Light custodians in natural composite Higgs models

Roberto Contino,[1] Leandro Da Rold,[2] and Alex Pomarol[3]

[1]*Dipartimento di Fisica, Università di Roma "La Sapienza" and INFN, Piazzale Aldo Moro 2, I-00185 Roma, Italy*
[2]*Instituto de Física, Universidade de São Paulo, Rua do Matão Travessa R, 187, 05508-900 SP, Brazil*
[3]*IFAE, Universitat Autònoma de Barcelona, 08193 Bellaterra, Barcelona*
(Received 11 January 2007; published 23 March 2007)

We present a class of composite Higgs models arising from a warped extra dimension that can satisfy all the electroweak precision tests in a significant portion of their parameter space. A custodial symmetry plays a crucial role in keeping the largest corrections to the electroweak observables below their experimental limits. In these models the heaviness of the top quark is not only essential to trigger the electroweak symmetry breaking, but it also implies that the lowest top resonance and its custodial partners, the custodians, are significantly lighter than the other resonances. These custodians are the trademark of these scenarios. They are exotic colored fermions of electromagnetic charges 5/3, 2/3, and $-1/3$, with masses predicted roughly in the range 500–1500 GeV. We discuss their production and detection at the CERN LHC.

DOI: 10.1103/PhysRevD.75.055014 PACS numbers: 12.60.Cn, 04.62.+v, 14.80.Cp

I. INTRODUCTION

Theories of warped extra dimensions, with their holographic interpretation in terms of 4D strongly coupled field theories [1,2], have recently given a new twist to the idea of Higgs compositeness [3,4]. Calculability is one of the main virtues of this new class of models. Differently from the old approach [5], physical quantities of central interest can be computed in a perturbative expansion. This opened up the possibility of building predictive and realistic theories of electroweak symmetry breaking (EWSB).

The minimal composite Higgs model (MCHM) of Ref. [4] is extremely simple to define based only on symmetry considerations. It consists in a 5D theory on AdS spacetime compactified by two boundaries, respectively called infrared (IR) and ultraviolet (UV) boundaries [6].[1] An $SO(5) \times U(1)_X \times SU(3)_c$ gauge symmetry in the bulk is broken down to $SO(4) \times U(1)_X \times SU(3)_c$ on the IR boundary (with $SO(4) \sim SU(2)_L \times SU(2)_R$), delivering four pseudo Goldstone bosons that transform as a **4** of SO(4) and are identified with the Higgs doublet. On the UV boundary the bulk symmetry is reduced to the standard model (SM) gauge group $G_{\rm SM} = SU(2)_L \times U(1)_Y \times SU(3)_c$, where hypercharge is defined as $Y = X + T_3^R$. Once the SO(5) bulk representations in which the SM fermions are embedded and their boundary conditions are chosen, the model is completely determined. One can write down the most general Lagrangian compatible with the above symmetries, compute the one-loop Higgs potential and determine the region of the parameter space with the correct EWSB and SM fermion masses.

In Ref. [4] the SM fermions were embedded in spinorial representations of SO(5). This choice leads generically to large corrections to the $Zb_L\bar{b}_L$ coupling, with the result that a sizable portion of the parameter space of the model ($\sim 95\%$) is ruled out [7]. However, it was recently realized [8] that the $Zb_L\bar{b}_L$ constraint is strongly relaxed if the fermions are embedded in fundamental (**5**) or antisymmetric (**10**) representations of SO(5), with b_L belonging to a $(\mathbf{2},\mathbf{2})$ of $SU(2)_L \times SU(2)_R$, and the boundary symmetry SO(4) is enlarged to O(4). In this case a subgroup of the custodial symmetry $O(3) \subset O(4)$ protects the $Zb_L\bar{b}_L$ coupling from receiving corrections.

In this paper we investigate the predictions of the MCHM with fermions in **5**'s or **10**'s of SO(5) and the IR symmetry enlarged to O(4). We will determine the region of the parameter space with successful EWSB, and study the constraints imposed by the electroweak precision tests. The most relevant constraint comes from the Peskin-Takeuchi S parameter, which excludes ~50%–75% of the parameter space of the model.[2] A sizable portion of the latter is still allowed, and awaits to be explored at the CERN LHC. An important prediction of the model is that the heaviness of the top quark implies that the lowest top Kaluza-Klein (KK) resonance and its O(4)-custodial partners, the "custodians," are significantly lighter than the other KK resonances. The custodians are color triplets and transform as $\mathbf{2_{7/6}}$ of $SU(2)_L \times U(1)_Y$ when the SM fermions are embedded in **5**'s of SO(5), and as $\mathbf{2_{7/6}} \oplus \mathbf{3_{2/3}} \oplus \mathbf{1_{5/3}} \oplus \mathbf{1_{-1/3}}$ in the case of **10**'s of SO(5). They have electromagnetic charges $Q_{\rm em} = 5/3, 2/3, -1/3$ and their masses are predicted roughly in the range 500–1500 GeV.

[1]Although these boundaries act as sharp cutoffs of the extra dimension, they can be considered as an effective description of some smoother configuration that can arise in a more fundamental (string) theory.

[2]This must be compared with the most popular supersymmetric models where experimental constraints have already excluded large portions of the parameter space ($\sim 99\%$ in the case of universal soft masses [9]).

The excitations of the SM gauge bosons are always heavier, with masses around 2–3 TeV. The Higgs mass is predicted in the range $m_{\rm Higgs} \simeq 115$–190 GeV. Its value is correlated with the mass of the custodians, since top loops give the largest contribution to the Higgs potential, and are thus responsible for triggering the EWSB. Being at the reach of the LHC, the custodians offer the best signature for distinguishing 5D composite Higgs from other scenarios of EWSB. We will discuss their most important production mechanisms and decay channels.

Other models of EWSB in which b_L is embedded in a $(\mathbf{2}, \mathbf{2})$ of $SU(2)_L \times SU(2)_R$ to protect $Zb\bar{b}$ have been proposed in Refs. [10–12].

II. HIGGS POTENTIAL AND EWSB

At the tree level, the massless spectrum of the bosonic sector of the MCHM consists of the SM gauge bosons, plus four real scalar fields that correspond to the SO(5)/SO(4) degrees of freedom of the fifth component of the 5D gauge field. The presence of these scalars is dictated by the symmetry breaking pattern of the model: they are pseudo Goldstone bosons and have the right quantum numbers to be identified with the SM Higgs, h^a ($a = 1, 2, 3, 4$). In addition to the massless sector, the theory also contains an infinite tower of massive resonances: the KK states. We can integrate out all the massive states and obtain an effective low-energy Lagrangian for the massless modes. We do this by following the holographic approach of Ref. [4]. The form of the effective Lagrangian for the gauge bosons is completely determined by the symmetries of the model. It can be found in Ref. [4], and it will not be repeated here.[3] We only report the following relations:

$$v \equiv \epsilon f_\pi = f_\pi \sin\frac{\langle h\rangle}{f_\pi} = 246\ \text{GeV},\qquad f_\pi = \frac{1}{L_1}\frac{2}{\sqrt{g_5^2 k}},\qquad m_\rho \simeq \frac{3\pi}{4L_1}, \tag{1}$$

where $h \equiv \sqrt{(h^a)^2}$. Here L_1 denotes the position of the IR boundary in conformal coordinates and sets the mass gap ($1/L_1 \sim$ TeV); g_5 is the SO(5) gauge coupling in the bulk; $1/k$ is the AdS_5 curvature radius; and m_ρ is the mass of the lightest gauge boson KK. Following Ref. [4], we define

$$\frac{1}{N} \equiv \frac{g_5^2 k}{16\pi^2} \tag{2}$$

as our expansion parameter. The fermionic sector of the model depends on our choice for the 5D bulk multiplets. We want to study the case in which the SM fermions are embedded in fundamental (**5**) or antisymmetric (**10**) representations of SO(5). To this aim, we consider two different choices of multiplets and boundary conditions. In the first choice, which we will refer to as the $MCHM_5$, the bulk fermions transform as **5**'s of SO(5) and are defined by Eq. (A1) of Appendix A. In the second choice, which we will denote as the $MCHM_{10}$, the bulk fermions are defined by Eq. (B1) and transform as **10**'s of SO(5). For all the technical details, we refer the reader to Appendixes A and B. Here it will suffice to say that in both cases the low-energy effective Lagrangian for the quarks can be written, in momentum space and at the quadratic order, as

$$\mathcal{L}_{\rm eff} = \bar{q}_L \not{p}\Big[\Pi_0^q(p^2) + \frac{s_h^2}{2}(\Pi_1^{q1}(p^2)\hat{H}^c\hat{H}^{c\dagger} + \Pi_1^{q2}(p^2)\hat{H}\hat{H}^\dagger)\Big]q_L + \bar{u}_R\not{p}\Big(\Pi_0^u(p^2) + \frac{s_h^2}{2}\Pi_1^u(p^2)\Big)u_R + \bar{d}_R\not{p}\Big(\Pi_0^d(p^2) + \frac{s_h^2}{2}\Pi_1^d(p^2)\Big)d_R + \frac{s_h c_h}{\sqrt{2}}M_1^u(p^2)\bar{q}_L\hat{H}^c u_R + \frac{s_h c_h}{\sqrt{2}}M_1^d(p^2)\bar{q}_L\hat{H} d_R + \text{H.c.} \tag{3}$$

Here $c_h \equiv \cos(h/f_\pi)$, $s_h \equiv \sin(h/f_\pi)$, and

$$\hat{H} = \frac{1}{h}\begin{bmatrix} h^1 - ih^2 \\ h^3 - ih^4 \end{bmatrix},\qquad \hat{H}^c = \frac{1}{h}\begin{bmatrix} -(h^3 + ih^4) \\ h^1 + ih^2 \end{bmatrix}. \tag{4}$$

The form factors $\Pi_{0,1}^i$ and M_1^i in Eq. (3) can be computed in terms of 5D propagators using the holographic approach of Ref. [4]. Their explicit form is given in Appendixes A and B. From Eq. (3) one can derive the SM up and down fermion masses, $m_{u,d}$. A reasonably good approximation to the exact expressions can be obtained by setting $p^2 = 0$ in the form factors, the error being of order $(m_{u,d}L_1)$. We obtain

$$m_u \simeq \frac{s_h c_h}{\sqrt{2}}\frac{M_1^u(0)}{\sqrt{Z_{u_L}Z_{u_R}}},\qquad m_d \simeq \frac{s_h c_h}{\sqrt{2}}\frac{M_1^d(0)}{\sqrt{Z_{d_L}Z_{d_R}}}, \tag{5}$$

where $Z_{u_L,d_L} = \Pi_0^q(0) + (s_h^2/2)\Pi_1^{q1,q2}(0)$ and $Z_{u_R,d_R} = \Pi_0^{u,d}(0) + (s_h^2/2)\Pi_1^{u,d}(0)$. An explicit version of Eq. (5) in terms of the 5D input parameters is given in Eqs. (A13) and (B8) of Appendixes A and B, respectively, for the $MCHM_5$ and the $MCHM_{10}$.

At the tree level the Higgs field is an exact Goldstone boson, and as such it has no potential. At the one-loop level, the virtual exchange of the SM fields transmits the explicit breaking of SO(5) and generates a potential for h.

[3]The only difference between the gauge sector of the model presented here and that of Ref. [4] is the symmetry on the IR boundary. Enlarging SO(4) to O(4) forbids the otherwise allowed IR-boundary kinetic term $\epsilon^{(ijkl)}F_{(ij)}^{\mu\nu}F_{(kl)\mu\nu}$, where i, j, k, l are SO(4) indices. Since this term was not included in Ref. [4], we can use the results for the gauge sector presented there.

The largest contribution comes from t_L, b_L, and t_R, and from the $SU(2)_L$ gauge bosons, since these are the fields that are most strongly coupled to the Higgs. They give

$$V(h) = \frac{9}{2}\int \frac{d^4p}{(2\pi)^4}\log\left(\Pi_0 + \frac{s_h^2}{4}\Pi_1\right) - 2N_c\int\frac{d^4p}{(2\pi)^4}\left\{\log\left(\Pi_0^q + \frac{s_h^2}{2}\Pi_1^{q2}\right) + \log\left[p^2\left(\Pi_0^u + \frac{s_h^2}{2}\Pi_1^u\right)\left(\Pi_0^q + \frac{s_h^2}{2}\Pi_1^{q1}\right) - \frac{s_h^2c_h^2}{2}(M_1^u)^2\right]\right\}. \tag{6}$$

Here, and from now on, the fermionic form factors Π_0^q, $\Pi_1^{q1,q2}$, $\Pi_{0,1}^u$, M_1^u stand for those of the 3rd quark family, $q_L = (t_L, b_L)$ and t_R. The gauge form factors $\Pi_{0,1}$ can be found in Ref. [4]. Since the Π_1^i's and M_1^u drop exponentially for $pL_1 \gg 1$, the logarithms in Eq. (6) can be expanded and the potential is well approximated by

$$V(h) \simeq \alpha s_h^2 - \beta s_h^2 c_h^2, \tag{7}$$

where α and β are integral functions of the form factors. In particular, α receives contributions from loops of the gauge fields and of q_L or t_R alone. We will denote these contributions, respectively, by $\alpha_{\rm gauge}$, α_L and α_R. On the other hand, β receives contributions from loops where both t_L and t_R propagate. For $\alpha < \beta$ and $\beta \geq 0$ we have that the electroweak symmetry is broken: $\epsilon \neq 0$. If $\beta > |\alpha|$, the minimum of the potential is at

$$s_h = \epsilon = \sqrt{\frac{\beta - \alpha}{2\beta}}, \tag{8}$$

while for $\beta < |\alpha|$ the minimum corresponds to $c_h = 0$, and the EWSB is maximal: $\epsilon = 1$. In this latter case the fermion masses vanish—see Eq. (5)—due to an accidental chiral symmetry that is restored in the limit $\epsilon \to 1$. The model is thus realistic only for $0 < \epsilon < 1$, i.e. $\beta > |\alpha|$. The coefficients α and β can be computed in terms of the relevant 5D parameters

$$N, \quad c_q, \quad c_u, \quad \tilde{m}_u, \quad \tilde{M}_u, \tag{9}$$

where c_q, c_u are the bulk masses (in units of k) of the 5D multiplets ξ_q, ξ_u that contain the SM q_L and t_R, and $\tilde{m}_u$, $\tilde{M}_u$ are mass terms localized on the IR boundary that mix ξ_q with ξ_u (see Appendixes A and B). The scale L_1 has been traded for v. The five parameters of Eq. (9) are not completely determined by the present experimental data. There are only two constraints coming from fixing the top quark mass to its experimental value, $m_t^{\rm pole} = 173$ GeV, and by requiring $0 < \epsilon < 1$.

Let us outline the viable region of this five-dimensional space of parameters. The large Yukawa coupling of the top quark is reproduced by having the wave functions of the t_L and t_R zero modes peaked towards the IR boundary, where the Higgs lives. This constrains the 5D bulk masses to lie in the interval $|c_{q,u}| < 1/2$; see Eqs. (A13) and (B8). In the 4D dual interpretation, this corresponds to saying that the elementary fields t_L and t_R couple to relevant operators $\mathcal{O}$ of the strongly coupled conformal field theory (CFT), with conformal dimension $3/2 < \dim[\mathcal{O}] < 5/2$ (see [13]). Since the operators $\mathcal{O}$ have the quantum numbers to excite the fermionic composite resonances, the elementary top will have a sizable mixing with these massive states. The stronger the mixing, the larger the degree of compositeness of the physical top quark will be. The requirement $|c_{q,u}| < 1/2$ is also necessary in order to have EWSB. In this region the top quark contribution to the Higgs potential dominates over the gauge one, which would otherwise align the vacuum in an $(SU(2)_L \times U(1)_Y)$-preserving direction (since $\alpha_{\rm gauge}$ is always positive). The conditions $\alpha < 0$ and $\beta > 0$ can then be easily satisfied thanks to the top contribution. In other words, the EWSB, in our model, is a direct consequence of the heaviness of the top.

To illustrate this point, we show in Fig. 1 the contour plots of ϵ in the planes (c_q, c_u) and $(\tilde{m}_u, \tilde{M}_u)$, respectively, for the $MCHM_{10}$ and the $MCHM_5$. The region with no EWSB ($\epsilon = 0$) is depicted in black, and the dashed black curve corresponds to $m_t^{\rm pole} = 173$ GeV. In each plot, we have set $N = 8$ and kept the remaining 5D parameters fixed. The condition $\beta > |\alpha|$, i.e. $0 < \epsilon < 1$, further selects a smaller region of the planes (c_q, c_u) and $(\tilde{m}_u, \tilde{M}_u)$. A naive estimate shows that $|\alpha_{L,R}|$ is parametrically larger than β by a factor $(1/4 - c_{u,q}^2)$. A reduction in α, however, can be obtained in the region where $\alpha_L \simeq -\alpha_R$. As already pointed out in Ref. [4], this is possible since α_L and α_R have generally opposite sign. In the case of the $MCHM_{10}$, the regions with smaller α are the two gray areas with the "boomerang" shape shown in the left plot of Fig. 1, plus a specular region under $c_q \to -c_q$ which is not shown. These two solutions correspond to q_L almost elementary ($c_q \simeq +1/2$) or almost composite ($c_q \simeq -1/2$). We found that the case of the $MCHM_5$ is analogous, but with the role of c_q and c_u interchanged: the two possible solutions are for t_R almost elementary ($c_u \simeq -1/2$) or almost composite ($c_u \simeq +1/2$).

A second circumstance in which $\beta > |\alpha|$ is when $\tilde{m}_u \simeq -1/\tilde{M}_u$. As one can directly check, by using their expressions in terms of 5D propagators, the fermionic form factors Π_1^{q1}, Π_1^u, and consequently $\alpha_{L,R}$,[4] identically vanish for $\tilde{m}_u = -1/\tilde{M}_u$ (both in the $MCHM_5$ and in the $MCHM_{10}$). Therefore, for $\tilde{m}_u \simeq -1/\tilde{M}_u$ one can have $\beta > |\alpha|$. This is shown for the $MCHM_5$ in Fig. 1, right plot, and

[4]Notice that $\Pi_1^{q2} = 2\Pi_1^{q1}$ in the $MCHM_{10}$, while in the $MCHM_5$ Π_1^{q2} is always suppressed and its contribution to α_L can be neglected; see Appendixes A and B.

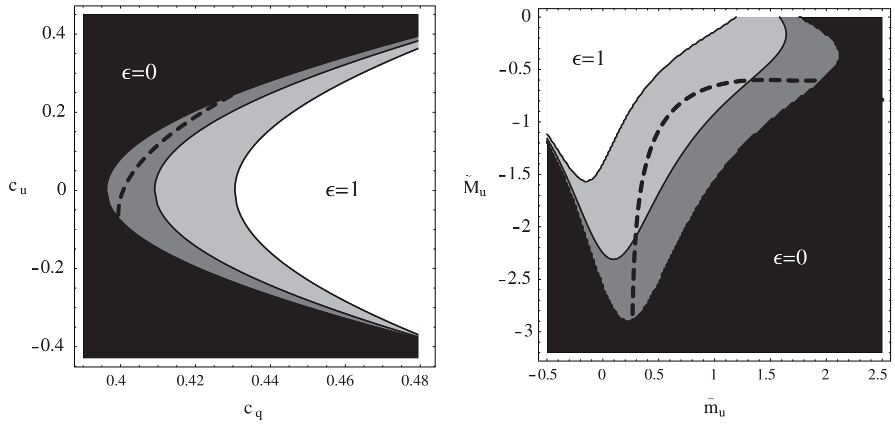


FIG. 1. Contour plots of ϵ in the plane (c_q, c_u) with $\tilde{m}_u = 1$, $\tilde{M}_u = -2$, $N = 8$ for the MCHM_{10} (left plot), and in the plane $(\tilde{m}_u, \tilde{M}_u)$ with $c_q = 0.35$, $c_u = 0.45$, $N = 8$ for the MCHM_5 (right plot). The two gray areas correspond to the region with EWSB and nonzero fermion masses, $0 < \epsilon < 1$. The lighter gray area is excluded when the bound $S \lesssim 0.3$ is imposed. The dashed black line represents the curve with $m_t^{\overline{\mathrm{MS}}}(2\ \mathrm{TeV}) = 150$ GeV, equivalent to $m_t^{\mathrm{pole}} = 173$ GeV.

a similar result holds for the MCHM_{10}. Even though one can reach the $0 < \epsilon < 1$ region by moving along the plane $(\tilde{m}_u, \tilde{M}_u)$ for almost any choice of c_q, c_u, the additional constraint of having the top quark mass equal to its experimental value (the black dashed line in the plots of Fig. 1) disfavors in most of the cases solutions with both q_L and t_R elementary (that is, with $c_q \simeq 1/2$ and $c_u \simeq -1/2$). This is especially true for the MCHM_{10}, since the formula for the top mass has an extra suppression factor $1/\sqrt{2}$ compared to the MCHM_5; see Eqs. (A13) and (B8) of Appendixes A and B.

Our investigation of the structure of the Higgs potential has then revealed that there are specific regions of the parameter space in which the electroweak symmetry is broken and the SM quarks get a mass. Part of these regions, however, is excluded by the precision tests. How large this portion is gives us a measure of the "degree of tuning" required in our model. This is the subject of the next section.

III. ELECTROWEAK PRECISION TESTS

There are two types of corrections to the electroweak observables that any composite Higgs model must address, since they are usually sizable: nonuniversal corrections to the $Zb\bar{b}$ coupling, and universal corrections to the gauge boson self-energies. The results of Ref. [8] show that for both our choices of fermionic 5D representations, Eqs. (A1) and (B1), nonuniversal corrections to $Zb\bar{b}$ are small, due to the custodial O(3) symmetry of the bulk and IR boundary. Therefore, we need to consider only universal effects, which can be parametrized in terms of four quantities: S, T, W, and Y [14]. The last two parameters are suppressed by a factor $\sim(g^2N/16\pi^2)$ compared to S and T, and can be neglected [4]. The parameter T is zero at tree level due to the custodial symmetry. Loop effects can be estimated to be small ($T \lesssim 0.3$), and explicit calculations in similar 5D models confirm this expectation [7,10].[5] We defer a full calculation of the T parameter in the MCHM_5 and MCHM_{10} to a future study.

The Peskin-Takeuchi S parameter gives the most robust and model-independent constraint. Neglecting a small correction from boundary kinetics terms, one has [4]

$$S = \frac{3}{8}\frac{N}{\pi}\epsilon^2. \tag{10}$$

The 99% CL experimental bound $S \lesssim 0.3$ [14][6] then translates into

$$\epsilon^2 \lesssim \frac{1}{4}\left(\frac{10}{N}\right). \tag{11}$$

For $N = 5$ ($N = 10$) this rules out the values $1/2 \lesssim \epsilon^2 < 1$ ($1/4 \lesssim \epsilon^2 < 1$), which we naively expect to correspond to $\sim 1/2$ ($\sim 3/4$) of the region with EWSB and nonzero fermion masses (the region $0 < \epsilon^2 < 1$). The exact numerical results—see Fig. 1 for a case with $N = 8$—reasonably agree with this rough estimate. This means that there is still a large portion of the parameter space which is not ruled out by the constraint from S, and no large fine-tuning is hence required. Notice that smaller values of N imply larger fractions of allowed parameter space, although N

[5] In Ref. [10] the SM fermions were also embedded in the **5** and **10** representations of SO(5), but with different boundary conditions from ours. This implies that, for example, while in the MCHM_5 there is one $SU(2)_L$-doublet KK state with hypercharge $Y = 7/6$ that becomes light in the limit of t_R composite, in the model of Ref. [10] this happens in the limit of t_R mostly elementary.

[6] In order to fully saturate the bound $S \lesssim 0.3$, a positive T of the same size is required. The results of Refs. [7,10] suggest that this could be possible in certain regions of the parameter space. Otherwise, the 99% CL experimental bound on S becomes stronger: $S \lesssim 0.2(0.1)$ for $T \lesssim 0.1(0)$.

cannot be too small if we want to remain in the perturbative regime.

IV. SPECTRUM OF FERMIONIC RESONANCES AND THE HIGGS MASS

An important prediction of our model is that the requirement of a large top mass always forces some of the fermionic KK states to be lighter than their gauge counterpart (a similar property is found in the model of Refs. [12,15]). The reason is the following. The embedding of t_L and t_R into SO(5) bulk multiplets implies that some of their SO(5) partners have $(\pm, \mp)$ boundary conditions, an assignment that is necessary to avoid extra massless states [see Eqs. (A1) and (B1)]. Consider, for example, the case in which the left-handed chirality of the 5D field is $(+, -)$ (hence the right-handed one is $(-, +)$): for values of the 5D mass $c_{i=u,q} > 1/2$, the lightest KK mode, denoted by q^*, has its left-handed chirality exponentially peaked on the UV boundary, while the right-handed one is localized on the IR boundary. This implies that the mass of q^* is exponentially suppressed. On the other hand, for $c_i < -1/2$ both chiralities are localized on the IR boundary and the mass of q^* is of the same order of that of the other KKs: $m_{q^*} \simeq m_\rho$. In the intermediate region $-1/2 < c_i < 1/2$, the one in which the large mass of the top can be reproduced, one finds that m_{q^*} is well interpolated by [4,7]

$$m_{q^*} \simeq \frac{\kappa}{L_1}\sqrt{\frac{1}{2} - c_i}, \tag{12}$$

where $\kappa \sim 2$ is a numerical coefficient with a mild dependence on the values of the boundary masses. This means that m_{q^*} is still parametrically smaller than m_ρ by a factor $\sqrt{1/2 - c_i}$. Analogous results hold for left-handed 5D fermions with a $(+, +)$ boundary condition if they mix with additional 4D fermions localized on the IR boundary. In the case of right-handed 5D fields with a $(+, -)$ boundary condition the same argument above also applies if $c_i \to -c_i$. Equation (12) has a clear interpretation in the 4D dual description of the theory where the left- and right-handed chiralities of the lightest massive state correspond, respectively, to an elementary and a composite state [13]. For $-1/2 < c_i < 1/2$, it can be shown (see also Appendix A) that the coupling of the elementary state to the CFT flows to a fixed-point value proportional to $\sqrt{-\gamma} = \sqrt{1/2 - c_i}$, where γ is the anomalous dimension of the CFT operator to which the elementary field couples. Naive dimensional analysis then immediately gives Eq. (12).

We will concentrate on the region $-1/2 < c_u < 1/2$ with c_q slightly smaller than $1/2$ (t_L mostly elementary). From the argument above and by inspecting Eqs. (A1) and (B1), one can deduce that the light KK modes are all the SO(5) partners of t_R inside ξ_u. In the case of the MCHM$_5$, ξ_u transforms as a **5** of SO(5) and the partners of t_R form a $(\mathbf{2}, \mathbf{2})$ of $SU(2)_L \times SU(2)_R$, equivalent to two $SU(2)_L$ doublets of hypercharge $Y = 1/6$ and $Y = 7/6$. The first is the lightest resonance with the quantum numbers of t_L, while the second is its O(4)-custodial companion. In the case of the MCHM$_{10}$, ξ_u transforms as a **10** of SO(5) and the light partners of t_R also include its own O(4) custodians, a $\mathbf{3_{2/3}} \oplus \mathbf{1_{5/3}} \oplus \mathbf{1_{-1/3}}$ of $SU(2)_L \times U(1)_Y$.

Figure 2 shows the spectrum of the lowest fermionic KK states in the MCHM$_5$ (upper plot) and in the MCHM$_{10}$ (lower plot). The light states are those predicted. Their mass is around 500–1500 GeV for $\epsilon = 0.5$ and $N = 8$, much smaller than that of the lightest gauge KK, $m_\rho = 2.6$ TeV, and of other fermionic excitations. The custodian

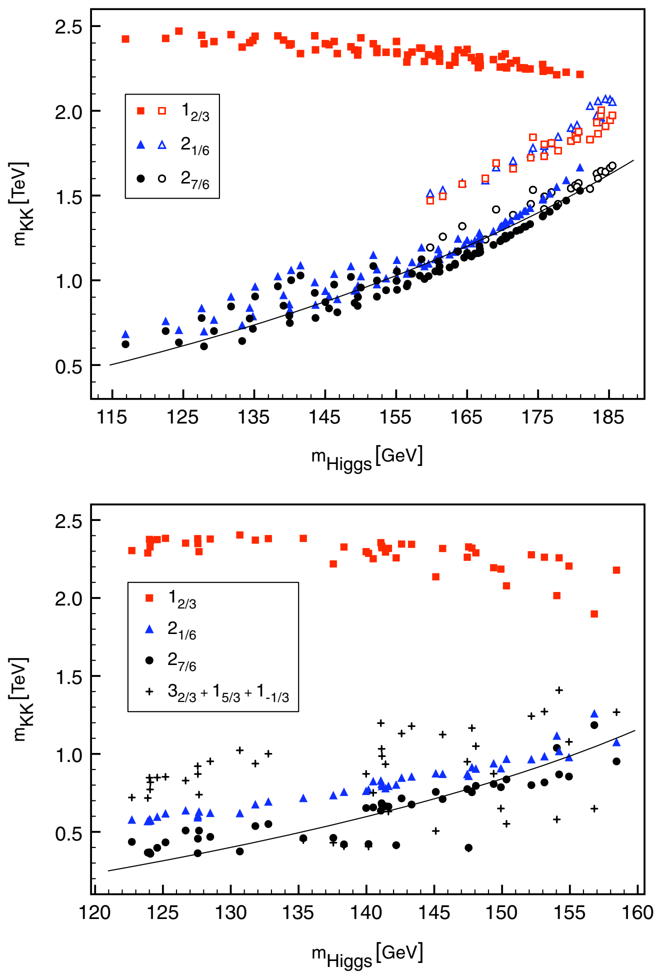


FIG. 2 (color online). Masses of the lightest colored KK fermions in the MCHM$_5$ (upper plot) and in the MCHM$_{10}$ (lower plot). Different symbols denote KKs with different quantum numbers under $SU(2)_L \times U(1)_Y$, as specified in the plots. Both plots are for $\epsilon = 0.5$, $N = 8$. In the upper one we have varied $0.28 < c_q < 0.38$, $0 < c_u < 0.41$, $0.32 < \tilde{m}_u < 0.42$, $-3.5 < \tilde{M}_u < -2.2$ (filled points), or $0.2 < c_q < 0.35$, $-0.25 < c_u < -0.42$, $-1.3 < \tilde{m}_u < 0.2$, $0.1 < \tilde{M}_u < 2.3$ (empty points). In the lower plot we have varied $0.36 < c_q < 0.45$, $0 < c_u < 0.38$, $0.8 < \tilde{m}_u < 3$, $-3 < \tilde{M}_u < -0.3$. The black continuous line is the fit to the mass of the lightest resonance according to Eqs. (15) and (18).

$\mathbf{2}_{7/6}$ turns out to be the lightest among all the light fermionic resonances and therefore the most accessible.

There is an alternative way to understand why in these types of models one expects light colored fermionic resonances. From Eq. (7), we have that the Higgs mass is given by

$$m_h^2 \simeq \frac{8\beta}{f_\pi^2} s_h^2 c_h^2, \tag{13}$$

where

$$\beta \simeq N_c \int \frac{d^4p}{(2\pi)^4} \frac{F(p)}{p^2},$$
$$F(p) \equiv \frac{(M_1^u)^2}{(\Pi_0^q + (s^2/2)\Pi_1^{q1})(\Pi_0^u + (s^2/2)\Pi_1^u)}. \tag{14}$$

Using Eq. (5), we have $m_t^2 = F(0)s_h^2 c_h^2/2$ and hence

$$m_h^2 \simeq \frac{N_c}{\pi^2} \frac{m_t^2}{v^2} \epsilon^2 \Lambda^2, \tag{15}$$

where we have defined

$$\Lambda^2 \equiv 2 \int_0^\infty dp\, p \frac{F(p)}{F(0)}. \tag{16}$$

Equation (15) shows the relation between the Higgs mass and Λ, which is, roughly speaking, the scale at which the momentum integral is cut off. On general grounds, one would expect this cutoff scale to be of the order of the mass of the lowest fermionic resonance[7]:

$$m_{q^*} \simeq \Lambda \simeq 900\ \mathrm{GeV}\left(\frac{m_h}{150\ \mathrm{GeV}}\right)\left(\frac{0.5}{\epsilon}\right), \tag{17}$$

where in the last equality Eq. (15) has been used. Equation (17) shows that in composite Higgs models with a light Higgs and no tuning ($\epsilon \sim 1$) colored resonances are expected to be not heavier than $\sim$1 TeV. In our model, the relation between the Higgs mass and the mass of the lowest fermionic KK turns out to be more complicated than that of Eq. (17). We find that the points of Fig. 2 are better reproduced by a relation of the form

$$\Lambda^2 = a_1 m_{q^*}^2 + a_2 m_{q^*} M + a_3 M^2, \tag{18}$$

where a_i are numerical coefficients, $M \equiv m_\rho$ parametrizes the mass of the heavier resonances, and by m_{q^*} we denote the mass of the KK weak doublet with hypercharge $Y = 7/6$ (the lightest among the fermionic KKs in Fig. 2). This means that in our model the integral $\int_0^\infty dp\, p[F(p)/F(0)]$ is not completely cut off at $p \sim m_{q^*}$, and that other (heavier) resonances also play a role. A fit to the points of Fig. 2 gives $a_{i=1,2,3} = (-0.10, 0.35, 0.007)$ for the MCHM$_5$ (upper plot) and $a_i = (-0.14, 0.24, 0.06)$ for the MCHM$_{10}$ (lower plot). The dispersion of the points around the fitted curve (shown in each plot) can be explained as follows. In Fig. 2 we have fixed $N = 8$, $\epsilon = 0.5$ and $m_t^{\mathrm{pole}} = 173$ GeV, which leaves two of the five parameters of Eq. (9) free to vary. If c_u is traded for m_{q^*}, we are left with one free parameter, for example, c_q. The coefficients a_i of Eq. (18) will thus depend on c_q. To generate the points in Fig. 2 we have scanned over the values $0.2 < c_q < 0.38$ (upper plot) and $0.36 < c_q < 0.45$ (lower plot) and therefore the fitted a_i given above should be considered as average values over these intervals of c_q.

[7]Using Eq. (8) we can rewrite Eq. (15) as

$$m_h^2 \simeq \frac{N_c}{\pi^2} \frac{m_t^2}{2v^2} \Lambda^2 - \frac{4c_h^2 \alpha}{f_\pi^2}.$$

The first term is the formula for the Higgs mass one obtains in the SM by defining $\Lambda^2/2 \equiv \int dp\, p$ in the top loop. The degree of cancellation between the first and second terms gives a measure of the degree of "tuning" needed in our model. This exactly corresponds to ϵ^2.

V. PRODUCTION AND DETECTION OF THE LIGHTEST FERMIONIC RESONANCES AT THE LHC

The most promising way to discover these models is by detecting their lowest fermionic KKs at the LHC. In particular, detecting the custodian with electric charge 5/3, $q_{5/3}^*$, that arises from the $\mathbf{2}_{7/6}$ multiplet of $SU(2)_L \times U(1)_Y$, would be the smoking-gun signature of the model. This exotic state is a direct consequence of the custodial symmetry required to forbid large corrections to $Zb\bar{b}$. For not-too-large values of its mass $m_{q_{5/3}^*}$, roughly below 1 TeV, this new particle will be mostly produced in pairs, via QCD interactions,

$$q\bar{q}, gg \to q_{5/3}^* \bar{q}_{5/3}^*, \tag{19}$$

with a cross section completely determined in terms of $m_{q_{5/3}^*}$ (see for example [16,17]). Once produced, $q_{5/3}^*$ will decay to a (longitudinally polarized) W^+ plus a t_R, with a coupling of order $4\pi/\sqrt{N}\sqrt{(c_u + 1/2)}$. Decays to SM light quarks will be strongly suppressed, with a coupling of order of the square root of their Yukawa couplings. In general, colored resonances will mostly decay to tops and bottoms, since these are the SM quarks more strongly coupled to them, as the result of the large top mass. The process of Eq. (19) then leads to a final state of four W's and two b jets:

$$q_{5/3}^* \bar{q}_{5/3}^* \to W^+ t W^- \bar{t} \to W^+ W^+ b W^- W^- \bar{b}. \tag{20}$$

The same final state also comes from pair production of KKs with charge $-1/3$. A way to discriminate between the two cases consists in reconstructing the electric charge of the resonance. For example, one could look for events with two highly energetic leptons of the same sign, coming from the leptonic decay of two of the four W's, plus at least six jets, two of which are tagged as b jets. Demanding that the invariant mass of the system of the two hadronically decaying W's plus one b jet equals $m_{q^*_{5/3}}$ then identifies the process and gives evidence for the charge $5/3$ of the resonance. Furthermore, indirect evidence in favor of $q^*_{5/3}$ would come from the nonobservation of the decays to Zb, Hb that are allowed for resonances of charge $-1/3$.

For increasing values of $m_{q^*_{5/3}}$ the cross section for pair production quickly drops, and single production might become more important. The relevant process is tW fusion [18], where a longitudinal W radiated from one proton scatters off a top coming from the other proton. The analogous process initiated by a bottom quark, bW fusion, has been studied in detail in the literature and shown to be an efficient way to singly produce heavy excitations of the top quark [16,17,19]. To prove that the same conclusion also applies to the case of tW fusion, a dedicated analysis is required. The main uncertainty and challenge comes from the small top quark content of the proton, which, however, can be compensated by the large coupling involved, especially in the case of t_R largely composite [4].

Besides $q^*_{5/3}$, the other components of the $\mathbf{2}_{1/6}$ and $\mathbf{2}_{7/6}$ multiplets of $SU(2)_L \times U(1)_Y$ are also predicted to be light in both our models; see Fig. 2. In the specific case of the $MCHM_{10}$ there are also other states transforming as $\mathbf{3}_{2/3}$, $\mathbf{1}_{5/3}$, $\mathbf{1}_{-1/3}$. These multiplets contain states with $Q_{\rm em} = 5/3$ whose phenomenology will be similar to that of the $q^*_{5/3}$ described above. The states of electric charge $2/3$ or $-1/3$ will also be produced in pairs via QCD interactions or singly via bW or tW fusion. They will decay to a SM top or bottom quark plus a longitudinally polarized W or Z, or a Higgs. When kinematically allowed, a heavier resonance will also decay to a lighter KK accompanied with a $W_{\rm long}$, $Z_{\rm long}$ or h. Decay chains could lead to extremely characteristic final states. For example, in the $MCHM_{10}$ the KK with charge $2/3$ from $\mathbf{3}_{2/3}$ is predicted to be generally heavier than $q^*_{5/3}$; see Fig. 2. If pair produced, it can decay to $q^*_{5/3}$ leading to a spectacular six W's plus two b jets final state:

$$q^*_{2/3}\bar{q}^*_{2/3} \to W^- q^*_{5/3} W^+ \bar{q}^*_{5/3} \to W^- W^+ W^+ b W^+ W^- W^- \bar{b}. \tag{21}$$

To fully explore the phenomenology of the fermionic resonances in our models, a detailed analysis is necessary. One could, for example, adopt the simplifying strategy proposed in Ref. [20], where a 4D effective theory has been introduced to consistently describe the SM fields and the first KK excitations of a large class of warped models. Existing studies in the literature have focused on the production and detection of $SU(2)_L$ singlets of hypercharge $Y = 2/3$ [17,21], since this is a typical signature of little Higgs theories. In our models, however, the singlet is not predicted to be light, except for specific regions of the parameter space. In conclusion, our brief discussion shows that there are characteristic signatures predicted by our models that will distinguish them from other extensions of the SM. While certainly challenging, these signals will be extremely spectacular, and will provide an indication of a new strong dynamics responsible for electroweak symmetry breaking.

ACKNOWLEDGMENTS

It is a pleasure to thank Kaustubh Agashe for collaboration in the early stages of this project and for many useful discussions and comments. R. C. also thanks G. Servant for useful discussions. The work of L. D. and A. P. was partly supported by the FEDER Research Project No. FPA2005-02211 and DURSI Research Project No. SGR2005-00916. The work of L. D. was partly supported by FAPESP. A. P. and R. C. thank the Galileo Galilei Institute for Theoretical Physics for hospitality and the INFN for partial support during the completion of this work. A. P. also thanks the theory group at CERN for hospitality.

APPENDIX A: DEFINING THE $MCHM_5$

Here we present in detail the fermion sector of our composite Higgs models. We have assumed that (i) each SM fermion is embedded in a different 5D field, and (ii) all three SM families have the same embedding. Following these rules, we construct what we think are the minimal models with fermions embedded in **5** or **10** representations of SO(5).

The quark sector of the $MCHM_5$ is defined in terms of 5D bulk multiplets transforming as fundamentals of SO(5). Each SM generation is identified with the zero modes of the 5D fields

$$\begin{aligned}
\xi_{q_1} &= \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{q_1}}_{\mathbf{L}} = \begin{bmatrix} q'_{1L}(-+) \\ q_{1L}(++) \end{bmatrix} & (\mathbf{2},\mathbf{2})^{\mathbf{q_1}}_{\mathbf{R}} = \begin{bmatrix} q'_{1R}(+-) \\ q_{1R}(--) \end{bmatrix} \\ (\mathbf{1},\mathbf{1})^{\mathbf{q_1}}_{\mathbf{L}}(--) & (\mathbf{1},\mathbf{1})^{\mathbf{q_1}}_{\mathbf{R}}(++) \end{bmatrix}, \qquad \xi_u = \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{L}}(+-) & (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}}(-+) \\ (\mathbf{1},\mathbf{1})^{\mathbf{u}}_{\mathbf{L}}(-+) & (\mathbf{1},\mathbf{1})^{\mathbf{u}}_{\mathbf{R}}(+-) \end{bmatrix}, \\
\xi_{q_2} &= \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{q_2}}_{\mathbf{L}} = \begin{bmatrix} q_{2L}(++) \\ q'_{2L}(-+) \end{bmatrix} & (\mathbf{2},\mathbf{2})^{\mathbf{q_2}}_{\mathbf{R}} = \begin{bmatrix} q_{2R}(--) \\ q'_{1R}(+-) \end{bmatrix} \\ (\mathbf{1},\mathbf{1})^{\mathbf{q_2}}_{\mathbf{L}}(--) & (\mathbf{1},\mathbf{1})^{\mathbf{q_2}}_{\mathbf{R}}(++) \end{bmatrix}, \qquad \xi_d = \begin{bmatrix} (\mathbf{2},\mathbf{2})^{\mathbf{d}}_{\mathbf{L}}(+-) & (\mathbf{2},\mathbf{2})^{\mathbf{d}}_{\mathbf{R}}(-+) \\ (\mathbf{1},\mathbf{1})^{\mathbf{d}}_{\mathbf{L}}(-+) & (\mathbf{1},\mathbf{1})^{\mathbf{d}}_{\mathbf{R}}(+-) \end{bmatrix},
\end{aligned} \tag{A1}$$

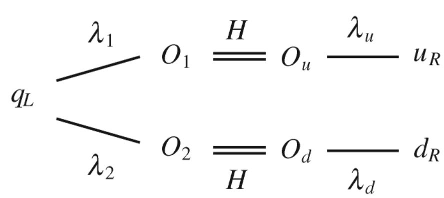
where ξ_{q_1}, ξ_u (ξ_{q_2}, ξ_d) transform as $\mathbf{5}_{2/3}$ ($\mathbf{5}_{-1/3}$) of $SO(5) \times U(1)_X$. A similar 5D embedding also works for the SM leptons, although with different $U(1)_X$ charges. Chiralities under the 4D Lorentz group have been denoted by L, R, and

$(\pm, \pm)$ is a shorthand notation to denote Neumann $(+)$ or Dirichlet $(-)$ boundary conditions on the two boundaries. We have grouped the fields of each multiplet ξ_i in representations of $SO(4) \sim SU(2)_L \times SU(2)_R$, and used the fact that a fundamental of SO(5) decomposes as $\mathbf{5} = \mathbf{4} \oplus \mathbf{1} = (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{1})$. Although each of ξ_{q_1} and ξ_{q_2} alone could account for the q_L zero mode, we need them both to give mass to both the up and down SM quarks. We thus identify the SM q_L field with the zero mode of the linear combination $(q_{1L} + q_{2L})$, and get rid of the extra zero mode by introducing a localized right-handed field on the UV boundary that has a mass mixing with the orthogonal combination. We denote by c_i, $i = q_1, q_2, u, d$, the bulk masses of each 5D field ξ_i in units of k. Localized on the IR boundary, we consider the most general set of mass terms invariant under $O(4) \times U(1)_X$:

$$\tilde{m}_u \overline{(\mathbf{2},\mathbf{2})}^{\mathbf{q_1}}_{\mathbf{L}} (\mathbf{2},\mathbf{2})^{\mathbf{u}}_{\mathbf{R}} + \tilde{M}_u \overline{(\mathbf{1},\mathbf{1})}^{\mathbf{q_1}}_{\mathbf{R}} (\mathbf{1},\mathbf{1})^{\mathbf{u}}_{\mathbf{L}} + \tilde{m}_d \overline{(\mathbf{2},\mathbf{2})}^{\mathbf{q_2}}_{\mathbf{L}} (\mathbf{2},\mathbf{2})^{\mathbf{d}}_{\mathbf{R}} + \tilde{M}_d \overline{(\mathbf{1},\mathbf{1})}^{\mathbf{q_2}}_{\mathbf{R}} (\mathbf{1},\mathbf{1})^{\mathbf{d}}_{\mathbf{L}} + \text{H.c.} \tag{A2}$$

The holographic description

The 5D field content of Eq. (A1) has a very simple holographic interpretation in terms of three elementary chiral fields, $q_L = (u_L, d_L)$, u_R and d_R, coupled to a CFT sector through composite operators O_i.[8] An important difference from the model of Ref. [4], and also from the MCHM_{10} discussed in the next section, is that here the elementary q_L couples to two different CFT operators, O_1 and O_2, with couplings λ_1, λ_2. This is the consequence of having two different bulk fields in Eq. (A1), q_{1L} and q_{2L}, mixed on the UV boundary. The elementary fields u_R and d_R, instead, couple to one operator each, O_u and O_d, with couplings λ_u and λ_d. The bulk gauge symmetry of the 5D model maps into an $SO(5) \times U(1)_X$ global symmetry of the CFT, and Eq. (A1) implies that O_1, O_u transform as $\mathbf{5}_{2/3}$, while O_2, O_d transform as $\mathbf{5}_{-1/3}$. Schematically,



The double line indicates that $(\bar{O}_1 O_u)$ and $(\bar{O}_2 O_d)$ have the correct quantum numbers to excite the Higgs and generate in this way the up and down Yukawa couplings of the 4D low-energy theory. Large hierarchies among the Yukawas can be explained naturally as the result of the renormalization group (RG) evolution of the couplings λ_i [4].

[8] We adopt a left-source holographic description for the fields ξ_{q_1} and ξ_{q_2}, and a right-source description for ξ_u and ξ_d. See Ref. [13].

Notice that the CFT dynamics alone do not mix O_1 and O_u with O_2 and O_d, since they have different $U(1)_X$ quantum numbers. Nevertheless, both O_1 and O_2 can couple to the external source q_L, since this latter coupling will only preserve the $SU(2)_L \times U(1)_Y$ elementary symmetry. This suggests that a hierarchy in the up and down Yukawa couplings, like in the case of the top and bottom quarks, can follow from the RG evolution if $\lambda_u \gg \lambda_d$ at low energy, as already pointed out for the model of Ref. [4], or alternatively if $\lambda_1 \gg \lambda_2$.

Quite interestingly, it is simple to show that λ_1 can grow much bigger (or smaller) than λ_2 in the infrared, even if both operators O_1 and O_2 are relevant. The argument goes as follows. The RG equations of the two couplings λ_1, λ_2 form a coupled system:

$$p\frac{d}{dp}\lambda_1(p) = \gamma_1\lambda_1 + \frac{N}{16\pi^2}(a_1\lambda_1^3 + a_{12}\lambda_1\lambda_2^2) + \ldots, \tag{A3}$$

$$p\frac{d}{dp}\lambda_2(p) = \gamma_2\lambda_2 + \frac{N}{16\pi^2}(a_2\lambda_2^3 + a_{21}\lambda_2\lambda_1^2) + \ldots. \tag{A4}$$

The dots stand for subleading terms in a $1/N$ expansion, where the number of CFT colors N is defined by Eq. (2). The duality with the 5D theory implies that the coefficients a_1 and a_2 are both positive (see [13]), and that the anomalous dimensions $\gamma_{1,2}$ are linear functions of the 5D bulk masses: $\gamma_{1,2} = |c_{q_1,q_2} + 1/2| - 1$. Furthermore, it is easy to show that the leading contribution to the coefficients a_{12} and a_{21} comes from the wave function renormalization, which in turn implies $a_{21} = a_1$, $a_{12} = a_2$ at leading order in $1/N$. Let us then consider the case in which the operator with the smallest anomalous dimension, say O_1, is relevant, that is $c_{q_1} < 1/2$, $c_{q_1} < c_{q_2}$ (i.e. $\gamma_1 < 0$, $\gamma_1 < \gamma_2$). Then λ_1 will grow faster than λ_2 in the infrared, and at some energy E_* it will reach a fixed-point value $\lambda_{1*} \simeq 4\pi/\sqrt{N}\sqrt{-\gamma_1/a_1}$. Below that energy, one can set $\lambda_1 = \lambda_{1*}$ in the RG equation for λ_2, Eq. (A4), which becomes

$$p\frac{d}{dp}\lambda_2(p) \simeq (\gamma_2 - \gamma_1)\lambda_2 + \ldots. \tag{A5}$$

Since $(\gamma_2 - \gamma_1) > 0$ by assumption, this implies that λ_2 will be suppressed at low energy,

$$\lambda_2(E) \sim \left(\frac{E}{E_*}\right)^{\gamma_2 - \gamma_1}, \tag{A6}$$

even if $\gamma_2 < 0$, that is, even if the operator O_2 is relevant.

The above argument shows that the small ratio m_b/m_t follows naturally in the MCHM_5 by having $c_{q_2} > c_{q_1}$, even when b_R is strongly coupled to the CFT sector. The large top mass, on the other hand, requires $|c_{q_1}|$, $|c_u| < 1/2$ for the third generation quarks, i.e. the operators O_1 and O_u need to both be relevant. In our numerical analysis of the MCHM_5 (hence in all the results presented in the text), we have set $c_{q_2} = 0.4$, $c_d = -0.55$ for the third generation, and varied $-1/2 < c_u < 1/2$, $-1/2 < c_{q_1} < c_{q_2}$. Choosing $c_{q_1} < c_{q_2}$ also implies that the contribution of ξ_{q_2} to the Higgs potential will be suppressed and hence negligible compared to that of ξ_{q_1} (see also below). This justifies the following identification:

$$\xi_q \equiv \xi_{q_1}, \qquad c_q \equiv c_{q_1}, \tag{A7}$$

where by ξ_q we mean the field responsible for the contribution of q_L to the Higgs potential to which we referred in the text.

Following the method of Ref. [4], one can derive the most general form of the holographic Lagrangian by introducing spurion fields and embedding the elementary sources in complete $\mathrm{SO}(5) \times \mathrm{U}(1)_X$ (chiral) multiplets. The fact that the elementary q_L couples to two different CFT operators implies that there are two different ways to embed it in a fundamental representation of SO(5), namely, as the $T^{3R} = +1/2$ or the $T^{3R} = -1/2$ component of the internal $(\mathbf{2}, \mathbf{2})$. More explicitly, grouping the entries of each $\mathbf{5}$ in $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R$ representations, one has

$$\Psi_{1L} = \left[\begin{pmatrix} q'_{1L} \\ q_L \end{pmatrix} \atop u'_L\right], \qquad \Psi_{2L} = \left[\begin{pmatrix} q_L \\ q'_{2L} \end{pmatrix} \atop d'_L\right],$$
$$\Psi_{uR} = \left[\begin{pmatrix} q^u_R \\ q'^u_R \end{pmatrix} \atop u_R\right], \qquad \Psi_{dR} = \left[\begin{pmatrix} q'^d_R \\ q^u_R \end{pmatrix} \atop d_R\right]. \tag{A8}$$

The multiplets Ψ_{1L}, Ψ_{uR} (Ψ_{2L}, Ψ_{dR}) have $\mathrm{U}(1)_X$ charge $+2/3$ $(-1/3)$, and all components other than q_L, u_R and d_R are nondynamical spurion fields. Therefore, the most general $(\mathrm{SO}(5) \times \mathrm{U}(1)_X)$-invariant holographic Lagrangian, at the quadratic order and in momentum space, is

$$\begin{aligned}\mathcal{L}^{(2)}_{\mathrm{holo}} = &\sum_{r=1,2} \bar{\Psi}^i_{rL} \not{p}(\delta^{ij}\hat{\Pi}^{rL}_0(p) + \Sigma^i\Sigma^j\hat{\Pi}^{rL}_1(p))\Psi^j_{rL} \\ &+ \sum_{r=u,d} \bar{\Psi}^i_{rR} \not{p}(\delta^{ij}\hat{\Pi}^{rR}_0(p) + \Sigma^i\Sigma^j\hat{\Pi}^{rR}_1(p))\Psi^j_{rR} \\ &+ \bar{\Psi}^i_{1L}(\delta^{ij}\hat{M}^{1L}_0(p) + \Sigma^i\Sigma^j\hat{M}^{1L}_1(p))\Psi^j_{uR} \\ &+ \bar{\Psi}^i_{2L}(\delta^{ij}\hat{M}^{2L}_0(p) + \Sigma^i\Sigma^j\hat{M}^{2L}_1(p))\Psi^j_{dR} + \mathrm{H.c.}\end{aligned} \tag{A9}$$

Here $i, j = 1, \ldots, 5$ are SO(5) indices, and Σ is the nonlinear realization of the Higgs field [4]:

$$\Sigma = \frac{s_h}{h}\left(h^1, h^2, h^3, h^4, h\frac{c_h}{s_h}\right). \tag{A10}$$

The form factors $\hat{\Pi}$, $\hat{M}$ can be computed using the holographic technique of Ref. [4]; the result is

$$\begin{aligned}\hat{\Pi}^{1L}_0 &= \Pi_{q_L}(c_{q_1}, c_u, \tilde{m}_u), \\ \hat{\Pi}^{1L}_1 &= \Pi_{Q_L}(c_{q_1}, c_u, \tilde{M}_u) - \Pi_{q_L}(c_{q_1}, c_u, \tilde{m}_u), \\ \hat{\Pi}^{2L}_0 &= \Pi_{q_L}(c_{q_2}, c_d, \tilde{m}_d), \\ \hat{\Pi}^{2L}_1 &= \Pi_{Q_L}(c_{q_2}, c_d, \tilde{M}_d) - \Pi_{q_L}(c_{q_2}, c_d, \tilde{m}_d), \\ \hat{\Pi}^{uR}_{0,1} &= \hat{\Pi}^{1L}_{0,1}(c_{q_1} \leftrightarrow c_u; L \leftrightarrow R), \\ \hat{\Pi}^{dR}_{0,1} &= \hat{\Pi}^{2L}_{0,1}(c_{q_2} \leftrightarrow c_d; L \leftrightarrow R), \\ \hat{M}^{1L}_0 &= M_q(c_{q_1}, c_u, \tilde{m}_u), \\ \hat{M}^{1L}_1 &= M_Q(c_{q_1}, c_u, \tilde{M}_u) - M_q(c_{q_1}, c_u, \tilde{m}_u), \\ \hat{M}^{2L}_0 &= M_q(c_{q_2}, c_d, \tilde{m}_d), \\ \hat{M}^{2L}_1 &= M_Q(c_{q_2}, c_d, \tilde{M}_d) - M_q(c_{q_2}, c_d, \tilde{m}_d),\end{aligned} \tag{A11}$$

where Π_{q_L,Q_L} and $M_{q,Q}$ are the form factors defined in the Appendix of Ref. [4]. After setting all nondynamical fields to zero, the Lagrangian (A9) reduces to that of Eq. (3) with

$$\begin{aligned}\Pi^q_0 &= \hat{\Pi}^{1L}_0 + \hat{\Pi}^{2L}_0, & \Pi^{q_1}_1 &= \hat{\Pi}^{1L}_1, & \Pi^u_1 &= -\tfrac{1}{2}\hat{\Pi}^{uR}_1, & M^u_1 &= \hat{M}^{1L}_1, \\ \Pi^u_0 &= \hat{\Pi}^{uR}_0 + \hat{\Pi}^{uR}_1, & \Pi^{q_2}_1 &= \hat{\Pi}^{2L}_1, & \Pi^d_1 &= -\tfrac{1}{2}\hat{\Pi}^{dR}_1, & M^d_1 &= \hat{M}^{2L}_1. \\ \Pi^d_0 &= \hat{\Pi}^{dR}_0 + \hat{\Pi}^{dR}_1, \end{aligned} \tag{A12}$$

Since we set $c_{q_2} > c_{q_1}$, $c_d < -1/2$, the form factors $\hat{\Pi}^{2L}_1$, $\hat{\Pi}^d_1$ are suppressed compared to $\hat{\Pi}^{1L}_1$, $\hat{\Pi}^u_1$, and their effect in the Higgs potential can be neglected.

The fermionic spectrum of the SM fields and heavy resonances of the MCHM_5 can be expressed in terms of poles and zeros of the form factors (see Ref. [7] for the gauge spectrum). Before EWSB, there are five towers of states:

(i) a tower of q_L's $[\mathbf{2}_{1/6}$ of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y]$ with masses given by $\mathrm{zeros}\{\not{p}\Pi^q_0\}$.
(ii) a tower of u_R's $[\mathbf{1}_{2/3}$ of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y]$ with masses given by $\mathrm{zeros}\{\not{p}\Pi^u_0\}$.
(iii) a tower of d_R's $[\mathbf{1}_{-1/3}$ of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y]$ with masses given by $\mathrm{zeros}\{\not{p}\Pi^d_0\}$.
(iv) a tower of $\mathbf{2}_{7/6}$ of $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$ with masses given by $\mathrm{poles}\{\not{p}(\Pi^u_0 + \frac{1}{2}\Pi^u_1)\}$.

(v) a tower of $\mathbf{2}_{-5/6}$ of $SU(2)_L \times U(1)_Y$ with masses given by poles$\{\not{p}(\Pi_0^d + \frac{1}{2}\Pi_1^d)\}$.

After EWSB the different towers are mixed and the final spectrum consists of

(i) a tower of charge $+2/3$ fermions with masses given by

$$\text{zeros}\left\{p^2\left(\Pi_0^q + \frac{\epsilon^2}{2}\Pi_1^{q_1}\right)\left(\Pi_0^u + \frac{\epsilon^2}{2}\Pi_1^u\right) - \frac{\epsilon^2(1-\epsilon^2)}{2}(M_1^u)^2\right\}.$$

(ii) a tower of charge $-1/3$ fermions with masses given by

$$\text{zeros}\left\{p^2\left(\Pi_0^q + \frac{\epsilon^2}{2}\Pi_1^{q_2}\right)\left(\Pi_0^d + \frac{\epsilon^2}{2}\Pi_1^d\right) - \frac{\epsilon^2(1-\epsilon^2)}{2}(M_1^d)^2\right\}.$$

(iii) a tower of charge $+5/3$ fermions with masses given by poles$\{\not{p}(\Pi_0^u + \frac{1}{2}\Pi_1^u)\}$.

(iv) a tower of charge $-4/3$ fermions with masses given by poles$\{\not{p}(\Pi_0^d + \frac{1}{2}\Pi_1^d)\}$.

From the formulas above for the fermions of charge $2/3$ and $-1/3$, one recovers Eq. (5) by approximating the form factors with their values at $p^2 = 0$. Further use of Eqs. (A11) and (A12) gives

$$m_u \simeq \frac{2}{L_1}\epsilon\sqrt{1-\epsilon^2}\frac{\sqrt{(1/4 - c_q^2)(1/4 - c_u^2)}\tilde{M}_u(1 - \tilde{m}_u\tilde{M}_u)}{[(1/2 + c_u)(1-\epsilon^2) + \tilde{M}_u^2((1/2 + c_q) + \epsilon^2\tilde{m}_u^2(1/2 + c_u))]^{1/2}} \times [\epsilon^2(1/2 - c_q) + \tilde{M}_u^2(2(1/2 - c_u) + \tilde{m}_u^2(1/2 - c_q)(2 - \epsilon^2))]^{-1/2}, \tag{A13}$$

and a similar formula for the down quark mass.

APPENDIX B: DEFINING THE MCHM$_{10}$

The quark sector of the MCHM$_{10}$ is defined in terms of 5D bulk multiplets transforming as antisymmetric representations of SO(5). Each SM generation is identified with the zero modes of three $\mathbf{10}_{2/3}$ of $SO(5) \times U(1)_X$,

$$\xi_q = \begin{bmatrix} (\mathbf{2},\mathbf{2})_\mathbf{L}^\mathbf{q} = \begin{bmatrix} q_L'(-+) \\ q_L(++) \end{bmatrix} & (\mathbf{2},\mathbf{2})_\mathbf{R}^\mathbf{q} = \begin{bmatrix} q_R'(+-) \\ q_R(--) \end{bmatrix} \\ (\mathbf{3},\mathbf{1})_\mathbf{L}^\mathbf{q}(--) & (\mathbf{3},\mathbf{1})_\mathbf{R}^\mathbf{q}(++) \\ (\mathbf{1},\mathbf{3})_\mathbf{L}^\mathbf{q}(--) & (\mathbf{1},\mathbf{3})_\mathbf{R}^\mathbf{q}(++) \end{bmatrix},$$
$$\xi_u = \begin{bmatrix} (\mathbf{2},\mathbf{2})_\mathbf{L}^\mathbf{u}(+-) & (\mathbf{2},\mathbf{2})_\mathbf{R}^\mathbf{u}(-+) \\ (\mathbf{3},\mathbf{1})_\mathbf{L}^\mathbf{u}(++) & (\mathbf{3},\mathbf{1})_\mathbf{R}^\mathbf{u}(--) \\ (\mathbf{1},\mathbf{3})_\mathbf{L}^\mathbf{u} = \begin{bmatrix} \chi_L^u(++) \\ u_L^{c\prime}(-+) \\ d_L^{c\prime}(++) \end{bmatrix} & (\mathbf{1},\mathbf{3})_\mathbf{R}^\mathbf{u} = \begin{bmatrix} \chi_R^u(--) \\ u_R(+-) \\ d_R'(--) \end{bmatrix} \end{bmatrix}, \tag{B1}$$
$$\xi_d = \begin{bmatrix} (\mathbf{2},\mathbf{2})_\mathbf{L}^\mathbf{d}(+-) & (\mathbf{2},\mathbf{2})_\mathbf{R}^\mathbf{d}(-+) \\ (\mathbf{3},\mathbf{1})_\mathbf{L}^\mathbf{d}(++) & (\mathbf{3},\mathbf{1})_\mathbf{R}^\mathbf{d}(--) \\ (\mathbf{1},\mathbf{3})_\mathbf{L}^\mathbf{d} = \begin{bmatrix} \chi_L^d(++) \\ u_L^{c\prime\prime}(++) \\ d_L^{c\prime\prime}(-+) \end{bmatrix} & (\mathbf{1},\mathbf{3})_\mathbf{R}^\mathbf{d} = \begin{bmatrix} \chi_R^d(--) \\ u_R'(--) \\ d_R(+-) \end{bmatrix} \end{bmatrix},$$

and an additional $[\widetilde{(\mathbf{3},\mathbf{1})}_R \oplus \widetilde{(\mathbf{1},\mathbf{3})}_R]$ [an irreducible representation of O(4)] localized on the IR boundary. Bulk and boundary fields mix through the most general set of O(4)-symmetric IR-boundary mass mixing terms:

$$[\overline{(\mathbf{3},\mathbf{1})}_\mathbf{L}^\mathbf{u,d}\widetilde{(\mathbf{3},\mathbf{1})}_\mathbf{R} + \overline{(\mathbf{1},\mathbf{3})}_\mathbf{L}^\mathbf{u,d}\widetilde{(\mathbf{1},\mathbf{3})}_\mathbf{R}] + \text{H.c.} \tag{B2}$$

and

$$\tilde{M}_{u,d}[\overline{(\mathbf{3},\mathbf{1})}_\mathbf{L}^\mathbf{u,d}(\mathbf{3},\mathbf{1})_\mathbf{R}^\mathbf{q} + \overline{(\mathbf{1},\mathbf{3})}_\mathbf{L}^\mathbf{u,d}(\mathbf{1},\mathbf{3})_\mathbf{R}^\mathbf{q}] + \tilde{m}_{u,d}\overline{(\mathbf{2},\mathbf{2})}_\mathbf{L}^\mathbf{q}(\mathbf{2},\mathbf{2})_\mathbf{R}^\mathbf{u,d} + \text{H.c.} \tag{B3}$$

We will denote by c_i, $i = q, u, d$, the bulk masses of each

5D field ξ_i in units of k. We have grouped the fields of each multiplet ξ_i in representations of $SO(4) \sim SU(2)_L \times SU(2)_R$, and used the fact that an antisymmetric of SO(5) decomposes as $\mathbf{10} = \mathbf{4} \oplus \mathbf{6} = (\mathbf{2}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{3}) \oplus (\mathbf{3}, \mathbf{1})$. A similar 5D embedding also works for the SM leptons, although with different $U(1)_X$ charges.

The holographic interpretation of the 5D theory defined above is that of three elementary fields q_L, u_R, d_R coupled to a CFT sector via the composite operators O_q, O_u, O_d.[9] This is the same 4D description of the model of Ref. [4], though in this case the operators O_i transform as $\mathbf{10}_{2/3}$ representations of the $SO(5) \times U(1)_X$ global symmetry of the CFT. Following the usual procedure, we can embed the elementary fields into complete $SO(5) \times U(1)_X$ multiplets [$\mathbf{10}_{2/3}$ of $SO(5) \times U(1)_X$ in this case],

$$\Psi_{qL} = \begin{bmatrix} \begin{pmatrix} q'_L \\ q_L \end{pmatrix} \\ (3,1)^q_L \\ (1,3)^q_L \end{bmatrix}, \qquad \Psi_{uR} = \begin{bmatrix} (2,2)^u_R \\ (3,1)^u_R \\ \begin{pmatrix} \chi^u_R \\ u_R \\ d'_R \end{pmatrix} \end{bmatrix}, \qquad \Psi_{dR} = \begin{bmatrix} (2,2)^d_R \\ (3,1)^d_R \\ \begin{pmatrix} \chi^d_R \\ u'_R \\ d_R \end{pmatrix} \end{bmatrix} \tag{B4}$$

and derive the most general holographic Lagrangian at the quadratic order and in momentum space[10]:

$$\mathcal{L}^{(2)}_{\rm holo} = \sum_{r=qL,uR,dR} [\mathrm{Tr}(\bar{\Psi}_r \not{p} \hat{\Pi}^r_0(p) \Psi_r) + \Sigma \bar{\Psi}_r \not{p} \hat{\Pi}^r_1(p) \Psi_r \Sigma^T] + \sum_{r=uR,dR} [\mathrm{Tr}(\bar{\Psi}_{qL} \hat{M}^r_0(p) \Psi_r) + \Sigma \bar{\Psi}_{qL} \hat{M}^r_1(p) \Psi_r \Sigma^T] + \mathrm{H.c.} \tag{B5}$$

The form factors $\hat{\Pi}$, $\hat{M}$ can be computed using the holographic technique of Ref. [4]:

$$\begin{aligned}
\hat{\Pi}^{qL}_0 &= \Pi_{Q_L}(c_q, c_u, \tilde{M}_u), \\
\hat{\Pi}^{qL}_1 &= 2[\Pi_{q_L}(c_q, c_u, \tilde{m}_u) - \Pi_{Q_L}(c_q, c_u, \tilde{M}_u)], \\
\hat{\Pi}^{uR}_0 &= \hat{\Pi}^{qL}_0(c_q \leftrightarrow c_u, L \leftrightarrow R), \\
\hat{\Pi}^{uR}_1 &= \hat{\Pi}^{qL}_1(c_q \leftrightarrow c_u, L \leftrightarrow R), \\
\hat{\Pi}^{dR}_0 &= \hat{\Pi}^{qL}_0(c_q \leftrightarrow c_d, L \leftrightarrow R), \\
\hat{\Pi}^{dR}_1 &= \hat{\Pi}^{qL}_1(c_q \leftrightarrow c_d, L \leftrightarrow R), \\
\hat{M}^{uR}_0 &= M_Q(c_q, c_u, \tilde{M}_u), \\
\hat{M}^{uR}_1 &= 2[M_q(c_q, c_u, \tilde{m}_u) - M_Q(c_q, c_u, \tilde{M}_u)], \\
\hat{M}^{dR}_0 &= M_Q(c_q, c_d, \tilde{M}_d), \\
\hat{M}^{dR}_1 &= 2\sqrt{2}[M_q(c_q, c_d, \tilde{m}_d) - M_Q(c_q, c_d, \tilde{M}_d)].
\end{aligned} \tag{B6}$$

Here Π_{q_L,Q_L} and $M_{q,Q}$ are the form factors defined in the Appendix of Ref. [4]. After setting all nondynamical fields to zero, the Lagrangian (B5) reduces to that of Eq. (3) with

$$\begin{aligned}
&\Pi^q_0 = \hat{\Pi}^{qL}_0 + \frac{1}{2}\hat{\Pi}^{qL}_1, \qquad \Pi^u_0 = \hat{\Pi}^{uR}_0, \qquad \Pi^d_0 = \hat{\Pi}^{dR}_0, \\
&\Pi^{q_1}_1 = -\frac{1}{2}\hat{\Pi}^{qL}_1, \qquad \Pi^{q_2}_1 = -\hat{\Pi}^{qL}_1, \qquad \Pi^u_1 = \frac{1}{2}\hat{\Pi}^{uR}_1, \\
&\Pi^d_1 = \frac{1}{2}\hat{\Pi}^{dR}_1, \qquad M^u_1 = \frac{1}{2\sqrt{2}}\hat{M}^{uR}_1, \qquad M^d_1 = \frac{1}{2}\hat{M}^{dR}_1.
\end{aligned} \tag{B7}$$

The fermionic spectrum of the MCHM_{10} can be expressed in terms of poles and zeros of the form factors. Before EWSB, there are six towers of states:

(i) a tower of q_L's [$\mathbf{2}_{1/6}$ of $SU(2)_L \times U(1)_Y$] with masses given by zeros$\{\not{p}\Pi^q_0\}$.
(ii) a tower of u_R's [$\mathbf{1}_{2/3}$ of $SU(2)_L \times U(1)_Y$] with masses given by zeros$\{\not{p}\Pi^u_0\}$.
(iii) a tower of d_R's [$\mathbf{1}_{-1/3}$ of $SU(2)_L \times U(1)_Y$] with masses given by zeros$\{\not{p}\Pi^d_0\}$.
(iv) a tower of $\mathbf{2}_{7/6}$ of $SU(2)_L \times U(1)_Y$ with masses given by poles$\{\not{p}\Pi^q_0\}$.
(v) a tower of $\mathbf{1}_{5/3}$ plus a tower of $\mathbf{3}_{2/3}$ of $SU(2)_L \times U(1)_Y$ with masses given by poles$\{\not{p}\Pi^u_0\}$.

The final spectrum after EWSB consists of

(i) a tower of charge $+2/3$ fermions with masses given by

$$\mathrm{zeros}\left\{p^2\left(\Pi^q_0 + \frac{\epsilon^2}{2}\Pi^{q_1}_1\right)\left(\Pi^u_0 + \frac{\epsilon^2}{2}\Pi^u_1\right) - \frac{\epsilon^2(1-\epsilon^2)}{2}(M^u_1)^2\right\}.$$

[9] We adopt a left-source holographic description for ξ_q and a right-source description for ξ_u and ξ_d. See Ref. [13].

[10] The operator $\Psi^{ij}\Psi^{kl}\Sigma^m\epsilon^{ijklm}$ is also SO(5) invariant, but its form factor identically vanishes due to the O(4) symmetry of the CFT. Also, we have omitted for simplicity a possible mixing term between Ψ_u and Ψ_d, since it can be safely neglected in our analysis due to the small coupling of b_R to the CFT needed to explain the bottom quark mass.

(ii) a tower of charge $-1/3$ fermions with masses given by

$$\text{zeros}\left\{p^2\left(\Pi_0^q+\frac{\epsilon^2}{2}\Pi_1^{q_2}\right)\left(\Pi_0^d+\frac{\epsilon^2}{2}\Pi_1^d\right)-\frac{\epsilon^2(1-\epsilon^2)}{2}(M_1^d)^2\right\}.$$

(iii) a tower of charge $+5/3$ fermions with masses given by poles$\{\not p\Pi_0^q\}$ and poles$\{\not p\Pi_0^u\}$.

From the formulas above for the fermions of charge $2/3$ and $-1/3$, one recovers Eq. (5) by approximating the form factors with their values at $p^2=0$. Further use of Eqs. (B6) and (B7) gives the same explicit formula valid for the MCHM_5, Eq. (A13), but a factor $\sqrt{2}$ smaller:

$$m_u|_{\text{MCHM}_{10}}\simeq\frac{1}{\sqrt{2}}m_u|_{\text{MCHM}_5}. \tag{B8}$$

A similar result is also valid for the down quark mass.

[1] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998); S. S. Gubser, I. R. Klebanov, and A. M. Polyakov, Phys. Lett. B **428**, 105 (1998); E. Witten, Adv. Theor. Math. Phys. **2**, 253 (1998).

[2] N. Arkani-Hamed, M. Porrati, and L. Randall, J. High Energy Phys. 08 (2001) 017.

[3] R. Contino, Y. Nomura, and A. Pomarol, Nucl. Phys. **B671**, 148 (2003).

[4] K. Agashe, R. Contino, and A. Pomarol, Nucl. Phys. **B719**, 165 (2005).

[5] D. B. Kaplan and H. Georgi, Phys. Lett. **136B**, 183 (1984).

[6] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[7] K. Agashe and R. Contino, Nucl. Phys. **B742**, 59 (2006).

[8] K. Agashe, R. Contino, L. Da Rold, and A. Pomarol, Phys. Lett. B **641**, 62 (2006).

[9] See, for example, G. F. Giudice and R. Rattazzi, Nucl. Phys. **B757**, 19 (2006).

[10] M. Carena, E. Ponton, J. Santiago, and C. E. M. Wagner, Nucl. Phys. **B759**, 202 (2006).

[11] G. Cacciapaglia, C. Csaki, G. Marandella, and J. Terning, Phys. Rev. D **75**, 015003 (2007).

[12] G. Cacciapaglia, C. Csaki, G. Marandella, and J. Terning, hep-ph/0611358.

[13] R. Contino and A. Pomarol, J. High Energy Phys. 11 (2004) 058.

[14] R. Barbieri, A. Pomarol, R. Rattazzi, and A. Strumia, Nucl. Phys. **B703**, 127 (2004).

[15] G. Panico, M. Serone, and A. Wulzer, Nucl. Phys. **B762**, 189 (2007).

[16] T. Han, H. E. Logan, B. McElrath, and L. T. Wang, Phys. Rev. D **67**, 095004 (2003).

[17] G. Azuelos *et al.*, Eur. Phys. J. C **39S2**, 13 (2005).

[18] S. S. D. Willenbrock and D. A. Dicus, Phys. Rev. D **34**, 155 (1986).

[19] M. Perelstein, M. E. Peskin, and A. Pierce, Phys. Rev. D **69**, 075002 (2004).

[20] R. Contino, T. Kramer, M. Son, and R. Sundrum, hep-ph/0612180.

[21] J. A. Aguilar-Saavedra, Phys. Lett. B **625**, 234 (2005); **633**, 792(E) (2006); Proc. Sci., TOP2006 (2006) 003; J. High Energy Phys. 12 (2006) 033.