Diquarks and the semileptonic decay of Λ_b in the hybrid scheme

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In this work we use the heavy-quark-light-diquark picture to study the semileptonic decay $\Lambda_b \to \Lambda_c + l + \bar{\nu}_l$ in the so-called hybrid scheme. Namely, we apply the heavy quark effective theory (HQET) for larger q^2 (corresponding to small recoil), which is the invariant mass square of $l + \bar{\nu}$, whereas the perturbative QCD approach for smaller q^2 to calculate the form factors. The turning point where we require the form factors derived in the two approaches to be connected, is chosen near $\rho_{\rm cut} = 1.1$. It is noted that the kinematic parameter ρ which is usually adopted in the perturbative QCD approach, is in fact exactly the same as the recoil factor $\omega = v \cdot v'$ used in HQET where v, v' are the four velocities of Λ_b and Λ_c respectively. We find that the final result is not very sensitive to the choice, so that it is relatively reliable. Moreover, we apply a proper numerical program within a small range around $\rho_{\rm cut}$ to make the connection sufficiently smooth and we parametrize the form factor by fitting the curve gained in the hybrid scheme. The expression and involved parameters can be compared with the ones gained by fitting the experimental data. In this scheme the end-point singularities do not appear at all. The calculated value is satisfactorily consistent with the data which is recently measured by the DELPHI collaboration within 2 standard deviations.

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I. INTRODUCTION

The general theory of QCD has been developed for more than 40 years, and at present, nobody ever doubts its validity. However, on the other side there is still not a reliable way to deal with the long-distance effects of QCD which are responsible for the quark confinement and hadronic transition matrix elements, because their evaluations cannot be done in perturbative approach. Thus, one needs to factorize the perturbative subprocesses and the nonperturbative parts which correspond to different energy scales. The perturbative parts are, in principle, calculable to any order within the framework of quantum field theory, whereas the nonperturbative part must be evaluated by either fitting data while its universality is assumed, or invoking concrete models.

The perturbative QCD method (PQCD) has been applied to study processes where transitions from heavy mesons or baryons to light hadrons are concerned [1–3], namely, the PQCD which includes the Sudakov resummation, is proved to be successful for handling processes with small 4-momentum transfer q^2. Indeed the processes involving heavy hadrons may provide us with an opportunity to study strong interaction, because compared to $\Lambda_{\rm QCD}$ there exist natural energy scales (heavy-quark masses) which can be used to factorize the perturbative contributions from the nonperturbative effects. On the other hand, for the processes involving heavy hadrons, at small recoil region, where $v \cdot v'$ is close to unity (v and v' denote the four-velocities of the initial and final hadrons), i.e. the momentum transfer q^2 is sufficiently large, the heavy-quark effective theory (HQET) works well due to an extra symmetry $SU_f(2) \otimes SU_s(2)$ [4]. Therefore the HQET and PQCD seem to apply at different regions of q^2. For a two-body decay, the momentum transfer is fixed by the kinematics, however, for a three-body decay, q^2 would span the two different regions.

Among all the processes, the semileptonic decay of hadrons plays an important role for probing the underlying principles and employed models because this process is relatively simple and less dependent on the nonperturbative QCD effects. Namely leptons do not participate in strong interaction, and there is no contamination from the crossed gluon-exchanges between quarks residing in different hadrons which are produced in the weak transitions, whereas such effects are important for the nonleptonic decays. Thus one might gain more model-independent information, such as extraction of the Cabibbo-Kobayashi-Maskawa matrix elements from data. In the semileptonic decays of heavy hadrons it is expected to factorize the perturbative and nonperturbative parts more naturally. Recently the DELPHI collaboration reported their measurement on the Λ_b decay form factor in the semileptonic process $\Lambda_b \to \Lambda_c + l + \bar{\nu}_l$ and determined the parameter $\hat{\rho}^2$ in the Isgur-Wise function $\xi(v \cdot v') = 1 - \hat{\rho}^2(1 - v \cdot v')$ [5].

There is a flood of papers to discuss the semileptonic decays of heavy mesons and the concerned factorization. By contraries, the studies on heavy baryons are much behind [6–8], because baryons consist of three constituent quarks and their inner structures are much more complicated than mesons. In this work, we are going to employ the one-heavy-quark-one-light-diquark picture for the heavy Λ_b and Λ_c to evaluate the form factors of this semileptonic transition $\Lambda_b \to \Lambda_c + l + \bar{\nu}$. Even though the subject of diquark is still in dispute, it is commonly believed that the quark-diquark picture may be a plausible description of baryons [9], especially for the heavy baryons which possess one or two heavy quarks.

The kinematic region for the semileptonic decay $\Lambda_b \to \Lambda_c + l + \bar{\nu}$ can be characterized by the quantity ρ, which is defined as

$$\rho \equiv \frac{p \cdot p'}{M_{\Lambda_b} M_{\Lambda_c}}, \tag{1}$$

where p, p' are the four-momenta of Λ_b and Λ_c respectively. It is noted that this parameter ρ which is commonly adopted in the PQCD approach is exactly the same as the recoil factor $\omega = v \cdot v'$ used in the HQET. The momentum transfer q^2 in the process is within the range of $(m_l + m_\nu)^2 \le q^2 \le (M_{\Lambda_b} - M_{\Lambda_c})^2$, equivalently, it is $1 \le \rho \le \frac{(M^2_{\Lambda_b} + M^2_{\Lambda_c})}{2M_{\Lambda_b} M_{\Lambda_c}} \equiv \rho_{\max}$. In the framework of the HQET [4], this process was investigated by some authors [6–8,10]. For larger q^2, the HQET works well, whereas one can expect that for smaller q^2, the PQCD approach applies. In this work, following Ref. [2], we calculate the contribution from the region with small q^2, i.e. $\rho \to \rho_{\max}$, to the amplitude in PQCD. One believes that the PQCD makes better sense in this region. Körner *et al.* discussed similar cases and suggested that the symmetry for smaller recoil is different from that for larger recoil, so they used the Isgur-Wise function to obtain the amplitude in the kinematic region of smaller recoil, but Brodsky-Lepage function for larger recoils [11]. Our strategy is similar that we apply the PQCD for small q^2 while apply HQET for large q^2 where the PQCD is no longer reliable, instead.

Concretely, when integrating the amplitude square from minimum to maximum of q^2 to gain the decay rate, we divide the whole kinematic region into two parts, small and large q^2 (ρ, equivalently). We phenomenologically adopt a turning point at a certain ρ value, to derive the form factors (defined below in the text) in terms of PQCD in the region from $\rho_{\max}$ to this point and then beyond it we use the HQET instead [8]. We let the two parts connect at the turning point. From Ref. [3], we notice that as $\rho \le 1.1$, the PQCD result is not reliable, so that we choose the turning point at vicinity of $\rho = 1.1$. To testify if the choice is reasonable, we slightly vary the values of the turning point as choosing $\rho = 1.05$, 1.10 and 1.15 to see how sensitive the result is to the choice. Moreover, it is noted that as $\rho_{\rm cut} = 1.1$ and 1.15 are chosen, the two parts connect almost smoothly. Even though, to make more sense, we adopt a proper numerical program to make the connection sufficiently smooth, namely, we let not only the two parts connect, but also the derivatives from two sides are exactly equal. In fact, small differences in the derivatives are easily smeared out by the program. Later in the text, we will explicitly show that the final result is not much sensitive to it, thus one can trust its validity. We name the scheme as the "hybrid" approach. We also parametrize the form factor with respect to ρ based on our numerical results. In fact, when we integrate over the whole kinematic range of ρ, we just use the parametrized expression.

Moreover, we not only reevaluate the form factors f_1 and g_1 of the exclusive process in the diquark picture, but also calculate the form factors f_2 and g_2 which were neglected in previous works [3]. So far, the data on the Λ_b semileptonic decay are only provided by the DELPHI collaboration [5] and not rich enough to single out the contributions from f_2 and g_2. More accurate measurement in the future may offer information about them. Our treatment has another advantage. In the pure PQCD approach, there is an end-point divergence at $\rho \to 1$, even though it is mild and the decay rate which includes an integration over the phase space of final states, i.e. over ρ, is finite. As calculating the contribution from the region with large q^2 ($\rho \to 1$)[1] to the form factors in terms of the HQET, the end-point divergence does not exist at all.

We organize our paper as follows, in Sec. II, we derive the factorization formula for $\Lambda_b \to \Lambda_c l \bar{\nu}$. Our numerical results are presented in Sec. III. Finally, Sec. IV is devoted to some discussions and our conclusion.

II. FORMULATIONS

The amplitude of $\Lambda_b \to \Lambda_c l \bar{\nu}$ decay process is written as

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cb} \bar{l} \gamma^\mu (1 - \gamma_5) \nu_l \langle \Lambda_c(p') \times | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p) \rangle, \tag{2}$$

where p and p' are the momenta of Λ_b and Λ_c respectively. According to its Lorentz structure, the hadronic transition matrix element can be parametrized as

$$\begin{aligned} \mathcal{M}_\mu &\equiv \langle \Lambda_c(p') | \bar{c} \gamma_\mu (1 - \gamma_5) b | \Lambda_b(p) \rangle \\ &= \bar{\Lambda}_c(p') \bigg[\gamma_\mu (f_1(q^2) + \gamma_5 g_1(q^2)) \\ &\quad + \sigma_{\mu\nu} \frac{q^\nu}{M_{\Lambda_b}} (f_2(q^2) + \gamma_5 g_2(q^2)) \\ &\quad + \frac{q_\mu}{M_{\Lambda_b}} (f_3(q^2) + \gamma_5 g_3(q^2)) \bigg] \Lambda_b(p), \end{aligned} \tag{3}$$

where $q \equiv p - p'$ and $\sigma_{\mu\nu} \equiv [\gamma_\mu, \gamma_\nu]/2$.

For the convenience of comparing with the works in literature, we rewrite the above equation in the following form according to Ref. [8]

[1] If m_l is not zero, ρ cannot be exactly 1, thus the superficial singularity does not exist at all, but the form factors are obviously proportional to $1/m_l$ which has the singular property.

$$\mathcal{M}_\mu = \bar{\Lambda}_c(p')\bigg[\gamma_\mu(F_1(q^2) + \gamma_5 G_1(q^2)) + \frac{p_\mu}{M_{\Lambda_b}}(F_2(q^2) + \gamma_5 G_2(q^2)) + \frac{p'_\mu}{M_{\Lambda_c}}(F_3(q^2) + \gamma_5 G_3(q^2))\bigg]\Lambda_b(p). \tag{4}$$

For the case of massless leptons,

$$q_\mu \bar{l}\gamma^\mu(1-\gamma_5)\nu_l = 0, \tag{5}$$

thus the form factors f_3 and g_3 result in null contributions. The contributions from f_2 and g_2 were neglected in previous literature [3], nevertheless in our work, we will consider their contributions to the matrix elements and calculate them in terms of the diquark picture and our hybrid scheme.

The kinematic variables are defined as follows. In the rest frame of Λ_b

$$p \equiv (p^+, p^-, \mathbf{p}_T) = \left(\frac{M_{\Lambda_b}}{\sqrt{2}}, \frac{M_{\Lambda_b}}{\sqrt{2}}, \mathbf{0}_T\right), \tag{6}$$

and

$$p' = \left(\frac{\rho + \sqrt{\rho^2-1}}{\sqrt{2}}M_{\Lambda_c}, \frac{\rho - \sqrt{\rho^2-1}}{\sqrt{2}}M_{\Lambda_c}, \mathbf{0}_T\right), \tag{7}$$

the diquark momenta inside Λ_b and Λ_c are parametrized, respectively, as

$$k_1 = \left(0, \frac{M_{\Lambda_b}}{\sqrt{2}}x_1, \mathbf{k_{1T}}\right), \qquad k_2 = \left(\frac{M_{\Lambda_c}}{\sqrt{2}}\xi_1 x_2, 0, \mathbf{k_{2T}}\right), \tag{8}$$

where $\rho \equiv \frac{p\cdot p'}{M_{\Lambda_b}M_{\Lambda_c}}$, $\xi_1 \equiv \rho + \sqrt{\rho^2-1}$, $x_1 \equiv k_1^-/p^-$ and $x_2 \equiv k_2^+/p'^+$. According to the factorization theorem [1–3,12,13], the hadronic matrix element is factorized in the b-space as

$$\mathcal{M}_\mu = \int_0^1 dx_1 dx_2 \int \frac{d^2\mathbf{b}_1}{(2\pi)^2}\frac{d^2\mathbf{b}_2}{(2\pi)^2}\bar{\Psi}_{\Lambda_c}(x_2, \mathbf{b}_2, p', \mu) \times \tilde{H}_\mu(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, r, M_{\Lambda_b}, \mu)\Psi_{\Lambda_b}(x_1, \mathbf{b}_1, p, \mu), \tag{9}$$

where $r \equiv M_{\Lambda_c}/M_{\Lambda_b}$. The renormalization group evolution of the hard amplitude $\tilde{H}_\mu$ is shown as follows [14]

$$\tilde{H}_\mu(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, r, M_{\Lambda_b}, \mu) = \exp\left[-4\int_\mu^{t_{a(b)}}\frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right] \times \tilde{H}_\mu(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, r, M_{\Lambda_b}, t_{a(b)}), \tag{10}$$

where $\gamma_q(\alpha_s(\bar{\mu}))$ is the anomalous dimension.

The wave function of Λ_b which has the heavy-quark and light-diquark structure, is given as [11,15]

$$\Psi_{\Lambda_b}(x_1, \mathbf{b}_1, p, \mu) = f^S_{\Lambda_b}\Phi^S_{\Lambda_b}(x_1, \mathbf{b}_1, p, \mu)\chi^S_{\Lambda_b}\Lambda_b(p, \lambda), \tag{11}$$

where $\Lambda_b(p, \lambda)$ is the baryon spinor, and the superscript S denotes scalar diquark (spin = 0, isospin = 0). $f^S_{\Lambda_b}$ is a constant introduced in literature. $\chi^S_{\Lambda_b}$ is the flavor component of the baryon, namely $\chi^S_{\Lambda_b} = b^\dagger S^\dagger_{[u,d]}|0\rangle$, where $b^\dagger$ and $S^\dagger_{[u,d]}$ are the creation operator of b-quark and the scalar diquark of ud-quarks.

The Λ_c distribution amplitude bears a similar form,

$$\Psi_{\Lambda_c}(x_2, \mathbf{b}_2, p', \mu) = f^S_{\Lambda_c}\Phi^S_{\Lambda_c}(x_2, \mathbf{b}_2, p', \mu)\chi^S_{\Lambda_c}\Lambda_c(p', \lambda_2). \tag{12}$$

Including the Sudakov evolution of hadronic wave functions, i.e. running the scale of wave function from μ down to $\omega_1(\omega_2)$ [14]:

$$\Phi^S_{\Lambda_b}(x_1, \mathbf{b}_1, p, \mu) = \exp\left[-s(\omega_1, (1-x_l)p^-) - 2\int_{\omega_1}^\mu \frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right]\Phi^S_{\Lambda_b}(x_1, \mathbf{b_1}),$$
$$\Phi^S_{\Lambda_c}(x_2, \mathbf{b}_2, p', \mu) = \exp\left[-s(\omega_2, (1-x_2)p'^+) - 2\int_{\omega_2}^\mu \frac{d\bar{\mu}}{\bar{\mu}}\gamma_q(\alpha_s(\bar{\mu}))\right]\Phi^S_{\Lambda_c}(x_2, \mathbf{b_2}), \tag{13}$$

where $\omega_i = 1/b_i (i = 1, 2)$.

In our work, the effective gluon-diquark vertices are defined as [11,15]

$$\begin{aligned} &SgS\colon\ ig_s t^\alpha(p_1+p_2)_\mu F_S(Q^2),\\ &F_S(Q^2) = \delta_s\frac{Q_0^2}{Q_0^2+Q^2},\\ &\quad \delta_s = \alpha_s(Q^2)/\alpha_s(Q_0^2)(if Q^2 \geq Q_0^2);\\ &\quad \delta_s = 1(if Q^2 < Q_0^2), \end{aligned} \tag{14}$$

where $Q^2 \equiv -(p_1-p_2)^2$.

According to the factorization scheme which is depicted in Fig. 1, it is straightforward to obtain the analytic expressions of the form factors F_1, G_1, F_2, G_2, F_3 and G_3, by comparing Eqs. (4) with (9). In the above derivation, the following transformation has been used:

$$k_1 = Ap + Bp', \qquad k_2 = Cp + Dp' \tag{15}$$

and the explicit expressions of A, B, C and D are given in the appendix.

Then we can obtain the analytical form of f_i and $g_i (i = 1, 2, 3)$ making use of the relations between them and $F_j, G_j (j = 1, 2, 3)$ listed below

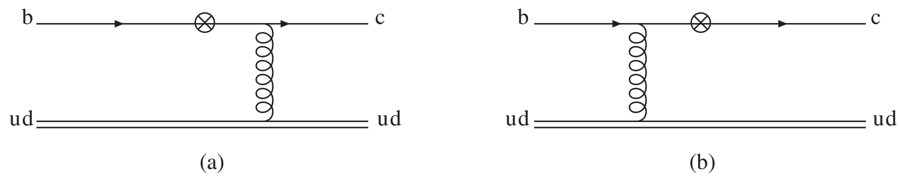

FIG. 1. The lowest order diagrams of the hard parts of the transition processes in the quark-diquark picture.

$$\begin{aligned} f_1 &= F_1 + \frac{1}{2}\left(\frac{F_2}{M_{\Lambda_b}} + \frac{F_3}{M_{\Lambda_c}}\right)(M_{\Lambda_b} + M_{\Lambda_c}), & g_1 &= G_1 - \frac{1}{2}\left(\frac{G_2}{M_{\Lambda_b}} + \frac{G_3}{M_{\Lambda_c}}\right)(M_{\Lambda_b} - M_{\Lambda_c}), \\ f_2 &= -\frac{1}{2}\left(\frac{F_2}{M_{\Lambda_b}} + \frac{F_3}{M_{\Lambda_c}}\right)M_{\Lambda_b}, & g_2 &= -\frac{1}{2}\left(\frac{G_2}{M_{\Lambda_b}} + \frac{G_3}{M_{\Lambda_c}}\right)M_{\Lambda_b}, \\ f_3 &= \frac{1}{2}\left(\frac{F_2}{M_{\Lambda_b}} - \frac{F_3}{M_{\Lambda_c}}\right)M_{\Lambda_b}, & g_3 &= \frac{1}{2}\left(\frac{G_2}{M_{\Lambda_b}} - \frac{G_3}{M_{\Lambda_c}}\right)M_{\Lambda_b}. \end{aligned} \tag{16}$$

We do not display the expressions of f_3 and g_3 for the reason given above. The form factors are integrations which convolute over three parts [12]: the hard-part kernel function, the Sudakov factor and the wave functions of the concerned hadrons as

$$\begin{aligned} f_i(g_i) = 4\pi C_F f_{\Lambda_b} f_{\Lambda_c} \int_0^1 dx_1 dx_2 \int_0^\infty b_1 db_1 b_2 db_2 \\ \times \int_0^{2\pi} d\theta \Phi_{\Lambda_c}(x_2, \mathbf{b}_2) \times \mathrm{kern}SgS_{f_i(g_i)} \Phi_{\Lambda_b}(x_1, \mathbf{b}_1) \\ \times \exp[-S(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, r, M_{\Lambda_b})], \end{aligned} \tag{17}$$

where the explicit expression of the kernel functions $\mathrm{kern}SgS_{f_i(g_i)}$ is given in the appendix for concision of the text. The explicit form of the Sudakov factor appearing in the above equations is given in Ref. [12] as

$$\begin{aligned} S(x_1, x_2, \mathbf{b}_1, \mathbf{b}_2, r, M_{\Lambda_b}) = s(\omega_1, (1 - x_l)p^-) \\ + s(\omega_2, (1 - x_2)p'^+) \\ + 2\int_{\omega_1}^{t_{a(b)}} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \\ + 2\int_{\omega_2}^{t_{a(b)}} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})), \end{aligned} \tag{18}$$

where $C_F = 4/3$ is the color factor.

The wave function $\Phi_{\Lambda_b}(x_1, \mathbf{k}_{1T})$ is [13,16]

$$\Phi_{\Lambda_b}(x_1, \mathbf{k}_{1T}) = \frac{2N_b x_1^6 (1 - x_1)^3}{\pi[(1 - a_b - x_1)^2 x_1^2 (1 - x_1) + \varepsilon_{pb}(1 - x_1)^2 x_1^2 + \mathbf{k}_{1T}^2]^3}, \tag{19}$$

and

$$\Phi_{\Lambda_b}(x_1, b_1) = \int d^2\mathbf{k}_{1T} \Phi_{\Lambda_b}(x_1, \mathbf{k}_{1T}) e^{i\mathbf{k}_{1T}\cdot\mathbf{b}_1} = \frac{N_b x_1^6 (1 - x_1)^3 b_1^2 K_2(\sqrt{(1 - a_b - x_1)^2 x_1^2 (1 - x_1) + \varepsilon_{pb}(1 - x_1)^2 x_1^2} b_1)}{2[(1 - a_b - x_1)^2 x_1^2 (1 - x_1) + \varepsilon_{pb}(1 - x_1)^2 x_1^2]}, \tag{20}$$

where K_2 is the modified Bessel function of the second kind. If neglecting the transverse momentum $\mathbf{k}_{1T}$, i.e. set $\mathbf{k}_{1T} \sim 0$, the wave function can be simplified as

$$\Phi_{\Lambda_b}(x_1) \simeq \frac{N_b x_1^2 (1 - x_1)}{[(1 - a_b - x_1)^2 + \varepsilon_{pb}(1 - x_1)]^2}. \tag{21}$$

The normalization conditions are set as [13]

$$\begin{aligned} \int_0^1 \Phi_{\Lambda_b}(x_1) dx_1 = 1, \qquad \int_0^1 \Phi_{\Lambda_b}(x_1) x_1 dx_1 = \frac{\bar{\Lambda}}{M_{\Lambda_b}}, \\ \int_0^1 \Phi_{\Lambda_b}(x_1) x_1^2 dx_1 = \left(\frac{\bar{\Lambda}^2}{M_{\Lambda_b}^2} + \frac{\lambda_1}{3M_{\Lambda_b}^2}\right). \end{aligned} \tag{22}$$

The first formula determines the normalization of the parton distribution of the baryon, whereas the second one is related to the effective mass of the light diquark $\bar{\Lambda} \sim M_{\Lambda_b} - m_b$, and the third formula reflects connection between hadronic matrix element of the kinematic operator $\lambda_1 = -\frac{1}{2M_{\Lambda_b}} \langle \Lambda_b | \bar{b}_v (iD_\perp)^2 b_v | \Lambda_b \rangle$ and hadronic distribution amplitude. To satisfy the above three normalization conditions, the parameters would take the following values $\bar{\Lambda} = 0.848$ GeV, $\lambda_1 = -0.76$ GeV^2, $a_b = 0.916$, $\varepsilon_{\mathrm{pb}} = 0.0051$ and $N_b = 0.0219$, and in the following numerical evaluation, we will use them as inputs.

For the wave function of Λ_c, the expressions are the same as that for Λ_b, while the corresponding parameters are $\bar{\Lambda} = 0.8849$ GeV, $\lambda_1 = -1.87$ GeV^2, $a_c = 1.48$,

$\varepsilon_{pc} = 0.080$ and $N_c = 12.14$. Thus we can write the differential decay width as

$$\frac{d\Gamma}{d\rho} = \frac{G_F^2|V_{cb}|^2}{24\pi^3} M_{\Lambda_b}^5 r^3 \sqrt{\rho^2 - 1}[|f_1|^2(\rho - 1)(3r^2 - 4\rho r + 2r + 3) + |g_1|^2(\rho + 1)(3r^2 - 4\rho r - 2r + 3) + 6f_1 f_2(r + 1)(\rho - 1)(r^2 - 2r\rho + 1) + 6g_1 g_2(r - 1)(r^2 - 2r\rho + 1) + |f_2|^2(\rho - 1) \times (3r^2 - 2\rho r + 4r + 3)(r^2 - 2\rho r + 1) + |g_2|^2(\rho + 1)(3r^2 - 2\rho r - 4r + 3)(r^2 - 2\rho r + 1)], \tag{23}$$

with

$$1 \le \rho \le \rho_{\max} = \frac{1}{2}\left(\frac{M_{\Lambda_b}}{M_{\Lambda_c}} + \frac{M_{\Lambda_c}}{M_{\Lambda_b}}\right). \tag{24}$$

III. NUMERICAL RESULTS

A. The results of PQCD

In the one-heavy-quark-one-light-diquark picture, the $(ud)_{\bar{3}}$ diquark in Λ_b is considered as a scalar of color antitriplet. To calculate the form factors in the framework of PQCD, one can adjust the product $f^S_{\Lambda_b} f^S_{\Lambda_c}$ to fit the empirical formula $f_1(\rho_{\max}) \sim 1.32/\rho_{\max}^{5.18}$ given by the authors of Ref. [3].[2] Then the corresponding parameters are obtained as

$$m_b \simeq M_{\Lambda_b} = 5.624 \text{ GeV}, \qquad m_c \simeq M_{\Lambda_c} = 2.2849 \text{ GeV}, \qquad V_{cb} = 0.040, \qquad f^S_{\Lambda_b} f^S_{\Lambda_c} = 0.0096 \text{ GeV}^2, \qquad Q_0^2 = 3.22 \text{ GeV}^2, \qquad \Lambda_{\rm QCD} = 0.2 \text{ GeV}. \tag{25}$$

Using these values, we can continue to numerically estimate the form factors f_1 and g_1. Figure 2(a) shows that the form factor f_1 is exactly equal to $|g_1|$ in the heavy-quark limit. The form factor f_2 and g_2 are much smaller than f_1 and $|g_1|$, thus they can in fact be safely neglected for the present experimental accuracy.

In Fig. 2(b), we plot the dependence of the differential width $d\Gamma/d\rho$ on ρ. Although both f_1 and g_1 have an end-point divergence at $\rho \to 1$ in PQCD approach, the differential decay rate is finite.

If one extrapolates the PQCD calculation to the region with smaller ρ values, we obtain the form factors $f_{1,2}$ and $g_{1,2}$ in that region where there are obvious end-point singularities at $\rho \to 1$. Redo the computations with the extrapolation (in the original work [3], the authors extend the tangent of the PQCD result at a small ρ value to $\rho = 1$, so the end-point singularity is avoided) and obtain $BR(\Lambda_b \to \Lambda_c l \nu)$ which is about 1.35% (slightly smaller than the value of 2% guessed by the authors of Ref. [3], because then no data were available.).

Obviously, the calculation in PQCD depends on the factor $f^S_{\Lambda_b} f^S_{\Lambda_c}$, which regularly must be obtained by fitting the data of semileptonic decays, so that the theoretical predictions are less meaningful. Instead, we will use our hybrid scheme where we do not need to obtain the factor $f^S_{\Lambda_b} f^S_{\Lambda_c}$ by fitting data, since the connection requirement substitutes the fitting procedure (see below for details).

[2]When first calculating the form factors, there were no data available, the authors of [3] used a reasonable estimate of $BR(\Lambda_b \to \Lambda_c l \nu)$ as about 2%. Nowadays, measurements have been done and with the data, we have made a new fit.

B. The results of HQET

The transition rate was evaluated in terms of HQET by the authors of Ref. [8]. According to the definitions given in Eq. (16), we recalculate f_1, f_2, g_1, g_2 while dropping out f_3 and g_3 and also obtain similar conclusion that f_1 and g_1 are the same in amplitude, but opposite in sign, as shown in Fig. 2(a), whereas f_2 is very small and g_2 is exactly zero in HQET. The theoretical prediction on the rate of the semileptonic decay $\Lambda_b \to \Lambda_c + l + \bar{\nu}$ in the HQET is

$$\begin{aligned} &\Gamma(\Lambda_b \to \Lambda_c l \bar{\nu}) = 1.54 \times 10^{-14} \text{ GeV}, \\ &\quad \text{without contributions from } f_2 \text{ and } g_2, \\ &\Gamma(\Lambda_b \to \Lambda_c l \bar{\nu}) = 1.56 \times 10^{-14} \text{ GeV}, \\ &\quad \text{with contributions from } f_2 \text{ and } g_2, \end{aligned} \tag{26}$$

and the branching ratio is 2.87%.

The transition rate of $\Lambda_b \to \Lambda_c l \bar{\nu}$ has recently been measured by the DELPHI Collaboration [5], and the value of $BR(\Lambda_b \to \Lambda_c l \nu)$ is $5.0^{+1.1}_{-0.8}(\text{stat})^{+1.6}_{-1.2}(\text{syst})\%$. It is noted that the result calculated in terms of HQET is only consistent with data within 2 standard deviations.

C. The hybrid scheme: Reconciling the two approaches

As widely discussed in literature, in the region with large q^2 (small ρ-values), the result of HQET is reliable, whereas for the region with small q^2 (larger ρ-values) the PQCD is believed to work well. Therefore, to reconcile the two approaches which work in different ρ regions, $1 \le \rho < \rho_{\max}$, we apply the HQET for small ρ, but use PQCD for larger ρ. Our strategy is that we let the form factors f_1, g_1 derived in the PQCD approach be equal to the value obtained in terms of HQET at the point $\rho_{\rm cut}$. The numerical results of the form factor $f_1 = |g_1|$ in the hybrid scheme are shown in Fig. 3 for three different $\rho_{\rm cut}$-values: 1.05, 1.10 and 1.15, respectively. It is noted that for $\rho_{\rm cut} = 1.1$ and 1.15, the left- and right-derivatives are very close and the connection is smooth, whereas, as $\rho_{\rm cut}$ is chosen as 1.05, a difference between the derivatives at the two sides of $\rho_{\rm cut} = 1.05$ obviously manifests. We then adopt a proper numerical program to smoothen the curve, namely, let the derivatives of the two sides meet each other for any ρ-value near the cut point. Figure 3 shows that such treat-

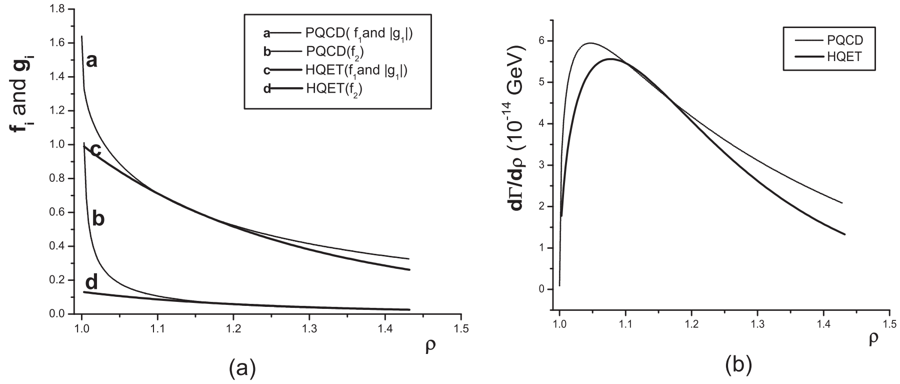

FIG. 2. (a) Form factors f_2, f_1 and $|g_1|$ (b) Differential decay width of $\Lambda_b \to \Lambda_c l\nu$.

ment in fact does not change the general form of the curve, but makes it sufficiently smooth for all ρ values including the selected cut point $\rho_{\rm cut}$.

Another advantage of adopting such a hybrid scheme is that there does not exist end-point singularity for the form factors at $\rho = 1$. Since at the turning point, we let the form factors derived in terms of PQCD be connected with that obtained in HQET, the product $f_{\Lambda_b} f_{\Lambda_c}$ is automatically determined by the connection. With the value, we calculate the form factors within the range of small q^2 in PQCD. In this hybrid scheme, one does not need to invoke the data on the semileptonic decay to fix the parameter $f_{\Lambda_b} f_{\Lambda_c}$ at all.

By our numerical results obtained in the hybrid scheme, the form factors f_1 (or g_1) can be parametrized in a satisfactory expression, here we only present the expression for $\rho_{\rm cut} = 1.10$ as

$$f_1(\rho) = 1 - 3.61(\rho - 1) + 7.24(\rho - 1)^2 - 5.83(\rho - 1)^3, \tag{27}$$

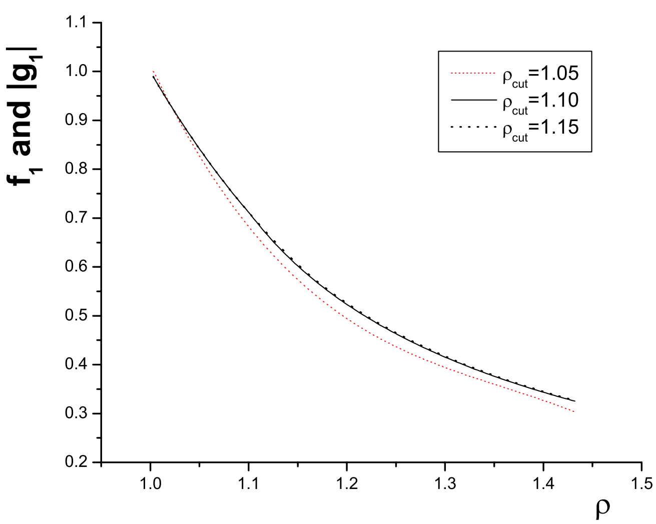

FIG. 3 (color online). Form factors $f_1 = |g_1|$ in the hybrid scheme.

and similar parametrized form factors were discussed in Ref. [17].

The expression can be described by only one "Isger-Wise function" for the transition $\Lambda_b \to \Lambda + l\bar{l}$ at the heavy-quark limit, and it is done by the DELPHI collaboration based on their data on $\Lambda_b \to \Lambda_c l\bar{\nu}$. It is parametrized as [5]

$$\xi(\omega) = 1 - \hat{\rho}^2(\omega - 1) + O((\omega - 1)^2), \tag{28}$$

where $\hat{\rho}^2 = 2.03 \pm 0.46({\rm stat})^{+0.72}_{-1.00}({\rm syst})$.

Obviously, this expression is only valid to the leading order, i.e. linearly proportional to $\omega - 1$ where ω exactly corresponds to the parameter ρ which is commonly adopted in the PQCD language. By contrast, our result includes higher power terms because the $1/M$ corrections are automatically taken into account in our work. It is noted that the coefficient of the linear term in our numerical result is reasonably consistent with the $\hat{\rho}^2$ obtained by fitting data.

To obtain the total decay width, we integrate over the whole range ρ from 1 to $\rho_{\max}$, the integrand is the parametrized form factor in Eq. (27). We obtain

$$\begin{aligned} \Gamma(\Lambda_b \to \Lambda_c l\nu) &= 1.65 \times 10^{-14}\ {\rm GeV}, \\ BR &= 3.08\% \quad \text{with} \quad \rho_{\rm cut} = 1.10, \\ f_{\Lambda_b} f_{\Lambda_c} &= 0.0149\ {\rm GeV}^2. \end{aligned} \tag{29}$$

For a comparison, we present the results corresponding to other two $\rho_{\rm cut}$ values where the smoothing treatment is employed, and they are

$$\begin{aligned} \Gamma(\Lambda_b \to \Lambda_c l\nu) &= 1.54 \times 10^{-14}\ {\rm GeV}, \qquad BR = 2.87\% \\ \rho_{\rm cut} &= 1.05, \qquad f_{\Lambda_b} f_{\Lambda_c} = 0.0142\ {\rm GeV}^2; \\ \Gamma(\Lambda_{\rm b} \to \Lambda_{\rm c} {\rm l}\nu) &= 1.67 \times 10^{-14}\ {\rm GeV}, \qquad {\rm BR} = 3.12\% \\ \rho_{\rm cut} &= 1.15; \qquad {\rm f}_{\Lambda_{\rm b}} {\rm f}_{\Lambda_{\rm c}} = 0.0150\ {\rm GeV}^2. \end{aligned} \tag{30}$$

One can notice that the factor $f_{\Lambda_b} f_{\Lambda_c}$ does not change much as $\rho_{\rm cut}$ varies and they are about 1.5 times larger than the value obtained in pure PQCD (Eq. (25)). When the turning point is chosen at $\rho_{\rm cut} = 1.05$, the branching ratio calculated in the hybrid scheme is very close to the result obtained in pure HQET, while for $\rho_{\rm cut} = 1.10$ and $\rho_{\rm cut} = 1.15$, the resultant branching ratio is slightly larger than that obtained in pure HQET, but more coincides with the data.

In our scenario, the HQET is applied for smaller ρ, and the values of the form factors at $\rho_{\rm cut}$ are fixed by the theory. The values can also determine $f_{\Lambda_b} f_{\Lambda_c}$ which will be used for the PQCD calculations for larger ρ. The HQET is an ideal theoretical framework, but there is an unknown function which is fully governed by the nonperturbative QCD effects, that is the famous Isgur-Wise function. The function can be either obtained by fitting data, or evaluated by concrete models. Various models would result in different slopes. The authors of Ref. [8] used the Drell-Yan type overlap integrals to obtain the slope which is what we employed to get the parametrization Eq. (27) and the slope is -3.6. Instead, the authors of [6] evaluated the slope in the Isgur-Wise function by means of the QCD sum rules. According to their result, we reparametrize the form factor $f_1(\rho)$ and have

$$f_1(\rho) = 1.01 - 1.57(\rho - 1) - 2.59(\rho - 1)^2 + 6.99(\rho - 1)^3. \tag{31}$$

Correspondingly, we obtain

$$\begin{aligned} \Gamma(\Lambda_b \to \Lambda_c l\nu) &= 2.38 \times 10^{-14}\ \text{GeV}, \\ BR &= 4.45\% \quad \text{with} \quad \rho_{\rm cut} = 1.10, \\ f_{\Lambda_b} f_{\Lambda_c} &= 0.0213\ \text{GeV}^2. \end{aligned} \tag{32}$$

The fitted slope by the DELPHI collaboration is $\hat{\rho}^2 = 2.03 \pm 0.46(\text{stat})^{+0.72}_{-1.00}(\text{syst})$ [5], which is between the two theoretical evaluated values. In these references, only linear term remained, due to uncertainties in the approximations the deviations are understandable. Therefore, one can note that there is a model-dependence which mainly manifests in the slope of the Isgur-Wise function. Even though they deviate from each other at the linear term, the high power terms would compensate the deviation slightly and the predicted values on the branching ratio in two approaches Eqs. (27) and (31) are qualitatively consistent with data. There indeed is a byproduct which brings in an advantage that more accurate measurements can help to make judgement on validity of the models by which the Isgur-Wise functions are evaluated.

To make more sense, we purposely present the ratio G_1/F_1 obtained in the hybrid scheme in Fig. 4, it is noted that the ratio is qualitatively consistent with that given in Ref. [11] which was shown on the left part of Fig. 3 of their paper [11].

The authors of Ref. [11] extended ρ into the unphysical region ($\rho > \rho_{\rm max}$ for $\Lambda_b \to \Lambda_c l\bar{\nu}$), while we only keep it within the physical region. It is noted that in the physical region, numerically our result is very close to that obtained

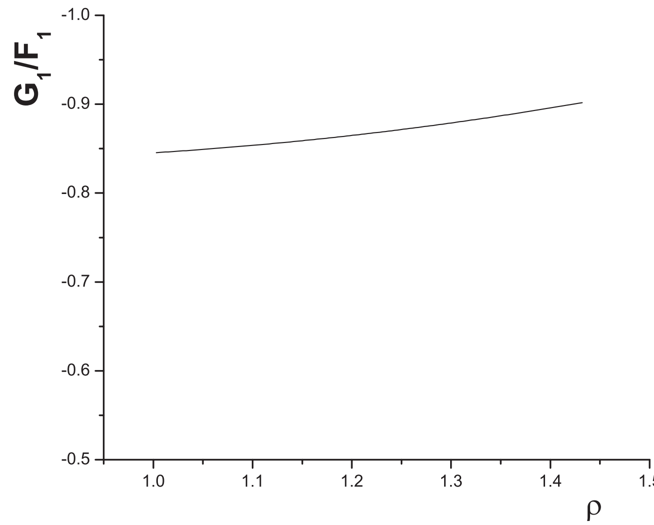

FIG. 4. The G_1/F_1 ratio obtained in our hybrid scheme.

in Ref. [11]. But if one extends the curve to larger ρ, he will notice that our curve is convex, but theirs is concave, namely, the coefficient of the quadratic term has an opposite sign, but the difference is too tiny to be observed or bring up substantial difference for the evaluation of the decay width.

IV. DISCUSSION AND CONCLUSION

In this work, we investigate the semileptonic decay $\Lambda_b \to \Lambda_c + l + \bar{\nu}_l$ in the so-called "hybrid scheme" and the diquark picture for heavy baryons Λ_b and Λ_c. The hybrid scheme means that for the range of smaller q^2 (larger ρ-values) we use the PQCD approach, whereas the HQET for larger q^2 (near $\rho = 1$), to calculate the form factors. We find that the form factors f_2 and g_2 can be safely neglected as suggested in the literature. Besides, we do not need to determine the phenomenological parameters f_{Λ_b} and f_{Λ_c} by fitting data in the hybrid scheme as one did with the pure PQCD approach. Our result is generally consistent with the newly measured branching ratio $\Lambda_b \to \Lambda_c + l + \bar{\nu}_l$ within 2 standard deviations and the end-point singularities existing in the PQCD approach are completely avoided. In fact, the final result somehow depends on the slope in the Isgur-Wise function of the HQET, which is obtained by model-dependent theoretical calculations. Therefore, we may only trust the obtained value to this accuracy, the further experimental data and development in theoretical framework will help to improve the accuracy of the theoretical predictions.

The quark-diquark picture seems to work well for dealing with the semileptonic decays of Λ_b, and we may expect that the quark-diquark picture indeed reflects the physical reality and is applicable to the processes where baryons are involved, at least for the heavy baryons [18]. This picture will be further tested in the nonleptonic decays of heavy baryons. We will employ the diquark picture and PQCD to further study the nonleptonic decay modes in our future work.

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APPENDIX: EXPLICIT EXPRESSIONS OF HARD KERNEL

$$\begin{aligned}
\mathrm{kern}SgS_{f_1} &= \{[(BC-AD)(1+r)-(C^2-A(2-C+Dr)+(B+D)r(Dr-2\rho)\\
&\quad +C(Br+2Br\rho+2Dr\rho-2))]hha+[(BC-AD)r(1+r)-(A^2+A(C-r(D+2\rho-2B\rho))\\
&\quad +r(B^2r-2(Dr+C\rho)-B(C+(-2+D)r+2C\rho)))]hhb\}M_{\Lambda_b}^2F_S(\eta),\\
\mathrm{kern}SgS_{g_1} &= \{[(BC-AD)(r-1)+(C^2+A(-2+C+Dr)+(B+D)r(Dr-2\rho)\\
&\quad -C(rB-2Dr\rho-2rB\rho+2))]hha+[(BC-AD)r(1-r)+(A^2+A(C+r(D-2\rho+2B\rho))\\
&\quad +r(B^2r-2(Dr+C\rho)+B((-2+D)r-C+2C\rho)))]hhb\}M_{\Lambda_b}^2F_S(\eta),\\
\mathrm{kern}SgS_{f_2} &= (AD-BC)(hha+rhhb)M_{\Lambda_b}^2F_S(\eta),\\
\mathrm{kern}SgS_{g_2} &= (AD-BC)(hha-rhhb)M_{\Lambda_b}^2F_S(\eta),
\end{aligned} \tag{A1}$$

where

$$\eta \equiv M_{\Lambda_b}M_{\Lambda_c}x_1x_2\xi_1. \tag{A2}$$

The explicit expressions of A, B, C and D

$$\begin{aligned}
A &= \frac{x_1\xi_1}{\xi_1-\xi_2}, \qquad B = -\frac{x_1M_{\Lambda_b}}{M_{\Lambda_c}(\xi_1-\xi_2)},\\
C &= -\frac{x_2M_{\Lambda_c}}{M_{\Lambda_b}(\xi_2-\xi_1)}, \qquad D = \frac{x_2\beta_1}{\xi_1-\xi_2},
\end{aligned} \tag{A3}$$

with

$$\xi_1 = \rho+\sqrt{\rho^2-1}, \qquad \xi_2 = \rho-\sqrt{\rho^2-1}. \tag{A4}$$

The explicit expressions of hha, hhb are

$$\begin{aligned}
hha &= \alpha_s(t_a)K_0(\sqrt{x_1x_2\xi_1M_{\Lambda_b}M_{\Lambda_c}}b_1)\\
&\quad \times K_0(\sqrt{x_2\xi_1M_{\Lambda_b}M_{\Lambda_c}}|\mathbf{b}_1+\mathbf{b}_2|),\\
hhb &= \alpha_s(t_b)K_0(\sqrt{x_1x_2\xi_1M_{\Lambda_b}M_{\Lambda_c}}b_2)\\
&\quad \times K_0(\sqrt{x_1\xi_1M_{\Lambda_b}M_{\Lambda_c}}|\mathbf{b}_1+\mathbf{b}_2|),
\end{aligned} \tag{A5}$$

with

$$\begin{aligned}
t_a &= \max(\sqrt{x_2\xi_1M_{\Lambda_b}M_{\Lambda_c}}, 1/|\mathbf{b}_1+\mathbf{b}_2|, 1/b_1, 1/b_2),\\
t_b &= \max(\sqrt{x_1\xi_1M_{\Lambda_b}M_{\Lambda_c}}, 1/|\mathbf{b}_1+\mathbf{b}_2|, 1/b_1, 1/b_2).
\end{aligned}$$

[1] H. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992); G. Korchemsky and G. Sterman, Phys. Lett. B **340**, 96 (1994); J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989).
[2] Y. Keum, H. Li, and A. I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001); C. Lü, K. Ukai, and M. Yang, Phys. Rev. D **63**, 074009 (2001).
[3] H. Shih, S. Lee, and H. Li, Phys. Rev. D **61**, 114002 (2000).
[4] N. Isgur and M. Wise, Phys. Lett. B **232**, 113 (1989); Nucl. Phys. **B348**, 276 (1991); H. Georgi, *ibid.* **B348**, 293 (1991).
[5] J. Abdallah *et al.* (DELPHI Collaboration), Phys. Lett. B **585**, 63 (2004).
[6] M. Huang, H. Jin, J. Körner, and C. Liu Phys. Lett. B **629**, 27 (2005).
[7] Y. Dai, C. Huang, M. Huang, and C. Liu, Phys. Lett. B **387**, 379 (1996); Y. Dai, C. Huang, C. Liu, and C. D. Lü, *ibid.* **371**, 99 (1996); C. Liu, Phys. Rev. D **57**, 1991 (1998); J. Lee, C. Liu, and H. Song, *ibid.* **58**, 014013 (1998).
[8] X. H. Guo and P. Kroll, Z. Phys. C **59**, 567 (1993).
[9] F. Wilczek, hep-ph/0409168.
[10] A. Flak and M. Neubert, Phys. Rev. D **47**, 2982 (1993); B. Holdom, M. Sutherland, and J. Mureika, *ibid.* **49**, 2359 (1994).
[11] J. Körner and P. Kroll, Z. Phys. C **57**, 383 (1993).
[12] H. Li, Phys. Rev. D **52**, 3958 (1995).
[13] H. Li and H. Yu, Phys. Rev. D **53**, 4970 (1996).

[14] H. Shih, S. Lee, and H. Li, Phys. Rev. D **59**, 094014 (1999); C. H. Chou, H. Shih, S. Lee, and H. Li, Phys. Rev. D **65**, 074030 (2002).

[15] R. Jakob, P. Kroll, M. Schuermann, and W. Schweiger, Z. Phys. A **347**, 109 (1993).

[16] C. Peterson, D. Schlatter, I. Schmitt, and P. Zerwas, Phys. Rev. D **27**, 105 (1983).

[17] H. Cheng, C. Cheung, and C. Hwang, Phys. Rev. D **69**, 074025 (2004).

[18] X. Guo and T. Muta, Phys. Rev. D **54**, 4629 (1996); X. Guo, A. Thomas, and A. Williams, *ibid.* **59**, 116007 (1999).