flat wormhole spacetime was found. Now, a fundamental ingredient of traversable wormholes is the violation of the null energy condition (NEC). However, in the context of braneworlds, it is the effective total stress-energy tensor that violates the NEC. Therefore, we have imposed that the stress-energy tensor confined on the brane, threading the wormhole, satisfied the NEC, and it was shown that in addition to the nonlocal corrections of the Weyl curvature in the bulk (as considered in Ref. [7]), local high-energy bulk effects, could leave a NEC violating signature on the brane, thus providing a natural scenario for the existence of traversable wormholes. The question of what 5-dimensional geometry produces this NEC imprint is much more difficult, and was not explored here. It is also important to emphasize another advantage of the analysis outlined in this paper, in generalizing the work of Ref. [7], namely, it is possible to consider a zero Weyl curvature term, and generalize the stress-energy tensor to incorporate an anisotropic pressure contribution on the brane. This latter consideration would also extend and generalize standard general relativistic wormholes with the inclusion of a high-energy contribution.

However, a few remarks are in order, namely, one may basically consider two strategies of obtaining solutions on the brane. First, the bulk spacetime may be given, by solving the full 5-dimensional equations, and the geometry of the embedded brane is then deduced. Second, due to the complexity of the 5-dimensional equations, one may follow the strategy outlined in this paper, by considering the intrinsic geometry on the brane, which encompasses the imprint from the bulk, and consequently evolve the metric off the brane. In principle, the second procedure may provide a well-determined set of equations, with the brane setting the boundary data. However, determining the bulk geometry proves to be an extremely difficult endeavor. Nevertheless, the behavior of the bulk should be analyzed, and found to be nonsingular to inspire any physical meaning in the models considered. Work along these lines is in progress.

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