PHYSICAL REVIEW D **75,** 045018 (2007)

Quark-lepton symmetry in five dimensions

A. Coulthurst,* K. L. McDonald,† and B. H. J. McKellar‡

School of Physics, Research Centre for High Energy Physics, The University of Melbourne, Victoria, 3010, Australia

(Received 20 November 2006; published 23 February 2007)

We construct a complete five dimensional quark-lepton symmetric model, with all fields propagating in the bulk. The extra dimension forms an $S^1/Z_2 \times Z_2'$ orbifold with the zero mode fermions corresponding to standard model quarks localized at one fixed point. Zero modes corresponding to left (right)-chiral leptons are localized at (near) the other fixed point. This localization pattern is motivated by the symmetries of the model. Shifting the right-handed neutrinos and charged leptons slightly from the fixed point provides a new mechanism for understanding the absence of relations of the type $m_e = m_u$ or $m_e = m_d$ in quark-lepton symmetric models. Flavor changing neutral currents resulting from Kaluza-Klein gluon exchange, which typically arise in the quark sector of split fermion models, are suppressed due to the localization of quarks at one point. The separation of quarks and leptons in the compact extra dimension also acts to suppress the proton decay rate. This permits the extra dimension to be much larger than that obtained in a previous construct, with the bound $1/R \gtrsim 30$ TeV obtained.

DOI: 10.1103/PhysRevD.75.045018 PACS numbers: 11.10.Kk, 11.15.Ex, 14.60.Pq, 14.60.St

I. INTRODUCTION

It is known that the evident differences between quarks and leptons may be no more than the low energy manifestation of a more symmetric underlying theory. In particular it has been shown that one may construct four dimensional models which are invariant under a discrete quark-lepton (QL) symmetry, whereby one interchanges all quarks and leptons in the Lagrangian [1–7]. This idea requires the introduction of leptonic color, an $SU(3)$ gauge group acting on a set of generalized leptons, and thus predicts new gauge bosons and fermions.

Recently the notion of a QL symmetry has been investigated in five dimensions [8]. In that work the scalar and gauge fields of the QL symmetric model were assumed to propagate in the bulk, while all quark and lepton fields were confined to a brane. The extra dimension was assumed to form an $S^1/Z_2 \times Z_2'$ orbifold, the construction of which reduced the five dimensional QL symmetric gauge group to one of its subgroups. The model required an order 10^{11} GeV cutoff to suppress the proton decay rate and the extra dimension was taken to be of order $1/R \sim 10^9$ GeV.

In the present work we construct a complete five dimensional QL symmetric model, with all fields assumed to propagate in a five dimensional spacetime. Placing fermions in the bulk provides new ways of addressing the problems encountered in the previous construct. Provided that the Standard Model (SM) quarks and leptons are localized at (or near) different ends of the compact extra dimension it may be as large as $1/R \sim 30$ TeV, permitting the observation of higher dimensional physics at future colliders.

A common problem encountered in QL symmetric models is the presence of tree level mass relations of the type $m_e = m_u$ or $m_e = m_d$. The higher dimensional framework provides a new way to understand the absence of such mass relations. The mass relations may be removed from the effective four dimensional theory if zero mode fermions corresponding to SM quarks and leptons have different profiles along the extra dimension. In our model this occurs as follows. All zero mode fermions corresponding to SM quarks are localized at one orbifold fixed point, with the varying widths of the fifth dimensional quark wave functions determining the degree of wave function overlap between left- and right-chiral SM quarks. This in turn determines the size of the effective four dimensional quark Yukawa couplings with the SM Higgs scalar. Leptons are localized at the opposite end of the extra dimension, with $SU_L(2)$ doublet leptons localized at the fixed point and right-chiral neutrinos and charged leptons shifted slightly into the bulk. Thus the fifth dimensional wave function overlaps in the lepton sector tend to be reduced relative to that of the quark sector, motivating the flavor differences observed between quarks and leptons. By shifting the right-chiral neutrinos further into the bulk than the right-chiral charged leptons, neutrino Dirac masses may be suppressed below the electroweak scale.

The symmetries of the model motivate the localization pattern of SM fermions outlined above. The localization of quarks at one point in the extra dimension allows one to evade the flavor changing neutral current bounds on the size of the extra dimension which arise generically in the quark sector of split fermion models [9]. These bounds may be quite severe, requiring $1/R$ to be as large as 5000 TeV^{-1} [10]. The flavor changing neutral current bounds which arise in the lepton sector are much weaker and permit the extra dimension to be relatively large.

We note that the concept of leptonic color has recently been generalized in [11] and studied within the context of unified theories in [12–17]. Split fermions have also been employed to remove quark-lepton mass relations in a different context in [18].

*Electronic address: a.coulthurst@physics.unimelb.edu.au
†Electronic address: k.mcdonald@physics.unimelb.edu.au
‡Electronic address: b.mckellar@physics.unimelb.edu.au

The layout of this paper is as follows. In Sec. II the gauge and scalar content of the model are discussed. Section III introduces fermions and we determine the orbifold parity assignments necessary to obtain a realistic zero mode fermion spectrum. The Yukawa sector of the model is introduced in Sec. IV while the issue of proton decay is addressed in Sec. V. Fermion masses are considered in Sec. VI, with the neutrino sector considered separately in Sec. VII. The size of the extra dimension is discussed in Sec. VIII while Sec. IX addresses the lifetime and cosmological bounds on the mass of the lightest SM singlet neutrino. A discussion of the expected scale for new physics and some experimental signatures is provided in Sec. X and the paper concludes in Sec. XI.

II. THE GAUGE AND SCALAR SECTORS

The QL symmetric gauge group is

$$\mathcal{G}_{ql} = SU_l(3) \otimes SU_c(3) \otimes SU_L(2) \otimes U_X(1), \quad (1)$$

where $SU_l(3)$ is the lepton color group and $SU_c(3)$ is the usual color group for quarks. The action of the group $SU_L(2)$ on the zero mode fermion spectrum will be identified with the usual weak group of the SM and $X \neq Y$, where Y is the SM hypercharge. Under the discrete QL symmetry the gauge fields transform as:

$$G_c^M \leftrightarrow G_l^M, \qquad W^M \leftrightarrow W^M, \qquad C^M \leftrightarrow -C^M, \quad (2)$$

where $G_{c,l}^M$ are the $SU_{c,l}(3)$ gauge bosons, W^M are the weak bosons and C^M is the $U_X(1)$ gauge boson. The five dimensional Lorentz index takes the values $M = \mu, 5$, where μ is the $3+1$ dimensional Lorentz index. The additional spatial dimension is taken as the orbifold $S^1/Z_2 \times Z_2'$, whose coordinate is labeled as y. The construction of the orbifold proceeds via the identification $y \to -y$ under Z_2 and $y' \to -y'$ under the Z_2' symmetry, where $y' = y + \pi R/2$. The physical region in y is given by the interval $[0, \pi R/2]$. The Lagrangian is required to be invariant under the discrete $Z_2 \times Z_2'$ symmetry, whose action in the space gauge fields is defined as follows:

$$\begin{aligned} W_\mu(x^\mu, y) &\to W_\mu(x^\mu, -y) = P W_\mu(x^\mu, y) P^{-1}, \\ W_5(x^\mu, y) &\to W_5(x^\mu, -y) = -P W_5(x^\mu, y) P^{-1}, \\ W_\mu(x^\mu, y') &\to W_\mu(x^\mu, -y') = P' W_\mu(x^\mu, y') P'^{-1}, \\ W_5(x^\mu, y') &\to W_5(x^\mu, -y') = -P' W_5(x^\mu, y') P'^{-1}, \end{aligned}$$

where W denotes a generic gauge field. We take P and P' to be trivial for the $SU_c(3)$, $SU_L(2)$ and $U_X(1)$ gauge bosons. For the $SU_l(3)$ gauge bosons we choose $P = \mathrm{diag}(1, 1, 1)$ and $P' = \mathrm{diag}(-1, 1, 1)$, which will reduce the leptonic color symmetry to $SU_l(2) \otimes U_{X'}(1)$ in the four dimensional effective theory. We write the five dimensional $SU_l(3)$ gauge bosons as

$$G_l = T_a G_l^a = \begin{pmatrix} -\frac{2}{\sqrt{3}} G^8 & \sqrt{2} Y^1 & \sqrt{2} Y^2 \\ \sqrt{2} Y^{1\dagger} & G^3 + \frac{1}{\sqrt{3}} G^8 & \sqrt{2} \tilde{G} \\ \sqrt{2} Y^{2\dagger} & \sqrt{2} \tilde{G}^\dagger & -G^3 + \frac{1}{\sqrt{3}} G^8 \end{pmatrix}, \quad (3)$$

and find their $Z_2 \times Z_2'$ parities to be

$$\begin{aligned} Y_\mu^1, Y_\mu^2, Y_\mu^{1\dagger}, Y_\mu^{2\dagger} &\to (+, -), \\ Y_5^1, Y_5^2, Y_5^{1\dagger}, Y_5^{2\dagger} &\to (-, +), \\ G_\mu^8, G_\mu^3, \tilde{G}_\mu, \tilde{G}_\mu^\dagger &\to (+, +), \\ G_5^8, G_5^3, \tilde{G}_5, \tilde{G}_5^\dagger &\to (-, -). \end{aligned} \quad (4)$$

One may expand the gauge bosons as a Fourier series in the compact extra dimension, with the $Z_2 \times Z_2'$ parities constraining the series as usual. One has

$$\begin{aligned} \psi_{(+,+)}(x^\mu, y) &= \frac{2}{\sqrt{\pi R}} \left(\frac{1}{\sqrt{2}} \psi_{(+,+)}^{(0)}(x^\mu) + \sum_{n=1}^{\infty} \psi_{(+,+)}^{(n)}(x^\mu) \times \cos\frac{2ny}{R} \right), \\ \psi_{(+,-)}(x^\mu, y) &= \frac{2}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi_{(+,-)}^{(n)}(x^\mu) \cos\frac{(2n-1)y}{R}, \\ \psi_{(-,+)}(x^\mu, y) &= \frac{2}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi_{(-,+)}^{(n)}(x^\mu) \sin\frac{(2n-1)y}{R}, \\ \psi_{(-,-)}(x^\mu, y) &= \frac{2}{\sqrt{\pi R}} \sum_{n=1}^{\infty} \psi_{(-,-)}^{(n)}(x^\mu) \sin\frac{2ny}{R}, \end{aligned} \quad (5)$$

where ψ represents a generic field. Thus the four dimensional charge 1/2 bosons Y_μ^1, Y_μ^2, $Y_\mu^{1\dagger}$ and $Y_\mu^{2\dagger}$ do not possess zero modes, with the nth mode possessing a mass of $(2n-1)/R$. The fields G_μ^8, G_μ^3, $\tilde{G}_\mu$ and $\tilde{G}_\mu^\dagger$ all have zero modes, with the higher modes possessing a mass of $2n/R$. After compactification the zero mode gauge group is

$$\mathcal{G}_{ql} \to SU_l(2) \otimes SU_c(3) \otimes SU_L(2) \otimes U_{X'}(1) \otimes U_X(1), \quad (6)$$

and the next stage of symmetry breaking requires

$$U_{X'}(1) \otimes U_X(1) \to U_Y(1), \quad (7)$$

which shall be achieved by the usual Higgs mechanism. The scalar content necessary to break $\mathcal{G}_{ql}$ is

$$\begin{aligned} &\chi \sim (3, 1, 1, 2/3), \\ &\chi' \sim (1, 3, 1, 2/3), \quad \text{and} \quad \phi \sim (1, 1, 2, 1), \end{aligned} \quad (8)$$

with the action of the discrete QL symmetry given by,

$$\phi \leftrightarrow \tilde{\phi}, \qquad \chi \leftrightarrow \chi', \quad (9)$$

where $\tilde{\phi} = \epsilon \phi^*$ and ϵ is the two dimensional antisymmetric tensor. The $Z_2 \times Z_2'$ parities of ϕ are trivial while for χ we have

$$\begin{aligned} \chi(x^\mu, y) &\to \chi(x^\mu, -y) = P\chi(x^\mu, y), \\ \chi(x^\mu, y') &\to \chi(x^\mu, -y') = -P'\chi(x^\mu, y'), \end{aligned} \tag{10}$$

with $P = \mathrm{diag}(1, 1, 1)$ and $P' = \mathrm{diag}(-1, 1, 1)$. For χ' we take

$$\begin{aligned} \chi'(x^\mu, y) &\to \chi'(x^\mu, -y) = P\chi'(x^\mu, y), \\ \chi'(x^\mu, y') &\to \chi'(x^\mu, -y') = P'\chi'(x^\mu, y'), \end{aligned} \tag{11}$$

with P and P' trivial. Under the symmetry reduction

$$SU_l(3) \to SU_l(2) \otimes U_{X'}(1), \tag{12}$$

one has

$$\chi \to \chi_2 \oplus \chi_1, \tag{13}$$

where $\chi_2 \sim (2, 1)$ and $\chi_1 \sim (1, -2)$ have the $Z_2 \times Z_2'$ parities

$$\chi_1 \to (+, +), \qquad \chi_2 \to (+, -). \tag{14}$$

The zero mode for χ_1 may develop a VEV to break the gauge symmetry as follows:

$$\begin{gathered} SU_l(2) \otimes SU_c(3) \otimes SU_L(2) \otimes U_{X'}(1) \otimes U_X(1) \\ \downarrow \\ SU_l(2) \otimes SU_c(3) \otimes SU_L(2) \otimes U_Y(1). \end{gathered} \tag{15}$$

At this stage the hypercharge generator may be identified as

$$Y = X + \frac{1}{\sqrt{3}} T_8, \tag{16}$$

where $T_8 = (1/\sqrt{3}) \times \mathrm{diag}(-2, 1, 1)$ is a diagonal generator of $SU_l(3)$. The final stage of symmetry breaking occurs when the neutral component of ϕ develops the VEV u to give

$$\begin{gathered} SU_l(2) \otimes SU_c(3) \otimes SU_L(2) \otimes U_Y(1) \\ \downarrow \\ SU_l(2) \otimes SU_c(3) \otimes U_Q(1). \end{gathered} \tag{17}$$

Assuming $g_S^2 \gg g_X^2, g_L^2$, where g_L $[g_X]$ is the $SU_L(2)$ $[U_X(1)]$ coupling constant and g_S denotes the common $SU_l(3)$ and $SU_c(3)$ coupling constant, one may write the neutral gauge boson mass eigenvalues as [8]

$$\begin{aligned} M_\gamma^{2(n)} &= \left(\frac{2n}{R}\right)^2, \\ M_Z^{2(n)} &\simeq \frac{1}{2}(g_X^2 + g_L^2)u^2 - \frac{g_X^2}{6g_S^2}u^2 + \left(\frac{2n}{R}\right)^2, \\ M_{Z'}^{2(n)} &\simeq \frac{2}{3} g_S^2 w^2 \left\{1 + \frac{g_X^2}{3g_S^2}\right\} + \left(\frac{2n}{R}\right)^2. \end{aligned} \tag{18}$$

Note that the zero modes possess the same masses as the neutral gauge bosons in the minimal four dimensional QL symmetric model [1,4]. These zero modes couple to fermions in exactly the same way as the neutral gauge bosons in the minimal QL symmetric model, making the phenomenology of these sates identical to that of the neutral gauge bosons studied in [4].

The zero modes consist of the massless photon, the Z boson with mass of order u, the electroweak scale, and an additional neutral boson Z' with mass of order w, the $U_{X'}(1) \otimes U_X(1)$ symmetry breaking scale. The phenomenological bound of $M_{Z'} > 720$ GeV obtained in [4] also applies to the zero mode Z' boson in the present model. Thus we obtain a lower bound on the $U_{X'}(1) \otimes U_X(1)$ symmetry breaking scale of $w \gtrsim 1$ TeV, which is low enough to permit observation of the Z' boson at the LHC.

The Kaluza-Klein (KK) tower of charged 1/2 bosons possess the mass

$$M_{Y^1}^{2(n)} = M_{Y^2}^{2(n)} = \frac{1}{2} g_S^2 w^2 + \left(\frac{2n-1}{R}\right)^2, \tag{19}$$

where the zero mode is absent. We shall discuss the bounds on R and the Y boson mass scales in Sec. VIII.

The mass of the W bosons are given by

$$M_W^{2(n)} = \frac{1}{2} g_L^2 u^2 + \left(\frac{2n}{R}\right)^2, \tag{20}$$

with the zero mode corresponding to the usual W boson.

III. FERMION PARITIES

Whilst the five dimensional theory is vectorlike and necessarily free from anomalies, we shall be introducing four dimensional chirality via the orbifold boundary conditions [19]. The effective four dimensional gauge theory is free from anomalies if all anomalies cancel amongst the zero mode fermions [20]. After compactification the gauge group is given in Eq. (6). To cancel anomalies involving $U_{X'}(1)$ factors one requires both the SM leptons and extra colored leptons to appear at the zero mode level. This may be achieved by doubling the fermion content of the QL symmetric model, in analogy with the doubling required in five dimensional left-right symmetric models [21,22]. We denote the fermions by

$$\begin{aligned} &Q_i \sim (1, 3, 2, 1/3), && L_i \sim (3, 1, 2, -1/3), \\ &U_i^c \sim (1, \bar{3}, 1, -4/3), && E_i^c \sim (\bar{3}, 1, 1, 4/3), \\ &D_i^c \sim (1, \bar{3}, 1, 2/3), && N_i^c \sim (\bar{3}, 1, 1, -2/3), \end{aligned} \tag{21}$$

where the quantum numbers under $\mathcal{G}_{ql}$ are shown and $i = 1, 2$ labels distinct fermion multiplets. The action of the QL symmetry on the fermions is

$$Q_i \leftrightarrow L_i, \qquad U_i^c \leftrightarrow E_i^c, \qquad D_i^c \leftrightarrow N_i^c. \tag{22}$$

The transformation rules for a fermion field under the $Z_2 \otimes Z_2'$ discrete symmetries are

$$\begin{aligned} F(x^\mu, y) &\to F(x^\mu, -y) = \pm\gamma_5 P F(x^\mu, y), \\ F(x^\mu, y') &\to F(x^\mu, -y') = \pm\gamma_5 P' F(x^\mu, y'), \end{aligned} \tag{23}$$

where F denotes a generic fermion field and the signs may be chosen independently for the distinct discrete symmetries. The matrices P and P' are identical to the ones used for the gauge sector in the last section, with the only nontrivial matrix being P' acting in leptonic color space. We demand that the lepton fields acquire the $Z_2 \otimes Z_2'$ quantum numbers:

$$L_1 = \begin{pmatrix} L^1_{1L}(+,+) \\ L^2_{1L}(+,-) \\ L^3_{1L}(+,-) \\ L^1_{1R}(-,-) \\ L^2_{1R}(-,+) \\ L^3_{1R}(-,+) \end{pmatrix}, \qquad L_2 = \begin{pmatrix} L^1_{2L}(+,-) \\ L^2_{2L}(+,+) \\ L^3_{2L}(+,+) \\ L^1_{2R}(-,+) \\ L^2_{2R}(-,-) \\ L^3_{2R}(-,-) \end{pmatrix},$$

$$N_1^c = \begin{pmatrix} n^{1c}_{1L}(+,+) \\ n^{2c}_{1L}(+,-) \\ n^{3c}_{1L}(+,-) \\ n^{1c}_{1R}(-,-) \\ n^{2c}_{1R}(-,+) \\ n^{3c}_{1R}(-,+) \end{pmatrix}, \qquad N_2^c = \begin{pmatrix} n^{1c}_{2L}(+,-) \\ n^{2c}_{2L}(+,+) \\ n^{3c}_{2L}(+,+) \\ n^{1c}_{2R}(-,+) \\ n^{2c}_{2R}(-,-) \\ n^{3c}_{1R}(-,-) \end{pmatrix},$$

$$E_1^c = \begin{pmatrix} e^{1c}_{1L}(+,+) \\ e^{2c}_{1L}(+,-) \\ e^{3c}_{1L}(+,-) \\ e^{1c}_{1R}(-,-) \\ e^{2c}_{1R}(-,+) \\ e^{3c}_{1R}(-,+) \end{pmatrix}, \qquad E_2^c = \begin{pmatrix} e^{1c}_{2L}(+,-) \\ e^{2c}_{2L}(+,+) \\ e^{3c}_{2L}(+,+) \\ e^{1c}_{2R}(-,+) \\ e^{2c}_{2R}(-,-) \\ e^{3c}_{2R}(-,-) \end{pmatrix}.$$

Here the numerical superscripts 1, 2, 3 label the different lepton colors and the numerical subscripts 1, 2 label the different five dimensional fields. As $SU_c(3)$ is not broken by the orbifold compactification the three quark colors all possess the same $Z_2 \otimes Z_2'$ parities. Suppressing the quark color index, we enforce the following orbifold parities for quarks:

$$Q_1 = \begin{pmatrix} Q_{1L}(+,+) \\ Q_{1R}(-,-) \end{pmatrix}, \qquad Q_2 = \begin{pmatrix} Q_{2L}(+,-) \\ Q_{2R}(-,+) \end{pmatrix},$$
$$U_1^c = \begin{pmatrix} u^c_{1L}(+,+) \\ u^c_{1R}(-,-) \end{pmatrix}, \qquad U_2^c = \begin{pmatrix} u^c_{2L}(+,-) \\ u^c_{2R}(-,+) \end{pmatrix},$$
$$D_1^c = \begin{pmatrix} d^c_{1L}(+,+) \\ d^c_{1R}(-,-) \end{pmatrix}, \qquad D_2^c = \begin{pmatrix} d^c_{2L}(+,-) \\ d^c_{2R}(-,+) \end{pmatrix}.$$

Only fermion fields with the orbifold parities $(+,+)$ appear at the zero mode level. We identify the zero modes of the fields L^1_{1L}, e^{1c}_{1L}, Q_{1L}, u^c_{1L} and d^c_{1L} with the SM fields L, e^c_R, Q_L, u^c_R and d^c_R respectively. The zero modes of the fields $L^{2,3}_{2L}$, $e^{2,3c}_{2L}$ and $n^{2,3c}_{2L}$ are the usual exotic leptons found in QL symmetric models (known as liptons in the literature [4]). The liptons form doublets under the remnant leptonic color symmetry $SU_l(2) \subset SU_l(3)$ and are confined into two particle bound states. These states form one of the key signatures of QL symmetric models, leading to an exotic spectrum of particles which may decay into SM fields by creation of electroweak gauge bosons [8,11]. We shall see below that the zero mode liptons acquire mass at the symmetry breaking scale w and may be observed at the LHC. Note that the zero mode liptons appear in different multiplets to the SM leptons. Consequently the lightest liptons will not couple directly to SM leptons via Y-boson exchange. This is one of the major phenomenological differences between the current model and that constructed in [8]. We explore the implications of this difference below.

The zero mode of n^{1c}_{1L} is an additional neutrino which is sterile under the electroweak gauge group. This state will play the role of the usual SM singlet neutrino and we shall label it as ν^c_R. We shall discuss the mass of this state in Sec. VII and issues relating to its cosmological evolution in Sec. IX.

IV. YUKAWA COUPLINGS

The Yukawa Lagrangian must remain invariant under both gauge and orbifold parity transformations. It is convenient to split the five dimensional Yukawa Lagrangian into two portions, with the nonelectroweak portion given by

$$\begin{aligned} \mathcal{L}_{\text{non-ew}} = \frac{1}{\sqrt{\Lambda}}\{ & h_1[L_1^T C_5^{-1} L_1 \chi + Q_1^T C_5^{-1} Q_1 \chi'] \\ & + h_2[L_2^T C_5^{-1} L_2 \chi + Q_2^T C_5^{-1} Q_2 \chi'] \\ & + h_1'[N_1^{cT} C_5^{-1} E_1^c \chi^\dagger + D_1^{cT} C_5^{-1} U_1^c \chi'^\dagger] \\ & + h_2'[N_2^{cT} C_5^{-1} E_2^c \chi^\dagger + D_2^{cT} C_5^{-1} U_2^c \chi'^\dagger] \\ & + \text{H.c.}\}. \end{aligned} \tag{24}$$

Here the h's are Yukawa coupling matrices in flavor space, $C_5 = \gamma^0\gamma^2\gamma^5$ is the five dimensional charge conjugation matrix and Λ is the cutoff. Contraction over $SU_{c,l}(3)$ color indices with the three dimensional antisymmetric tensor $\epsilon^{\alpha\beta\gamma}$ is implied in (24). When the zero mode of χ develops a VEV $\langle\chi^{(0)}\rangle = w$ the h_2 and h_2' terms in (24) generate an order w mass for the liptons. All SM fields remain massless at this stage of symmetry breaking.

The electroweak portion of the Yukawa Lagrangian is

$$\begin{aligned} \mathcal{L}_{\text{ew}} = \frac{1}{\sqrt{\Lambda}}\{ & \lambda_1[E_1^c C_5^{-1} L_1 \phi^\dagger + U_1^c C_5^{-1} Q_1 \tilde{\phi}^\dagger] \\ & + \lambda_2[E_2^c C_5^{-1} L_2 \phi^\dagger + U_2^c C_5^{-1} Q_2 \tilde{\phi}^\dagger] \\ & + \lambda_1'[N_1^c C_5^{-1} L_1 \tilde{\phi}^\dagger + D_1^c C_5^{-1} Q_1 \phi^\dagger] \\ & + \lambda_2'[N_2^c C_5^{-1} L_2 \tilde{\phi}^\dagger + D_2^c C_5^{-1} Q_2 \phi^\dagger] + \text{H.c.}\}, \end{aligned} \tag{25}$$

where the λ's are flavor space Yukawa coupling matrices. This Lagrangian generates fermion Dirac mass terms when

$\phi^{(0)}$ develops a VEV, with the SM fermions acquiring mass through the λ_1, λ_1' terms. If the SM fermions have uniform profiles along the extra dimension, then the troublesome tree level mass relations $m_e = m_u$ and $m_\nu = m_d$ arise. However, if one is able to generate different five dimensional wave function profiles for quarks and leptons, the troublesome tree level relations will not persist in the effective four dimensional theory. We shall have more to say on this matter in Sec. VI, but it will prove useful to consider the stability of the proton within our model first.

V. PROTON DECAY

At the five dimensional level, the lowest dimension nonrenormalizable operator which leads to proton decay in our model is

$$\frac{f}{\Lambda^{9/2}}\epsilon_{\alpha\beta\gamma}Q_2^\alpha Q_2^\beta Q_2^\gamma \chi_{\bar{\alpha}}^\dagger L_1^{\bar{\alpha}}. \tag{26}$$

where α, β, γ are quark color indices, $\bar{\alpha}$ is the lepton color index and f is a dimensionless coupling constant. If the SM quarks and leptons have uniform profiles across the extra dimension, proton decay occurs in the effective four dimensional theory via the operator

$$\frac{f}{(\Lambda\pi R)^{3/2}}\frac{w}{\Lambda^3}\epsilon_{\alpha\beta\gamma}Q^\alpha Q^\beta Q^\gamma L, \tag{27}$$

where Q and L denote four dimensional quark and lepton fields, respectively. Current experimental bounds require the lifetime of the proton to be in excess of 1.6×10^{33} years [23]. With $w\sim 1$ TeV and $\Lambda R\sim 100$ one requires $\Lambda\sim 5\times 10^{10}$ GeV to suppress the proton decay rate.

It is possible to reduce the scale of the cutoff by localizing quarks and leptons at different points in the extra dimension [24]. If quarks and leptons are separated in the extra dimension, the diminished overlap of the quark and lepton wave functions in the fifth dimension serves to reduce the proton decay rate. To suppress the proton decay rate we shall localize quarks and leptons at different fixed points. To illustrate our idea it will suffice to consider an S^1/Z_2 orbifold. Let us add a gauge singlet, bulk scalar field Σ which transforms as

$$\Sigma(x^\mu, y)\to\Sigma(x^\mu,-y) = -\Sigma(x^\mu, y), \tag{28}$$

under the Z_2 symmetry and has the potential

$$V(\Sigma) = \frac{\kappa}{4\Lambda}(\Sigma^2 - v'^2)^2, \tag{29}$$

where κ (v') is a dimensionless (dimensionful) constant. The minimum of the potential clashes with the orbifold boundary conditions (28) and results in the vacuum profile [19] (see also [22]),

$$\langle\Sigma\rangle(y)\approx\frac{v}{\sqrt{2\pi R}}\tanh[\xi(-\pi R - y)]\tanh[\xi y], \tag{30}$$

where $\xi^2 = \kappa v^2/2$ and we have introduced $v = \sqrt{2\pi R}v'$. The points $y = \pm\pi R$ are identified under the orbifold construction. One can simplify the analysis when $\kappa v^2(2\pi R)^2 \gg 1$ by treating the VEV profile of Σ as a step function [25],

$$\langle\Sigma\rangle(y) = \frac{v}{\sqrt{2\pi R}}h(y), \tag{31}$$

where,

$$h(y) = \begin{cases} +1 & \pi R > y > 0 \\ -1 & -\pi R < y < 0. \end{cases} \tag{32}$$

In five dimensions, gauge invariance permits the Yukawa couplings

$$-\frac{h_F}{\sqrt{\Lambda}}\bar{F}F\Sigma, \tag{33}$$

for all fermion fields, where h_F is a Yukawa coupling constant. The shape of zero mode fermions in the extra dimension, when $h_F v > 0$, is subsequently given by [25]

$$F_L^{(0)} = \sqrt{\frac{2|h_F v|}{1 - e^{-2|h_F v|\pi R}}}e^{-|h_F v|y}, \tag{34}$$

or via the replacement $y\to(\pi R - y)$ if $h_F v < 0$, demonstrating the localization of zero mode fermions at different fixed points on the S^1/Z_2 orbifold, depending on the sign of the coupling constant h_F.

We wish to suppress the proton decay rate by localizing quarks and leptons at different fixed points. Let us take Σ to be odd under the discrete QL symmetry [26]. The resulting Yukawa Lagrangian is,

$$\mathcal{L}_\Sigma = \frac{1}{\sqrt{\Lambda}}\sum_{i=1,2}\{h_{D_i}[D_i^{c2} - N_i^{c2}] + h_{U_i}[U_i^{c2} - E_i^{c2}] + h_{Q_i}[Q_i^2 - L_i^2]\}\Sigma, \tag{35}$$

where h_{U_i}, h_{D_i} and h_{Q_i} are Yukawa coupling constants. We use an obvious notation with $F^2 = \bar{F}F$ and we have suppressed family indices. If one takes all Yukawa couplings to be greater than zero, quarks and leptons are automatically localized at different fixed points.

After integrating over the extra dimension, the operator (26) produces a proton decay inducing operator in the four dimensional theory. The approximate form of this operator is

$$\frac{f'}{\sqrt{\Lambda R}}\frac{wv}{\Lambda^2}\exp\{-cv\pi R\}\frac{Q^3L}{\Lambda^2}, \tag{36}$$

where f' and c are dimensionless constants and Q and L are four dimensional quark and lepton fields, respectively. For Λ of order 100–500 TeV and an order TeV QL symmetry breaking scale, the lower bound on the lifetime of the proton requires $v\pi R > 40$ if one takes c to be order unity.

VI. FERMION MASS

We have seen that one is able to understand the long lifetime of the proton if quarks and leptons are localized at different fixed points. In Sec. IV we noted that despite the Yukawa coupling relations induced by the QL symmetry in Eq. (25), one may understand the absence of relations of the type $m_e = m_u$ in the effective four dimensional theory if quarks and leptons have different wave function profiles in the extra dimension.

Inspection of Eqs. (34) and (35) reveals that the methods employed to separate quarks and leptons in the extra dimension induce identical fifth dimensional wave function profiles for fermions related by the QL symmetry. Upon integrating over the fifth dimension the troublesome tree level mass relationships persist, despite quarks and leptons being localized at different fixed points. We could in principle add a second SM Higgs doublet to the model and generate enough parameter freedom to remove the unwanted mass relations [1]. However this will not allow us to understand the lightness of the known neutrinos relative to the electroweak scale.

In this section we shall apply a purely higher dimensional mechanism to remove the undesirable tree level mass relations. The full construction of a theory of flavor is beyond the scope of the present work. We shall sketch our ideas in what follows and to this end it is helpful to recall some key results obtained in previous studies involving split fermions.

The work of Arkani-Hamed and Schmaltz (AS) [24] demonstrated that four dimensional flavor could be addressed in terms of fifth dimensional wave function overlaps. AS spatially separated the left- and right-chiral fermion fields in an extra dimension, with the size of the spatial separation influencing the degree of wave function overlap. The amount of overlap then determined the size of the effective four dimensional Yukawa couplings to the SM Higgs. However one need not separate the left- and right-chiral fields to address flavor in terms of fifth dimensional wave function overlaps. Indeed one can localize all the fermions at one point in the extra dimension, provided the left- and right-chiral fermions have fifth dimensional wave functions with different widths [9].

We shall employ each of these mechanisms to realize flavor. In particular we shall localize all quark fields at one point in the extra dimension, with the different widths of the left- and right-chiral quark fields determining the effective four dimensional Yukawa couplings in the quark sector. For the leptons we shall separate the left- and right-chiral fields in the extra dimension, thus motivating the observed flavor differences between the quark and lepton sectors.

AS separated fermion fields by introducing distinct bulk Dirac mass terms for the different bulk fermions. In an orbifold theory, however, the fermion transformations necessary to introduce four dimensional chirality preclude bulk Dirac mass terms. One may localize fermions at nonfixed points in an orbifold theory by introducing a second localizing scalar [27]. This works as follows [28]. With one localizing scalar the chiral zero mode of a fermion F is localized at one of the orbifold fixed points. The point of localization is determined by the sign of the product $h_F v$ as mentioned already in Sec. V, which amounts to the sign of h_F when $v > 0$. If the field F couples to a second bulk scalar with an opposite sign Yukawa coupling, the second scalar tends to localize the zero mode at the opposite fixed point. When one scalar is dominant the fermion is found localized at the fixed point preferred by that scalar. In general, however, an interplay between the two scalars results in a compromise which sees the fermion localized in the bulk. More technical details may be found in [27].

Let us now investigate this idea in the QL symmetric framework, assuming an S^1/Z_2 orbifold again for simplicity. We introduce a second bulk scalar, σ, which transforms as

$$\sigma(x^\mu, y) \to \sigma(x^\mu, -y) = -\sigma(x^\mu, y), \tag{37}$$

under the orbifold Z_2 symmetry and transforms trivially under the QL symmetry. The Yukawa Lagrangian for σ is,

$$\mathcal{L}_\Sigma = \frac{1}{\sqrt{\Lambda}} \sum_{i=1,2} \{ f_{D_i}[D_i^{c2} + N_i^{c2}] + f_{U_i}[U_i^{c2} + E_i^{c2}] + f_{Q_i}[Q_i^2 + L_i^2]\}\sigma. \tag{38}$$

We achieved quark-lepton separation in the last section by demanding

$$h_{U_i}, h_{D_i}, h_{Q_i} > 0. \tag{39}$$

Let us further demand that

$$f_{U_i}, f_{D_i}, f_{Q_i} > 0, \tag{40}$$

so that all quark Yukawa couplings to Σ and σ are positive.

Consider first the effects of the second scalar σ on the quark sector. Inspection of (35) and (38) reveals that both Σ and σ will attempt to localize the quarks at the same fixed point. Thus for all positive values of the Yukawa couplings, which we generically denote as h_F and f_F, the quark fields will be localized at one fixed point. Note that the SM fields Q, u_R and d_R will each have, in general, different profiles in the extra dimension, allowing one to reproduce the necessary quark flavor spectrum along the lines of [9].

Let us now consider the lepton sector. This sector is more complicated because Σ and σ attempt to localize a given chiral zero mode lepton at different fixed points. We have shown in the last section that the long lifetime of the proton may be understood if quarks and leptons are localized at different fixed points. To retain this result we shall require Σ to dominate over σ for the SM fields L and e_R. The situation with ν_R is not as clear. We shall discuss neutrino masses further in the next section, but for now let us estimate the degree of separation required between the field L and the field e_R to obtain an order MeV electron mass. It was shown in [27] that, in the decoupling limit of the two localizing scalars, one may arrange zero mode fermions to be localized in the bulk of an orbifold theory with fifth dimensional profiles of the form

$$G^{(0)}(y) = N \exp\left\{-\frac{k}{2}\frac{v^2}{\Lambda}(y-y_m)^2\right\}. \tag{41}$$

Here N is a normalization factor, v denotes the VEV of one of the localizing scalars and y_m is the location of the wave function maxima. We show in Sec. X that we expect $v \geq 420$ TeV and use this lower bound in what follows. To leading order the dimensionless constant k depends on the largest fermion-bulk scalar Yukawa coupling and the dimensionless scalar quartic self-coupling $k \approx h_F\sqrt{2\kappa}$. Note that naive dimensional analysis permits values of $k \sim 24\pi^3\sqrt{2} \sim 10^3$ if the underlying theory has strong couplings at the cutoff [25,29]. However we shall be able to obtain acceptable suppression of the electroweak scale with significantly lower k values [25].

The charged lepton mass terms arise from the couplings

$$\frac{\lambda_1}{\sqrt{\Lambda}} E_1^c C_5^{-1} L_1 \phi^\dagger, \tag{42}$$

which lead to Dirac masses of the form

$$\frac{\lambda_1 u}{\sqrt{\Lambda \pi R}} \int_0^{\pi R} e_R^{(0)}(y) L^{(0)}(y) dy, \tag{43}$$

where $e_R^{(0)}(y)$ is the electron wave function in the extra dimension which takes the form of $G^{(0)}(y)$ in Eq. (41). The localization of the zero mode of the field L is assumed to be dominated by the scalar Σ such that $L^{(0)}(y)$ has the exponential form of Eq. (34).

Taking λ, $k \sim 1$ in (43) reveals that a charged lepton Dirac mass of order MeV is obtained if e_R is localized a distance of $\pi R/15$ from the fixed point at which L is localized. Thus one is not required to shift the zero modes corresponding to the SM right-chiral charged leptons far to obtain a realistic charged lepton spectrum. This slight shift from the fixed point should not produce an observable enhancement of the proton decay rate.

VII. NEUTRINO MASSES

Neutrinos acquire Dirac masses at the electroweak symmetry breaking scale via the coupling

$$\frac{\lambda_1'}{\sqrt{\Lambda}} N_1^c C_5^{-1} L_1 \tilde{\phi}^\dagger. \tag{44}$$

We wish to suppress the electroweak contribution to the neutrino mass by separating ν_R and ν_L in the extra dimension. Given that neutrino masses are required to be less than an eV it is clear that one must localize the SM neutrinos away from the quarks to suppress the proton decay mode $p \to \nu_L \pi$. We shall shift ν_R into the bulk to suppress the electroweak contribution to the neutrino mass, however the acceptable proximity of ν_R to the quarks is not as clear. We must consider the mass scale of this field to determine its acceptable proximity to the quarks, and this requires a consideration of the nonrenormalizable contributions to the neutrino mass sector.

Consider the nonrenormalizable operators which induce Majorana mass for the neutrinos in our model. The operator

$$\mathcal{O}_{\nu_L} = \frac{g}{\Lambda^5}(\chi^\dagger L_1 \tilde{\phi}^\dagger)^2, \tag{45}$$

written in terms of five dimensional fields, with g a dimensionless constant, has dimension ten and in the effective four dimensional theory induces the operator

$$\mathcal{O}_{\nu_L}^{\rm eff} = \frac{g}{(\Lambda\pi R)^2}\frac{(\chi^{(0)\dagger} L^{(0)} \tilde{\phi}^{\dagger(0)})^2}{\Lambda^3}, \tag{46}$$

which has dimension seven. When $\chi^{(0)}$ and $\phi^{(0)}$ develop VEV's, $\mathcal{O}_{\nu_L}^{\rm eff}$ produces Majorana masses for the SM neutrinos ν_L. These masses take the form

$$m_{\nu_L} = \frac{g}{(\Lambda\pi R)^2}\frac{(wu)^2}{\Lambda^3}. \tag{47}$$

We require $m_{\nu_L} \sim g \times 1$ eV so that this mass is in the right range to accommodate the atmospheric and solar neutrino oscillation data. The neutrinos ν_R also acquire mass at the nonrenormalizable level. At the five dimensional level, one has the operator

$$\mathcal{O}_{\nu_R} = \frac{g'}{\Lambda^2}(\chi N_1^c)^2, \tag{48}$$

where g' is a dimensionless coupling constant. This produces a Majorana mass for ν_R in the effective four dimensional theory,

$$m_{\nu_R} = \frac{g'}{(\Lambda\pi R)}\frac{w^2}{\Lambda}. \tag{49}$$

We intend to reduce the effective Dirac mass between ν_L and ν_R in the four dimensional theory below the electroweak scale by separating L and ν_R in the extra dimension. To estimate the degree of suppression required of the effective Dirac mass we shall take m_{ν_R} to be O (GeV) (we shall see in Sec. IX that masses in this range are compatible with our framework). Consider the complete neutrino mass matrix in the Majorana basis for one generation,

$$\begin{pmatrix} m_{\nu_L} & m_D \\ m_D & m_{\nu_R} \end{pmatrix}, \tag{50}$$

where m_D is the effective four dimensional Dirac mass. Under the usual see-saw hierarchy

$$m_{\nu_L} \ll m_D \ll m_{\nu_R}, \tag{51}$$

the mass eigenvalues take the approximate form

$$m_1 \approx m_{\nu_L} - m_D^2/m_{\nu_R}, \tag{52}$$

$$m_2 \approx m_{\nu_R}. \tag{53}$$

The seesaw contribution to the light neutrino mass eigenvalue, m_D^2/m_{ν_R}, is required to be order eV or less to ensure that m_1 is not too heavy. Hence one requires $m_D \lesssim 1$ keV for $m_{\nu_R} \sim 1$ GeV to ensure that $m_D^2/m_{\nu_R} \lesssim 10^{-1}$ eV. This requires ν_R to be localized a distance $\gtrsim \pi R/12$ from the boundary at which L is localized. We show in Sec. IX that m_{ν_R} is required to be at least of order GeV. Thus proton decays into ν_R are kinematically forbidden and localizing ν_R further into the bulk does not effect the stability of the proton, a result which holds even if ν_R is localized at the "quark end" of the extra dimension.

VIII. THE SIZE OF THE EXTRA DIMENSION

We now consider the bounds on R, the compactification scale. In Sec. II it was shown that the mass of the lightest Y boson has a contribution inversely proportional to R (the usual KK contribution). In 4D [4] and 5D [8] QL models explored previously, bounds on the Y boson mass have been obtained by considering the decay $\mu \to 3e$, which proceeds radiatively by the creation of virtual Y bosons. This results in an approximate bound of $M_Y \geq 5$ TeV. In [8] this led to a bound on R, however the current model requires a reevaluation of this bound.

We note from Sec. III that the liptons which form $SU_l(3)$ multiplets with SM leptons do not possess zero modes. Vertices in the effective 4D Lagrangian which couple SM leptons to liptons via Y boson exchange will generally take the form

$$\mathcal{L} \sim K_{lL'} Y l L', \tag{54}$$

where Y, L' and l denote Y boson, lipton and SM lepton operators, respectively, and $K_{lL'}$ is a numerical factor representing the wave function overlap of Y, l and L' in the extra dimension. The lightest lipton that couples to a SM lepton l via Y exchange is one of the $n = 1$ KK liptons in the same $SU_l(3)$ multiplet as l. However the bulk scalars employed to localize chiral zero mode fermions in Secs. V and VI also alter the mass and 5D profiles of $n > 0$ KK modes. When one localizing scalar is employed it has been shown that the low lying KK modes (with $n \neq 0$) also become localized, albeit with broader 5D wave functions [30]. Furthermore the odd and even KK modes tend to be localized at opposite fixed points on an S^1/Z_2 model. Given that we localize SM leptons at one boundary of the extra dimension, the $n = 1$ KK liptons will be found at the other boundary. Thus $K_{lL'} \sim \exp\{-\upsilon \pi R\}$ for $n = 1$ liptons, rendering the associated vertex vanishingly small. Consequently the radiative decay $\mu \to 3e$ will occur only via the coupling of SM charged leptons to even n KK mode liptons, with the dominant contribution arising from the $n = 2$ mode.

It is difficult to evaluate $K_{lL'}$ exactly for an $n = 2$ lipton L' in our model, given the two localizing scalars employed. This would require a determination of the fermion 5D wave function profiles for a two localizing scalar scenario, which is beyond the scope of the present study. To approximate $K_{lL'}$ we use the one bulk scalar results of [30] where analytic expressions for the KK tower 5D wave functions were obtained. We find that the overlap is very small for a range of parameter values, with $K_{lL'} \sim 10^{-3}$ for a zero mode lepton l and an $n = 2$ lipton L'. Assuming similar values for our model we find that the decay $\mu \to 3e$ is highly suppressed due to the factor of $K_{lL'}^4$ in the rate. The associated bound on $M_Y^{(1)}$, and thus R, is exceptionally weak.

A stronger bound on R may be obtained by considering flavor changing neutral currents (FCNCs). It is a generic feature of models involving split fermions that FCNCs arise [10]. Zero mode gauge bosons possess uniform profiles in the extra dimension and thus couple uniformly to localized fermions. However the $n > 0$ KK mode gauge bosons are not uniform along the extra dimension and thus, in general, couple to different localized fermions with different strengths, resulting in FCNCs.

The most stringent bounds from FCNCs in split fermion models arise in the quark sector, where exchange of KK mode gluons can lead to bounds of $1/R > 5000$ TeV [10]. We have localized all quarks at one fixed point, aiming to realize quark flavor through varying width 5D quark wave functions. This significantly reduces the bounds from FCNCs in the quark sector to $1/R \gtrsim 2$–5 TeV [9]. In the lepton sector we have localized all left-chiral SM leptons at one fixed point, while the SM right-chiral leptons are localized near the fixed point, with different points of localization for different fields. Thus only gauge bosons which couple to right-chiral fields, namely, the hypercharge and $U_{X'}(1) \subset SU_l(3)$ gauge bosons, will have markedly different couplings to different leptons and thus give rise to FCNCs in the lepton sector.

By considering the contribution of the KK tower for the electrically neutral $SU_L(2)$ gauge boson to $\mu \to 3e$ a bound of $1/R \gtrsim 30$ TeV has previously been obtained [10]. In our model the KK tower for the Z' gauge bosons will contribute to $\mu \to 3e$ and we expect this bound to apply, even though KK $SU_L(2)$ gauge bosons will not mediate this decay. Thus we require $1/R \gtrsim 30$ TeV to avoid trouble with FCNCs in the model. We note that if all right-chiral SM leptons were localized at one point the lower bound of $1/R \gtrsim 2$–5 TeV would apply, though this would have to be arbitrarily imposed on the model.

IX. COSMOLOGY OF THE STERILE NEUTRINO

We have seen in Sec. VI that the neutrinos will acquire Majorana masses at the nonrenormalizable level. The cosmological density of the neutrino ν_R must be considered to ensure that this state does not spoil any of the successful features of the standard cosmological model. The neutrinos ν_R will remain in thermal equilibrium in the early universe.

This results from annihilation's involving Z' bosons, namely:

$$\nu_R \bar{\nu}_R \leftrightarrow Z' \leftrightarrow l\bar{l}, \tag{55}$$

where l denotes a SM lepton. At some temperature the state ν_R will freeze out and the remaining cosmological abundance will contribute to the energy density of the universe. The perturbations to the usual scenario induced by ν_R depend on its mass and lifetime. It is known from 4D models that right-chiral neutrinos may decay via photon emission or via $SU_l(2) \subset SU_l(3)$ glueball emission.

Before we enter into the specifics, let us summarize the findings of this section for readers not interested in the more technical details. Below we show that in our 5D model the neutrino decay $\nu_R \to \nu_L \gamma$ is expected to dominate the decays involving glueball emission. Known cosmological bounds between the mass of heavy neutrinos and the lifetime for radiative decays then require $m_{\nu_R} \geq 100$ GeV. This result is obtained under the assumption that the neutrino Dirac mass m_D is suppressed to keV energies by localizing ν_R in the bulk. However the lightest ν_R effectively becomes stable if m_D is suppressed by localizing ν_R at the quark end of the extra dimension. In this case the usual bound for stable massive neutrinos applies and one requires $m_{\nu_R} \gtrsim 5$ GeV. Let us now consider these decay mechanisms in more detail.

A. Radiative heavy neutrino decays

Consider the decay $\nu_R \to \nu_L \gamma$, where ν_L denotes a SM neutrino. In QL models this occurs radiatively via the creation of a Y boson and a lipton (see Fig. 1). The rate for these decays can be obtained by modifying existing results for similar neutrino decays. In particular Ref. [31] studied radiative neutrino decays of the type $\nu_\mu \to \nu_e \gamma$ when exotic leptons and gauge bosons exist. Equation (2) of that work identifies the rate for $\nu_\mu \to \nu_e \gamma$ to be

$$\Gamma(\nu_\mu \to \gamma \nu_e) = \frac{G_F^2 \alpha B^2 M_L^2 M_{\nu_\mu}^3}{128 \pi^4}, \tag{56}$$

where G_F is the Fermi constant, α is the fine structure constant, B is a numerical vertex factor and M_L (M_{ν_μ}) is the mass of an exotic lepton (muon neutrino). This expression may be applied to the decay $\nu_R \to \gamma \nu_L$ in the 5D QL model by making the following replacements

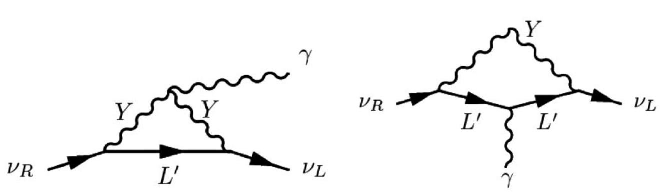

FIG. 1. The graphs for $\nu_R \to \nu_L + \gamma$ in quark-lepton symmetric models.

$$\alpha \to \tfrac{1}{4}\alpha, \tag{57}$$

$$M_{\nu_\mu} \to m_{\nu_R}, \tag{58}$$

$$B^2 G_F^2 \to K_1^2 K_2^2 G_Y^2 \sin^2\theta (\sqrt{2})^4. \tag{59}$$

Here (57) is required as the Y bosons and liptons carry electric charge $1/2$. The factor of $\sqrt{2}$ is due to the enhanced coupling of the $n > 0$ KK mode Y boson and G_Y is the equivalent of the Fermi constant for the Y bosons,

$$\left(\frac{G_Y}{\sqrt{2}}\right)^2 = \left(\frac{g_S^2}{8 M_Y^{2(1)}}\right)^2. \tag{60}$$

The angle $\sin\theta$ represents the mixing between a lipton which couples to ν_R and a lipton which couples to ν_L. This mixing is induced by electroweak symmetry breaking and vanishes in the limit $u \to 0$. The angle takes the approximate form

$$\sin\theta \approx \frac{K_3 u}{M_L}, \tag{61}$$

where K_3 is the 5D wave function overlap of the ν_R lipton with the ν_L lipton. We shall comment further on K_3 shortly. The low lying $n > 0$ KK mode liptons will have mass of order v, the bulk scalar symmetry breaking scale [30]. Note that the interplay of the M_L^2 factor in the numerator of (56) and the M_L^{-1} dependence of θ renders the rate for $\nu_R \to \gamma \nu_L$ independent of v. We expect the $n = 2$ KK lipton to be the lightest state to contribute significantly to the decay $\nu_R \to \gamma \nu_L$. The $n = 1$ state is not localized at the same fixed point as the $n = 0$ chiral modes [30] and thus its contribution to the decay will be suppressed by the K_3 factor.

The factors K_1 and K_2 in Eq. (59) represent the wave function overlap between $n = 0$ chiral mode neutrinos, the $n = 2$ lipton which couples to this neutrino and the $n = 1$ Y gauge boson. They take the form

$$\begin{aligned} K_1 &\sim \int_0^{\pi R/2} \nu_R^{(0)}(y) L'^{(2)}(y) \cos\frac{y}{R} dy, \\ K_2 &\sim \int_0^{\pi R/2} L^{(0)}(y) L'^{(2)}(y) \cos\frac{y}{R} dy, \end{aligned} \tag{62}$$

where $\nu_R^{(0)}(y)$ and $L^{(0)}(y)$ are zero mode SM singlet neutrino and electroweak lepton doublet 5D wave functions, respectively. $L'^{(2)}$ represents an $n = 2$ lipton 5D wave function and the Y boson fifth dimensional profile is given by the cosine factor. The absence of analytic expressions for the 5D fermion KK tower profiles in two bulk scalar models and the dependence on free parameters makes it difficult to determine these factors exactly. We approximate these using the results obtained for the one localizing scalar case in [30] and find that typical values are $K_{1,2} \sim 10^{-3}$.

To estimate K_3 we consider the case where ν_R and ν_L are localized by one bulk scalar, but the potential trapping ν_R in the bulk has a minimum shifted away from the fixed point. Noting that ν_R is shifted some distance away from the fixed point such that the effective Dirac mass coupling ν_R and ν_L in the 4D theory is m_D and using the analytic $n = 2$ KK mode wave functions of [30] we find that typically $K_3 u \sim 10^2 m_D$. The Dirac mass coupling the $n = 2$ liptons is expected to be larger than the associated $n = 0$ mode Dirac mass m_D as the 5D wave functions for the higher KK modes generally have broader wave functions in the extra dimension than the zero modes.

Putting all this together we find that

$$\tau(\nu_R \to \gamma\nu_L) = 5 \times 10^{-11}\ (\mathrm{s\,GeV}) \frac{M_Y^{4(1)}}{m_D^2 m_{\nu_R}^3}. \qquad (63)$$

Taking $m_D \sim 1$ keV and $1/R \sim 30$ TeV gives $\tau \sim 10^{22}$ s ($\tau \sim 10^{19}$ s) for $m_{\nu_R} = 10^2$ MeV (1 GeV). Such long lived neutrinos would still be decaying and would contribute to the diffuse photon background. Studies of the diffuse photon background require heavy neutrinos to satisfy the bound

$$\tau \geq (10^{25}\ \mathrm{s\,GeV}^2) \times m^{-2} \qquad (64)$$

if their lifetime exceeds $\tau \gtrsim 3 \times 10^{17}$ s [32]. Thus one requires $\tau \geq 10^{27}$ s (10^{25} s) for $m_{\nu_R} = 10^2$ MeV (1 GeV), which is not satisfied by the radiative ν_R decays. Note that increasing m_{ν_R} decreases the radiative lifetime of ν_R. For $m_{\nu_R} \geq 10$ GeV one finds that $\tau \leq 10^{17}$ s. If the lifetime of a heavy neutrino lies in the range $t_{\mathrm{rec}} \leq \tau \leq t_U$, where t_{rec} (t_U) is the time of recombination (age of the universe), a different relationship between the neutrinos mass and lifetime must be satisfied. This relationship takes the form [32]

$$m_{\nu_R} \gtrsim 8 \times 10^{-3} \tau_{\mathrm{sec}}^{1/3}\ \mathrm{GeV}. \qquad (65)$$

Using (63) gives

$$m_{\nu_R} \gtrsim (8 \times 10^{-3})^{1/2} \left(\frac{5 \times 10^{-11} M_Y^{4(1)}}{m_D^2} \right)^{1/6}, \qquad (66)$$

where all masses should be given in GeV. Using the lower bound for $1/R$ and $m_D = 1$ keV (100 keV) gives $m_{\nu_R} \gtrsim 170$ GeV (40 GeV) so that an order 10 GeV ν_R is disallowed. For masses $m_{\nu_R} \geq 10^2$ GeV the bound (66) may be satisfied and thus consideration of the radiative decay of ν_R into light neutrinos in a cosmological context leads to the bound $m_{\nu_R} \geq 10^2$ GeV. As mentioned already, the neutrino ν_R may also decay via emission of an $SU_l(2) \subset SU_l(3)$ glueball in QL symmetric models. Before concluding that cosmological considerations demand $m_{\nu_R} \geq 10^2$ GeV we must further investigate this decay mode.

B. Glueball mediated heavy neutrino decays

The interaction eigenstate right-chiral neutrino is an $SU_l(2)$ singlet and as such does not couple directly to the $SU_l(2)$ gluons (henceforth referred to as "stickons," adopting the notation of [13]). However the physical mass eigenstate ν_R does couple to the stickons. Recall that $SU_l(2)$ is predicted to be an unbroken symmetry and is thus expected to be confining. Stickon exchange leads to bound state fermions formed by the liptons and the Y bosons. Interestingly these bound state fermions possess the same quantum numbers as the SM leptons [33]. Consequently they mix with the known leptons and the resulting physical leptons contain small admixtures of states which couple to the stickons.

Of importance to us is the mixing of ν_R with the SM singlet bound state fermions (henceforth bound state neutrinos). It has been shown that the neutrino-bound state neutrino mixing leads to heavy neutrino decays of the type $\nu_R \to G\nu_L$, where G is an $SU_l(2)$ glueball (henceforth a stickball). The stickballs have a mass set by the $SU_l(2)$ strong interaction scale $\Lambda_{SU_l(2)}$, which has been estimated in [33] to be of order 10 MeV. The lightest stickball is expected to be a Lorentz scalar G_s and the neutrino ν_R may decay by emitting a real stickball if $m_{\nu_R} > \Lambda_{SU_l(2)}$. If $m_{\nu_R} < \Lambda_{SU_l(2)}$ the stickball must be virtual and the decay mode is $\nu_R \to 3\nu_L$.

The mixing between ν_R and the bound state neutrinos was quantified in [33] in terms of two mixing angles $\theta_{1,2}$. It was found that a maximal value of $\sin^2\theta_{1,2} \sim 10^{-6}$ is expected for 4D QL models. The rate for $\nu_R \to G_s \nu_L$ then depends on $\sin^4\theta_{1,2}$.

In the 5D theory additional factors arise. Again, because Y boson exchange only couples ν_R to $n > 0$ KK mode liptons a factor of K_1^2 must be included. Also the study of [33] assumed Dirac neutrinos. We have Majorana neutrinos and the decay rate for stickball emission will depend on the mixing between ν_L and ν_R^c, which takes the standard seesaw form of $\sin\theta_s \sim m_D/m_{\nu_R}$ in our model. Putting this together we find that the replacement

$$\sin^2\theta_{1,2} \to K_1^2 \sin^2\theta_{1,2} \left(\frac{m_D}{m_{\nu_R}} \right), \qquad (67)$$

is required to utilize the results of the 4D study. In [33] it was found that the lifetime for a 17 keV neutrino to decay by stickball emission could be $\leq 10^5$ s. Employing the replacement (67) one finds that the lifetime for stickball emission is $\leq 10^{25}$ s for a heavy neutrino with mass $m_{\nu_R} = 10^2$ MeV in the 5D model. The lifetime is inversely proportional to m_{ν_R} so that larger mass values decrease the lifetime. However the dominant decay mode will be $\nu_R \to \nu_L \gamma$ for $m_{\nu_R} > 10^2$ MeV, so that the bound of $m_{\nu_R} \geq 10^2$ GeV still applies.

Interestingly both the stickball and photon decay modes for ν_R depend on the Dirac mass coupling ν_L and ν_R. For

$m_D \to 0$ these decays do not occur and ν_R becomes stable. In this limit it is important to ensure that the density of ν_R particles which remain at the freeze-out temperature of $T_* \sim m_{\nu_R}/15$ does not overclose the universe. This leads to the well-known Lee-Weinberg bound of $m_{\nu_R} \gtrsim 5$ GeV for a massive Majorana neutrino. This is much weaker than the bound obtained when considering the radiative decay. In the present model the limit $m_D \to 0$ corresponds to the limit in which ν_R is localized at the opposite boundary to ν_L. We note that this limit does not lead to rapid proton decays of the type $p \to \nu_R \pi$ as kinematic considerations of any neutrino which satisfies the Lee-Weinberg bound will preclude proton decay via this channel. Given that this is the lowest bound on the mass of ν_R we will consider this scenario in what remains. This setup has the added advantage of allowing for an improved understanding of the hierarchy between quark and lepton masses in a QL symmetric framework, a matter which is currently under investigation [34].

X. BOUNDS ON THE REMAINING SCALES AND EXPERIMENTAL SIGNATURES

Having obtained the lower bound on the mass of ν_R in the preceding section we may now consider the bounds on the remaining scales in the theory. The QL symmetry breaking scale must satisfy the bound $w \gtrsim 1$ TeV, due to leptonic annihilations involving the Z' bosons. If we take $1/R = 30$ TeV in accordance with the lower bound obtained by considering KK contributions to FCNCs, then the lower bound on m_{ν_R} and the upper bound of 1 eV on m_{ν_L} translate into lower bounds on w and Λ. Using (47) and (49) gives:

$$w = \left(\frac{u^4 m_{\nu_R}^5 \pi R}{m_{\nu_L}^2}\right)^{1/6}, \qquad \Lambda = \left(\frac{m_{\nu_R} u^2}{m_{\nu_L} \pi R}\right)^{1/3},$$

so that the lower bound on m_{ν_R} translates into the bounds

$$w \gtrsim 5 \text{ TeV}, \qquad \Lambda \gtrsim 460 \text{ TeV},$$

for $1/R = 30$ TeV. Furthermore the demand that the proton does not decay too rapidly translates into a bound on v, which we find to be $v \gtrsim 420$ TeV. This enables us to summarize the mass scales for the exotic fields in our construction and some of the associated phenomenology.

With $1/R = 30$ TeV and $m_{\nu_R} = O$ (GeV) in accordance with the lower bounds, we find the zero mode Z' boson mass to be $M_{Z'}^{(0)} = 5$ TeV. The heaviest liptons acquire an order w mass and all liptons will appear at energies $\lesssim w$. These liptons do not couple directly to the known leptons via Y exchange. They appear in different $SU_l(3)$ multiplets and the $Z_2 \times Z_2'$ parities preclude a direct coupling of the type YL_1L_2. This is a major distinction between this model and previous QL symmetric constructs.

Assuming order one Yukawa couplings the lightest liptons will possess masses less than w. The lower bound of $w \gtrsim 5$ TeV permits these liptons to appear at TeV energies and thus these states may be observed at the LHC. Although they do not couple directly to leptons via Y boson exchange they will couple with the known fermions through electroweak interactions and through Z' exchange. Thus the $n = 0$ liptons may appear at the LHC via interactions of the type $p\bar{p} \to \gamma, Z \to L\bar{L}$ etc. A key signature for this construct would be the appearance of liptons at the LHC. One could then discriminate this model from other QL symmetric models by studying the coupling of the liptons to leptons at the proposed ILC. The liptons of this model do not couple to leptons via Y boson exchange and thus only electroweak and Z' interactions would couple e^+e^- pairs to the liptons.

We remind the reader that the unbroken $SU_l(2)$ symmetry confines liptons into two particle bound states. Having the lightest liptons in different $SU_l(3)$ multiplets to the SM leptons does not alter the stability of the lightest lipton bound states. These decay via the electromagnetic or weak interactions. As the electroweak bosons have uniform profiles in the fifth dimension their couplings to the liptons are the same as the 4D case. Thus the lifetimes of the lightest confined liptons are the same as the 4D case and these bound states present no cosmological concern.

The liptons which do couple to the SM leptons are $n > 0$ members of KK towers. These states possess mass of order $v \sim 420$ TeV. Their couplings to SM leptons via Y exchange are highly suppressed due to the localization methods employed in this work, thus the associated phenomenology (like the rare decay $\mu \to 3e$) is also suppressed.

Whilst new physics may appear at TeV energies in this model, it is not until energies of order 30 TeV that the higher dimensional nature of the theory reveals itself. At these scales the KK excitations for the neutral gauge bosons Z and Z', the photon, the gluons the W bosons appear. The Y bosons also appear at this scale, though at these energies they will only manifest themselves in precision experiments through couplings to the other gauge bosons.

XI. CONCLUDING REMARKS

We have constructed a complete five dimensional QL symmetric model. This model differs from a previous five dimensional QL symmetric model [8] in that all fermions are assumed to propagate in the bulk. Placing fermions in the bulk provides the following advantages over the earlier framework. Namely:

(i) The longevity of the proton is readily understood by localizing quarks and leptons at (or near) different fixed points in the extra dimension.

(ii) The extra dimension may be as large as $R = 1/(30 \text{ TeV})$ allowing the phenomenology associated with the KK towers of the gauge sector to be observed at future colliders.

(iii) The higher dimensional framework allows one to understand the absence of mass relations of the type $m_e = m_u$ or $m_e = m_d$ in a QL symmetric framework due to the different profiles of quark and lepton wave functions in the extra dimension.

(iv) The five dimensional model permits a purely higher dimensional mechanism whereby one may suppress the neutrino mass scale relative to the electroweak scale by spatially separating left- and right-chiral neutrino fields.

The bounds on the mass of ν_R depend on its localization point. We find that $m_{\nu_R} \geq 100$ GeV is required if ν_R is localized near ν_L, while ν_R can be of order GeV if it is localized at the "quark end" of the extra dimension.

The model as it stands has some features which are introduced in a somewhat arbitrary fashion and it would be pleasing to uncover deeper reasons for their implementation. In particular it would be satisfying to understand why the five dimensional fermion L_1 couples more strongly to the bulk scalar Σ than do N^c_{R1} and E^c_{R1}. It would also be pleasing to discover a connection between the fermions that undergo $SU(3)$ symmetry breaking and the fermions which couple with opposite sign Yukawa coupling constants to the bulk scalars Σ and σ. In four dimensional QL symmetric models one starts with two sets of fermions which are indistinguishable at high energies. At low energies leptons are defined as those fermions which experience $SU(3)$ symmetry breaking. In our five dimensional model there are two independent features which distinguish quarks from leptons. Leptons are defined to be those fermions which experience $SU(3)$ symmetry breaking *and* couple to σ and Σ with opposite signs. It would be pleasing to develop a mechanism which ensures that the fermions which undergo $SU(3)$ symmetry breaking *must* also couple to the two bulk scalars with opposite signs.

We note also that some interesting steps towards understanding flavor in a five dimensional left-right symmetric model have been made in [35]. An intriguing direction for further study would be to attempt to combine the methods employed in [35] with those of the present study.

ACKNOWLEDGMENTS

The authors thank Ray Volkas, Alison Demaria and Damien George for discussions and A. Perez-Lorenzana for a helpful correspondence. This work was supported in part by the Australian Research Council.

[1] R. Foot and H. Lew, Phys. Rev. D **41**, 3502 (1990).
[2] R. Foot and H. Lew, Phys. Rev. D **42**, 945 (1990).
[3] R. Foot and H. Lew, Mod. Phys. Lett. A **5**, 1345 (1990).
[4] R. Foot, H. Lew, and R. R. Volkas, Phys. Rev. D **44**, 1531 (1991).
[5] Y. Levin and R. R. Volkas, Phys. Rev. D **48**, 5342 (1993).
[6] D. S. Shaw and R. R. Volkas, Phys. Rev. D **51**, 6490 (1995).
[7] R. Foot and R. R. Volkas, Phys. Lett. B **358**, 318 (1995).
[8] K. L. McDonald and B. H. J. McKellar, Phys. Rev. D **74**, 056005 (2006).
[9] B. Lillie, J. High Energy Phys. 12 (2003) 030.
[10] A. Delgado, A. Pomarol, and M. Quiros, J. High Energy Phys. 01 (2000) 030.
[11] R. Foot and R. R. Volkas, Phys. Lett. B **645**, 345 (2007).
[12] G. C. Joshi and R. R. Volkas, Phys. Rev. D **45**, 1711 (1992).
[13] K. S. Babu, E. Ma, and S. Willenbrock, Phys. Rev. D **69**, 051301 (2004).
[14] S. L. Chen and E. Ma, Mod. Phys. Lett. A **19**, 1267 (2004).
[15] A. Demaria, C. I. Low, and R. R. Volkas, Phys. Rev. D **72**, 075007 (2005).
[16] A. Demaria, C. I. Low, and R. R. Volkas, Phys. Rev. D **74**, 033005 (2006).
[17] A. Demaria and K. L. McDonald, hep-ph/0610346.
[18] P. Q. Hung, Nucl. Phys. **B720**, 89 (2005).
[19] H. Georgi, A. K. Grant, and G. Hailu, Phys. Rev. D **63**, 064027 (2001).
[20] N. Arkani-Hamed, A. G. Cohen, and H. Georgi, Phys. Lett. B **516**, 395 (2001).
[21] R. N. Mohapatra and A. Perez-Lorenzana, Phys. Rev. D **66**, 035005 (2002).
[22] G. Perez, Phys. Rev. D **67**, 013009 (2003).
[23] P. Nath and P. F. Perez, hep-ph/0601023.
[24] N. Arkani-Hamed and M. Schmaltz, Phys. Rev. D **61**, 033005 (2000).
[25] D. E. Kaplan and T. M. P. Tait, J. High Energy Phys. 11 (2001) 051.
[26] A. Coulthurst, K. L. McDonald, and B. H. J. McKellar, Phys. Rev. D **74**, 127701 (2006).
[27] Y. Grossman and G. Perez, Phys. Rev. D **67**, 015011 (2003).
[28] Y. Grossman and G. Perez, Pramana **62**, 733 (2004).
[29] Z. Chacko, M. A. Luty, and E. Ponton, J. High Energy Phys. 07 (2000) 036.
[30] P. Q. Hung and N. K. Tran, Phys. Rev. D **69**, 064003 (2004).
[31] T. Goldman and G. J. Stephenson, Phys. Rev. D **16**, 2256 (1977).
[32] E. W. Kolb and M. S. Turner, Front. Phys. **69**, 1 (1990).
[33] R. Foot, H. Lew, and R. R. Volkas, Int. J. Mod. Phys. A **8**, 983 (1993).
[34] A. Coulthurst and K. L. McDonald (unpublished).
[35] P. Q. Hung, Phys. Rev. D **67**, 095011 (2003).