$B \to \eta^{(\prime)}(l^- \bar{\nu}_l, l^+ l^-, K, K^*)$ decays in the quark-flavor mixing scheme

A. G. Akeroyd,[1,2,*] Chuan-Hung Chen,[1,2,†] and Chao-Qiang Geng[3,4,‡]

[1]*Department of Physics, National Cheng-Kung University, Tainan 701, Taiwan*
[2]*National Center for Theoretical Sciences, Taiwan*
[3]*Department of Physics, National Tsing-Hua University, Hsinchu 300, Taiwan*
[4]*Theory Group, TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada*

(Received 8 January 2007; published 1 March 2007)

In the quark-flavor mixing scheme, η and η' are linear combinations of flavor states $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$ with the masses of m_{qq} and m_{ss}, respectively. Phenomenologically, m_{ss} is strictly fixed to be around 0.69, which is close to $\sqrt{2m_K^2 - m_\pi^2}$ by the approximate flavor symmetry, while m_{qq} is found to be 0.18 ± 0.08 GeV. For a large allowed value of m_{qq}, we show that the branching ratios (BRs) for $B \to \eta^{(\prime)} X$ decays with $X = (\ell^- \bar{\nu}_\ell, \ell^+ \ell^-)$ are enhanced. We also illustrate that $\mathrm{BR}(B \to \eta X) > \mathrm{BR}(B \to \eta' X)$ in the mechanism without the flavor-singlet contribution. Moreover, we demonstrate that the decay branching ratios for $B \to \eta^{(\prime)} K^{[*]}$ are consistent with the data. In particular, the puzzle of the large $\mathrm{BR}(B \to \eta' K)$ can be solved. In addition, we find that the *CP* asymmetry for $B^\pm \to \eta K^\pm$ can be as large as -30%, which agrees well with the data. However, we cannot accommodate the *CP* asymmetries of $B \to \eta K^*$ in our analysis, which could indicate the existence of some new *CP* violating sources.

DOI: 10.1103/PhysRevD.75.054003 PACS numbers: 13.20.He

I. INTRODUCTION

The branching ratio (BR) of $B^0 \to \eta' K^0$ was first observed by the CLEO collaboration with $(89^{+18}_{-16} \pm 9) \times 10^{-6}$ [1], which is much larger than $(20 - 40) \times 10^{-6}$ estimated by the factorization ansatz [2]. With more data accumulated, this incomprehensible value becomes a real puzzle now that the measurements from Belle and *BABAR* depart from the theoretical estimations, where the former has observed $\mathrm{BR}(B^+ \to \eta K^+) = (1.9 \pm 0.3^{+0.2}_{-0.1}) \times 10^{-6}$ [3], $\mathrm{BR}(B^+ \to \eta' K^+) = (69.2 \pm 2.2 \pm 3.7) \times 10^{-6}$, and $\mathrm{BR}(B^0 \to \eta' K^0) = (58.9^{+3.6}_{-3.5} \pm 4.3) \times 10^{-6}$ [4], while the latter has measured $\mathrm{BR}(B^+ \to \eta K^+) = (3.3 \pm 0.6 \pm 0.3) \times 10^{-6}$ [5], $\mathrm{BR}(B^+ \to \eta' K^+) = (68.9 \pm 2.0 \pm 3.2) \times 10^{-6}$, and $\mathrm{BR}(B^0 \to \eta' K^0) = (67.4 \pm 3.3 \pm 3.2) \times 10^{-6}$ [6]. To unravel the mystery, many solutions have been proposed, such as the intrinsic charm in η' [7], the gluonium state [8], the spectator hard scattering mechanism [9], and the flavor-singlet component in η' [10]. Nevertheless, there are still no conclusive solutions yet.

Recently, the *BABAR* collaboration [11] has also measured the semileptonic decays with the data as follows:

$$
\begin{aligned}
\mathrm{BR}(B^+ \to \eta \ell^+ \nu_\ell) &= (0.84 \pm 0.27 \pm 0.21) \times 10^{-4} \\
&< 1.4 \times 10^{-4} (90\%\mathrm{C.L.}), \\
\mathrm{BR}(B^+ \to \eta' \ell^+ \nu_\ell) &= (0.33 \pm 0.60 \pm 0.30) \times 10^{-4} \\
&< 1.3 \times 10^{-4} (90\%\mathrm{C.L.}). \qquad (1)
\end{aligned}
$$

Although the significance of the former in Eq. (1) is 2.55σ, the central value is a factor of 2 larger than 0.4×10^{-4} calculated by the light-cone sum rules (LCSRs) [12]. Because of these results, we speculate that the mechanism to enhance the BRs of $B \to \eta' K$ may also affect the semileptonic decays of $B^- \to \eta^{(\prime)} \ell \bar{\nu}_\ell$. After surveying various proposed mechanisms, one finds that only the flavor-singlet mechanism (FSM) [10] could have direct influence on the BRs of semileptonic decays [12,13]. In this paper, inspired by the measurements of the semileptonic decays, we would like to propose another possible mechanism within the quark-flavor mixing scheme to study the decays of $B \to \eta^{(\prime)}(\ell^- \bar{\nu}_\ell, \ell^+ \ell^-, K, K^*)$. We will also compare our results with those in the FSM [10,12–15] and explore the differences between the two mechanisms, which could be tested in future B experiments.

The paper is organized as follows. In Sec. II, we review the quark-flavor mixing scheme. In Sec. III, we carry out a general analysis for the decay amplitudes and form factors. Numerical results and discussions are presented in Sec. IV. Our conclusions are given in Sec. V.

II. THE QUARK-FLAVOR MIXING SCHEME

It is known that the physical states η and η' are composed of the flavor octet η_8 and singlet η_1, in which the flavor wave functions are denoted as $\eta_8 = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$ and $\eta_1 = (u\bar{u} + d\bar{d} + s\bar{s})/\sqrt{3}$, respectively. Because of the $U_A(1)$ anomaly, it is understood that the mass of η' is much larger than that of η. To satisfy the current experimental data, usually one needs to introduce two angles to the mixing matrix, defined by $\eta = \cos\theta_8 \eta_8 - \sin\theta_1 \eta_1$ and $\eta' = \sin\theta_8 \eta_8 + \cos\theta_1 \eta_1$ [2,16], to describe the connection between physical and flavor states. However, it is known that by using the two-angle scheme, we will encounter a divergent problem in some B decays [17], such as $B \to \eta' K$. To illustrate this problem,

*Electronic address: akeroyd@mail.ncku.edu.tw
†Electronic address: physchen@mail.ncku.edu.tw
‡Electronic address: geng@phys.nthu.edu.tw

we notice that in these decays, the factorized parts are associated with the matrix element $\langle 0|\bar{s}i\gamma_5 s|\eta_1\rangle$. From the equation of motion, one has $\langle 0|\partial^\mu \bar{s}\gamma_\mu\gamma_5 s|\eta_1\rangle = \langle 0|2m_s\bar{s}i\gamma_5 s|\eta_1\rangle = m^2_{\eta_1} f_{\eta_1}$, leading to $\langle 0|\bar{s}i\gamma_5 s|\eta_1\rangle = m^2_{\eta_1} f_{\eta_1}/2m_s$, where $f_{\eta_1}(m_{\eta_1})$ is the decay constant (mass) of η_1. In the chiral limit of $m_s \to 0$, the matrix element diverges because $m_{\eta_1} \neq 0$. To explicitly display the chiral limit, it is better to use the quark-flavor scheme, defined by [18,19]

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}, \tag{2}$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$. From the definition of $\langle 0|\bar{q}'\gamma_\mu\gamma_5 q'|\eta_{q'}(p)\rangle = if_{\eta_{q'}} p_\mu$ $(q' = q, s)$, the masses of $\eta_{q,s}$ can be expressed by

$$\begin{aligned} m^2_{qq} &= \frac{\sqrt{2}}{f_{\eta_q}} \langle 0|m_u \bar{u}i\gamma_5 u + m_d \bar{d}i\gamma_5 d|\eta_q\rangle, \\ m^2_{ss} &= \frac{2}{f_{\eta_s}} \langle 0|m_s \bar{s}i\gamma_5 s|\eta_s\rangle. \end{aligned} \tag{3}$$

Clearly, in terms of the quark-flavor basis, m_{qq} and m_{ss} are zero in the chiral limit. We note that m_{qq} and m_{ss} are unknown parameters and their values can be obtained by fitting with the data, such as the masses of $\eta^{(\prime)}$ and the decay rates of some relevant B decays. Note that $m_{qq,ss}$ are related to $m^0_{\eta_q,\eta_s,K}$ by $m^0_{\eta_q} = m^2_{qq}/(m_u + m_d)$, $m^0_{\eta_s} = m^2_{ss}/2m_s$ and $m^0_K = m^2_K/(m_s + m_q)$. From the divergences of the axial vector currents

$$\partial^\mu \bar{q}'\gamma_\mu\gamma_5 q' = \frac{\alpha_s}{4\pi} G\tilde{G} + 2m_{q'}\bar{q}' i\gamma_5 q', \tag{4}$$

where $G = G^{a\mu\nu}$ are the gluonic field strength and $\tilde{G} = \tilde{G}^{a\mu\nu} \equiv \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta}$, one obtains the $\eta_{q,s}$ masses as

$$\begin{aligned} \begin{pmatrix} M^2_{qq} & M^2_{qs} \\ M^2_{sq} & M^2_{ss} \end{pmatrix} &= \begin{pmatrix} \langle 0|\partial^\mu J^q_{\mu 5}|\eta_q\rangle/f_q & \langle 0|\partial^\mu J^s_{\mu 5}|\eta_q\rangle/f_s \\ \langle 0|\partial^\mu J^q_{\mu 5}|\eta_s\rangle/f_q & \langle 0|\partial^\mu J^s_{\mu 5}|\eta_s\rangle/f_s \end{pmatrix} \\ &= \begin{pmatrix} m^2_{qq} + 2a^2 & \sqrt{2}ya^2 \\ \sqrt{2}ya^2 & m^2_{ss} + y^2a^2 \end{pmatrix} \end{aligned} \tag{5}$$

with $a^2 = \langle 0|\alpha_s G\tilde{G}|\eta_q\rangle/(4\sqrt{2}\pi f_q)$ and $y = f_q/f_s$. Furthermore, by using the mixing matrix introduced in Eq. (2), we have [20]

$$\begin{aligned} \sin\phi &= \left[\frac{(m^2_{\eta'} - m^2_{ss})(m^2_\eta - m^2_{qq})}{(m^2_{\eta'} - m^2_\eta)(m^2_{ss} - m^2_{qq})}\right]^{1/2}, \\ y &= \left[2\frac{(m^2_{\eta'} - m^2_{ss})(m^2_{ss} - m^2_\eta)}{(m^2_{\eta'} - m^2_{qq})(m^2_\eta - m^2_{qq})}\right]^{1/2}, \\ a^2 &= \frac{1}{2}\frac{(m^2_{\eta'} - m^2_{qq})(m^2_\eta - m^2_{qq})}{m^2_{ss} - m^2_{qq}}, \end{aligned} \tag{6}$$

where $m_{\eta^{(\prime)}}$ is the mass of $\eta^{(\prime)}$.

According to the relations in Eq. (3), it is interesting to see that the parameter $m_{qq(ss)}$ is involved in the distribution amplitude of the $\eta_{q(s)}$ state, which is defined by [21]

$$\langle 0|\bar{q}''(0)_j q''_k(z)|\eta_{q'}(p)\rangle = \frac{i}{\sqrt{2N_c}} \int_0^1 dx e^{-ixp\cdot z}[(\not{p}\gamma_5)_{jk}\phi_{\eta_{q'}}(x) + (\gamma_5)_{jk} m^0_{\eta_{q'}} \phi^p_{\eta_{q'}}(x) + m^0_{\eta_{q'}}[\not{n}_-\not{n}_+ - 1]_{jk}\phi^\sigma_{\eta_{q'}}(x)], \tag{7}$$

where $q'' = u, d$ and s, $q' = q$ and s, $\phi_{\eta_{q'}}(x)$ and $\phi^{p(\sigma)}_{\eta_{q'}}(x)$ denote the twist-2 and twist-3 wave functions of the $\eta_{q'}$ state, respectively, x is the momentum fraction, $m^0_{\eta_{q'}}$ stands for the chiral symmetry breaking parameter, and $n_+ = (1, 0, \mathbf{0})$ and $n_- = (0, 1, \mathbf{0})$ are defined in the light-cone coordinates. On substituting Eq. (7) into Eq. (3), we obtain $m^0_{\eta_q} = m^2_{qq}/(m_u + m_d)$ and $m^0_{\eta_s} = m^2_{ss}/2m_s$. In the next section, it will be clear that the value of m_{qq} is crucial for the determination of the $B \to \eta^{(\prime)}$ transition form factors, which play important roles in the decay branching ratios of $B \to \eta^{(\prime)} X$ with $X = (\ell^-\bar{\nu}_\ell, \ell^+\ell^-, K)$.

III. DECAY AMPLITUDES AND FORM FACTORS

We first study the semileptonic decays of $B^- \to \eta^{(\prime)}\ell^-\nu_\ell$ and $\bar{B} \to \eta^{(\prime)}\ell^+\ell^-$ by writing the effective Hamiltonians at quark level in the SM as

$$\mathcal{H}_I = \frac{G_F V_{ub}}{\sqrt{2}} \bar{u}\gamma_\mu(1 - \gamma_5)b\bar{\ell}\gamma^\mu(1 - \gamma_5)\nu_\ell, \tag{8}$$

$$\mathcal{H}_{II} = \frac{G_F \alpha_{\rm em}\lambda^{q'}_t}{\sqrt{2}\pi}[H_{1\mu}L^\mu + H_{2\mu}L^{5\mu}], \tag{9}$$

respectively, with

$$\begin{aligned} H_{1\mu} &= C^{\rm eff}_9(\mu)\bar{q}'\gamma_\mu P_L b - \frac{2m_b}{q^2} C_7(\mu)\bar{q}' i\sigma_{\mu\nu}q^\nu P_R b, \\ H_{2\mu} &= C_{10}\bar{q}'\gamma_\mu P_L b, \\ L^\mu &= \bar{\ell}\gamma^\mu\ell, \\ L^{5\mu} &= \bar{\ell}\gamma^\mu\gamma_5\ell, \end{aligned} \tag{10}$$

where $\alpha_{\rm em}$ is the fine structure constant, V_{ij} denote the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements, $\lambda^{q'}_t = V_{tb}V^*_{tq'}$, C_i are the Wilson coefficients (WCs) with their explicit expressions given in Ref. [22], m_b is the current b-quark mass, q is the momentum transfer, and $P_{L(R)} = (1 \mp \gamma_5)/2$. Note that the long-distance effects of $c\bar{c}$ bound states have been included in $C^{\rm eff}_9$, given by [23]

$$C_9^{\rm eff}(\mu) = C_9(\mu) + (3C_1(\mu) + C_2(\mu))\left(h(z,s) - \frac{3}{\alpha_{\rm em}^2}\sum_{V=\Psi,\Psi'} k_V \frac{\pi\Gamma(V\to\ell^+\ell^-)M_V}{M_V^2 - q^2 - iM_V\Gamma_V}\right),$$
$$h(z,s) = -\frac{8}{9}\ln\frac{m_b}{\mu} - \frac{8}{9}\ln z + \frac{8}{27} + \frac{4}{9}x - \frac{2}{9}(2+x)|1-x|^{1/2}\begin{cases} \ln\left|\frac{\sqrt{1-x}+1}{\sqrt{1-x}-1}\right| - i\pi, & \text{for } x\equiv 4z^2/s<1, \\ 2\arctan\frac{1}{\sqrt{x-1}}, & \text{for } x\equiv 4z^2/s>1, \end{cases} \tag{11}$$

where $h(z, s)$ describes the one-loop matrix elements of operators $O_1 = \bar{s}_\alpha\gamma^\mu P_L b_\beta \bar{c}_\beta \gamma_\mu P_L c_\alpha$ and $O_2 = \bar{s}\gamma^\mu P_L b \bar{c}\gamma_\mu P_L c$ [22] with $z = m_c/m_b$ and $s = q^2/m_b^2$, M_V (Γ_V) are the masses (widths) of intermediate states. The hadronic matrix elements for the $B \to P$ transition are parametrized as

$$\langle P(p_P)|\bar{q}'\gamma^\mu b|\bar{B}(p_B)\rangle = f_+^P(q^2)\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right) + f_0^P(q^2)\frac{P\cdot q}{q^2}q_\mu,$$
$$\langle P(p_P)|\bar{q}' i\sigma_{\mu\nu}q^\nu b|\bar{B}(p_B)\rangle = \frac{f_T^P(q^2)}{m_B + m_P}[P\cdot q q_\mu - q^2 P_\mu] \tag{12}$$

with P representing the pseudoscalar, $P_\mu = (p_B + p_P)_\mu$, $q_\mu = (p_B - p_P)_\mu$ and $f_\alpha^P(q^2)$ are form factors. Consequently, the transition amplitudes associated with the interactions in Eqs. (8) and (9) can be expressed as

$$\mathcal{M}_I = \frac{\sqrt{2}G_F V_{ub}}{\pi} f_+^P(q^2)\bar{\ell}\not{p}_P\ell, \tag{13}$$

$$\mathcal{M}_{II} = \frac{G_F\alpha_{\rm em}\lambda_t^{q'}}{\sqrt{2}\pi}[\tilde{m}_{97}\bar{\ell}\not{p}_P\ell + \tilde{m}_{10}\bar{\ell}\not{p}_P\gamma_5\ell] \tag{14}$$

for $\bar{B} \to P\ell^-\bar{\nu}_\ell$ and $\bar{B} \to P\ell^+\ell^-$, respectively, where

$$\tilde{m}_{97} = C_9^{\rm eff} f_+^P(q^2) + \frac{2m_b}{m_B + m_P}C_7 f_T^P(q^2),$$
$$\tilde{m}_{10} = C_{10} f_+^P(q^2). \tag{15}$$

The differential decay rates for $B^- \to P\ell^-\bar{\nu}_\ell$ and $\bar{B}_d \to P\ell^+\ell^-$ as functions of q^2 are given by [12]

$$\frac{d\Gamma_I}{dq^2} = \frac{G_F^2|V_{ub}|^2 m_B^3}{3\times 2^6\pi^3}\sqrt{(1-s+\hat{m}_P^2)^2 - 4\hat{m}_P^2}(f_+^P(q^2)\hat{P}_P)^2, \tag{16}$$

$$\frac{d\Gamma_{II}}{dq^2} = \frac{G_F^2\alpha_{\rm em}^2 m_B^3}{3\times 2^9\pi^5}|\lambda_t^{q'}|^2\sqrt{(1-s+\hat{m}_P^2)^2 - 4\hat{m}_P^2} \times \hat{P}_P^2(|\tilde{m}_{97}|^2 + |\tilde{m}_{10}|^2), \tag{17}$$

respectively, with $\hat{P}_P = 2\sqrt{s}|\vec{p}_P|/m_B = \sqrt{(1-s-\hat{m}_P^2)^2 - 4s\hat{m}_P^2}$. Since we concentrate on the production of the light leptons, we have neglected the terms explicitly related to the lepton mass. We note that due to $C_9 \gg C_7$, the effect associated with the form factor of $f_T^P(q^2)$ in Eq. (15) is small. From Eqs. (16) and (17), we see clearly that the semileptonic decays are only sensitive to the form factor $f_+^P(q^2)$. By this property, we can use the data of $B^- \to \eta\ell^-\bar{\nu}_\ell$ to constrain the unknown parameters in the calculations of the form factors. The constrained parameters could make some predictions for the decays $B \to \eta^{(\prime)}\ell^+\ell^-$ and $B \to \eta^{(\prime)}K$.

In the large recoil region, i.e. $q^2 \to 0$, the form factors can be evaluated by the perturbative QCD (PQCD) [24,25] approach, in which the transverse momenta of valence quarks are included to remove the end point singularities when $x \to 0$. Hence, in terms of Eq. (7) and the flavor diagrams shown in Fig. 1, the form factors $f_+^P(q^2)$, $f_-^P(q^2)$, and $f_T^P(q^2)$ for $B \to P$ can be formulated as [23]

$$f_+^P(q^2) = f_1^P(q^2) + f_2^P(q^2),$$
$$f_0^P(q^2) = f_1^P(q^2)\left(1 + \frac{q^2}{m_B^2}\right) + f_2^P(q^2)\left(1 - \frac{q^2}{m_B^2}\right), \tag{18}$$

where

$$f_1^P(q^2) = 8\pi C_F m_B^2 r_P \int_0^1 [dx]\int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1,b_1)[\phi_P^p(x_2) - \phi_P^t(x_2)]E(t^{(1)})h(x_1,x_2,b_1,b_2),$$
$$f_2^P(q^2) = 8\pi C_F m_B^2 \int_0^1 [dx]\int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1,b_1)\left\{\left[(1+x_2\xi)\phi_P(x_2) + 2r_P\left(\left(\frac{1}{\xi} - x_2\right)\phi_P^t(x_2) - x_2\phi_P^p(x_2)\right)\right]\right.$$
$$\left.\times E(t^{(1)})h(x_1,x_2,b_1,b_2) + 2r_P\phi_P^p(x_2)E(t^{(2)})h(x_2,x_1,b_2,b_1)\right\}, \tag{19}$$

and

$$f_T^P(q^2) = 8\pi C_F m_B^2 (1 + m_P/m_B) \int_0^1 [dx] \int_0^\infty b_1 db_1 b_2 db_2 \phi_B(x_1, b_1) \left\{ \left[\phi_P(x_2) - r_P x_2 \phi_P^p(x_2) + r_P\left(\frac{2}{\xi} + x_2\right)\phi_P^t(x_2) \right] \times E(t^{(1)}) h(x_1, x_2, b_1, b_2) + 2 r_P \phi_P^p(x_2) E(t^{(2)}) h(x_2, x_1, b_2, b_1) \right\}, \tag{20}$$

with $C_F = 4/3$, $\xi = 1 - q^2/m_B^2$, and $r_P = m_P^0/m_B$. From Eq. (18), we find that $f_+(0) = f_0(0)$. The evolution factor is given by $E(t) = \alpha_s(t)\exp(-S_B(t) - S_P(t))$ where the Sudakov exponents $S_{B(P)}$ can be found in Ref. [26]. The hard function h is written as

$$h(x_1, x_2, b_1, b_2) = S_t(x_2) K_0(\sqrt{x_1 x_2 \xi} m_B b_1)[\theta(b_1 - b_2) K_0(\sqrt{x_2\xi} m_B b_1) I_0(\sqrt{x_2\xi} m_B b_2) + \theta(b_2 - b_1) K_0(\sqrt{x_2\xi} m_B b_2) I_0(\sqrt{x_2\xi} m_B b_1)], \tag{21}$$

where the threshold resummation effect is described by $S_t(x) = 2^{1+2c}\Gamma(\frac{3}{2} + c)[x(1-x)]^c/\sqrt{\pi}\Gamma(1+c)$ with $c = 0.3$ [25]. The hard scales $t^{(1,2)}$ are chosen to be [27]

$$t^{(1)} = \max(\sqrt{m_B^2 \xi x_2}, 1/b_1, 1/b_2, \bar{\Lambda}),$$
$$t^{(2)} = \max(\sqrt{m_B^2 \xi x_1}, 1/b_1, 1/b_2, \bar{\Lambda}),$$

where $\bar{\Lambda}$ is used to exclude the effects from nonperturbative contributions. To get the BRs for the three-body semileptonic decays, besides the values of the form factors at $q^2 = 0$, we also need to know their q^2 dependences. To obtain them, we adopt the fitting results calculated by the light-cone sum rules (LCSR) [28], given by

$$f^P_{+(T)}(q^2) = \frac{f^P_{+(T)}(0)}{(1 - q^2/m_{B^*}^2)(1 - \alpha_{+(T)} q^2/m_{B^*}^2)} \tag{22}$$

with $\alpha_{+(T)} = 0.52(0.84)$ and $m_{B^*} = 5.32$ GeV. In terms of the quark-flavor mixing scheme, we will calculate the $B \to \eta_{q,s}$ form factors, which are related to those of $B \to \eta^{(\prime)}$ by

$$f^\eta_{+(T)}(q^2) = \frac{\cos\phi}{\sqrt{2}} f^{\eta_q}_{+(T)}(q^2),$$
$$f^{\eta'}_{+(T)}(q^2) = \frac{\sin\phi}{\sqrt{2}} f^{\eta_q}_{+(T)}(q^2). \tag{23}$$

For the nonleptonic decays of $B \to \eta^{(\prime)} K$, we will assume the color transparency [29], i.e., no rescattering effects in B decays. The effective interaction for the $b \to s$ transition at the quark level is given by [22]

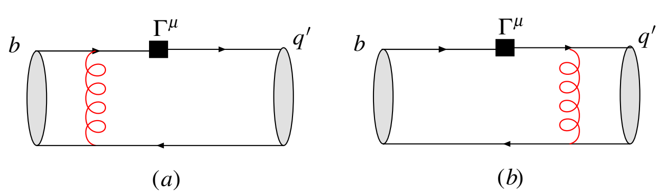

FIG. 1 (color online). Flavor diagrams for the $B \to P$ transition with $\Gamma^\mu = (\gamma^\mu, i\sigma^{\mu\nu}q_\nu)$.

$$H_{\rm eff} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[C_1(\mu) O_1^{(q)}(\mu) + C_2(\mu) O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right], \tag{24}$$

where $V_q = V_{qs}^* V_{qb}$ are the CKM matrix elements and the operators O_1-O_{10} are defined as

$$\begin{aligned}
O_1^{(q)} &= (\bar{q}'_\alpha q_\beta)_{V-A} (\bar{q}_\beta b_\alpha)_{V-A},\\
O_2^{(q)} &= (\bar{q}'_\alpha q_\alpha)_{V-A} (\bar{q}_\beta b_\beta)_{V-A},\\
O_3 &= (\bar{q}'_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A},\\
O_4 &= (\bar{q}'_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A},\\
O_5 &= (\bar{q}'_\alpha b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V+A},\\
O_6 &= (\bar{q}'_\alpha b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A},\\
O_7 &= \frac{3}{2}(\bar{q}'_\alpha b_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V+A},\\
O_8 &= \frac{3}{2}(\bar{q}'_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V+A},\\
O_9 &= \frac{3}{2}(\bar{q}'_\alpha b_\alpha)_{V-A} \sum_q e_q (\bar{q}_\beta q_\beta)_{V-A},\\
O_{10} &= \frac{3}{2}(\bar{q}'_\alpha b_\beta)_{V-A} \sum_q e_q (\bar{q}_\beta q_\alpha)_{V-A},
\end{aligned} \tag{25}$$

with α and β being the color indices. In Eq. (24), O_1-O_2 are from the tree level of weak interactions, O_3-O_6 are the so-called gluon penguin operators, and O_7-O_{10} are the electroweak penguin operators, while C_i ($i = 1, 2, \cdots, 10$) are the corresponding WCs. Using the unitarity condition, the CKM matrix elements for the penguin operators O_3-O_{10} can also be expressed as $V_u + V_c = -V_t$. To study the nonleptonic decays, we will encounter the transition matrix elements such as $\langle P_1 P_2 | H_{\rm eff} | B\rangle = \langle P_1 P_2 | V_q C_i O_i | B\rangle$. To describe the B decay amplitudes, we have to know not only the relevant effective weak interactions but also all possible topologies for the specific process. In Fig. 2, we display the flavor diagrams for $B_d \to \eta_{q(s)} K$ decays, in which 2(a)–2(c), 2(d) and 2(e), and 2(f) illustrate penguin emission, penguin annihilation, and tree emission topologies, respectively. Since the b-quark is dictated by the weak charged current, its chirality is always left handed. However, the chiralities for $q\bar{q}$ pairs, produced by gluon, Z-boson, and photon penguins, could be both left and right handed, resulting in processes containing both $V - A$ and $V + A$ currents. In Fig. 2, we have explicitly labeled the associated type of currents except the diagram 2(f) which is from the tree and only has the left-handed interaction. Note that although we use the states $\eta_{q,s}$ as our basis, the physical states can be easily obtained by using Eq. (2). For the charged B decays, besides the flavor diagrams displayed in Fig. 2, three more diagrams arising from tree emission and annihilation topologies need to be included as shown in Fig. 3. From Figs. 2 and 3, the decay amplitudes for $B^{0,+} \to \eta_q K^{(*)0,+}$ and $B \to \eta_s K^{(*)0,+}$ are given by

$$\begin{aligned} A_q^0 &= V_t(F_{Pa}^0 + N_{Pa}^0 + F_{Pc}^0 + N_{Pc}^0 + F_{Pd}^0 + N_{Pd}^0) \\ &\quad - V_u(F_{Tf}^0 + N_{Tf}^0), \\ A_s^0 &= V_t(F_{P(b+c)}^0 + N_{P(b+c)}^0 + F_{Pe}^0 + N_{Pe}^0) \end{aligned} \tag{26}$$

and

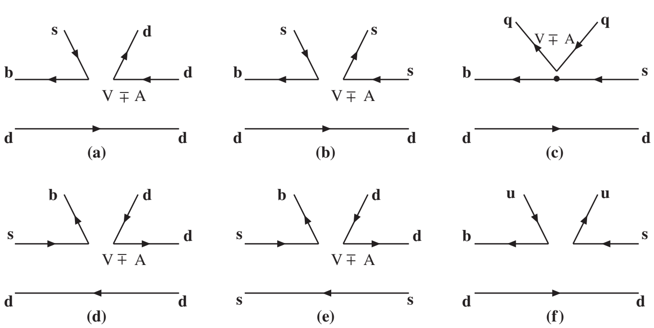

FIG. 2. Flavor diagrams for $B_d \to \eta^{(\prime)} K^0$ decays: (a)–(e) stand for the penguin contributions while (f) is the tree contribution, where $V \mp A$ denote the left-handed and right-handed currents, respectively.

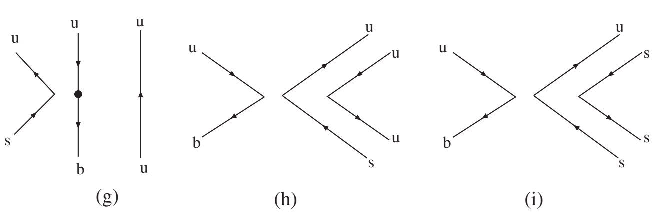

FIG. 3. Flavor diagrams arising from tree emission and annihilation for charged B decays.

$$\begin{aligned} A_q^+ &= V_t(F_{Pa}^+ + N_{Pa}^+ + F_{Pc}^+ + N_{Pc}^+ + F_{Pd}^+ + N_{Pd}^+) \\ &\quad - V_u(F_{T(f+g)}^+ + N_{T(f+g)}^+ + F_{Th}^+ + N_{Th}^+), \\ A_s^+ &= V_t(F_{P(b+c)}^+ + N_{P(b+c)}^+ + F_{Pe}^+ + N_{Pe}^+) \\ &\quad - V_u(F_{Ti}^+ + N_{Ti}^+), \end{aligned} \tag{27}$$

where $V_t = V_{tb}V_{ts}^* = -A\lambda^2$ and $V_u = V_{ub}V_{us}^* = A\lambda^4 R_b e^{-i\phi_3}$, $F_{Pk}^{0,+}$ and $N_{Pk}^{0,+}$ represent the penguin factorized and nonfactorized contributions for the topology k, and $F_{Tk}^{0,+}$ and $N_{Tk}^{0,+}$ are the tree factorized and nonfactorized effects, respectively. The lengthy formulas for various factorizable and nonfactorizable parts can be found in Refs. [26,30]. We note that for simplicity we have used the same notations for the $\eta_{q,s}K$ and $\eta_{q,s}K^*$ modes. Furthermore, from Eq. (2) the physical decays can be written as

$$\begin{aligned} A(B^{0,+} \to \eta K^{(*)}) &= \frac{\cos\phi}{\sqrt{2}} A_q^{0,+} - \frac{\sin\phi}{\sqrt{2}} A_s^{0,+}, \\ A(B^{0,+} \to \eta' K^{(*)}) &= \frac{\sin\phi}{\sqrt{2}} A_q^{0,+} + \frac{\cos\phi}{\sqrt{2}} A_s^{0,+}. \end{aligned} \tag{28}$$

The decay BRs and CP asymmetries (CPAs) are given by

$$\begin{aligned} &\mathrm{BR}(B^{0,+} \to \eta^{(\prime)} K^{[*]0,+}) \\ &\quad = \frac{G_F^2 |\vec{p}| m_B^2 \tau_{B^{0,+}}}{16\pi} |A(B^{0,+} \to \eta^{(\prime)} K^{[*]0,+})|^2, \end{aligned} \tag{29}$$

$$\begin{aligned} &A_{CP}(B \to \eta^{(\prime)} K^{[*]}) \\ &\quad = \frac{\mathrm{BR}(\bar{B} \to \eta^{(\prime)} \bar{K}^{[*]}) - \mathrm{BR}(B \to \eta^{(\prime)} K^{[*]})}{\mathrm{BR}(\bar{B} \to \eta^{(\prime)} \bar{K}^{[*]}) + \mathrm{BR}(B \to \eta^{(\prime)} K^{[*]})}, \end{aligned} \tag{30}$$

which can be evaluated in terms of Eqs. (26)–(28), where $|\vec{p}| = \sqrt{E_K^2 - m_K^2}$ and $E_K = (m_B^2 - m_{\eta^{(\prime)}}^2 + m_K^2)/2m_B$.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In the PQCD approach, if we regard the meson wave functions as known objects, the remaining unknown theoretical quantities are the chiral symmetry breaking parameters of states $\eta_{q,s}$ and K, denoted by $m^0_{\eta_q,\eta_s,K}$, and the meson decay constants $f_{B,\eta_{q,s},K}$. It is known that f_K has been determined quite precisely to be around 0.16 GeV by experiment, while the lattice QCD calculations give $f_B = 0.216 \pm 0.022$ GeV [31], which is consistent with the extracted value from the decay $B^- \to \tau\bar{\nu}_\tau$ measured by Belle [32]. By low-energy experiments, the decay constants of $\eta_{q,s}$ are found to be $f_{\eta_q} = (1.07 \pm 0.02) f_\pi$ and $f_{\eta_s} = (1.34 \pm 0.06) f_\pi$ [19], respectively. Basically, the undetermined parameters in our considerations are the parameters m_{qq} and m_{ss}. To obtain the allowed range for $m_{qq,ss}$ in a model-independent way, we adopt the phenomenological approach. The parameters in Eq. (6) are limited

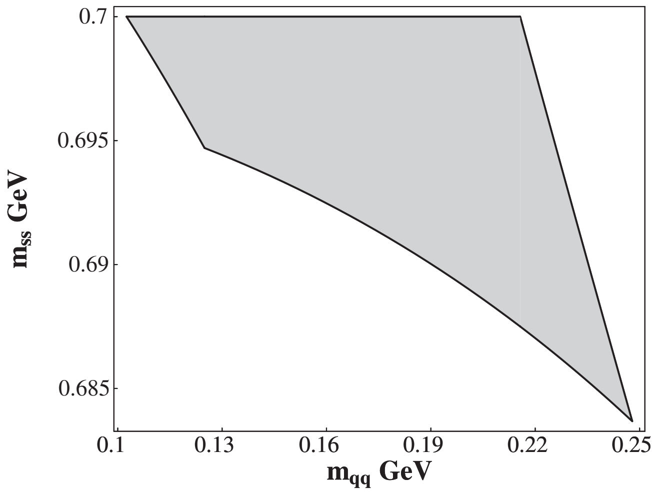

FIG. 4. The allowed ranges for m_{qq} and m_{ss}.

to be $\phi = 39.3° \pm 1.0°$, $y = 0.81 \pm 0.03$, and $a^2 = 0.265 \pm 0.010$ [19]. With these values, the allowed ranges for m_{qq} and m_{ss} are presented in Fig. 4. From the figure, we find that m_{ss} has a narrow allowed window around 0.69 GeV, which can be understood in terms of the flavor symmetry, given by $m_{ss} = \sqrt{2m_K^2 - m_\pi^2}$ [20]. However, m_{qq} is relatively broader, given by 0.18 ± 0.08 GeV. To do the numerical estimations, we take $f_B = 0.19$ GeV, $f_{\eta_q} = 0.14$ GeV, and $\phi = 39.3°$ as the input values. For the nonperturbative wave functions, we use the results derived by the LCSR for the light mesons [28], while for the B meson wave function, we use

$$\phi_B(x) = N_B x^2(1-x)^2 \exp\left[-\frac{m_B^2 x^2}{2\omega_B^2}\right]\exp\left[-\frac{\omega_B^2 b^2}{2}\right] \tag{31}$$

with $N_B = 111.2$ and $\omega_B = 0.38$ [26]. Accordingly, we get the $B \to K$ form factor of $f_+^K(0)$, defined in Eq. (12), to be 0.36. From Eq. (23), we show the form factors $f_{+,T}^{\eta^{(\prime)}}(0)$ in Table I. From the table, we see clearly that they will be enhanced with increasing m_{qq}. In addition, it is easy to understand that the behavior $f_{+,T}^{\eta}(0) > f_{+,T}^{\eta'}(0)$ is always satisfied as seen from Eq. (23) due to $\cos\phi > \sin\phi$ with $\phi \sim 39.3°$. This property is different from that in the FSM, given by [10]

$$\begin{aligned} f_i^{\eta}(0) &= \frac{\cos\phi}{\sqrt{2}}\frac{f_q}{f_\pi}f_i^{\pi}(0) + \frac{1}{\sqrt{3}}\left(\sqrt{2}\cos\phi\frac{f_q}{f_\pi} - \sin\phi\frac{f_s}{f_\pi}\right)f_i^{\rm sing}(0), \\ f_i^{\eta'}(0) &= \frac{\sin\phi}{\sqrt{2}}\frac{f_q}{f_\pi}f_i^{\pi}(0) + \frac{1}{\sqrt{3}}\left(\sqrt{2}\sin\phi\frac{f_q}{f_\pi} + \cos\phi\frac{f_s}{f_\pi}\right)f_i^{\rm sing}(0), \end{aligned} \tag{32}$$

TABLE I. $f_+^{\eta^{(\prime)}}(0)$ and $f_T^{\eta^{(\prime)}}(0)$ with three allowed values of m_{qq}.

m_{qq} (GeV)	$f_+^{\eta}(0)$	$f_T^{\eta}(0)$	$f_+^{\eta'}(0)$	$f_T^{\eta'}(0)$
0.14	0.14	0.14	0.12	0.11
0.18	0.21	0.20	0.18	0.17
0.22	0.29	0.29	0.24	0.24

where $f_i^{\rm sing}(0)(i = +, T)$ correspond to the new form factors due to the flavor-singlet state. Based on $f_+^{\pi}(0) \approx f_T^{\pi}(0) \approx 0.26$ calculated by the LCSRs [28], we present the numerical results of Eq. (32) in Table II. From the table, we see that $f_{+,T}^{\eta}(0) < f_{+,T}^{\eta'}(0)$ in the FSM. Furthermore, by using $|V_{ub}| = 3.5 \times 10^{-3}$, $|V_{td}| = 8.1 \times 10^{-3}$ [33], Eqs. (16), (17), and (22), and the values in Tables I and II, we show the semileptonic decay BRs in Table III. From the table, we find that the results in both approaches could be consistent with the data of $B^- \to \eta^{(\prime)}\ell\nu_\ell$. On the other hand, in our approach, we always predict $\mathrm{BR}(B^- \to \eta\ell^+\bar\nu_\ell) > \mathrm{BR}(B^- \to \eta'\ell^+\bar\nu_\ell)$, whereas the inequality is reversed in the FSM. A similar conclusion can be also drawn for the processes of $B_d \to \eta^{(\prime)}\ell^+\ell^-$. We note that the BRs are insensitive to the parametrizations displayed in Eq. (22) [12].

We now give our numerical analysis for the nonleptonic decays $B \to \eta^{(\prime)}K^{[*]}$. By using the PQCD approach, the values of factorized and nonfactorized contributions for the B decays are shown in Table IV. Based on these values and $V_{ts} = -0.041$ and $V_{ub} = 4.6 \times 10^{-3}e^{-i\phi_3}$ with $\phi_3 = 72°$, the predictions for $\mathrm{BR}(B \to \eta^{(\prime)}K^{[*]})$ and $A_{CP}(B \to \eta^{(\prime)}K^{[*]})$ are given in Tables V and VI, respectively. Our results can be summarized as follows:

(i) From Table V, we see clearly that with $m_{qq} = 0.22$ GeV, the BRs for $B \to \eta^{(\prime)}K^{[*]}$ are consistent with the world average (WA) data. It is interesting to note that by increasing m_{qq}, $\mathrm{BR}(B \to \eta^{(\prime)}K)$ tend to be small (large), while $\mathrm{BR}(B \to \eta^{(\prime)}K^*)$ tend to be large (small), favored by the experiments.

(ii) As seen from Table V, with the same value of m_{qq}, $\mathrm{BR}(B \to \eta K) < O(10^{-1})\mathrm{BR}(B \to \eta' K)$, while $\mathrm{BR}(B \to \eta K^*) > BR(B \to \eta' K^*)$. The phenomena could be ascribed to the signs in the amplitudes of $B \to (\eta_q, \eta_s)K^{(*)}$ by comparing Eqs. (26)–(28) with the specific values of $F_{Pa}^{0,+}$ and $F_{P(b+c)}^{0,+}$ in Table IV.

TABLE II. $f_{+,T}^{\eta^{(\prime)}}(0)$ with various values of $f_i^{\rm sing}(0)$ in the FSM.

$f_i^{\rm sing}(0)$	$f_{+,T}^{\eta}(0)$	$f_{+,T}^{\eta'}(0)$
0.0	0.15	0.13
0.1	0.20	0.25
0.2	0.25	0.38

TABLE III. BRs of $B^- \to \eta^{(\prime)} \ell \bar{\nu}_\ell$ (in units of 10^{-4}) and $\bar{B}_d \to \eta^{(\prime)} \ell^+ \ell^-$ (in units of 10^{-7}) with $m_{qq} = 0.14$, 0.18, and 0.22 GeV in our mechanism and $f_+^{\rm sing}(0) = 0.0$, 0.1, and 0.2 in the FSM. $\phi = 39.3°$.

m_{qq} (GeV)	$B^- \to \eta \ell \bar{\nu}_\ell$	$B^- \to \eta' \ell \bar{\nu}_\ell$	$\bar{B}_d \to \eta \ell^+ \ell^-$	$\bar{B}_d \to \eta' \ell^+ \ell^-$
0.14	0.30	0.15	0.02	0.01
0.18	0.67	0.35	0.04	0.02
0.22	1.27	0.62	0.07	0.04
$f_+^{\rm sing}(0)$	$B^- \to \eta \ell \bar{\nu}_\ell$	$B^- \to \eta' \ell \bar{\nu}_\ell$	$\bar{B}_d \to \eta \ell^+ \ell^-$	$\bar{B}_d \to \eta' \ell^+ \ell^-$
0.0	0.38	0.18	0.05	0.03
0.1	0.47	0.64	0.07	0.10
0.2	0.58	1.39	0.08	0.21
Exp.	$0.84 \pm 0.27 \pm 0.21 (<1.4)$	$0.33 \pm 0.60 \pm 0.30 (<1.3)$	—	—

TABLE IV. Factorizable and nonfactorizable parts for the decays $B \to \eta_{q,s} K^{[*]}$ with $m_{qq} = 0.22$ GeV, where the values in the square brackets are for $B \to \eta_{q,s} K^*$.

$F^0_{Pa}10^2$	$N^0_{Pa}10^5$	$F^0_{P(b+c)}10^2$	$N^0_{P(b+c)}10^4$	$F^0_{Pc}10^2$	$N^0_{Pc}10^4$
-1.10	$6.36 - i2.06$	-0.55	$-0.33 + i1.22$	0.26	$-7.17 + i3.39$
$[-0.42]$	$[3.89 - i2.37]$	$[0.45]$	$[-0.89 + i1.74]$	$[0.19]$	$[-7.88 + i2.56]$
$F^0_{Pd}10^3$	$N^0_{Pd}10^5$	$F^0_{Pe}10^3$	$N^0_{Pe}10^5$	$F^0_{Tf}10^2$	$N^0_{Tf}10^3$
$-0.61 + i2.43$	$-5.77 - i9.62$	$-0.44 + i1.25$	$-0.51 - i4.62$	-0.61	$3.61 - i1.57$
$[-0.19 + i2.37]$	$[-5.05 - i3.36]$	$[0.30 - i1.86]$	$[-5.02 - i9.06]$	$[-0.41]$	$[4.00 - i1.29]$
$F^+_{Pa}10^2$	$N^+_{Pa}10^5$	$F^+_{P(b+c)}10^2$	$N^+_{P(b+c)}10^4$	$F^+_{Pc}10^2$	$N^+_{Pc}10^4$
-1.05	$3.55 - i0.27$	-0.54	$-3.56 + i1.71$	0.21	$-6.24 + i1.70$
$[-0.43]$	$[-1.86 - i2.61]$	$[0.45]$	$[-0.89 + i1.74]$	$[0.188]$	$[-7.64 + i3.53]$
$F^+_{Pd}10^3$	$N^+_{Pd}10^5$	$F^+_{Pe}10^3$	$N^+_{Pe}10^5$	$F^+_{Tf}10^2$	$N^+_{Tf}10^3$
$-0.63 + i2.20$	$-2.86 - i4.46$	$-0.50 + i1.60$	$-1.59 - i2.98$	-0.45	$3.27 - i0.90$
$[-0.09 + i2.37]$	$[-2.21 - i0.72]$	$[0.29 - i1.83]$	$[-2.83 - 4.07]$	$[-0.41]$	$[3.85 - i1.74]$
$F^+_{Tg}10^2$	$N^+_{Tg}10^3$	$F^+_{Th}10^3$	$N^+_{Th}10^3$	$F^+_{Ti}10^3$	$N^+_{Ti}10^3$
10.03	$-1.16 + i0.18$	$2.38 + i0.02$	$0.99 + i1.38$	$-1.02 - i0.02$	$0.27 + i1.13$
$[11.60]$	$[-1.55 + i0.06]$	$[-2.19 - i1.14]$	$[1.09 + i0.70]$	$[1.61 + i0.95]$	$[0.89 + i1.55]$

(iii) From Table VI, we find that for $m_{qq} = 0.22$ GeV, $A_{CP}(B_u \to \eta K^+)$ is as large as -30%, which agrees well with the data, whereas the other two sets of m_{qq} lead to positive and small asymmetries. In addition, our prediction for $A_{CP}(B_d \to \eta K^{*0})$ is too small, while that of $A_{CP}(B_u \to \eta K^{*+})$ is too large, in comparison with the data. If future experiments display the current tendencies for these CPAs, such phenomena will become new puzzles.

Finally, we remark that in the quark-flavor scheme, as the errors in the decay constants of f_q and f_s are only 2%

TABLE V. $\text{BR}(B \to \eta^{(\prime)} K^{[*]})$ (in units of 10^{-6}) with $m_{qq} = 0.14$, 0.18, and 0.22 GeV as well as the WA values [34].

m_{qq}	$B_d \to \eta K^0$	$B_d \to \eta' K^0$	$B_u \to \eta K^+$	$B_u \to \eta' K^+$
0.14	3.01	31.44	5.66	34.60
0.18	0.28	44.04	1.26	47.36
0.22	1.43	62.69	1.52	65.04
WA	<1.9	64.9 ± 3.5	2.2 ± 0.3	$69.7^{+2.8}_{-2.7}$
m_{qq}	$B_d \to \eta K^{*0}$	$B_d \to \eta' K^{*0}$	$B_u \to \eta K^{*+}$	$B_u \to \eta' K^{*+}$
0.14	11.54	8.21	11.74	10.06
0.18	15.91	5.76	15.94	8.12
0.22	22.31	3.35	22.13	6.38
WA	16.1 ± 1.0	3.8 ± 1.2	$19.5^{+1.6}_{-1.5}$	$4.9^{+2.1}_{-1.9}$

TABLE VI. $A_{CP}(B \to \eta^{(\prime)} K^{[*]})$ (in units of 10^{-2}) with $m_{qq} = 0.14$, 0.18, and 0.22 GeV as well as the WA values [34].

m_{qq}	$B_d \to \eta K^0$	$B_d \to \eta' K^0$	$B_u \to \eta K^+$	$B_u \to \eta' K^+$
0.14	-2.10	0.69	5.62	-5.28
0.18	-2.47	0.57	5.88	-6.19
0.22	4.41	0.48	-30.64	-6.88
WA	—	—	-29 ± 11	3.1 ± 2.1
m_{qq}	$B_d \to \eta K^{*0}$	$B_d \to \eta' K^{*0}$	$B_u \to \eta K^{*+}$	$B_u \to \eta' K^{*+}$
0.14	0.79	-0.82	-15.79	8.39
0.18	0.67	-0.98	-20.51	8.83
0.22	0.57	-1.30	-24.57	4.60
WA	19 ± 5	-8 ± 25	2 ± 6	30^{+33}_{-37}

TABLE VII. BRs (in units of 10^{-6}) and CPAs (in units of 10^{-2}) for $B \to \eta^{(\prime)} K^{[*]}$ decays with $m_{qq} = 0.22$ GeV and $\phi = 39.3° \pm 1.0°$.

Obs.	$B_d \to \eta K^0$	$B_d \to \eta' K^0$	$B_u \to \eta K^+$	$B_u \to \eta' K^+$
BR	$1.43^{+0.34}_{-0.31}$	$62.69^{+0.30}_{-0.34}$	$1.52\pm^{+0.16}_{-0.13}$	$65.04^{+0.12}_{-0.15}$
A_{CP}	$4.41^{+0.57}_{-0.44}$	0.48 ± 0.009	$-30.64^{+4.12}_{-2.87}$	$-6.88^{+0.13}_{-0.12}$
Obs.	$B_d \to \eta K^{*0}$	$B_d \to \eta' K^{*0}$	$B_u \to \eta K^{*+}$	$B_u \to \eta' K^{*+}$
BR	$22.31^{+0.28}_{-0.29}$	$3.35^{+0.29}_{-0.27}$	$22.13^{+0.26}_{-0.27}$	6.38 ± 0.26
A_{CP}	0.57 ± 0.011	-1.30 ± 0.08	$-24.57^{+0.72}_{-0.27}$	$4.60^{+1.16}_{-1.32}$

and 4%, respectively, their effects on BRs and CPAs are mild. However, the influence from the mixing angle ϕ could be larger. We present the results with the error of ϕ in Table VII.

V. CONCLUSIONS

Because of the current experimental limits on the mixing parameters of the η and η' mesons, we have studied the phenomenologically allowed ranges for m_{ss} and m_{qq}. Explicitly, we have found that m_{ss} is around 0.69 GeV and $m_{qq} = 0.18 \pm 0.08$ GeV. We have shown that the semileptonic decays of $B^- \to \eta^{(\prime)} \ell \bar{\nu}_\ell$ are sensitive to m_{qq} and thus they can provide strong constraints on its value. In addition, our mechanism based on the quark-flavor mixing scheme naturally leads to $f_+^{\eta'}(0) < f_+^{\eta}(0)$ as well as $\mathrm{BR}(B^- \to \eta \ell^- \bar{\nu}_\ell) > \mathrm{BR}(B^- \to \eta' \ell^- \bar{\nu}_\ell)$, in contrast with the reversed inequalities in the FSM due to the flavor-singlet contribution [12,13]. Similar conclusions can also be drawn for the decays $B_d \to \eta^{(\prime)} \ell^+ \ell^-$. It is interesting to note that the future measurements on $\mathrm{BR}(B^- \to \eta^{(\prime)} \ell \bar{\nu}_\ell)$ and $\mathrm{BR}(B_d \to \eta^{(\prime)} \ell^+ \ell^-)$ can be used to distinguish the two flavor mechanisms. Moreover, we have shown that $\mathrm{BR}(B \to \eta^{(\prime)} X)$ with $X = (\ell^- \bar{\nu}_\ell, \ell^+ \ell^-)$ are enhanced and, in particular, the puzzle of the large $\mathrm{BR}(B \to \eta' K)$ can be solved with a reasonable large value of m_{qq}. We have also demonstrated that $A_{CP}(B^\pm \to \eta K^\pm)$ can be as large as -30% and $\mathrm{BR}(B \to \eta^{(\prime)} K^*)$ are consistent with the current data. Finally, we remark that our results for $A_{CP}(B \to \eta K^*)$ do not agree with the experimental values. According to our analysis, currently, they are the most incomprehensible phenomena. Other mechanisms as well as more precise measurements are needed for a complete description of all the above decays.

ACKNOWLEDGMENTS

The authors would like to thank Professor Hai-Yang Cheng and Professor Hsiang-Nan Li for useful discussions. This work is supported in part by the National Science Council of R.O.C. under Grant No. NSC-95-2112-M-006-013-MY2 and No. NSC-95-2112-M-007-059-MY3.

[1] T.E. Browder *et al.* (CLEO Collaboration), Phys. Rev. Lett. **81**, 1786 (1998).
[2] A. Ali and C. Greub, Phys. Rev. D **57**, 2996 (1998).
[3] K. Abe *et al.* (Belle Collaboration), hep-ex/0608033.
[4] J. Schümann *et al.* (Belle Collaboration), Phys. Rev. Lett. **97**, 061802 (2006).
[5] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **95**, 131803 (2005).
[6] B. Aubert *et al.* (*BABAR* Collaboration), Phys. Rev. Lett. **94**, 191802 (2005).
[7] I. Halperin and A. Zhitnitsky, Phys. Rev. D **56**, 7247 (1997); A.A. Petrov, Phys. Rev. D **58**, 054004 (1998); T. Feldmann and P. Kroll, Eur. Phys. J. C **5**, 327 (1998); T. Feldmann, P. Kroll, and B. Stech, Phys. Lett. B **449**, 339 (1999).
[8] D. Atwood and A. Soni, Phys. Lett. B **405**, 150 (1997); M.R. Ahmady *et al.*, Phys. Rev. D **58**, 014015 (1998); M.R. Ahmady and E. Kou, Phys. Rev. D **59**, 054014 (1999).
[9] M.Z. Yang and Y.D. Yang, Nucl. Phys. **B609**, 469 (2001).
[10] M. Beneke and M. Neubert, Nucl. Phys. **B651**, 225 (2003).
[11] B. Aubert *et al.* (*BABAR* Collaboration), hep-ex/0607066.
[12] C.H. Chen and C.Q. Geng, Phys. Lett. B **645**, 197 (2007).
[13] C.S. Kim, S. Oh, and C. Yu, Phys. Lett. B **590**, 223 (2004).
[14] A.R. Williamson and J. Zupan, Phys. Rev. D**74**, 014003 (2006); **74**, 039901(E) (2006).
[15] Y.Y. Charng, T. Kurimoto, and H.N. Li, Phys. Rev. D **74**, 074024 (2006).
[16] H. Leutwyler, Nucl. Phys. B, Proc. Suppl. **64**, 223 (1998).
[17] A. Ali *et al.*, Phys. Rev. D **58**, 094009 (1998).
[18] J. Schechter, A. Subbaraman, and H. Weigel, Phys. Rev. D **48**, 339 (1993).
[19] T. Feldmann, P. Kroll, and B. Stech, Phys. Rev. D **58**, 114006 (1998).
[20] T. Feldmann and P. Kroll, Phys. Scr. **T99**, 13 (2002).
[21] T. Kurimoto, H.N. Li, and A.I. Sanda, Phys. Rev. D **65**, 014007 (2001).
[22] G. Buchalla, A.J. Buras, and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).
[23] C.H. Chen and C.Q. Geng, Phys. Rev. D **66**, 094018 (2002).
[24] G.P. Lepage and S.J. Brodsky, Phys. Lett. B **87**, 359 (1979); Phys. Rev. D **22**, 2157 (1980); H.N. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992); G. Sterman, Phys. Lett. B **179**, 281 (1986); Nucl. Phys. **B281**, 310 (1987); S. Catani and L. Trentadue, Nucl. Phys. **B327**, 323 (1989); **B353**, 183 (1991).

[25] T. W. Yeh and H. N. Li, Phys. Rev. D **56**, 1615 (1997); H. N. Li, Phys. Rev. D **64**, 014019 (2001); **66**, 094010 (2002).
[26] C. H. Chen and H. N. Li, Phys. Rev. D **63**, 014003 (2000).
[27] C. H. Chen, hep-ph/0601019.
[28] P. Ball and R. Zwicky, Phys. Rev. D **71**, 014015 (2005).
[29] J. D. Bjorken, Nucl. Phys. B, Proc. Suppl. **11**, 325 (1989).
[30] E. Kou and A. I. Sanda, Phys. Lett. B **525**, 240 (2002).
[31] A. Gray *et al.* (HPQCD Collaboration), Phys. Rev. Lett. **95**, 212001 (2005).
[32] K. Ikado *et al.* (Belle Collaboration), Phys. Rev. Lett. **97**, 251802 (2006).
[33] W. M. Yao *et al.* (Particle Data Group), J. Phys. G **33**, 1 (2006).
[34] E. Barberio *et al.* (Heavy Flavor Averaging Group), hep-ex/0603003; online update at http://www.slac.stanford.edu/xorg/hfag.