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Seeding of primordial perturbations during a decelerated expansion

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A scalar field with a modified dispersion relation may seed, under certain conditions, the primordial perturbations during a decelerated expansion. In this paper we examine whether and how these perturbations can be responsible for the structure formation of the observable universe. We discuss relevant difficulties and possible solutions.

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I. INTRODUCTION

Recently, lots of observations have paid attention to the nature of primordial perturbations that gave rise to the inhomogeneities observed in the Universe. The results of these measurements are consistent with an adiabatic and nearly scale invariant spectrum of primordial perturbations, as predicted by the simplest models of inflation [1]. However, due to the central role of primordial perturbations on the formation of cosmological structure, it is still very interesting and might also be desirable to explore various and possible origins of primordial perturbations.

The inflation stage is supposed to have taken place at the earlier moments of the Universe [2,3], which superluminally stretched a tiny patch to become our observable universe today, and in the meantime makes the quantum fluctuations in the horizon leave the horizon to become the primordial perturbations responsible for the formation of cosmological structure [4,5]. This is one of the remarkable successes of inflation, see also the superinflation, e.g. Refs. [6–9], in which the null energy condition is broken. In Ref. [6], it was first noticed that there is an interesting case in the generating phases of primordial perturbations, in which the scale factor and thus the wavelengths of perturbations grow very slowly but the Hubble length rapidly shrinks. The inflation can be generally regarded as an accelerated or superaccelerated stage, and so may be defined as an epoch when the comoving Hubble length decreases. This length starts out very large, and then the inflation forces it to shrink enough so that the perturbations can be generated causally. In a decelerated expanding background the comoving Hubble length is increased, thus in this case it seems hardly possible to causally explain the origin of primordial perturbations. The variable speed of light [10,11] has been considered, however, also see Ref. [12] for a reexamination. Note that it has been illustrated in Ref. [13] that the existence of adiabatic perturbations on scales much larger than the Hubble radius implies that either inflation occurred in the past, the perturbations were there as initial conditions, or causality is broken. Thus if we want to obtain the primordial perturbations in a decelerated expanding phase, we have to require that the scalar field responsible for the perturbations should have some special or modified dispersion relation. Though there have been many detailed descriptions how the required dispersion relations are obtained from the effective field theory [14], it will be still significant to examine the feasibility of this seeding mechanism matched to the observable cosmology.

The outline of this paper is as follows. In Sec. II, we will show how the primordial perturbation may be generated during a decelerated expanding phase when the dispersion relation is modified. In this case, the perturbation spectrum is calculated in Sec. III. We discuss relevant difficulties in matching the spectrum to observable cosmology and possible solutions. Finally, we summarize and discuss our results, as well as give some comments on future issues.

II. GENERATION OF SPECTRUM

In this section we will begin with a general discussion on the generation of causal primordial perturbations. The generation of primordial perturbations requires that the perturbation modes can leave the horizon during their generation and then reenter the horizon at late time. Thus it may be convenient to define

$$\mathcal{N} \equiv \ln\left(\frac{k_e}{k}\right) \equiv \left(\frac{a_e h_e}{ah}\right), \tag{1}$$

which measures the efolding number of mode with some scale $\sim k^{-1}$ which leaves the horizon before the end of the generating phase of perturbations, see Ref. [15], where k is the comoving wave number, and the subscript 'e' denotes the end time of the generating phase of perturbations; thus k_e is the last mode to be generated, and $h \equiv \dot{a}/a$ is the Hubble parameter, where the dot denotes the derivative with respect to the cosmic time. When taking $ah = a_0 h_0$, where the subscript "0" denotes the present time, we will obtain the efolding number required by observable cosmology. In this case, Eq. (1) is actually the ratio of the physical wavelength corresponding to the present observable scale to that at the end of the generating phase of perturbations.

The evolution of the scale factor in the expanding background can be simply taken as $a(t) \sim t^n$, $(t \to \infty)$ and $a(t) \sim (-t)^n$, $(t \to 0_-)$. We will assume that n is a constant for simplicity. In the conformal time η, we obtain $a(\eta) \sim (-\eta)^{n/(1-n)}$. Thus we have

$$a \sim \left(\frac{n}{(n-1)ah}\right)^{n/(1-n)}. \tag{2}$$

To produce the efolding number, i.e. $\mathcal{N} > 0$, ah must increase with time, i.e. $\ddot{a} > 0$. This suggests that $n > 1$ for $a(t) \sim t^n$, which corresponds to the accelerated expanding phases, in which $\dot{h} < 0$, and $n < 0$ for $a(t) \sim (-t)^n$, which corresponds to the superaccelerated expanding phases, in which $\dot{h} > 0$, e.g. see Ref. [16].[1] Taking the logarithm in both sides of (2), we obtain

$$\ln\left(\frac{1}{ah}\right) = \left(\frac{1-n}{n}\right)\ln a. \tag{3}$$

This equation can be also applied to the expansion with arbitrary constant n. We plot Fig. 1, in which, as well as in the whole paper, we have assumed that after the generating phases of primordial perturbations ends, the "reheating" will rapidly occur and then bring the Universe back to the usual Friedmann-Robertson-Walker (FRW) evolution.[2] Here the reheating means a transition that the fields or fluids dominating the background decay into radiation.

We can see in Fig. 1 that in principle one cannot obtain the primordial perturbations in the decelerated expanding phases in which $0 < n < 1$, however, introducing the modified dispersion relation can change this point. The usual dispersion relation may be generally expected to receive some corrections with the increase of energy, which in some sense can be regarded as a phenomenological description of high energy new physics, see Refs. [19,20]. These modifications have been applied to the early universe, especially the inflation cosmology, in which the modified dispersion relation can significantly affect the spectrum of primordial perturbations generated during inflation [21,22]. The modified dispersion relation can also naturally arise from a generally covariant scalar field [23]. Following Ref. [14], in the conformal time, one can introduce a projector $h_{\mu\nu}$, which projects onto the space orthogonal to a timelike vector $u_\mu = (1/a, 0, 0, 0)$ and satisfies $h_{\mu\nu}h^\nu_\rho = h_{\mu\rho}$ and $h_{\mu\nu}u^\nu = 0$. Thus one may write $h_{\mu\nu}$ as $h_{\mu\nu} = a^2 \cdot (0, 1, 1, 1)$. By the help of the projection tensor, we can define a spatial derivative as $\mathcal{D}_\mu = h^\nu_\mu \nabla_\nu$, which is orthogonal to u_μ and so has only spatial components, and a time derivative $(\delta^\nu_\mu - h^\nu_\mu)\nabla_\nu$. In principle, by combining these two generally covariant derivatives, one can obtain any combination of time and spatial derivatives, so can any dispersion relation when acting on a scalar field. For example, if the spatial component of the scalar field Lagrangian is as follows $\sim\varphi(\mathcal{D}_\mu\mathcal{D}^\mu)^q\varphi/a^p$, where p and q are constant, we will have the dispersion relation

$$\omega = \frac{k^q}{a^p}. \tag{4}$$

Here for our purpose we will not pay more attention to the relevant discussions on the modified dispersion relation. In the following we will focus on the primordial perturbations of the scalar field with the dispersion relation (4) in a decelerated expanding background.

We first begin with a simple modification as follows $\omega = k/a^p$, where p is the constant, and when $p = 0$, it recovers to the normal one. Note also that this case in some sense is similar to that of the fluid with the decaying sound

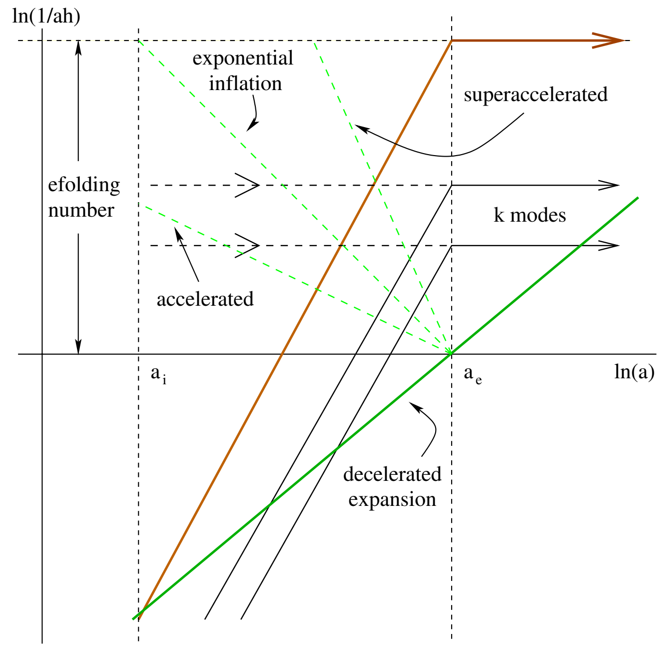

FIG. 1 (color online). The sketch of the evolution of $\ln(1/ah)$ with respect to the scale factor $\ln a$ during the generation of perturbations. The left side of a_e is the generating phase of perturbations, in which in the region above the $\ln a$ axis the dashed line corresponding to the usual exponential inflation with $|n| \to \infty$ divides this region into the superaccelerated phase (upper right) in which $n < 0$ and the accelerated phase (lower left) in which $n > 1$. The perturbation modes (dashed lines) with the wave number k can leave the Hubble horizon during their generations and then reenter the horizon during the radiation/matter domination at late time. In principle it is hardly possible to obtain the causal primordial perturbations in a decelerated expanding phase in which $0 < n < 1$, see the region below the $\ln a$ axis, since there is nothing leaving the horizon during their evolutions. However, when we introduce the scalar field with the modified dispersion relation $\omega = k/a^p$, the similar case to inflation can be imitated in a decelerated expanding phases. In this case the comoving wave number of perturbations will not unchange any more. This makes the evolution of their physical wavelengths able to be faster than that of $1/h$ when the condition $n(p+1) - 1 > 0$ is satisfied.

[1]The limit case of $n \simeq 0_-$ can be very interesting [6] and was studied in detail in the island universe model [17,18].

[2]The superaccelerated phase will generally evolve to a big rip at late time, unless there is not an exit mechanism or reheating.

speed [24,25]. To make a match to the observable cosmology, it is convenient to define an equivalent "efolding number." Note that with the above modification to the dispersion relation, the effective comoving wave number ω will not unchange anymore during its evolution and has an extra suppression lead by a^p, and thus an increasing $\sim a^p$ of the effective comoving wavelength, which directly affects the physical wavelength of the correspondent mode. What the efolding number required by observable cosmology actually reflects is the ratio of the physical wavelength corresponding to the present observable scale to that at the end of the generating phase of perturbations, thus the change of physical wavelength induced by the shift of the comoving wave number must be included in the definition of the efolding number. Thus similar to Eq. (1), the equivalent efolding number can be written as

$$e^{\mathcal{N}} \equiv \left(\frac{a_e}{a}\right)^p \cdot \left(\frac{k_e}{k}\right). \tag{5}$$

From Eq. (2), we have $\ln(k_e/k) = (n-1)\ln(h/h_e)$. Thus substituting it and Eq. (2) into Eq. (5), we can obtain $\mathcal{N} = (np + n - 1)\ln(h/h_e)$. For an expanding universe with $0 < n < 1$, we can generally have $h > h_e$. Thus to make $\mathcal{N} > 0$, which is required by the generation of primordial perturbations, $n(p+1) - 1 > 0$ must be satisfied. This can be reduced to $n > 1$ for $p = 0$, which corresponds to the usual inflationary cases. In the expanding process, for the field with the normal dispersion relation, the physical wavelength of its modes $\sim a$, and only when the evolution of a is faster than that of $1/h$, can the primordial perturbations be generated, which can only be implemented in the cases of $n > 1$ and $n < 0$ (here $h < h_e$). However, for the field with the modified dispersion relation k/a^p, the physical wavelength of correspondent modes is $\sim a \cdot a^p$, thus even if for $0 < n < 1$ it is also possible that the evolution of physical wavelength is faster than that of $1/h$, which means that the correspondent modes can leave the horizon and thus the generation of primordial perturbation, see Fig. 1 for an illustration. In fact the condition $n(p+1) - 1 > 0$ means that in Fig. 1 the lines of k modes must have been intersected with that of $\ln(1/ah)$, in the past. If $n(p+1) - 1 \simeq 0$, the intersection will be expected to be in an infinite far position of the lower left side of Fig. 1, and in this case the scale of h will be required to be very high. Note that h can be taken to the Planck scale at most and in principle h_e has also a lower limit, thus generally $n(p+1) - 1$ should be far away from 0 in order to obtain enough efolding number.

III. CALCULATIONS OF SPECTRUM

In this section we will calculate the primordial perturbations spectrum of the scalar field with modified dispersion relation. We assume that this scalar field φ does not affect the evolution of the background. In the momentum space, the motion equation of φ is given by

$$u_k'' + (\omega^2 - f(\eta))u_k = 0, \tag{6}$$

where u_k is related to the perturbation of φ by $u_k \equiv a\varphi_k$, the prime denotes the derivative with respect to η, and ω is given by Eq. (4) with $q = 1$. Generally since $\omega = k/a^p \sim k/(-\eta)^{n/(1-n)}$, which is different from the usual case with $\omega = k$ constant, by using the mathematics handbook about the deformed Bessel equation, $f(\eta)$ is required to be written as

$$f(\eta) \equiv \frac{\upsilon^2 r^2 - 1/4}{\eta^2}, \tag{7}$$

where υ is generally required to be nearly constant so that Eq. (6) is solvable, and is determined by the evolution of the background and the details of the φ field, such as its mass and its coupling to the background, and r is determined by the behavior of $\omega(\eta)$. Conventionally, the dispersion relation of the scalar field is $\omega = k$, which corresponds to $q = 1$ and $p = 0$, and thus $r = 1$. When $p \neq 0$, we have

$$r \equiv \frac{n(p+1) - 1}{n - 1}. \tag{8}$$

The general solutions of this equation are the Hankel functions with the order υ and the variable $\omega\eta$. In the regime $\omega\eta \gg 1$, the modes can be regarded as adiabatic. Note that the reason is that $\omega'/\omega \sim 1/\eta$, thus the adiabatic condition $\omega'/\omega^2 \ll 1$ is equivalent to $\omega\eta \gg 1$. Note also that $\eta \sim 1/(ah)$, we have $\omega\eta \sim \omega/(ah) \gg 1$, and thus we obtain $a\omega^{-1} \ll 1/h$, which corresponds to the case that the effective physical wavelength is very deep into the horizon. Thus in this regime we may take

$$u_k \simeq \frac{1}{\sqrt{2\omega(k,\eta)}} \exp\left(-i\int^{\eta} \omega(k,\eta)d\eta\right) \tag{9}$$

as an approximate solution of Eq. (6), which in some sense is similar to the case in which the initial condition can be taken as the usual Minkowski vacuum. Note that $\omega\eta$ will decrease with the expansion. Thus at late time, we can expect $\omega\eta \ll 1$, i.e. $a\omega^{-1} \gg 1/h$. The expansion of Hankel functions to the leading term of $\omega\eta$ gives

$$k^{3/2}|\varphi_k| \sim k^{3/2-\upsilon}, \tag{10}$$

where the other factors without k have been neglected. Thus to obtain the scale invariant spectrum, $\upsilon = 3/2$ is required.

A. Massless case

For the massless scalar field, we have $f(\eta) \equiv a''/a$. Note that $a \sim \eta^{n/(1-n)}$, and then using Eqs. (7) and (8), we can obtain

$$\upsilon = \frac{1}{2}\left|\frac{3n-1}{n(p+1)-1}\right|. \tag{11}$$

If $p = 0$, Eq. (11) will be reduced to the normal case, in which when $|n| \to \infty$ we can obtain $\upsilon = 3/2$ and thus a scale invariant spectrum, which is a familiar result in the inflation. From the above discussions we have known that $n(p+1) - 1$ is required to be larger than 0 for the generation of primordial perturbations. Thus in Eq. (11) taking $\upsilon = 3/2$ required by the scale invariant spectrum, we have

$$n(p+1) - 1 = |n - \tfrac{1}{3}|. \quad (12)$$

Substituting it to $\mathcal{N}$ lead by Eq. (5), we obtain

$$\mathcal{N} = \left| n - \frac{1}{3} \right| \ln\left(\frac{h_i}{h_e}\right), \quad (13)$$

where the subscript 'i' denotes the beginning time of the generating phase of perturbations. Note that in Eq. (13), the efolding number is not related to p, which is also actually valid for the case with arbitrary spectrum, though we are constrained to that with the scale invariant spectrum and then obtain Eq. (13). Thus since $0 < n < 1$, we see that to obtain enough efolding number required by observable cosmology, the ratio h_i to h_e must be large enough. Note that h_i has an upper limit, i.e. the Planck scale, thus it seems that h_e must be taken very low.

The efolding number required is generally determined by the evolution after reheating. In principle the lower the energy density when the generating phase of perturbations is over, the smaller the efolding number required. For an idealistic case, in which after the generating phases of perturbations ends the Universe will rapidly be linked to a usual FRW evolution, one can have $\mathcal{N} \simeq 68.5 + (1/2) \times \ln(h_e/m_p)$ [26]. Instituting it into (13), we can cancel $\mathcal{N}$ and obtain a relation between h_e and n. We plot Fig. 2 in which for various n in the region $0.4 < n < 1$, the $\log(h_i/h_e)$ required are given. We can see that when taking the initial energy scale as a Planck scale and the end scale as a nucleosynthesis scale, in which we have $h_i/h_e \sim 10^{40}$, in order to obtain enough efolding number, $n > 0.6$ is required, while when the end scale lies in the TeV scale, we have $n > 0.8$. We may also consider slightly red spectrum, i.e. $\upsilon > 3/2$, as was favored mildly by Wilkinson Microwave Anisotropy Probe (WMAP) [1]. From Eqs. (5) and (11), we see that the value of n will be required to be larger. These results indicate that generally it seems slightly difficult to satisfy simultaneously the conditions required by enough of an efolding number and the scale invariant spectrum, since the h_e required must be very low and in the meantime n is generally constrained in a cabined region. This makes some familiar phases, e.g. the radiation phase and the matter phase, hardly included in possible applications.

We may introduce the more general dispersion relation, e.g. change the power of k, as in Eq. (4) with $q \neq 1$. However, it seems unhelpful to relax the above difficult, since this only equals bringing a factor proportional to $1/q$ in $\mathcal{N}$ obtained from Eq. (5) and the denominator of Eq. (11) simultaneously. They will be generally set off in the calculations obtaining Eq. (13). Thus to solve the above problem, we must assure that the modifications introduced do not change $\mathcal{N}$ obtained from Eq. (5) and the denominator of Eq. (11) simultaneously.

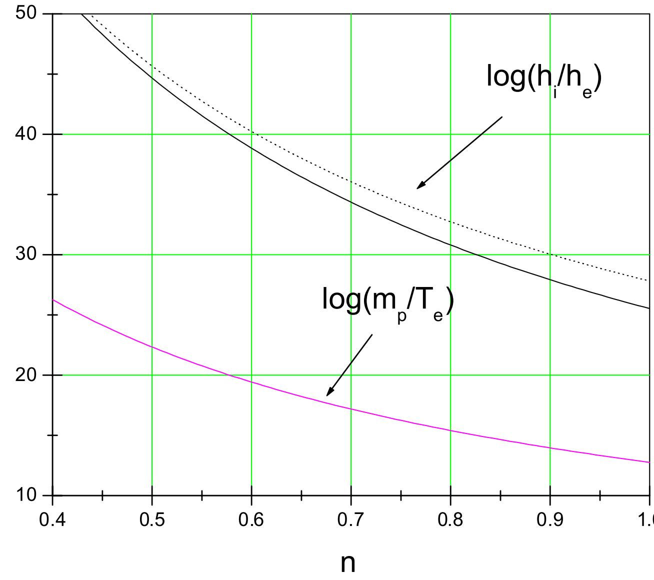

FIG. 2 (color online). The value of the $\log(h_i/h_e)$ with respect to n in order to obtain enough efolding number. The horizontal axis is n. The solid line is the case of $h_i \sim m_p$ and the dashed line is that of $h_i \sim 10^{-4} m_p$. The region above the corresponding line is that with enough efolding number. The lower line corresponds to the maximal reheating temperature at the h_e epoch where h_i is taken as Planck scale.

B. Massive case

When the scalar field is massive, $f(\eta)$ is given by $f(\eta) = \frac{a''}{a} - \mu(ah)^2$, where $\mu \equiv \frac{m_\varphi^2}{h^2}$ has been defined, see Ref. [27] for details on the spectrum of the massive scalar field with normal dispersion relation. Note that $a \sim \eta^{n/(1-n)}$, thus we can obtain

$$f(\eta) = \left[\frac{1}{4}\left(\frac{3n-1}{n-1}\right)^2 - \left(\frac{n}{n-1}\right)^2 \cdot \mu - \frac{1}{4}\right] \cdot \frac{1}{\eta^2}, \quad (14)$$

where $ah(n-1)/n = \eta$ has been used. Note that $h \sim 1/t$, i.e. it generally changes with the time in the decelerated expanding phase, thus μ is generally not constant and so is the numerator of Eq. (14), which will make it very difficult for us to obtain the analytic solution of Eq. (6). Thus we need to fix μ constant, which can be done by introducing a nonminimally coupling $\sim R\varphi^2$ between φ and gravity, where $R \sim h^2$ is the Ricci curvature scalar. In this case, we will have that $m_\varphi^2 \sim R \sim h^2$, and so μ can be a constant. Thus Eq. (6) becomes solvable exactly. From Eq. (14), and then using Eqs. (7) and (8), we can obtain

$$v_m^2 = v^2 - \left(\frac{n}{n(p+1)-1}\right)^2 \cdot \mu \tag{15}$$

which is used to replace v in Eq. (10), where the subscript m denotes the value of v for the massive scalar field. The scale invariance of the spectrum requires $v_m = 3/2$, thus with Eq. (15), we can obtain

$$n(p+1) - 1 = \sqrt{\left(n - \frac{1}{3}\right)^2 - \left(\frac{2n}{3}\right)^2 \cdot \mu}. \tag{16}$$

Note that the term inside the square root in Eq. (16) should be larger than 0, which suggests $\mu < (3/2 - 1/(2n))^2$. For $0 < n < 1$, the range of $(3/2 - 1/(2n))^2$ lies between 0 and ∞. Thus to obtain enough efolding number, $\mu < 0$ is generally required, which corresponds to introducing a scalar field with negative mass term. Substituting Eq. (15) into $\mathcal{N}$ obtained from Eq. (5), we can obtain

$$\mathcal{N} = \left(\sqrt{\left(n - \frac{1}{3}\right)^2 - \left(\frac{2n}{3}\right)^2 \cdot \mu}\right) \ln\left(\frac{h_i}{h_e}\right). \tag{17}$$

Thus one can see that to satisfy the requirements of observable cosmology, for the decelerated expanding phase with $0 < n < 1$, enough efolding number may be obtained by properly selecting the value of μ in the case with fixed n in Eq. (17), while when n and μ are fixed by Eq. (17), the scale invariant spectrum may be obtained by properly matching the value of p in Eq. (15). We plot Fig. 3, in which for arbitrary n in the region $0.4 < n < 1$, in order to obtain enough efolding number, the $-\mu$ required with respect to n is given. We can see that for a different value of n, $-\mu$ lies in an acceptable region and in the meantime h_i/h_e is not required to be very large, which is from the case with the massless scalar field. Note that both cases in Fig. 3 cannot be implemented in the massless field, in which the value of h_i/h_e in Fig. 3 is not so large that it ensures enough efolding number.

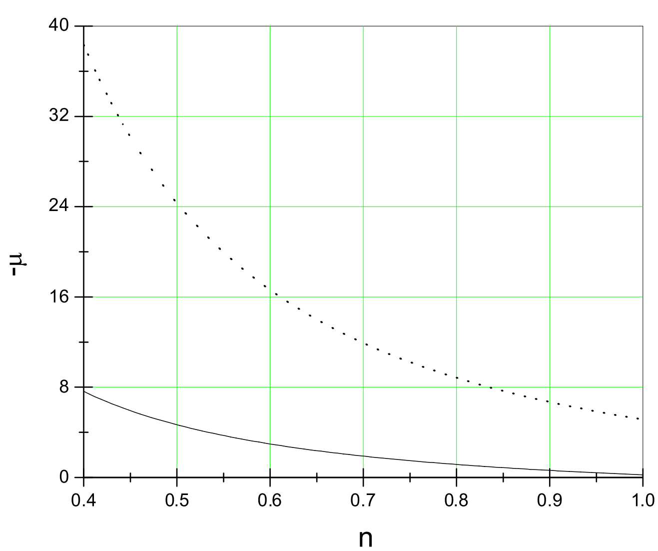

FIG. 3 (color online). The value of $-\mu$ with respect to n in order to obtain enough efolding number. The vertical axis is $-\mu$ and the horizontal axis is n. The solid line corresponds to the case of $h_i \sim 10^{-4} m_p$ and $h_e \sim$ Tev, and the dashed line is that of $h_i \sim m_p$ and $h_e \sim 10^{-9} m_p$.

IV. SUMMARY AND DISCUSSION

The modification of the dispersion relation of the scalar field brings a possibility generating the primordial perturbations in a decelerated expanding background, however, we find that in order to generate a nearly scale invariant spectrum, it will be slightly difficult to obtain enough efolding number required by the observable cosmology in a simple case. But when we consider more general cases, e.g. the massive scalar field, the problem can be relaxed. Thus though the conditions required look slightly special, it seems possible to seed the nearly scale invariant primordial perturbations within a conventional evolution not involving the inflation. These perturbations may be transferred to the curvature perturbations at late time by some mechanisms, e.g. as in Refs. [28,29], thus it may be interesting and responsible for the structure formation of an observable universe. Note that current observations actually favor a red tilt spectrum [1], but not an exact scale invariant one. However, this does not pose any problem here, since we can always set any value of p in Eq. (11) or (15) to obtain the v or v_m required by the red tilt of the spectrum, or even the blue tilt. In principle there is not the generation of the primordial gravitational wave, since the seeding of scalar perturbation occurs during a decelerated expansion, unless the sound speed of the gravitational wave is also time dependent, as in the case of scalar perturbation. Non-Gaussianity is expected to be small. However, we still need a detailed discussion on the gravitational wave and non-Gaussianity in order to match the coming observable tests. In addition, since the energy scale when the generating phase of primordial perturbations ends may be very low to TeV, even the big bang nucleosynthesis (BBN) scale, it is also interesting to study whether there are some observable effects on e.g. baryogenesis. Thus it seems that many significant issues related to this work remain. We expect to come back to these studies in the future.

Finally, it should be pointed out that such a decelerated evolution of the early universe cannot solve all problems of standard cosmology, as has been explained in the inflation models. For example, here initially the homogeneity in the super-Hubble scale must be imposed. Be that as it may, however, this work displays an "unnatural" but possible example seeding a phenomenologically realistic spectrum of primordial perturbations in a nonaccelerated expanding background, which to some extent highlights the fact again that identifying the origin of primordial perturbations may be a much more subtle task than expected.

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[1] D. N. Spergel *et al.*, astro-ph/0603449.
[2] A. H. Guth, Phys. Rev. D **23**, 347 (1981).
[3] A. D. Linde, Phys. Lett. B **108**, 389 (1982); A. A. Albrecht and P. J. Steinhardt, Phys. Rev. Lett. **48**, 1220 (1982).
[4] J. M. Bardeen, P. J. Steinhardt, and M. S. Turner, Phys. Rev. D **28**, 679 (1983).
[5] V. Mukhanov and G. Chibisov, JETP Lett. **33**, 549 (1981); A. H. Guth and S.-Y. Pi, Phys. Rev. Lett. **49**, 1110 (1982); S. W. Hawking, Phys. Lett. B **115**, 295 (1982); A. A. starobinsky, Phys. Lett. B **117**, 175 (1982).
[6] Y. S. Piao and E Zhou, Phys. Rev. D **68**, 083515 (2003).
[7] Y. S. Piao and Y. Z. Zhang, Phys. Rev. D **70**, 063513 (2004).
[8] P. F. Gonzalez-Diaz and J. A. Jimenez-Madrid, Phys. Lett. B **596**, 16 (2004).
[9] M. Baldi, F. Finelli, and S. Matarrese, Phys. Rev. D **72**, 083504 (2005).
[10] J. W. Moffat, Int. J. Mod. Phys. D **2**, 351 (1993).
[11] A. Albrecht and J. Magueijo, Phys. Rev. D **59**, 043516 (1999); J. D. Barrow and J. Magueijo, Phys. Lett. B **443**, 104 (1998).
[12] G. F. R. Ellis and J.-P. Uzan, Am. J. Phys. **73**, 240 (2005).
[13] A. Liddle, Phys. Rev. D **51**, R5347 (1995).
[14] C. Armendariz-Picon, J. Cosmol. Astropart. Phys. 10 (2006) 010.
[15] J. Khoury, P. J. Steinhardt, and Neil Turok, Phys. Rev. Lett. **91**, 161301 (2003).
[16] Y. S. Piao and Y. Z. Zhang, Phys. Rev. D **70**, 043516 (2004); Y. S. Piao, Phys. Lett. B **606**, 245 (2005).
[17] Y. S. Piao, Phys. Rev. D **72**, 103513 (2005).
[18] Y. S. Piao, Phys. Rev. D **74**, 043509 (2006).
[19] W. G. Unruh, Phys. Rev. D **51**, 2827 (1995).
[20] S. Corley and T. Jacobson, Phys. Rev. D **54**, 1568 (1996).
[21] J. Martin and R. H. Brandenberger, Phys. Rev. D **63**, 123501 (2001).
[22] R. H. Brandenberger and J. Martin, Mod. Phys. Lett. A **16**, 999 (2001).
[23] T. Jacobson and D. Mattingly, Phys. Rev. D **63**, 041502 (2001).
[24] P. P. Avelino and C. J. A. P. Martins, Phys. Rev. Lett. **85**, 1370 (2000); Phys. Rev. D **62**, 103510 (2000).
[25] C. Armendariz-Picon and E. A. Lim, J. Cosmol. Astropart. Phys. 12 (2003) 002.
[26] A. R Liddle and S. M. Leach, Phys. Rev. D **68**, 103503 (2003).
[27] Y. S. Piao and Y. Z. Zhang, Phys. Rev. D **75**, 027301 (2007).
[28] G. Dvali, A. Gruzinov, and M. Zaldarriaga, Phys. Rev. D **69**, 023505 (2004); **69**, 083505 (2004).
[29] L. Kofman, astro-ph/0303614.