PHYSICAL REVIEW D **75,** 056001 (2007)

Twist-3 distribution amplitudes of scalar mesons from QCD sum rules

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(Received 30 December 2006; revised manuscript received 5 February 2007; published 7 March 2007)

We study the twist-3 distribution amplitudes for scalar mesons made up of two valence quarks based on QCD sum rules. By choosing the proper correlation functions, we derive the moments of the scalar mesons up to the first two orders. Making use of these moments, we then calculate the first two Gegenbauer coefficients for twist-3 distribution amplitudes of scalar mesons. It is found that the second Gegenbauer coefficients of scalar density twist-3 distribution amplitudes for K_0^* and f_0 mesons are quite close to that for a_0, which indicates that the SU(3) symmetry breaking effect is tiny here. However, this effect could not be neglected for the forth Gegenbauer coefficients of scalar twist-3 distribution amplitudes between a_0 and f_0. Besides, we also observe that the first two Gegenbauer coefficients corresponding to the tensor current twist-3 distribution amplitudes for all the a_0, K_0^* and f_0 are very small. The renormalization group evolution of condensates, quark masses, decay constants and moments are considered in our calculations. As a by-product, it is found that the masses for isospin $I = 1, \frac{1}{2}$ scalar mesons are around $1.27 \sim 1.41$ GeV and $1.44 \sim 1.56$ GeV respectively, while the mass for isospin state composed of $\bar{s}s$ is $1.62 \sim 1.73$ GeV.

DOI: 10.1103/PhysRevD.75.056001 PACS numbers: 11.55.Hx, 14.40.Cs

I. INTRODUCTION

Although the quark model has achieved great successes for several decades, the fundamental structures of scalar mesons are still controversial. So far, there is not a definite answer on whether they are two-quark states, multiquark states or even glueball, molecule states among the light scalars yet [1–4]. Much efforts have been given to the study of decay and production of these mesons. However, many theoretical predictions on properties of scalar mesons, in particular, on the production of them in exclusive heavy flavor hadron decays [5,6] have large uncertainties due to the complicated nonperturbative effects.

It is no doubt that the hadronic light-cone distribution amplitudes are the important ingredients when applying factorization theorem to analyze these exclusive processes. The distribution amplitudes which are governed by the renormalization group equation can be obtained by integrating out the transverse momenta of quarks in hadron for hadronic wave functions. Unfortunately, only twist-2 light-cone distribution amplitudes of scalar mesons have been calculated in Ref. [6] in the framework of QCD sum rules [7]. So the unknown twist-3 distribution amplitudes will bring obvious uncertainties to the final results. In this work, we investigate twist-3 distribution amplitudes of scalar mesons in order to improve the accuracy of theoretical predictions of the scalar mesons.

The calculation of moments for distribution amplitudes making use of QCD sum rules was presented in much detail in the pioneer work of [8]. Once the moments are known, we can construct various models to obtain the distribution amplitudes for hadrons. Following the same method, we will calculate the first two nonzero moments of twist-3 distribution amplitudes for $a_0(\bar{u}d)$, $K_0^*(\bar{d}s)$ and $f_0(\bar{s}s)$ respectively based on renormalization group improved QCD sum rules. Besides, we will expand the twist-3 distribution amplitudes of scalar mesons according to Gegenbauer polynomials as usual and use the moments obtained to determine the first two Gegenbauer coefficients. As for a_0 and f_0 meson, the odd moments for both of the two twist-3 distribution amplitudes (see definition in Eq. (1)) are zero due to conservation of charge parity and isospin symmetry. However, the odd moments for K_0^* meson do not vanish when including SU(3) symmetry breaking effects.

The structure of this paper is as below: After this introduction, we derive the general sum rules of moments for twist-3 distribution amplitudes of scalar mesons in Sec. II. Then we will give the inputs used in our work and present the numerical results of the first two moments for the above three scalar mesons in Sec. III. The last section is devoted to our conclusions.

II. FORMULATION

In the valence quark model, there are two twist-3 light-cone distribution amplitudes for scalar mesons which are defined as [6]

$$\langle S(p)|\bar{q}_2(y)q_1(x)|0\rangle = m_S\bar{f}_S\int_0^1 du e^{i(up\cdot y+\bar{u}p\cdot x)}\phi_S^s(u,\mu),$$
$$\langle S(p)|\bar{q}_2(y)\sigma_{\mu\nu}q_1(x)|0\rangle = -m_S\bar{f}_S(p_\mu z_\nu - p_\nu z_\mu)\times\int_0^1 du e^{i(up\cdot y+\bar{u}p\cdot x)}\frac{\phi_S^\sigma(u,\mu)}{6}, \tag{1}$$

with $z = y - x$, $\bar{u} = 1 - u$ and u being the momentum fraction carried by the q_2 quark in the scalar meson. Here the scalar density meson decay constant $\bar{f}_S$ is defined by $\langle S(p)|\bar{q}_2 q_1|0\rangle = m_S \bar{f}_S$. This scalar decay constant $\bar{f}_S$ can be connected with the vector current decay constant f_S which is defined as $\langle S(p)|\bar{q}_2\gamma_\mu q_1|0\rangle = f_S p_\mu$:

$$\mu_S f_S = \bar{f}_S, \qquad \mu_S = \frac{m_S}{m_2(\mu) - m_1(\mu)}. \tag{2}$$

with m_1, m_2 and m_S being the mass of q_1, q_2 and scalar meson, respectively. The normalization of these two twist-3 light-cone distribution amplitudes are $\int_0^1 du \phi_S^s(u) = \int_0^1 du \phi_S^\sigma(u) = 1$. In general, they have the following form:

$$\phi_S^s(u,\mu) = 1 + \sum_{m=1}^{\infty} a_m(\mu) C_m^{1/2}(2u-1), \tag{3}$$

$$\phi_S^\sigma(u,\mu) = 6u(1-u)\left[1 + \sum_{m=1}^{\infty} b_m(\mu) C_m^{3/2}(2u-1)\right], \tag{4}$$

with the Gegenbauer polynomials $C_1^{1/2}(t) = t$, $C_2^{1/2}(t) = \frac{1}{2}(3t^2-1)$, $C_4^{1/2}(t) = \frac{1}{8}(35t^4 - 30t^2 + 3)$, $C_1^{3/2}(t) = 3t$, $C_2^{3/2}(t) = \frac{3}{2}(5t^2-1)$, $C_4^{3/2}(t) = \frac{15}{8}(21t^4 - 14t^2 + 1)$, etc.

Now we are ready to calculate the moments of the two twist-3 distribution amplitudes defined in Eq. (1) making use of background field method in QCD [9–11]. From Eq. (1), one can easily find

$$\begin{aligned} &\langle 0|\bar{q}_1(0)(iz\cdot \overleftrightarrow{D})^n q_2(0)|S(p)\rangle = m_S\bar{f}_S(p\cdot z)^n\langle \xi_s^n\rangle, \\ &\langle 0|\bar{q}_1(0)(iz\cdot \overleftrightarrow{D})^{n+1}\sigma_{\mu\nu} q_2(0)|S(p)\rangle \\ &\quad = -i\frac{n+1}{3} m_S\bar{f}_S(p_\mu z_\nu - p_\nu z_\mu)(p\cdot z)^n\langle \xi_\sigma^n\rangle, \end{aligned} \tag{5}$$

with

$$\begin{aligned} \langle \xi_s^n\rangle &= \int_0^1 du(2u-1)^n \phi_S^s(u,\mu), \\ \langle \xi_\sigma^n\rangle &= \int_0^1 du(2u-1)^n \phi_S^\sigma(u,\mu). \end{aligned} \tag{6}$$

In order to calculate the above scalar moments $\langle \xi_s^n\rangle$ and tensor moments $\langle \xi_\sigma^n\rangle$, we consider the following two different correlation functions, respectively

$$\begin{aligned} &i\int d^4x e^{iq\cdot x}\langle 0|T\{\bar{q}_1(x)(iz\cdot\overleftrightarrow{D})^n q_2(x), \bar{q}_2(0)q_1(0)\}|0\rangle \\ &\quad = -(z\cdot q)^n I_s^{(n,0)}(q^2), \end{aligned} \tag{7}$$

$$\begin{aligned} &i\int d^4x e^{iq\cdot x}\langle 0|T\{\bar{q}_1(x)\sigma_{\mu\nu}(iz\cdot\overleftrightarrow{D})^{n+1} q_2(x), \bar{q}_2(0)q_1(0)\}|0\rangle \\ &\quad = i(q_\mu z_\nu - q_\nu z_\mu)(z\cdot q)^n I_\sigma^{(n,0)}(q^2). \end{aligned} \tag{8}$$

In the deep Euclidean region ($-q^2 \gg 0$), the correlation functions (7) and (8) can be computed using operator product expansion at quark level. The results with power corrections to operators up to dimension-six and lowest order of α_s corrections are displayed as:

$$\begin{aligned} I_s^{(2n,0)}(q^2)_{\rm QCD} = &-\frac{3}{8\pi^2}\frac{1}{2n+1}\ln\frac{-q^2}{\mu^2}(2m_1m_2 - q^2) + \frac{\alpha_s}{8\pi}\frac{1}{q^2}\langle 0|G^2|0\rangle \\ &+ \frac{1}{q^2}\left[\left(\frac{2n+1}{2}m_1 + m_2\right)\langle\bar{q}_1q_1\rangle + \left(\frac{2n+1}{2}m_2 + m_1\right)\langle\bar{q}_2q_2\rangle\right] \\ &+ \frac{1}{2}g_s\frac{1}{q^4}\left\{\left[m_2 + n\left(\frac{8n+11}{6}m_1 + 2m_2\right)\right]\langle\bar{q}_1\sigma Gq_1\rangle + \left[m_1 + n\left(\frac{8n+11}{6}m_2 + 2m_1\right)\right]\langle\bar{q}_2\sigma Gq_2\rangle\right\} \\ &- \frac{4\pi\alpha_s}{81}(8n^2 - 16n - 21)\frac{\langle\bar{q}_1q_1\rangle^2 + \langle\bar{q}_2q_2\rangle^2}{q^4} + \frac{48\pi\alpha_s}{9}\frac{\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle}{q^4}, \end{aligned} \tag{9}$$

for the scalar density even moments,

$$\begin{aligned} I_s^{(1,0)}(q^2)_{\rm QCD} = &-\frac{3}{8\pi^2}(m_1^2 - m_2^2)\ln\frac{-q^2}{\mu^2} + \frac{\alpha_s}{4\pi}(m_1^2 - m_2^2)\left[\ln\frac{-q^2}{\mu^2} + \gamma_E\right]\frac{1}{q^4}\langle 0|G^2|0\rangle + (m_1 + m_2)\frac{\langle\bar{q}_1q_1\rangle - \langle\bar{q}_2q_2\rangle}{q^2} \\ &+ \frac{10\pi\alpha_s}{9}\frac{\langle\bar{q}_1q_1\rangle^2 - \langle\bar{q}_2q_2\rangle^2}{q^4} + \frac{g_s}{2}\frac{1}{q^4}\left\{\left(\frac{5}{4}m_1 + 2m_2\right)\langle\bar{q}_1\sigma Gq_1\rangle - \left(\frac{5}{4}m_2 + 2m_1\right)\langle\bar{q}_2\sigma Gq_2\rangle\right\}, \end{aligned} \tag{10}$$

for the first moment of scalar density, and

$$I_\sigma^{(2n,0)}(q^2)_{\rm QCD} = \frac{3}{16\pi^2}\frac{q^2}{2n+3}\left[1+2\ln\frac{-q^2}{\mu^2}\right] + \frac{2n+1}{2}\frac{1}{q^2}(m_1\langle\bar q_1 q_1\rangle + m_2\langle\bar q_2 q_2\rangle) + \frac{16n^2+14n+5}{24}\frac{g_s}{q^4}[m_1\langle\bar q_1\sigma G q_1\rangle + m_2\langle\bar q_2\sigma G q_2\rangle] - \frac{32n^2+18n-35}{81}\frac{\pi\alpha_s}{q^4}(\langle\bar q_1 q_1\rangle^2 + \langle\bar q_2 q_2\rangle^2) + \frac{\alpha_s}{24\pi}\frac{1}{q^2}\langle 0|G^2|0\rangle - \frac{8\pi\alpha_s}{9}\frac{1}{q^4}\langle\bar q_1 q_1\rangle\langle\bar q_2 q_2\rangle\delta_{n,0}, \tag{11}$$

for the tensor even moments, and

$$I_\sigma^{(1,0)}(q^2)_{\rm QCD} = \frac{\alpha_s}{6\pi}(m_1^2-m_2^2)\left[2\ln\frac{-q^2}{\mu^2}+2\gamma_E-3\right]\frac{\langle 0|G^2|0\rangle}{q^4} - \frac{1}{4\pi^2}(m_1^2-m_2^2)\left[1+\ln\frac{-q^2}{\mu^2}\right] + \frac{3}{4}g_s\frac{m_1\langle\bar q_1\sigma G q_1\rangle - m_2\langle\bar q_2\sigma G q_2\rangle}{q^4} + \frac{m_1\langle\bar q_1 q_1\rangle - m_2\langle\bar q_2 q_2\rangle}{q^2} + \frac{2\pi\alpha_s}{9}\frac{\langle\bar q_1 q_1\rangle^2 - \langle\bar q_2 q_2\rangle^2}{q^4}, \tag{12}$$

first moment of tensor sum rule. Since we are concerned with only the first two Gegenbauer coefficients, we do not display the explicit forms of sum rules for other odd moments for simplification. When n is equal to 0, the Eq. (9) is in accord with the results shown in Ref. [12]. On the other hand, the correlation functions (7) and (8) can also be calculated at hadron level by inserting a complete set of quantum states $\Sigma|n\rangle\langle n|$, which are written as

$${\rm Im}I_s^{(2n,0)}(q^2)_{\rm had} = -\pi\delta(q^2-m_S^2)m_S^2\bar f_S^2\langle\xi_s^{2n}\rangle + \pi\frac{3}{8\pi^2}\frac{1}{2n+1}(2m_1m_2-q^2)\times\theta(q^2-s_S), \tag{13}$$

$${\rm Im}I_s^{(1,0)}(q^2)_{\rm had} = -\pi\delta(q^2-m_S^2)m_S^2\bar f_S^2\langle\xi_s^{1}\rangle + \pi\frac{3}{8\pi^2}(m_1^2-m_2^2)\theta(q^2-s_S), \tag{14}$$

$${\rm Im}I_\sigma^{(2n,0)}(q^2)_{\rm had} = -\pi\delta(q^2-m_S^2)\frac{2n+1}{3}m_S^2\bar f_S^2\langle\xi_\sigma^{2n}\rangle - \pi\frac{3}{8\pi^2}\frac{1}{2n+3}q^2\theta(q^2-s_S^\sigma), \tag{15}$$

$${\rm Im}I_\sigma^{(1,0)}(q^2)_{\rm had} = -\pi\delta(q^2-m_S^2)\frac{2}{3}m_S^2\bar f_S^2\langle\xi_\sigma^{1}\rangle + \pi\frac{1}{4\pi^2}(m_1^2-m_2^2)\theta(q^2-s_S^\sigma). \tag{16}$$

Here the quark-hadron duality has been used to obtain the above equations. Then we can match these two different representations of correlation functions (7) and (8) calculated in quark and hadron level by the dispersion relation

$$\frac{1}{\pi}\int ds\frac{{\rm Im}I(s)_{\rm had}}{s-q^2} = I(q^2)_{\rm QCD}. \tag{17}$$

In order to suppress the contributions from the excited resonances and continuum states, we apply the Borel transformation to both sides of the above equation. On the other hand, this transformation can also remove the arbitrary polynomials in q^2. Then we obtain

$$\frac{1}{\pi}\frac{1}{M^2}\int ds e^{-s/M^2}\,{\rm Im}I(s)_{\rm had} = \mathcal{B}_{M^2}I(q^2)_{\rm QCD}, \tag{18}$$

where M is the Borel parameter, and $\mathcal{B}_{M^2}$ is the operator of Borel transformation which is defined as [7,13]

$$\mathcal{B}_{M^2} = \lim_{\substack{-q^2,n\to\infty \\ -q^2/n=M^2}}\frac{(-q^2)^{(n+1)}}{n!}\left(\frac{d}{dq^2}\right)^n. \tag{19}$$

Finally, substituting Eqs. (9)–(16) into the Eq. (18), we have the scalar density even moments

$$-m_S^2\bar f_S^2 e^{-m_S^2/M^2}\langle\xi_s^{2n}\rangle = \frac{3}{8\pi^2}\frac{1}{2n+1}\int_0^{s_S}(2m_1m_2-s)e^{-s/M^2}ds - \frac{\alpha_s}{8\pi}\langle 0|G^2|0\rangle - \left[\left(\frac{2n+1}{2}m_1+m_2\right)\langle\bar q_1 q_1\rangle + \left(\frac{2n+1}{2}m_2+m_1\right)\langle\bar q_2 q_2\rangle\right] + \frac{g_s}{2M^2}\left\{\left[m_2+n\left(\frac{8n+11}{6}m_1+2m_2\right)\right]\langle\bar q_1\sigma G q_1\rangle + \left[m_1+n\left(\frac{8n+11}{6}m_2+2m_1\right)\right]\langle\bar q_2\sigma G q_2\rangle\right\} + \frac{48\pi\alpha_s}{9M^2}\langle\bar q_1 q_1\rangle\langle\bar q_2 q_2\rangle - \frac{4\pi\alpha_s}{81}(8n^2-16n-21)\frac{1}{M^2}(\langle\bar q_1 q_1\rangle^2 + \langle\bar q_2 q_2\rangle^2), \tag{20}$$

the first moment of scalar density

$$-m_S^2\bar{f}_S^2 e^{-m_S^2/M^2}\langle\xi_s^1\rangle = \frac{3}{8\pi^2}(m_1^2-m_2^2)\int_0^{s_S} ds e^{-s/M^2} - (m_1+m_2)(\langle\bar{q}_1q_1\rangle - \langle\bar{q}_2q_2\rangle) + (m_1^2-m_2^2)\frac{1-\ln\frac{\mu^2}{M^2}}{4M^2}\left\langle\frac{\alpha_s}{\pi}G^2\right\rangle + \frac{10\pi\alpha_s}{9M^2}(\langle\bar{q}_1q_1\rangle^2 - \langle\bar{q}_2q_2\rangle^2) + \frac{g_s}{2M^2}\left[\left(\frac{5}{4}m_1+2m_2\right)\langle\bar{q}_1\sigma Gq_1\rangle - \left(\frac{5}{4}m_2+2m_1\right)\langle\bar{q}_2\sigma Gq_2\rangle\right], \tag{21}$$

tensor even moments

$$-\frac{2n+1}{3}m_S^2\bar{f}_S^2 e^{-m_S^2/M^2}\langle\xi_\sigma^{2n}\rangle = -\frac{3}{8\pi^2}\frac{1}{2n+3}\int_0^{s_S^\sigma} s e^{-s/M^2}ds - \frac{8\pi\alpha_s}{9}\frac{\langle\bar{q}_1q_1\rangle\langle\bar{q}_2q_2\rangle}{M^2}\delta_{n,0} - \frac{2n+1}{2}(m_1\langle\bar{q}_1q_1\rangle + m_2\langle\bar{q}_2q_2\rangle) - \frac{\alpha_s}{24\pi}\langle 0|G^2|0\rangle + \frac{16n^2+14n+5}{24}g_s\frac{m_1\langle\bar{q}_1\sigma Gq_1\rangle + m_2\langle\bar{q}_2\sigma Gq_2\rangle}{M^2} - \frac{(32n^2+18n-35)\pi\alpha_s}{81}\frac{\langle\bar{q}_1q_1\rangle^2 + \langle\bar{q}_2q_2\rangle^2}{M^2}, \tag{22}$$

and the first moment of tensor current

$$-\frac{2}{3}m_S^2\bar{f}_S^2 e^{-m_S^2/M^2}\langle\xi_\sigma^1\rangle = \frac{1}{4\pi^2}(m_1^2-m_2^2)\int_0^{s_S^\sigma} e^{-s/M^2}ds - (m_1\langle\bar{q}_1q_1\rangle - m_2\langle\bar{q}_2q_2\rangle) - \frac{\alpha_s}{6\pi}(m_1^2-m_2^2)\frac{1+2\ln\frac{\mu^2}{M^2}}{M^2}\langle 0|G^2|0\rangle + \frac{3}{4}g_s\frac{m_1\langle\bar{q}_1\sigma Gq_1\rangle - m_2\langle\bar{q}_2\sigma Gq_2\rangle}{M^2} + \frac{2\pi\alpha_s}{9}\frac{\langle\bar{q}_1q_1\rangle^2 - \langle\bar{q}_2q_2\rangle^2}{M^2}. \tag{23}$$

Here the vacuum saturation approximation [13,14] has been used to describe the four quark condensate, i.e.,

$$\langle 0|\bar{q}^A_{\alpha a}(x)\bar{q}^B_{\beta b}(y)q^C_{\gamma c}q^D_{\delta d}|0\rangle = \tfrac{1}{144}[\delta_{AD}\delta_{BC}\delta_{\alpha\delta}\delta_{\beta\gamma}\delta_{ad}\delta_{bc} - \delta_{AC}\delta_{BD}\delta_{\alpha\gamma}\delta_{\beta\delta}\delta_{ac}\delta_{bd}]\langle\bar{q}^Aq^A\rangle\langle\bar{q}^Bq^B\rangle. \tag{24}$$

Here α, β, γ, δ are the spinor indices, a, b, c, d are the color indices, and A, B, C, D denote the flavor of quarks. Besides, the flavor indices in the right hand side of the above equation do not mean the sum of all flavors.

It is noted that all the parameters in the above sum rules are fixed at scale of Borel mass M. The renormalization group equations of decay constant, quark mass and condensate are given as [15]

$$\begin{aligned}
\bar{f}_S(M) &= \bar{f}_S(\mu)\left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right)^{4/b},\\
m_{q,M} &= m_{q,\mu}\left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right)^{-4/b},\\
\langle\bar{q}q\rangle_M &= \langle\bar{q}q\rangle_\mu\left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right)^{4/b},\\
\langle g_s\bar{q}\sigma Gq\rangle_M &= \langle g_s\bar{q}\sigma Gq\rangle_\mu\left(\frac{\alpha_s(\mu)}{\alpha_s(M)}\right)^{-2/3b},\\
\langle\alpha_s G^2\rangle_M &= \langle\alpha_s G^2\rangle_\mu,
\end{aligned} \tag{25}$$

with $b = (33-2n_f)/3$, n_f is the number of active quark flavors. Making use of the orthogonality of Gegenbauer polynomials

$$\begin{aligned}
&\int_0^1 dx C_n^{1/2}(2x-1)C_m^{1/2}(2x-1) = \frac{1}{2n+1}\delta_{mn},\\
&\int_0^1 dx x(1-x)C_n^{3/2}(2x-1)C_m^{3/2}(2x-1)\\
&\quad = \frac{(n+2)(n+1)}{4(2n+3)}\delta_{mn},
\end{aligned} \tag{26}$$

the Gegenbauer moments a_l, b_l can be related to moments, $\langle\xi_s^k\rangle$, $\langle\xi_\sigma^k\rangle$ for example:

$$\begin{aligned}
&a_1 = 3\langle\xi_1\rangle, \qquad a_2 = \tfrac{5}{2}(3\langle\xi_2\rangle - 1),\\
&a_4 = \tfrac{9}{8}(35\langle\xi_4\rangle - 30\langle\xi_2\rangle + 3), \qquad b_1 = \tfrac{5}{3}\langle\xi_1\rangle,\\
&b_2 = \tfrac{7}{12}(5\langle\xi_2\rangle - 1), \qquad b_4 = \tfrac{11}{24}(21\langle\xi_4\rangle - 14\langle\xi_2\rangle + 1).
\end{aligned} \tag{27}$$

The renormalization group equations of Gegenbauer moments are given as

$$\langle a_n(\mu)\rangle = \langle a_n(\mu_0)\rangle\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\gamma_n^S/b},$$
$$\langle b_n(\mu)\rangle = \langle b_n(\mu_0)\rangle\left(\frac{\alpha_s(\mu_0)}{\alpha_s(\mu)}\right)^{-\gamma_n^T/b}, \tag{28}$$

where the one-loop anomalous dimensions are [16]

$$\gamma_n^S = C_F\left(1 - \frac{8}{(n+1)(n+2)} + 4\sum_{j=2}^{n+1}\frac{1}{j}\right),$$
$$\gamma_n^T = C_F\left(1 + 4\sum_{j=2}^{n+2}\frac{1}{j}\right), \tag{29}$$

with $C_F = 4/3$.

III. NUMERICAL RESULTS AND DISCUSSIONS

The input parameters used in this paper are taken as [6,13,17,18]

$$\begin{aligned}
\langle \bar{s}s\rangle &= (0.8 \pm 0.1)\langle \bar{u}u\rangle,\\
\langle \bar{u}u\rangle &\cong \langle \bar{d}d\rangle \cong -(1.65 \pm 0.15)\times 10^{-2}\ \text{GeV}^3,\\
\left\langle \frac{\alpha_s}{\pi}G^a_{\mu\nu}G^{a\mu\nu}\right\rangle &= (0.005 \pm 0.004)\ \text{GeV}^4,\\
\langle g_s\bar{u}\sigma G u\rangle &\cong \langle g_s\bar{d}\sigma G d\rangle = m_0^2\langle \bar{u}u\rangle,\\
m_u(1\ \text{GeV}) &= 2.8\ \text{MeV},\\
\langle g_s\bar{s}\sigma G s\rangle &= (0.8 \pm 0.1)\langle g_s\bar{u}\sigma G u\rangle,\\
m_d(1\ \text{GeV}) &= 6.8\ \text{MeV},\\
\alpha_s(1\ \text{GeV}) &= 0.517,\\
m_s(1\ \text{GeV}) &= 142\ \text{MeV},\\
m_0^2 &= (0.8 \pm 0.2)\ \text{GeV}^2.
\end{aligned} \tag{30}$$

Here all the values for vacuum condensates are adopted at the scale $\mu = 1$ GeV. Next we are ready to analyze the sum rules for the scalar meson nonet one by one.

A. Mass, decay constant and moments for a_0 meson

1. Determination of mass, decay constant and scalar moments $\langle \xi^{2(4)}_{s,a_0}\rangle$ of a_0 from sum rules in (20)

Here a_0 is the scalar meson with quark contents $\bar{q}_1 q_2 = \bar{d}u$. A common way to obtain the sum rules of meson mass from Eq. (20) is taking the logarithm of both sides of this equation, and then applying the differential operator $M^4\partial/\partial M^2$ to them. However, firstly we need to fix the value of threshold parameter and Borel parameter in order to obtain the value of the mass. As far as the threshold parameter s_S is concerned, its value should be adopted so that the Borel window is stable enough which indicates that the mass is independent of the choice for M^2 in some region. For the choice of the Borel parameter, one requires that the contribution from continuum states is less than 30% and the contribution from dimension-six condensates is less than 10%. In view of the above requirements, we choose the threshold parameter $s_S = (4.5 \pm 0.3)\ \text{GeV}^2$, such that the stable Borel window is in the range $M^2 \in [1.60, 1.80]\ \text{GeV}^2$ which is shown in Fig. 1. From this figure, we can observe that the mass for $\bar{q}_1 q_2 = \bar{d}u$ scalar ground state is $m_{a_0} = (1320 \sim 1410)$ MeV. This is very close to the physical state $a_0(1450)$ [19]. It should be pointed out that the possibility of the existence of light scalar resonance near 1.4 GeV was firstly predicted by Ref. [20] as the first radial excitations of $a_0(980)$ according to the so called "linear dual models" on the assumptions of $\bar{q}q$ structure of $a_0(980)$. The decay constant of a_0 can be easily read from the sum rules in Eq. (20) as soon as the mass is known. The decay constant within the Borel window is also plotted in Fig. 1 as the second diagram. It is easy to find that the decay constant is quite stable within the Borel window $M^2 \in [1.30, 1.60]\ \text{GeV}^2$ when the contribution from continuum states and the dimension-six condensate is less than 30% and 10%, respectively. Therefore, we obtain the decay constant as $\bar{f}_{a_0}(1\ \text{GeV}) = (322 \sim 341)$ MeV. In the following subsections, all the values of decay constants and moments are calculated at scale of 1 GeV unless explicitly pointed out.

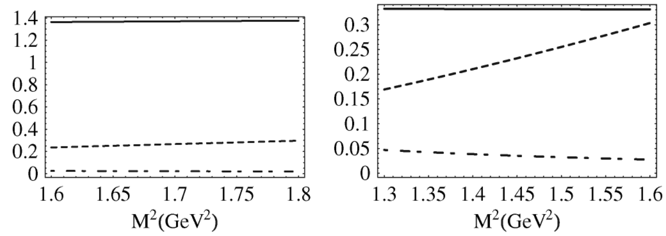

FIG. 1. Mass (left solid line) and decay constant (right solid line) of a_0 from scalar sum rule in Eq. (20) with $s_S = 4.5\ \text{GeV}^2$ as a function of Borel parameter M^2. The dashed line denotes the contribution from continuum states in the total sum rules and the dot-dashed line is the ratio of dimension-six condensates contribution in the total sum rules.

From the definition of twist-3 distribution amplitudes for a_0, it can be found that only even Gegenbauer moments are nonzero due to conservation of charge parity and isospin symmetry as mentioned in the introduction. Next we are going to consider the second and fourth moments of $\phi^s_{a_0}$ from scalar density sum rules for a_0 meson. Just as the determination of mass and decay constant, one should find a stable window for the sum rule of each moment. The contributions of continuum states and dimension-six condensates are plotted in Fig. 2, where the moments $\langle \xi^{2(4)}_{s,a_0}\rangle$ within the Borel window $M^2 \in [1.15, 1.45]\ \text{GeV}^2$ ($[1.25, 1.55]\ \text{GeV}^2$) are also included. For the second (fourth) moments, the contributions from both continuum states and the dimension-six condensates are less than 30%

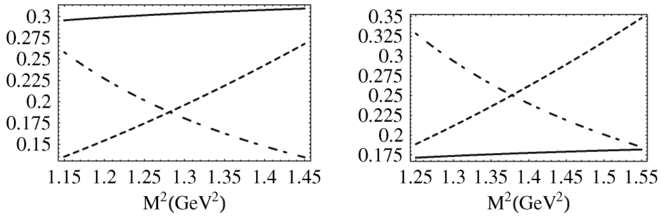

FIG. 2. $\langle \xi^2_{s,a_0}\rangle$ (left solid line) and $\langle \xi^4_{s,a_0}\rangle$ (right solid line) from scalar sum rules in Eq. (20) with $s_S = 4.5\ \mathrm{GeV}^2$ as a function of Borel parameter M^2. The dashed and the dot-dashed lines are the ratio of contribution from continuum states and dimension-six condensates, respectively.

(35%). Then we have $\langle \xi^2_{s,a_0}\rangle = 0.29 \sim 0.31$ and $\langle \xi^4_{s,a_0}\rangle = 0.16 \sim 0.19$.

2. Determination of mass, decay constant and tensor moments $\langle \xi^{2(4)}_{\sigma,a_0}\rangle$ of a_0 from sum rules in (22)

In the above, we have got the mass and decay constant for a_0 meson from scalar density sum rules in Eq. (20). Similarly, we can also extract them from tensor sum rules in Eq. (22). Moreover, the values of mass and decay constant may not be exactly the same between these two sum rules due to different correlation functions adopted for them. Following the similar procedure, one can get the mass and decay constant: $m_{a_0} = (1270 \sim 1390)$ MeV, $\bar{f}_{a_0} = (325 \sim 350)$ MeV, which are very close to the range we got from the sum rules of Eq. (20) in previous subsection. The mass (decay constant) is obtained under the condition that the contributions from both continuum states and the dimension-six condensates should be less than 30% (25%), respectively, in total sum rules. The threshold parameter s_S^σ is still adopted as $(4.5 \pm 0.3)\ \mathrm{GeV}^2$, while the Borel windows are $M^2 \in [1.60, 1.80]\ \mathrm{GeV}^2$ and $[1.20, 1.50]\ \mathrm{GeV}^2$, respectively. Making use of the mass and decay constant, we can determine the second and fourth moments $\langle \xi^2_{\sigma,a_0}\rangle$, $\langle \xi^4_{\sigma,a_0}\rangle$ for the tensor twist-3 distribution amplitude of a_0 meson within the Borel window $M^2 \in [1.20, 1.50]\ \mathrm{GeV}^2$ and $[1.15, 1.45]\ \mathrm{GeV}^2$ as shown in Fig. 3. Here the contributions from continuum states and the dimension-six condensate are no more than 30%, which indicate that the sum rules for these two moments are reliable. Hence, the results for $\langle \xi^2_{\sigma,a_0}\rangle$ and $\langle \xi^4_{\sigma,a_0}\rangle$ are $0.20 \sim 0.22$ and $0.093 \sim 0.12$, respectively, within the given Borel window and threshold parameter.

B. Mass, decay constant and moments for K_0^* meson

1. Determination of mass, decay constant and moments $\langle \xi^{1(2)}_{s,K_0^*}\rangle$ of K_0^* from scalar density sum rules

As explained before, here the scalar meson K_0^* is made up of $\bar{u}s$ quarks. Different from the a_0 meson, both odd and even moments of distribution amplitudes for K_0^* are nonzero. The mass and decay constant of K_0^* can be derived from scalar density sum rules in Eq. (20) following the same method as done for a_0 case. The threshold value is chosen as $s_S = (5.4 \pm 0.3)\ \mathrm{GeV}^2$ in the sum rules of Eq. (20) for K_0^* meson in order to gain the stable Borel window $M^2 \in [1.90, 2.10]\ \mathrm{GeV}^2$ and $[1.30, 1.70]\ \mathrm{GeV}^2$ for mass and decay constant, respectively. Then we can obtain the value of mass (decay constant) of K_0^* as $m_{K_0^*} = (1460 \sim 1560)$ MeV ($\bar{f}_{K_0^*} = (344 \sim 368)$ MeV) with the requirement that the contributions from both continuum states and dimension-six operator are less than 30% (25%).

Then we try to calculate the first and second moment for K_0^* meson scalar twist-3 distribution amplitude according to sum rules (20) and (21) respectively. For the first moment $\langle \xi^1_{s,K_0^*}\rangle$ of scalar density, we require that the contributions from both the continuum states and dimension-six condensates should be less than 15% in order to obtain stable Borel window. As for the sum rules of the second moment $\langle \xi^2_{s,K_0^*}\rangle$, the contributions from both the continuum states and dimension-six operators are less than 20%. From the Fig. 4, we can read out the results of the first scalar moment $\langle \xi^1_{s,K_0^*}\rangle$ as $(0.61 \sim 1.42) \times 10^{-2}$ within the Borel window $M^2 \in [1.90, 2.20]\ \mathrm{GeV}^2$, and the second moment $\langle \xi^2_{s,K_0^*}\rangle$ as $0.29 \sim 0.33$ within the Borel window $[1.20, 1.50]\ \mathrm{GeV}^2$. The threshold parameter is fixed at $s_S = (5.4 \pm 0.3)\ \mathrm{GeV}^2$.

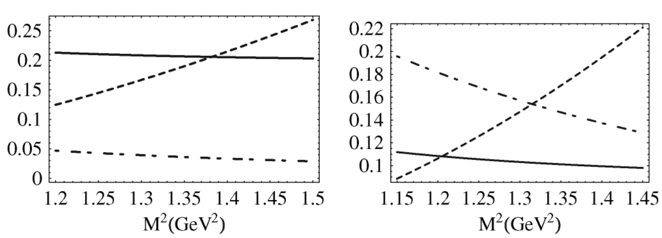

FIG. 3. $\langle \xi^2_{\sigma,a_0}\rangle$ (left solid line) and $\langle \xi^4_{\sigma,a_0}\rangle$ (right solid line) from tensor sum rules in Eq. (22) with $s_S^\sigma = 4.5\ \mathrm{GeV}^2$ as a function of Borel parameter M^2. The dashed and the dot-dashed line represent the ratio of contribution from the continuum states and dimension-six condensates.

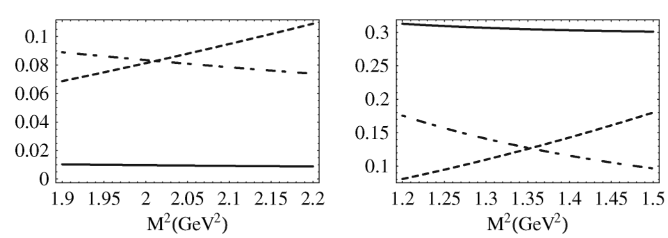

FIG. 4. $\langle \xi^1_{s,K_0^*}\rangle$ (left solid line) from scalar sum rules (21) and $\langle \xi^2_{s,K_0^*}\rangle$ (right solid line) from sum rules (20) with $s_S = 5.4\ \mathrm{GeV}^2$ as a function of Borel parameter M^2. The dashed and the dot-dashed lines indicate the ratio of continuum states and dimension-six condensates to the total sum rules, respectively.

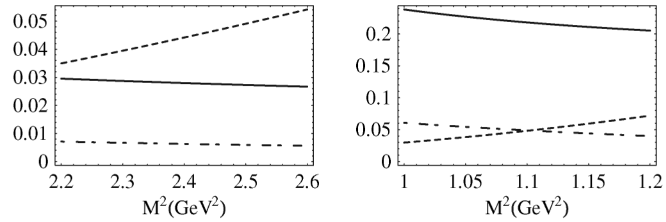

FIG. 5. $\langle \xi^1_{\sigma,K_0^*} \rangle$ (left solid line) from tensor sum rules in (23) and $\langle \xi^2_{\sigma,K_0^*} \rangle$ (right solid line) from sum rules in (22) with $s_S^\sigma = 5.4\ \mathrm{GeV}^2$ as a function of Borel parameter M^2. The dashed and the dot-dashed lines represent the ratio corresponding to the contribution from continuum states and dimension-six condensates in the total sum rules, respectively.

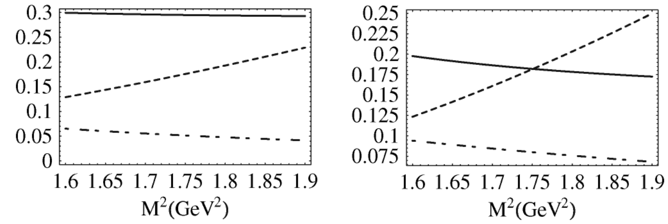

FIG. 6. $\langle \xi^2_{s,f_0} \rangle$ (left solid line) and $\langle \xi^4_{s,f_0} \rangle$ (right solid line) from scalar sum rules in Eq. (20) with $s_S = 6.5\ \mathrm{GeV}^2$ as a function of Borel parameter M^2. The dashed and the dot-dashed lines reflect the ratio of continuum states and dimension-six condensates to the total sum rules, respectively.

2. Determination of mass, decay constant and moments $\langle \xi^{1(2)}_{\sigma,K_0^*} \rangle$ of K_0^* from tensor current sum rules

Similarly, we can also derive the results of mass and decay constant from the tensor operator sum rules in Eq. (22). Here we will only show our values of them as $m_{K_0^*} = (1440 \sim 1550)$ MeV and $\bar{f}_{K_0^*} = (349 \sim 375)$ MeV within the Borel window $M^2 \in [2.00, 2.20]\ \mathrm{GeV}^2$ and $[1.30, 1.60]\ \mathrm{GeV}^2$. The threshold parameter is set the same as before, $s_S^\sigma = (5.4 \pm 0.3)\ \mathrm{GeV}^2$. The contributions from both the continuum states and dimension-six condensates are required to be less than 30% (20%) for mass (decay constant) sum rules, respectively. The mass of K_0^* determined here are consistent with the one determined in the previous subsection from sum rules (20), which is quite close to the physical state $K_0^*(1430)$. The first and second moments $\langle \xi^1_{\sigma,K_0^*} \rangle$, $\langle \xi^2_{\sigma,K_0^*} \rangle$ of the tensor twist-3 distribution amplitude can be computed following the same method, which have been plotted in Fig. 5. It is found that the sum rules for these two moments are quite stable within the Borel window $M^2 \in [2.20, 2.60]\ \mathrm{GeV}^2$ and $[1.00, 1.20]\ \mathrm{GeV}^2$, respectively, since contributions from both continuum states and dimension-six condensates are less than 10%. The results for them are shown as: $\langle \xi^1_{\sigma,K_0^*} \rangle = (2.2 \sim 3.3) \times 10^{-2}$, $\langle \xi^2_{\sigma,K_0^*} \rangle = 0.20 \sim 0.25$.

C. Mass, decay constant and moments for f_0 meson

1. Determination of mass, decay constant and scalar moments $\langle \xi^{2(4)}_{s,f_0} \rangle$ of f_0 from sum rules in (20)

Here f_0 refers to the scalar meson which is made up of $\bar{s}s$ quark. The sum rules for f_0 are much the same as for the a_0 meson. The odd moments vanish due to conservation of C parity. Therefore, we will only consider the first two even moments, $\langle \xi^2_{s,f_0} \rangle$ and $\langle \xi^4_{s,f_0} \rangle$ of the scalar twist-3 distribution amplitude for the f_0 meson. Since the calculations are similar as that for a_0 meson, the mass and decay constant for f_0 are given straightforward as $m_{f_0} = (1640 \sim 1730)$ MeV and $\bar{f}_{f_0} = (369 \sim 391)$ MeV within the Borel window $M^2 \in [2.50, 2.70]\ \mathrm{GeV}^2$, $[1.70, 2.00]\ \mathrm{GeV}^2$ respectively. The threshold parameter is set as $s_S = (6.5 \pm 0.3)\ \mathrm{GeV}^2$. Here we also require that the contributions from both the continuum states and dimension-six condensates are less than 30% (20%) for mass (decay constant) sum rules, respectively. As for the second and forth moments $\langle \xi^2_{s,f_0} \rangle$, $\langle \xi^4_{s,f_0} \rangle$, the results within the same Borel window $M^2 \in [1.60, 1.90]\ \mathrm{GeV}^2$ are plotted in Fig. 6. The number of $\langle \xi^2_{s,f_0} \rangle$, $\langle \xi^4_{s,f_0} \rangle$ are $0.29 \sim 0.31$ and $0.17 \sim 0.20$ respectively within the given Borel window and threshold parameter. The requirement that the contributions from the continuum states and dimension-six operator are less than 25% (30%) for the second (forth) scalar moments has been used.

2. Determination of mass, decay constant and tensor moments $\langle \xi^{2(4)}_{\sigma,f_0} \rangle$ of f_0 from sum rules in (22)

The mass and decay constant of f_0 can also be derived from tensor sum rules in Eq. (22). Adopting the same threshold parameter as the scalar density sum rules, we obtain the results as $m_{f_0} = (1620 \sim 1710)$ MeV and $\bar{f}_{f_0} = (381 \sim 426)$ MeV within the Borel window $M^2 \in [2.50, 2.70]\ \mathrm{GeV}^2$, $[1.20, 1.60]\ \mathrm{GeV}^2$ respectively. Here we require that contributions from both continuum states and dimension-six operators are less than 30% (10%) for mass (decay constant) sum rules. The mass we get here from tensor sum rules and also that from the scalar density sum rules (20) in previous subsection is close to the physical state $f_0(1710)$. The second and forth moment $\langle \xi^2_{\sigma,f_0} \rangle$, $\langle \xi^4_{\sigma,f_0} \rangle$ of tensor twist-3 distribution amplitude are also displayed in Fig. 7 within the Borel window $M^2 \in [1.50, 1.80]\ \mathrm{GeV}^2$ and $[1.60, 1.90]\ \mathrm{GeV}^2$ respectively. The condition that the contributions from the continuum states and dimension-six operators are less than 25% (30%) is adopted for the second (forth) tensor moment. The value of $\langle \xi^2_{\sigma,f_0} \rangle$ and $\langle \xi^4_{\sigma,f_0} \rangle$ are $0.15 \sim 0.17$ and $0.057 \sim 0.082$ within the given Borel window and threshold parameter.

Now we have finished the calculation of the moments $\langle \xi^n_{s(\sigma)} \rangle$ of twist-3 distribution amplitudes for scalar mesons a_0, K_0^* and f_0 in the framework of QCD sum rules. With the results of $\langle \xi^n_{s(\sigma)} \rangle$, it is straightforward to derive the

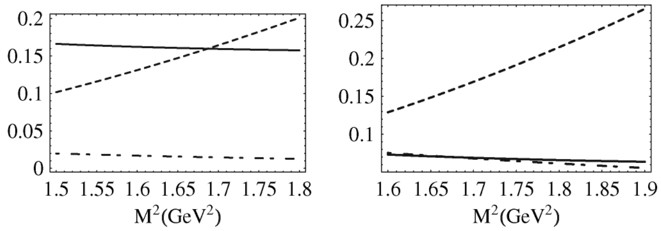

FIG. 7. $\langle \xi^2_{\sigma,f_0} \rangle$ (left solid line) and $\langle \xi^4_{\sigma,f_0} \rangle$ (right solid line) from tensor sum rules in Eq. (22) with $s_S^\sigma = 6.5\ \mathrm{GeV}^2$ as a function of Borel parameter M^2. The dashed and the dot-dashed line represent the ratio of contribution from the continuum states and dimension-six condensates.

Gegenbauer moments a_m and b_m in Eq. (3) and (4) using Eq. (27). The results for the first nonzero Gegenbauer moments at 1 GeV and 2.1 GeV scales are shown in Table I and II. They can be applied to various approaches involving light-cone distribution amplitudes of hadrons, such as perturbative QCD approach [21], QCD factorization approach [22] and light-cone sum rules [23] etc. As mentioned above, the odd moments of twist-3 distribution amplitudes for scalar mesons a_0 and f_0 are zero due to conservation of charge parity and flavor symmetry as explained in the introduction. As a by-product, we also collect the masses and decay constants of scalar mesons in Table I and II. These masses indicate that the ground state of $\bar{q}q$ scalars are probably $a_0(1450)$, $K_0^*(1430)$ and $f_0(1710)$.

In Ref. [24], the authors also studied the mass and decay constant of scalar meson K_0^*. Their results are $m_{K_0^*} = (1410 \pm 49)$ MeV and $f_{K_0^*} = (427 \pm 85)$ MeV, which are consistent with our results within error bar.

IV. SUMMARY

In this work, we have studied the masses, decay constants and twist-3 distribution amplitudes of scalar mesons based on the renormalization group improved QCD sum rules. It is shown that the mass sum rules for scalar mesons are not very satisfied, since the Borel windows are a bit narrow for all the three scalar mesons. Our results for the scalar meson masses show that the physical states $a_0(1450)$, $K_0^*(1430)$ and $f_0(1710)$ are preferred to be the ground state of scalar mesons. The sum rules for decay constants of these three scalar mesons are very stable in a much broader Borel window. The second and forth scalar moments of a_0 can be obtained with 30% and 35% uncertainties, respectively, while both the second and forth tensor moments of a_0 can be derived within 30% uncertainties. As for the K_0^* meson case, the first and second moments of scalar density twist-3 distribution amplitude $\phi^s_{K_0^*}$ are obtained under 15% and 20% uncertainties, respectively. The uncertainties can be reduced to 10% for both the results of the first and second moments of tensor twist-3 distribution amplitude $\phi^\sigma_{K_0^*}$. For the case of f_0 meson, the second moment for both of scalar twist-3 distribution amplitude $\phi^s_{f_0}$ and tensor twist-3 distribution amplitude $\phi^\sigma_{f_0}$ each has 25% uncertainties. Besides, the fourth moment for each of these two distribution amplitudes could be obtained within 30% uncertainties. It is also

TABLE I. Masses, decay constants and Gegenbauer moments from the scalar density sum rules (20) and (21) at the scale $\mu = 1$ GeV and 2.1 GeV (shown in the second line of each meson).

state	m (MeV)	$\bar{f}$ (MeV)	$a_1(\times 10^{-2})$	a_2	a_4
a_0	$1320 \sim 1410$	$322 \sim 341$	0	$-0.33 \sim -0.18$	$-0.11 \sim 0.39$
		$391 \sim 414$		$-0.26 \sim -0.14$	$-0.075 \sim 0.27$
K_0^*	$1460 \sim 1560$	$344 \sim 368$	$1.8 \sim 4.2$	$-0.33 \sim -0.025$	$\cdots$
		$418 \sim 447$	$1.6 \sim 3.8$	$-0.26 \sim -0.020$	
f_0	$1640 \sim 1730$	$369 \sim 391$	0	$-0.33 \sim -0.18$	$0.28 \sim 0.79$
		$448 \sim 475$		$-0.26 \sim -0.14$	$0.19 \sim 0.54$

TABLE II. Masses, decay constants and Gegenbauer moments from the tensor sum rules (22) and (23) at the scale $\mu = 1$ GeV and 2.1 GeV (shown in the second line of each meson).

state	m (MeV)	$\bar{f}$ (MeV)	$b_1(\times 10^{-2})$	b_2	b_4
a_0	$1270 \sim 1390$	$325 \sim 350$	0	$0 \sim 0.058$	$0.070 \sim 0.20$
		$395 \sim 425$		$0 \sim 0.041$	$0.045 \sim 0.13$
K_0^*	$1440 \sim 1550$	$349 \sim 375$	$3.7 \sim 5.5$	$0 \sim 0.15$	$\cdots$
		$424 \sim 456$	$2.8 \sim 4.2$	$0 \sim 0.11$	
f_0	$1620 \sim 1710$	$381 \sim 426$	0	$-0.15 \sim -0.088$	$0.044 \sim 0.16$
		$463 \sim 518$		$-0.11 \sim -0.062$	$0.028 \sim 0.10$

worthwhile to emphasize that the correlation functions are calculated to leading α_s power based on operator product expansion in this work, which will bring some additional uncertainties to mass, decay constants and Gegenbauer coefficients.

It is found that the second Gegenbauer coefficients of scalar density twist-3 distribution amplitudes for K_0^* and f_0 mesons are quite close to that for a_0, which indicates that the SU(3) symmetry breaking effect is tiny here. However, this effect could not be neglected for the forth Gegenbauer coefficients of scalar twist-3 distribution amplitudes between a_0 and f_0. Furthermore, one can also observe that the first two Gegenbauer coefficients corresponding to tensor current twist-3 distribution amplitudes for all the a_0, K_0^* and f_0 are very small. As is well known, the light-cone distribution amplitudes play a critical role for hadronic decay processes in the framework of factorization theorem where it describes the bound state effect of hadrons. The available twist-3 distribution amplitudes of scalar mesons allow us to improve the accuracy of the theoretical predictions on the properties of scalar mesons, in particular, for the heavy flavor hadron decays to scalar mesons; so that it is very helpful for us to understand the structure of scalar mesons and strong interactions.

ACKNOWLEDGMENTS

This work is partly supported by National Science Foundation of China under Grant Nos. 10475085 and 10625525. The authors would like to thank H. Y. Cheng, T. Huang, X. H. Wu, K. C. Yang, M. Z. Yang, Y. Li, Y. L. Shen and W. Wang for helpful discussions. C. D. Lü thank DESY theory group and Hamburg University for the hospitality during his visit at Hamburg.

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