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Reheating of the universe after inflation with $f(\phi)R$ gravity

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We show that reheating of the universe occurs spontaneously in a broad class of inflation models with $f(\phi)R$ gravity (ϕ is the inflaton). The model does not require explicit couplings between ϕ and bosonic or fermionic matter fields. The couplings arise spontaneously when ϕ settles in the vacuum expectation value (vev) and oscillates, with coupling constants given by derivatives of $f(\phi)$ at the vev and the mass of resulting bosonic or fermionic fields. This mechanism allows inflaton quanta to decay into any fields which are not conformally invariant in $f(\phi)R$ gravity theories.

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Inflation is an indispensable building block of the standard model of cosmology [1,2], and has passed a number of stringent observational tests [3]. Any inflation models must contain a mechanism by which the universe reheats after inflation [4]. The reheating mechanism requires detailed knowledge of interactions (e.g., Yukawa coupling) between inflaton fields and their decay products. Since the physics behind inflation is beyond the standard model of elementary particles, the precise nature of inflaton fields is currently undetermined, and the coupling between inflaton and matter fields is often put in by hand. On the other hand, inflaton and matter fields are coupled through gravity. Of course the gravitational coupling is suppressed by the Planck mass and thus too weak to yield interesting effects [1]; however, we shall show that the reheating occurs spontaneously if inflaton is coupled to gravity nonminimally, i.e., the gravitational action is given not by the Einstein-Hilbert form, R, but by $f(\phi)R$, where ϕ is an inflaton field. Even if matter fields do not interact with ϕ directly, they ought to interact via gravitation whose perturbations lead to Yukawa-type interactions. A similar idea was put forward by [5], who considered *preheating* with a nonminimal coupling between matter and gravity. Here, we do not consider preheating, but focus only on the perturbative reheating arising from $f(\phi)R$ gravity. Therefore, we can calculate the resulting reheating temperature after inflation.

Why study $f(\phi)R$ gravity? There are a number of motivations [6,7]. The strongest motivation comes from the fact that almost any candidate theories of fundamental physics which involve compactification of extra dimensions yield $f(\phi)R$ with the form of $f(\phi)$ depending on models. One illuminating example would be string moduli with $f(\phi) \propto e^{-\alpha\phi}$. Zee's induced gravity theory [8] has $f(\phi) = \xi\phi^2$, and renormalization in the curved spacetime yields other more complicated higher derivative terms [9]. Classic scalar-tensor theories, originally motivated by Mach's principle [10], also fall into this category. It has been shown that inflation occurs naturally in these generalized gravity models [11–16], and the spectrum of scalar curvature perturbations [17] as well as of tensor gravity wave perturbations [18] can be affected by the presence of $f(\phi)R$, thereby allowing us to constrain $f(\phi)$ from the cosmological data. We show that limits on the reheating temperature (e.g., gravitino problem) provide additional, totally independent constraints on $f(\phi)$. This argument opens up new possibilities that one can constrain a broad class of inflation models from the reheating temperature. Reheating in Starobinsky's R^2 inflation has been considered by [16,19].

Interactions between ϕ and matter stem from a mixing between metric and scalar field perturbations through $f(\phi)R$ [6]. It is this ''gravitational decay channel'' from ϕ to matter that makes the universe reheat after inflation. We realize this in a simple Lagrangian given by

$$\mathcal{L} = \sqrt{-g}[\tfrac{1}{2}f(\phi)R - \tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)] + \mathcal{L}_{\rm m}, \tag{1}$$

where $\mathcal{L}_{\rm m}$ is the matter Lagrangian. Note that there is no explicit coupling between ϕ (inflaton) and $\mathcal{L}_{\rm m}$. To guarantee the ordinary Einstein gravity at low energy, we must have $f(v) = M_{\rm Pl}^2$, where v is the vacuum expectation value (vev) of ϕ at the end of inflation and $M_{\rm Pl} = (8\pi G)^{-1/2} = 2.436 \times 10^{18}$ GeV is the reduced Planck mass. This Lagrangian satisfies the weak equivalence principle: ϕ does not couple to matter directly, $\nabla_\mu T_{\rm m}^{\mu\nu} = 0$. This, however, does not imply that inflaton *quanta*, i.e., fluctuations around the vev, cannot decay into matter. The gravitational field equation from Eq. (1) is

$$f(\phi)G_{\mu\nu} = T_{\mu\nu}^{\rm m} + \partial_\mu\phi\partial_\nu\phi - \tfrac{1}{2}g_{\mu\nu}(\partial\phi)^2 - g_{\mu\nu}V(\phi) - (g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f(\phi). \tag{2}$$

Before reheating completes the energy density of the uni-

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verse was dominated by ϕ; thus, we treat $T^{\rm m}_{\mu\nu}$ as perturbations. As usual we decompose $g_{\mu\nu}$ into the background, $\bar{g}_{\mu\nu}$, and perturbations, $h_{\mu\nu}$, as $g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$. The first-order perturbation in $G_{\mu\nu}$ is given by

$$\delta G_{\mu\nu} = \tfrac{1}{2}[-\Box h_{\mu\nu} + \nabla_\lambda \nabla_\mu h^\lambda_\nu + \nabla_\lambda \nabla_\nu h^\lambda_\mu - \nabla_\mu \nabla_\nu h - \bar{g}_{\mu\nu}(\nabla_\rho \nabla_\sigma h^{\rho\sigma} - \Box h - \bar{R}_{\rho\sigma} h^{\rho\sigma}) - \bar{R} h_{\mu\nu}], \tag{3}$$

where $h^\mu_\nu \equiv \bar{g}^{\mu\lambda} h_{\lambda\nu}$ and $h \equiv \bar{g}^{\mu\nu} h_{\mu\nu}$.

Reheating occurs at the potential minimum of ϕ where ϕ oscillates about v. We thus expand ϕ as $\phi = v + \sigma$, where σ represents inflaton quanta which decay into matter fields. Treating σ as perturbations, one obtains the linearized field equation

$$\frac{M^2_{\rm Pl}}{2}[-\Box h_{\mu\nu} + \cdots] + f'(v)(\bar{g}_{\mu\nu}\Box - \nabla_\mu \nabla_\nu)\sigma = T^{\rm m}_{\mu\nu}, \tag{4}$$

where $f'(v)$ means $\partial f/\partial \phi|_{\phi=v}$. This equation contains both $\Box h_{\mu\nu}$ and $\Box \sigma$, and thus wave modes are mixed up together. To diagonalize the wave mode we define a new field as [6]

$$\tilde{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu} h - \frac{f'(v)}{M^2_{\rm Pl}} \bar{g}_{\mu\nu}\sigma, \tag{5}$$

where the tilde denotes the operation defined by this equation, which essentially corresponds to the infinitesimal conformal transformation. The inverse operation is

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\tilde{h} - \frac{f'(v)}{M^2_{\rm Pl}} \bar{g}_{\mu\nu}\sigma. \tag{6}$$

Using the harmonic gauge (analog of the Lorenz gauge) conditions, $\nabla_\lambda \tilde{h}^\lambda_\nu = 0$, one can show that $\tilde{h}_{\mu\nu}$ indeed obeys the linearized field equation for the wave mode (see, e.g., Eq. (35.64) in [20]; Eq. (10.9.4) in [21])

$$\Box \tilde{h}_{\mu\nu} - 2\bar{R}_{\mu\lambda\rho\nu}\tilde{h}^{\lambda\rho} - 2\bar{R}_{\rho(\mu}\tilde{h}^\rho_{\nu)} - \bar{g}_{\mu\nu}\bar{R}_{\lambda\rho}\tilde{h}^{\lambda\rho} + \bar{R}\tilde{h}_{\mu\nu} = -\frac{2}{M^2_{\rm Pl}} T^{\rm m}_{\mu\nu}. \tag{7}$$

One may also impose the traceless condition on $\tilde{h}_{\mu\nu}$ to make sure that $\tilde{h}_{\mu\nu}$ describes tensor (spin two) gravity waves, $\tilde{h} = -h - 4f'(v)\sigma/M^2_{\rm Pl} = 0$, which relates a trace part of the original metric perturbations to σ as $h = -4f'(v)\sigma/M^2_{\rm Pl}$. In this sense σ describes the "scalar (spin zero) gravity waves", which are common in scalar-tensor theories of gravity. For generality we keep $\tilde{h}$ explicitly throughout the paper.

We consider both fermionic, ψ, and bosonic, χ, fields as matter: $\mathcal{L}_{\rm m} = \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\rm int}$. First, let us write $\mathcal{L}_\psi$ as

$$\mathcal{L}_\psi = -\sqrt{-g}\bar{\psi}[\not{D} + m_\psi]\psi, \tag{8}$$

where $\not{D} \equiv e^{\mu\alpha}\gamma_\alpha D_\mu$, $e^{\mu\alpha}$ is a tetrad (vierbein) field, and $D_\mu \equiv \partial_\mu - \Gamma_\mu$ is the covariant derivative for spinor fields [9,21]. Γ_μ is a spinor connection and $\Sigma^{\alpha\beta}$ are generators of the Lorentz group given by $\Gamma_\mu(x) \equiv -\frac{1}{2}\Sigma^{\alpha\beta} e^\lambda_\alpha \nabla_\mu e_{\beta\lambda}$ and $\Sigma^{\alpha\beta} = -\Sigma^{\beta\alpha} = \frac{1}{4}[\gamma^\alpha, \gamma^\beta]$, respectively. Here $\alpha, \beta, \ldots$ denote Lorentz indices while $\mu, \nu, \ldots$ denote general coordinate indices. Note that we have not antisymmetrized the first term in Eq. (8) to make the expression simpler. The antisymmetrized term yields the same result. Note also that we shall ignore the existence of torsion.

We expand the tetrad and metric into the background and perturbations as $e^{\mu\alpha} \simeq \bar{e}^{\mu\alpha} - \bar{e}^{\lambda\alpha} h^\mu_\lambda/2$, and $\sqrt{-g} = \bar{e} + \delta e \simeq \bar{e}(1 + \bar{g}^{\mu\nu} h_{\mu\nu}/2)$, respectively. The Lagrangian becomes (with Eq. (6))

$$\mathcal{L}_\psi \simeq -\bar{e}\bar{\psi}[\bar{e}^{\mu\alpha}\gamma_\alpha D_\mu + m_\psi]\psi + \tfrac{1}{2}\bar{e}\bar{\psi}[\bar{e}^{\nu\alpha}\gamma_\alpha(\tilde{h}^\mu_\nu + \tfrac{1}{2}\tilde{h}\delta^\mu_\nu)D_\mu + \tilde{h} m_\psi]\psi + \bar{e} g_\psi \sigma\bar{\psi}\psi. \tag{9}$$

The terms that are proportional to $\tilde{h}$ may be set to vanish by gauge transformation. Here we have used the background Dirac equation, $\bar{e}^{\mu\alpha}\gamma_\alpha D_\mu \psi = -m_\psi \psi$, to obtain the last term and ignored the second order terms as well as thermal mass induced by quantum corrections in thermal bath [22,23]. The last term in Eq. (9) is a Yukawa interaction term with a coupling constant given by

$$g_\psi \equiv \frac{f'(v) m_\psi}{2M^2_{\rm Pl}}. \tag{10}$$

Therefore, σ can decay into ψ and $\bar{\psi}$. Note that the Yukawa interaction vanishes when $f'(v) = 0$ or $m_\psi = 0$, which is consistent with previous work [13]: as massless fermions are conformally invariant, no peculiar effects can be caused by $f(\phi)R$ gravity at the tree level. Quantum corrections such as a conformal, or trace, anomaly might open additional decay channels.

Next, let us consider the bosonic matter, χ, given by

$$\mathcal{L}_\chi = \sqrt{-g}[-\tfrac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi)], \tag{11}$$

which may be expanded as

$$\mathcal{L}_\chi \simeq \sqrt{-\bar{g}}\Big[-\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - U(\chi) + \frac{1}{2}\tilde{h}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi + \frac{1}{2}\tilde{h}U(\chi) + \frac{f'(v)}{2M^2_{\rm Pl}}(4\sigma U(\chi) + \sigma\bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi)\Big], \tag{12}$$

where we have used $\sqrt{-g} \simeq \sqrt{-\bar{g}}(1 - \tilde{h}/2 - 2\sigma f'(v)/M^2_{\rm Pl})$ and $g^{\mu\nu} \simeq \bar{g}^{\mu\nu} - \tilde{h}^{\mu\nu} + \bar{g}^{\mu\nu}\tilde{h}/2 + \bar{g}^{\mu\nu}\sigma f'(v)/M^2_{\rm Pl}$. Again, the terms that are proportional to $\tilde{h}$ may be set to vanish by gauge transformation. For simplicity we assume that χ is a massive field with self-interaction, $U(\chi) = m^2_\chi\chi^2/2 + \lambda\chi^4/4$. The last term in Eq. (12) then yields the following interactions:

$\sqrt{-\bar{g}}[\sigma f'(v)/2M_{\rm Pl}^2](2m_\chi^2\chi^2+\lambda\chi^4+\bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi)$. We rewrite these interactions as

$$\sqrt{-\bar{g}}\frac{\sigma f'(v)}{2M_{\rm Pl}^2}(m_\chi^2\chi^2+\bar{g}^{\mu\nu}\bar{\nabla}_\mu(\chi\partial_\nu\chi))\approx\sqrt{-\bar{g}}g_\chi\sigma\chi^2, \tag{13}$$

where we have defined a coupling constant,

$$g_\chi\equiv\frac{f'(v)}{2M_{\rm Pl}^2}\left(m_\chi^2+\frac{m_\sigma^2}{2}\right), \tag{14}$$

which will give a rate of σ decaying into two χs. To derive Eq. (13) we used $\bar{g}^{\mu\nu}\partial_\mu\chi\partial_\nu\chi=-\chi U'(\chi)+\bar{g}^{\mu\nu}\bar{\nabla}_\mu(\chi\partial_\nu\chi)$, where $\bar{\nabla}_\mu$ is the covariant derivative on the background metric, and estimated as $\sigma\bar{\nabla}_\mu(\chi\partial^\mu\chi)\approx\sigma m_\sigma^2\chi^2/2$. This estimation is valid in the zero temperature limit (see [24] for the high temperature limit). While the first term in Eq. (14) is absent when $m_\chi=0$, σ can still decay through the second term as minimally coupled massless scalar fields are not conformally invariant.

What is the physical interpretation of these couplings? Through the field mixing that can be seen in Eq. (5) the inflaton quanta are coupled to the trace (spin zero) part of $h_{\mu\nu}$, which describes the scalar gravity waves. The scalar waves are then coupled to the matter fields. If inflaton quanta are at least twice as heavy as ψ or χ, they decay into $\psi\bar{\psi}$ or two χs.

The decay rates of $\sigma\to\psi\bar{\psi}$ from the last term in Eq. (9) and $\sigma\to\chi\chi$ from Eq. (13), $\Gamma_{\rm tot}=\Gamma_{\sigma\bar{\psi}\psi}+\Gamma_{\sigma\chi\chi}+\ldots$, are given by [25]

$$\Gamma_{\sigma\bar{\psi}\psi}=\frac{g_\psi^2m_\sigma}{8\pi}\left(1-\frac{4m_\psi^2}{m_\sigma^2}\right)^{3/2}\tanh\left(\frac{m_\sigma}{4T}\right)C_\psi, \tag{15}$$

$$\Gamma_{\sigma\chi\chi}=\frac{g_\chi^2}{8\pi m_\sigma}\left(1-\frac{4m_\chi^2}{m_\sigma^2}\right)^{1/2}\coth\left(\frac{m_\sigma}{4T}\right)C_\chi, \tag{16}$$

where $m_\sigma^2\equiv V''(v)$ and we have included suppression of decay rates due to thermal mass, C_ψ and C_χ, which are unity in the low temperature limit ($T\ll m_\sigma$) in which thermal mass is small, but can be quite small otherwise. The exact form of C depends on how decay products are thermalized. Yokoyama has calculated C_χ from thermalization due to $\Delta U=\lambda\chi^4/4$: for $T\gg m_\sigma/\sqrt{\lambda}$ he finds $C_\chi=\lambda/(8\pi^2)$ for decay via $\sigma\chi^2$ interaction [23] and $C_\chi=\lambda/(1024\pi^2)$ for $\sigma(\partial\chi)^2$ (which corresponds to the last term in Eq. (12); if this term dominates g_χ is given by $g_\chi=f'(v)\lambda T^2/(2M_{\rm Pl}^2)$ because of dominance of thermal mass, $\sqrt{\lambda}T$, over the intrinsic mass of χ) [24], while C_ψ has not been calculated explicitly yet.

As usual we use the condition, $3H(t^*)=\Gamma_{\rm tot}$, to define the reheating time, t^*, at which radiation begins to dominate the energy density of the universe. From the Friedmann equation, $H^2=\rho/(3M_{\rm Pl}^2)$, we get $\rho(t^*)=\Gamma_{\rm tot}^2M_{\rm Pl}^2/3=\frac{\pi^2}{30}g_*(T_{\rm rh})T_{\rm rh}^4$, where the last equality holds if all the decay products interact with each other rapidly enough to achieve thermodynamical equilibrium [22]. If some fraction of inflaton energy density is converted into the species that never interact with the visible sector, the reheating temperature may be lowered. $g_*(T_{\rm rh})$ is the effective number of relativistic degrees of freedom at the reheating time. Since $\Gamma_{\rm tot}>\Gamma_{\sigma\bar{\psi}\psi}+\Gamma_{\sigma\chi\chi}$, the reheating temperature is bounded from below:

$$T_{\rm rh}=\frac{\sqrt{\Gamma_{\rm tot}M_{\rm Pl}}}{(10\pi^2)^{1/4}}\left(\frac{g_*(T_{\rm rh})}{100}\right)^{-1/4}$$
$$>\sqrt{g_\psi^2+\frac{g_\chi^2}{m_\sigma^2}}\frac{\sqrt{m_\sigma M_{\rm Pl}}}{4\pi(40)^{1/4}}\left(\frac{g_*(T_{\rm rh})}{100}\right)^{-1/4}, \tag{17}$$

where we have used Eq. (15) and (16) assuming for simplicity that decay products are much lighter than σ (m_ψ, $m_\chi\ll m_\sigma$), the reheating temperature is much smaller than m_σ ($T\ll m_\sigma$), and therefore suppression due to thermal mass is unimportant ($C_\psi=1=C_\chi$). Generalization to the other limits is straightforward. One may reverse the argument and set a limit on $f'(v)$ as

$$|f'(v)|<8\pi(40)^{1/4}T_{\rm rh}\left(\frac{M_{\rm Pl}}{m_\sigma}\right)^{3/2}\left(\frac{g_*(T_{\rm rh})}{100}\right)^{1/4}, \tag{18}$$

which provides nontrivial constraints on inflation models with $f(\phi)R$ gravity. Let us consider a nonminimal coupling model, $f(\phi)=M^2+\xi\phi^2$, for instance (M^2 is given by the condition $f(v)=M_{\rm Pl}^2$). We obtain $|\xi|<4\pi(40)^{1/4}\times(T_{\rm rh}/v)(M_{\rm Pl}/m_\sigma)^{3/2}(g_*(T_{\rm rh})/100)^{1/4}$, which provides a new constraint on ξ, independent of the existing constraints from homogeneity and isotropy [12] and curvature as well as tensor perturbations [18].

Finally, let us show that one can obtain the same results by performing the following conformal transformation, $\hat{g}_{\mu\nu}=\Omega^2g_{\mu\nu}$, with

$$\Omega^2=\frac{f(\phi)}{M_{\rm Pl}^2}=1+\frac{f'(v)\sigma}{M_{\rm Pl}^2}+\frac{f''(v)\sigma^2}{2M_{\rm Pl}^2}+\mathcal{O}(\sigma^3). \tag{19}$$

Hereafter we put hats on variables in the Einstein frame (E frame). We use Maeda's transformation formula [26] to obtain the Lagrangian in the E frame,

$$\mathcal{L}=\sqrt{-\hat{g}}\left[\frac{M_{\rm Pl}^2}{2}\hat{R}-\frac{1}{2}\hat{g}^{\mu\nu}\partial_\mu\hat{\phi}\partial_\nu\hat{\phi}-\hat{V}(\hat{\phi})\right]+\mathcal{L}_{\rm m}, \tag{20}$$

where $\hat{R}$ is calculated from $\hat{g}_{\mu\nu}$, a transformed potential is given by $\hat{V}(\hat{\phi})\equiv\Omega^{-4}V(\phi)$, and a new scalar field, $\hat{\phi}$, is defined such that it has the canonical kinetic term. It is related to the original field by $d\hat{\phi}/d\phi=\sqrt{\frac{3}{2}(f'(\phi)/f(\phi))^2+M_{\rm Pl}^2/f(\phi)}$. As $\hat{\phi}$ is minimally coupled to $\hat{R}$, there is no mixing between $\hat{g}_{\mu\nu}$ and $\hat{\phi}$.

A transformed Lagrangian for ψ in the E frame is

$$\mathcal{L}_{\psi} = -\hat{e}\,\hat{\bar{\psi}}[(\hat{\partial\!\!\!/} - \hat{\overleftarrow{\partial\!\!\!/}})/2 + \Omega^{-1}m_{\psi}]\hat{\psi}, \tag{21}$$

where $\hat{\psi} = \Omega^{-3/2}\psi$, $\hat{\bar{\psi}} = \Omega^{-3/2}\bar{\psi}$, $\hat{e}^{\mu\alpha} = \Omega^{-1}e^{\mu\alpha}$, and $\hat{e} = \Omega^4 e$. While no Yukawa interaction arises from the kinetic term, the mass term yields a Yukawa interaction:

$$-\hat{e}\frac{m_{\psi}}{\Omega}\hat{\bar{\psi}}\hat{\psi} \simeq -\hat{e}m_{\psi}\hat{\bar{\psi}}\hat{\psi} + \hat{e}\frac{f'(v)m_{\psi}}{2M_{\rm Pl}^2}\sigma\hat{\bar{\psi}}\hat{\psi}. \tag{22}$$

Thus, Yukawa coupling constants agree precisely.

A transformed Lagrangian for χ in the E frame is

$$\mathcal{L}_{\chi} = -\tfrac{1}{2}\sqrt{-\hat{g}}\hat{g}^{\mu\nu}\mathcal{D}_{\mu}\hat{\chi}\mathcal{D}_{\nu}\hat{\chi} - \sqrt{-\hat{g}}\hat{U}(\hat{\chi}), \tag{23}$$

where $\hat{\chi} \equiv \Omega^{-1}\chi$ and we have followed the procedure in [6] to define a "covariant derivative" and potential in the E frame as $\mathcal{D}_{\mu} \equiv \partial_{\mu} + \partial_{\mu}(\ln\Omega)$, and $\hat{U}(\hat{\chi}) \equiv \Omega^{-4}U(\chi)$, respectively. In the case of $U(\chi) = m_{\chi}^2\chi^2/2 + \lambda\chi^4/4$, the second term of Eq. (23) becomes

$$-\sqrt{-\hat{g}}\left[\frac{1}{2}m_{\chi}^2\hat{\chi}^2 + \frac{\lambda}{4}\hat{\chi}^4 - \frac{f'(v)m_{\chi}^2}{2M_{\rm Pl}^2}\sigma\hat{\chi}^2\right], \tag{24}$$

to the linear order in σ; the last term agrees with the first term in Eq. (13). The covariant derivative yields a coupling, $-\sqrt{-\hat{g}}\hat{\chi}\hat{g}^{\mu\nu}(\partial_{\mu}\hat{\chi})(\partial_{\nu}\ln\Omega)$. After integration by parts and $\ln\Omega \simeq [f'(v)/(2M_{\rm Pl}^2)]\sigma$ this term yields

$$\frac{f'(v)}{2M_{\rm Pl}^2}\sigma\partial_{\nu}(\sqrt{-\hat{g}}\hat{\chi}\hat{g}^{\mu\nu}\partial_{\mu}\hat{\chi}) = \sqrt{-\hat{g}}\frac{f'(v)}{2M_{\rm Pl}^2}\sigma\hat{g}^{\mu\nu}\hat{\nabla}_{\mu}(\hat{\chi}\partial_{\nu}\hat{\chi}), \tag{25}$$

which agrees with the second term in Eq. (13) precisely.

One can calculate the other interaction terms such as $\sigma^2\chi^2$, $\sigma^2\bar{\psi}\psi$, $\sigma^2(\partial\chi)^2$, etc., with known coupling constants in the E frame easily. These interactions, whose coupling constants are proportional to the higher order derivatives such as $f''(v)$, $f'''(v)$, etc., actually dominate if $f(\phi)$ also has a minimum at the vev, $f'(v) = 0$. While we have ignored a parametric resonance (preheating) [27] entirely, it would be interesting to study how preheating might occur in the present context.

In summary, we have presented a natural mechanism for reheating of the universe after inflation without introducing any explicit couplings between inflaton and fermionic or bosonic matter fields. This mechanism allows inflaton quanta to decay into *any fields* which are present at the end of inflation and are not conformally invariant, when inflaton settles in the vacuum expectation value and oscillates. Reheating therefore occurs spontaneously in *any theories* of $f(\phi)R$ gravity.

We have calculated the reheating temperature from this mechanism. We argue that one must always check that the reheating temperature in any $f(\phi)R$ inflation models is reasonable, e.g., the reheating temperature does not exceed the critical temperature above which too many gravitinos would be produced thermally. This mechanism might also allow ϕ to decay into gravitinos. How our mechanism is related to an inflaton decay through supergravity effects [28] merits further investigation. Both of these effects should provide nontrivial constraints on $f(\phi)$ from the cosmological data.

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