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Dark energy in hybrid inflation

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The situation that a scalar field provides the source of the accelerated expansion of the Universe while rolling down its potential is common in both the simple models of the primordial inflation and the quintessence-based dark energy models. Motivated by this point, we address the possibility of causing the current acceleration via the primordial inflation using a simple model based on hybrid inflation. We trigger the onset of the motion of the quintessence field via the waterfall field, and find that the fate of the Universe depends on the true vacuum energy determined by choosing the parameters. We also briefly discuss the variation of the equation of state and the possible implementation of our scenario in supersymmetric theories.

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I. INTRODUCTION

The discovery of the acceleration of the Universe by the measurements of the luminosity-redshift relation for type Ia supervonae [1], combined with the observations of the anisotropies in the cosmic microwave background (CMB) [2], confirmed that about 70% of the density of the Universe is made up of an unknown form of energy. This component, usually referred to as "dark energy", is unclumped and smoothly distributed, with an exotic property that it has negative pressure to cause the expansion of the Universe to accelerate. The simplest candidate of dark energy is a nonvanishing, positive cosmological constant [3]. We have to, however, explain why it is nonzero but vanishingly small ($\sim 10^{-12}$ eV^4). It is possible that we can resort to some yet unknown fundamental symmetry which makes the cosmological constant vanishingly small but nonzero, or we might invoke an anthropic consideration that the observed small value of the cosmological constant allows life and that is why we observe it.[1]

An alternative form of dark energy is a slowly rolling scalar field, called quintessence, which has not yet relaxed at its ground state [6]. For recent years, there have been considerable developments in the dynamics of the quintessence field. For example, in the context of so-called tracker solutions [7], the simplest case arises for a potential of the form $V = M^{4+n}\phi^{-n}$, where n is positive and M is an adjustable constant. By choosing M suitably, it is possible to make a transition from an early matter-dominated universe to a later quintessence-dominated universe, free from fine tuning of the initial conditions. Another interesting possibility, k-essence, is to introduce a noncanonical kinetic term for the scalar field [8], which makes the evolution of the scalar field dependent on the background equation of state, explaining why dark energy is dominating now. Anyway, apart from the details, the situation that a scalar field is slowly rolling down its potential is reminiscent of the primordial inflation [9], where a scalar field (the inflaton) provides the vacuumlike energy density ($p \approx -\rho$) necessary for a phase of the accelerated expansion by slowly rolling down its flat potential.

Hence one may naturally raise a question of how we can couple the early accelerated expansion, the primordial inflation, and the current one together [10]. Is it possible to cause the acceleration we observe recently by the primordial inflation? In this paper, we are going to discuss this possibility using a simple model based on the hybrid inflation [11], which arises naturally in many string-inspired inflation models, in particular, in potentials for moduli fields. This paper is organized as follows: in Sec. II, we present a simple model of dark energy and analyze its dynamics in detail, presenting several conditions which should be satisfied for our model to work properly. Unlike the conventional lore that the true minimum of the quintessence potential is presumed to vanish, we find that positive, zero, or negative vacuum energy is possible as we choose a different set of parameters. In Sec. III, we discuss the variation of the equation of state w which might be detected in future observations such as HETDEX [12]. Also we briefly address the possibility of realizing our model in supersymmetric theories and obstacles to overcome. We briefly summarize this paper in Sec. IV.

II. A MODEL OF DARK ENERGY

In this section, we discuss a simple model of dark energy based on the hybrid model of inflation [11]. Perhaps the simplest way to combine the onset of present acceleration of the Universe with the primordial inflation is to directly couple the inflaton with the quintessence field. In this case, however, first we should ensure that inflation lasts for a

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[1]Interestingly, this idea is easily fulfilled in modern string theory landscape [4]. In string theory, there exist many (more than 10^{1500} [5]) different vacua, and some of them may be suitable for some kind of intelligent observers like us.

long enough time (at least 60 e-foldings) without being disturbed by the interaction with the quintessence field. Moreover, the quintessence potential must remain extremely flat after the inflaton reaches its minimum and decays to reheat the Universe. Rather, it is more plausible to couple the quintessence field with some different field which plays no role during the inflaton rolls down its potential and only works to finish inflation. This is what the waterfall field in hybrid inflation model does. Hence we can write the effective potential as

$$V(\phi, \psi, \sigma) = \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\psi^2 - \frac{h^2}{2}\psi^2\sigma^2 + \frac{1}{4\lambda}(M_\psi^2 - \lambda\psi^2)^2 + \frac{1}{4\mu}(M_\sigma^2 + \mu\sigma^2)^2, \quad (1)$$

where we take ϕ as the inflaton field, ψ as the waterfall field, and σ as the quintessence field. As can be seen from the coupled terms above, we will experience two phase transitions, which distinguish different stages of the evolution of the Universe. In the following subsections, we will discuss each stage in detail.

A. First stage: primordial inflation

The effective masses squared of ψ and σ are $g^2\phi^2 - M_\psi^2$ and $-h^2\psi^2 + M_\sigma^2$, respectively. Hence, for $\psi < \psi_c \equiv M_\sigma/h$, the only minimum of the effective potential, Eq. (1), is at $\sigma = 0$. Also, with $\sigma = 0$, for $\phi > \phi_c \equiv M_\psi/g$, the only minimum of the effective potential is at $\psi = 0$. Thus, at the early stage of the evolution of the Universe, ψ and σ are trapped at 0 while ϕ remains much larger than ϕ_c for a long time. This stage lasts until $\phi = \phi_c$, and at that moment we assume that the vacuum energy density $V(0, 0, 0) = M_\psi^4/(4\lambda) + M_\sigma^4/(4\mu)$ is much larger than the potential energy density of the inflaton field $m^2\phi_c^2/2 = m^2M_\psi^2/(2g^2)$, so that the Hubble parameter is given by

$$H^2 \simeq \frac{1}{12m_{\rm Pl}^2}\left(\frac{M_\psi^4}{\lambda} + \frac{M_\sigma^4}{\mu}\right), \quad (2)$$

where $m_{\rm Pl}^2 = (8\pi G)^{-1}$ is the reduced Planck mass. We can additionally assume that the vacuum energy $V(0, 0, 0)$ is dominated by M_ψ^4/λ, i.e.,

$$\frac{M_\psi^4}{\lambda} \gg \frac{M_\sigma^4}{\mu}, \quad (3)$$

so that the subsequent evolutions of the Universe after inflation, e.g., reheating, becomes identical to the usual hybrid model. This immediately gives

$$m \ll \frac{g}{\sqrt{2\lambda}}M_\psi. \quad (4)$$

Now, combining Eqs. (2) and (3), the slow-roll condition for m gives

$$M_\psi^2 \gg 6\sqrt{\lambda}mm_{\rm Pl}. \quad (5)$$

Note that this gives another bound on m as $m \ll M_\psi^2/(6\sqrt{\lambda}m_{\rm Pl})$, and the tightest bound on m depends on the comparison between the prefactors of this expression and Eq. (4), i.e., g and $M_\psi/(3\sqrt{2}m_{\rm Pl})$.

B. Second stage: phase transition between ϕ and ψ

As discussed above, when ϕ becomes smaller than ϕ_c, a phase transition with the symmetry breaking for ψ occurs. To finish inflation as soon as ϕ reaches the critical value ϕ_c, i.e., for the so-called "waterfall", we first require that the absolute value of the effective mass squared of ψ be much larger than H^2. During inflation, using the slow-roll equation of motion $3H\dot{\phi} + m^2\phi = 0$ and Eqs. (2) and (3), we can find that when the inflaton ϕ reaches ϕ_c, it decreases by

$$\Delta\phi = \frac{4\lambda m^2 m_{\rm Pl}^2}{gM_\psi^3} \quad (6)$$

during the time interval H^{-1}. At that time, the absolute value of the effective mass squared of ψ is given by

$$|m_\psi^2| = \frac{8\lambda m^2 m_{\rm Pl}^2}{M_\psi^2}, \quad (7)$$

which is much greater than H^2 for

$$M_\psi^3 \ll 12\sqrt{2}\lambda mm_{\rm Pl}^2. \quad (8)$$

Also, we demand that the time scale for ϕ to roll down from ϕ_c to 0 be much shorter than H^{-1}. Let us denote Δt as the time ϕ takes as it moves from ϕ_c to 0. Then, from

$$\frac{\partial V}{\partial \phi} = m^2\phi + g^2\phi\psi^2 \simeq \frac{8g^2m^2m_{\rm Pl}^2\phi}{M_\psi^2}, \quad (9)$$

where we have used Eq. (7) and the slow-roll equation of ϕ, we have

$$\left|\frac{H}{\Delta t}\right| = \frac{8g^2m^2m_{\rm Pl}^2}{3M_\psi^2} \gg H^2, \quad (10)$$

which gives another condition

$$M_\psi^3 \ll 4\sqrt{2\lambda}gmm_{\rm Pl}^2. \quad (11)$$

Note that the parameters with respect to ϕ-ψ transition are further constrained from the observed COBE amplitude of density perturbations, therefore we obtain [11]

$$M_\psi^3 \ll 5 \times 10^{-5}\lambda g^{-1}mm_{\rm Pl}, \quad (12)$$

which looks similar to Eqs. (8) and (11). Combining Eqs. (8), (11), and (12), we can extract the valid range of the parameters m, M_ψ, g and λ; e.g. if we take $g^2 \sim \lambda \sim 10^{-1}$, $m \sim 10^2$ GeV and $M_\psi \sim 10^{11}$ GeV, all the conditions are satisfied. After this phase transition, the waterfall field ψ oscillates at the minimum and decays so that the Universe is reheated.[2].

With these conditions, as soon as ϕ reaches ϕ_c, inflation ends within a Hubble time, H^{-1}. Also note that when such a rapid phase transition occurs within observationally interesting range, still we can calculate the power spectrum and the spectral index for the density perturbations [14] by generalizing the standard perturbative method [15]. Generally, the power spectrum shows a scale dependent oscillations after the phase transition [16].

C. Third stage: phase transition between ψ and σ

While ψ is rolling down to $\psi_0 \equiv M_\psi/\sqrt{\lambda}$, the effective mass squared of σ might become negative then another phase transition occurs. For this to happen, we want $\psi_c < \psi_0$, unless no instability for σ would develop. This is equivalent to the condition[3]

$$\frac{M_\sigma}{h} < \frac{M_\psi}{\sqrt{\lambda}}. \tag{13}$$

Note that we can obtain the same condition by requiring that the minimum along σ direction, σ_0 given by

$$\left.\frac{\partial V}{\partial \sigma}\right|_{\sigma_0} = -h^2\psi^2\sigma_0 + M_\sigma^2\sigma_0 + \mu\sigma_0^3 = 0, \tag{14}$$

be real, i.e.,

$$\sigma_0^2 = \frac{h^2 M_\psi^2 - \lambda M_\sigma^2}{\lambda\mu} > 0. \tag{15}$$

Once the above condition is satisfied and σ rolls down the effective potential, after all the fields are settled at

$$\phi = 0, \qquad \psi = \psi_0, \quad \text{and} \quad \sigma = \sigma_0, \tag{16}$$

where the potential becomes

$$V_0 = \frac{h^2 M_\psi^2}{4\lambda^2\mu}(2\lambda M_\sigma^2 - h^2 M_\psi^2). \tag{17}$$

1. Positive vacuum density

When $V_0 > 0$, Eq. (17) is greater than zero and it corresponds to a nonzero, positive vacuum energy. Hence no matter σ evolves quickly or not, we are provided with the source of the current acceleration of the Universe in this case, and this acceleration will last forever. The Universe will eventually behave as a de Sitter space. Note that the possible maximum value of V_0, $h^2 M_\psi^2 M_\sigma^2/(4\lambda\mu)$, could be at most as large as the current critical density, $\rho_{\rm crit} \sim (10^{-12}\ \text{GeV})^4$. However, an extreme fine tuning is required to match this value; if we restrict our interest to this case only, our model is no better than the conventional ΛCDM because the latter is simpler and hence preferable. Anyway here we do not try to improve this situation. A study of alleviating this fine tuning problem is very important and interesting, but is outside the scope of the present paper.

2. Zero vacuum density

For the case

$$2\lambda M_\sigma^2 = h^2 M_\psi^2, \tag{18}$$

Eq. (17) is exactly zero and no vacuum energy exists. This case corresponds to the usual quintessential inflation [10], where the cosmological constant Λ is assumed to be zero due to some unknown symmetry, and the observed dark energy is supplied by e.g. another scalar field, here σ. To be able to explain the acceleration of the Universe, we require that still σ be rolling down so that the potential is nonzero. That is, σ should roll extremely slowly. Note that although σ begins to evolve only after ψ reaches ψ_c, for the most time during σ rolls down the effective potential to σ_0, ψ has already settled at ψ_0 within a time $\Delta t \ll H^{-1}$ as soon as $\phi \sim \phi_c$. Thus, we just set $\psi = \psi_0$ throughout the evolution of σ.

Under this circumstance, when σ begins to roll, H is given by

$$H^2 \simeq \frac{M_\sigma^4}{12\mu m_{\rm Pl}^2}. \tag{19}$$

For σ to slowly evolve, we require that $|m_\sigma^2|$, the absolute value of the effective mass squared of σ, be much smaller than H^2, from which we obtain a condition

$$M_\sigma \gg 6\sqrt{\mu}\, m_{\rm Pl}, \tag{20}$$

where we have used Eq. (18). Indeed, the same bound could be found when we also require that σ roll very slowly so that for σ to move by an infinitesimal displacement $\Delta\sigma$, it takes much longer time than H^{-1}. The effective potential is given by

$$\begin{aligned} V(\phi = 0, \psi = \psi_0, \sigma) &= \frac{M_\sigma^4}{4\mu} + \frac{1}{2}\left(M_\sigma^2 - \frac{h^2}{\lambda}M_\psi^2\right)\sigma^2 \\ &\quad + \frac{1}{4}\mu\sigma^4 \\ &= \frac{\mu}{4}\left(\sigma^2 - \frac{M_\sigma^2}{\mu}\right)^2, \end{aligned} \tag{21}$$

where we have used Eq. (18). Hence, combining with the

[2]At this point, although we require that small enough be the coupling h^2 of σ to ψ, and in turn to the thermal bath, the oscillation of ψ may affect the dynamics of σ. However, we note that because of the negative sign of their interaction term in Eq. (1), the pattern of instability is quite different from the usual parametric resonance [13] even though the interaction is strong enough. We will study this issue in more detail separately.

[3]It should be guaranteed that the effective potential is bounded from below so that ψ is settled at ψ_0. When ψ moves along $\psi = \sigma$ direction, this is equivalent to the condition $\lambda > 2h^2$. This means, combined with Eq. (13), M_ψ is (much) bigger than M_σ. We are grateful to Andrei Linde for pointing out this.

slow-roll equation

$$3H\dot{\sigma} + \frac{\partial V}{\partial \sigma} \simeq 0, \tag{22}$$

we find

$$\frac{\dot{\sigma}}{\sigma} \simeq \frac{M_\sigma^2}{3H}, \tag{23}$$

i.e. $\delta\sigma/\sigma \simeq M_\sigma^2/(3H^2)$ for $\delta t \simeq H^{-1}$. Requiring $\delta\sigma/\sigma \ll 1$ gives

$$M_\sigma \gg 6\sqrt{\mu} m_{\rm Pl}, \tag{24}$$

which is the same as Eq. (20).

3. Negative vacuum density

Recent observations find that the vacuum energy, or cosmological constant, is very small and positive, with its value being of order 10^{-12} eV^4. The case of negative cosmological constant is not observationally justified, which seems to support the claim that the true minimum of the quintessence potential is zero. The vacuum state with negative energy is, nevertheless, an interesting theoretical possibility, especially in string theories where anti de Sitter solutions are popular. Hence a negative potential may play an important role in cosmology motivated from string theory or M theory, such as cyclic universe model [17].

We obtain $V_0 < 0$ when $V(\phi = 0, \psi = \psi_0, \sigma = \sigma_t) = 0$ has a real solution and that this solution is smaller than σ_0. If we take the limit $h^2 M_\psi^2/\lambda \gg M_\sigma^2$, the effective mass squared of σ is given by

$$m_\sigma^2 \simeq -\frac{h^2 M_\psi^2}{\lambda}. \tag{25}$$

Imposing the slow-roll condition, this gives

$$M_\sigma \gg 6\sqrt{\mu} m_{\rm Pl}, \tag{26}$$

the same as Eqs. (20) and (24).

Since the dark energy observed now is positive definite, σ should be still evolving and not have crossed σ_t, thus we again require that σ roll down very slowly, from which we obtain the same conditions as the previous section, Eqs. (20) and (24). However, unlike the cases before, the true vacuum energy is negative and after a (tremendously) long time the Universe will collapse eventually [18], no matter how small the magnitude is.

III. DISCUSSIONS

A. The equation of state

Generally, for evolving dark energy models the equation of state w is not a constant but a slowly evolving function. To provide an acceleration of the Universe it is constrained to be smaller than $-1/3$, usually taken to be $w < -2/3$. Recent combined observations of WMAP and SDSS constrains w to be -1, with uncertainties at the 20% level [19]. For the first case discussed in the previous section where $V_0 > 0$, this is easily satisfied because w is very slowly varying as σ rolls down its potential, finally becoming exactly -1 at the absolute minimum V_0. When $V_0 = 0$, w will be eventually 0 after σ settles at σ_0. For the third case, however, it is not as trivial as the other cases. Naively, we may take $\dot{\sigma}$ to be nonzero although very small, then we can write

$$w = \frac{1 - V/K}{1 + V/K}, \tag{27}$$

where K stands for the kinetic energy, $\dot{\sigma}^2/2$. At first look, it seems that w is divergent at $V = -K$ and a discontinuity appears and moreover a phantom phase $w < -1$ exists after that point. Let us more closely see this.

The simplest way is to solve the relevant equations numerically. We choose

$$\ddot{\sigma} + 3\frac{\dot{a}}{a}\dot{\sigma} + \frac{\partial V}{\partial \sigma} = 0, \qquad \frac{\ddot{a}}{a} = \frac{1}{3m_{\rm Pl}^2}(V - \dot{\sigma}^2), \tag{28}$$

to solve for $\sigma(t)$ and $a(t)$. In Fig. 1, we plot the resulting equation of state w where, as is shown, it oscillate. To understand this behavior first let us consider the expansion of the Universe with such a negative potential [18]; the Universe keeps expanding until the moment $\rho = \dot{\sigma}^2/2 + V = 0$ where the scale factor reaches its largest value, and the Universe shrinks afterwards. From the Friedmann equation describing a flat universe $H^2 = \rho/(3m_{\rm Pl}^2)$ which has no solution with $\rho < 0$,[4] we can see that to maintain a real value of H no matter positive or negative, ρ should remain positive. Hence, when V is negative, the kinetic energy must compensate this negative potential energy and consequently these energies oscillate out of phase. For example, at the negative bottom of the potential, the field is moving fastest so that we have maximum kinetic energy here and w becomes very large. This is the reason why the equation of state w is oscillating[5] and sometimes becomes greater than 1, the "stiff" fluid. We can easily find the relation with respect to the redshift z, by making use of the relation $a \propto (1+z)^{-1}$ by solving the equations

$$\begin{aligned}
&\frac{\dot{a}}{a}(1+z)a' = -\dot{a}, \qquad &&\frac{\dot{a}}{a}(1+z)(\dot{a})' = \frac{a}{3m_{\rm Pl}^2}(\dot{\sigma}^2 - V),\\
&\frac{\dot{a}}{a}(1+z)\sigma' = -\dot{\sigma}, \qquad &&\frac{\dot{a}}{a}(1+z)(\dot{\sigma})' = -3\frac{\dot{a}}{a}\dot{\sigma} - \frac{\partial V}{\partial \sigma},
\end{aligned} \tag{29}$$

where a prime denotes the derivative with respect to z, we can obtain w in terms of z. A typical relation between w and z is shown in the right panel of Fig. 1.

[4] This is the reason why we cannot reach an anti de Sitter universe dominated by a negative cosmological constant [20].

[5] Note that this oscillation occurs only when the potential is bounded from below. If this is not the case, e.g. when $V = V_0 - m^2\phi^2/2$, we do not see such an oscillation.

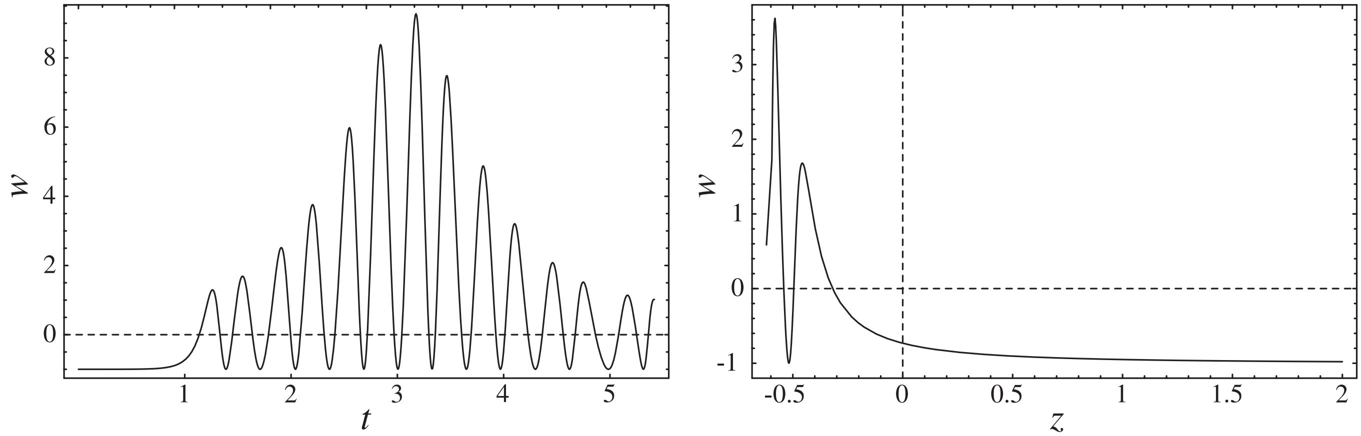

FIG. 1. (Left) plot of w versus time in the unit of t_0 which corresponds to $z = 0$, and (Right) w versus redshift z. As σ approaches σ_t where $V = 0$, w increases and becomes positive. When σ reaches the minimum of its potential which is negative, σ oscillates around this minimum and w shows the corresponding oscillatory behavior. When the Universe expands no more and begins to contract, the amplitude of this oscillation decreases.

B. Supersymmetric realization

Supersymmetry is believed to be the most promising candidate related to the fundamental problems in particle physics, such as the hierarchy problem and the gauge coupling unification. Since we have not yet observed any supersymmetric partner of known particles, supersymmetry is broken. In the primordial universe where the energy scale is much higher than the present one, however, the rich structure of supersymmetric and supergravity theories should have played an important role. Hence it is natural to try to implement inflationary scenario within supersymmetry. There has been an encouraging progress on the hybrid inflation scenario in the context of supersymmetric theories [21–23]. For example, consider a simple superpotential [23]

$$W = \phi(\lambda\psi_1\psi_2 - \mu^2), \tag{30}$$

where ψ_1 and ψ_2 are a pair of superfields in nontrivial representations of some gauge group under which ϕ is neutral. Then, in a globally supersymmetric theory, the effective potential is given by

$$V = \lambda^2|\phi|^2(|\psi_1|^2 + |\psi_2|^2) + |\lambda\psi_1\psi_2 - \mu^2|^2. \tag{31}$$

Here, the absolute supersymmetric minimum appears at $\phi = 0$, $\psi_1 = \psi_2 = \mu/\sqrt{\lambda}$. However, for $\phi > \phi_c = \mu/\sqrt{\lambda}$, ψ_1 and ψ_2 obtain positive masses squared and hence are confined at the origin. By simply adding a mass term $m^2\phi^2/2$ which softly breaks supersymmetry, we see that the hybrid inflation scenario is possible. If we take into account radiative corrections instead, the total effective potential including one-loop corrections is

$$V = \mu^4\left[1 + \frac{\lambda^2}{8\pi^2}\ln\left(\frac{\phi}{\phi_c}\right) + \cdots\right] \tag{32}$$

when $\phi \gg \phi_c$, and we can see that still inflation is possible.

Similarly, we can hope to implement our scenario within supersymmetry. The simplest possibility should be D-terms since, as we can see from Eq. (1), the coupled term of ψ and σ has a negative sign, so that an instability for σ could be developed as ψ, initially confined at zero, rolls away from the origin. This is easily achieved by assuming that ψ and σ are oppositely charged under some gauge symmetry. If it is a $U(1)$ symmetry, we can write the D-term contribution as

$$\frac{\alpha^2}{2}(-|\psi|^2 + |\sigma|^2 + \xi_1)^2, \tag{33}$$

where we assumed that ψ and σ have charges equal to -1 and $+1$ respectively, α is the gauge coupling, and ξ_1 is a Fayet-Iliopoulos D-term. We may also include another gauge symmetry under which ϕ and ψ are charged with the same sign, then D-term is given by

$$\frac{\beta^2}{2}(|\phi|^2 + |\psi|^2 + \xi_2)^2, \tag{34}$$

where we have introduced another gauge coupling β and Fayet-Iliopoulos term ξ_2. Taking only Eqs. (33) and (34) into account, the coupled terms are given by

$$[\beta^2|\phi|^2 - (\alpha^2\xi_1 - \beta^2\xi_2)]|\psi|^2 + (-\alpha^2|\psi|^2 + \alpha^2\xi_1)|\sigma|^2, \tag{35}$$

and we can reproduce the same forms for ψ^2 and σ^2 terms as Eq. (1) as long as $\alpha^2\xi_1 - \beta^2\xi_2$ is positive. This sheds some light on the realization of Eq. (1).

However, this is far from a realistic possibility. First of all, we have not considered any F-term contributions. We do not want to couple ϕ and σ to guarantee their desired behaviors on the flat enough effective potential, but it seems difficult to achieve this through the contributions from F-term. More fundamentally, in globally supersymmetric theories the scalar potential is either positive or zero, which does not include the interesting case of $V_0 < 0$. Also, it is thought that below the Planck scale particle physics is described by an effective $N = 1$ supergravity theory derived from string theory. Hence, a more detailed analysis of our model and associated problems should be addressed in the context of supergravity. For example, the mass of the field should be very small, of order the present Hubble parameter $H_0 \sim 10^{-42}$ GeV, so that it is still rolling toward its true minimum. However, usually scalar fields acquire masses of order the gravitino mass $m_{3/2}^2$, much heavier than H_0. One way to evade this difficulty is to use pseudo Nambu-Goldstone bosons[6] [24], e.g., string/M theory axion, as σ field [25]. We will leave such an analysis as a challenging future work.

IV. SUMMARY

We have investigated a simple dark energy model based on hybrid inflation. The quintessence field σ is coupled to the waterfall field ψ so that as ψ rolls towards $\psi_0 = M_\psi/\sqrt{\lambda}$, σ begins to move along the effective potential provided that Eq. (13) is satisfied. A number of bounds on the parameters of the model are found, such as Eqs. (3)–(5), (8), and (11). An interesting point is that the true minimum of the effective potential, Eq. (17), depends on our choice of the parameters, allowing the vacuum state with positive, negative and zero energy. When it is negative, the Universe will eventually collapse, showing an oscillation in the equation of state w for dark energy which we can hope to detect in future observations. This model could be realized in supersymmetric theories via D-term contributions, but including F-term parts and supergravity effects makes this model not easy to be achieved.

[6]Note that this also naturally solves the η problem associated with generic scalar field potentials in supergravity.

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