Modifying gravity with the aether: An alternative to dark matter

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There is evidence that Newton and Einstein's theories of gravity cannot explain the dynamics of a universe made up solely of baryons and radiation. To be able to understand the properties of galaxies, clusters of galaxies and the universe on the whole it has become commonplace to invoke the presence of dark matter. An alternative approach is to modify the gravitational field equations to accommodate observations. We propose a new class of gravitational theories in which we add a new degree of freedom, the Aether, in the form of a vector field that is coupled covariantly, but nonminimally, with the space-time metric. We explore the Newtonian and non-Newtonian limits, discuss the conditions for these theories to be consistent and explore their effect on cosmology.

I. INTRODUCTION

Despite the tremendous successes of contemporary cosmology, there is a nagging problem that refuses to go away. If we try to measure the total gravitational field in the universe it far surpasses what we would expect from the baryonic mass we can see. This is true on a wide range of scales. On Kiloparsec scales it is well known that the velocity of objects in the outer reaches of galaxies are moving around the central core at much greater speeds than what one expect from Keplerian motion due to the stars and gas. On Megaparsec scales it has been established that the random motion of galaxies in clusters is too large for these systems to remain gravitationally bound due simply to the visible mass. And on tens to hundreds of Megaparsecs there is evidence for structure in the distribution of galaxies which should in principle have been erased by dissipational damping at recombination, when the universe was a few hundred thousand years old.

There is a solution to this problem. One can invoke the existence of an exotic form of matter that does not couple to light. It is cold and clumps easily to form bound structures. The dark matter [1] will enhance the energy density of galaxies and clusters and can be modeled to fit almost all observations. It will also sustain gravitational potential wells through recombination and reinforce structure on large scales. A cosmological theory based on the existence of dark matter has emerged over the past 20 years with remarkable successes and predictive power [2,3]. Laboratory searches are under way to find tangible evidence for dark matter candidates which go beyond their gravitational effects.

One can take a different point of view. At the moment, all evidence for dark matter comes from its dynamical effect on visible objects. We see dark matter through its gravitational field. Could it be that our understanding of the gravitational field is lacking? This possibility has been mooted before. It has been proposed that the Newton-Poisson equation, $\nabla^2\Phi = 4\pi G\rho$ (where Φ is the gravita-

tional potential, ρ is the energy density and G is Newton's constant) should be modified to $\nabla \cdot [f(|\nabla\Phi|/a_0)\nabla\Phi] = 4\pi G\rho$ where $f(x) = 1$ in the strong field regime and $f(x) \simeq x$ in the weak-field regime. In regions of low acceleration, gravity is boosted above the standard Newtonian prediction and an f can be chosen to fit galactic rotation curves [4]. Such a theory, dubbed Modified Newtonian Dynamics (MOND) has proven very effective and it has recently has been proposed that such a behavior can emerge from the low-energy, nonrelativistic limit of a fully covariant theory (see [5,6] for various approaches).

MOND is not without problems. It has been shown that it is less effective at resolving the missing mass problem on the scale of clusters of galaxies. Indeed it has been shown by Sanders [7] that to correctly account for the mismatch between luminous and dynamical mass in clusters one must invoke a small fraction of massive neutrinos, with a mass of approximately 2 eV. This result has recently been reconfirmed with weak lensing data presented by Clowe *et al* [8] and the subsequent analysis by Angus *et al* [9]. Given that neutrinos exist, are massive and the mass required falls within the allowed range constrained by laboratory measurements, this solution to the missing mass problem in clusters is not outlandish.

In this paper we show that it is possible to modify gravity by introducing a dynamical *Aether* (or timelike vector field) with noncanonical kinetic terms. Our proposal builds on the extensive analysis of Einstein-Aether theories undertaken by Jacobson, Mattingley, Carroll, Lim and collaborators [10,11] and follows along a long series of proposals by others [12]. As the Aether vector field has a nonvanishing expectation value it will dynamically select a preferred frame at each point in space-time (i.e. the frame in which the time co-ordinate basis vector ∂_t aligns with the direction of the Aether field **A**). This violates local Lorentz invariance (and gauge invariance). Consequently, Aether theories traditionally have been used as phenomenological probes of possible Lorentz violation in quantum gravity.

As there has been recent interest in modifying gravity by using additional scalar and vector fields it is worth initially comparing our approach to other attempts. Another group of theories retains local Lorentz invariance but introduces a vector field, the action of which breaks U(1) symmetry through a nongauge invariant potential or kinetic term. These theories have variously been found to be able to model inflation [13] and dark energy [14], the latter invoking a set of three identical vectors along mutually orthogonal spacial directions (the "cosmic triad"). Such a vector field, coupled to scalar fields, has also been considered in the context of alternatives to dark matter [15]. Quite distinctly, it has been suggested that the recent acceleration of the universe could be accounted for by allowing for noncanonical kinetic terms in the action of the electromagnetic field [16]. Recently the possibility of a scalar field coupled to the Aether as a cause of inflation has been examined [17].

We will first lay out the formalism for our proposal, with the full field equations. We will then proceed to analyze them in the nonrelativistic regime and show that it is possible to naturally obtain modifications to Newtonian gravity. The physical consistency of the theory is discussed in the weak-field regime as are constraints from the Solar system. We then briefly explore the possible impact on the expansion of the Universe, showing that this modification of gravity can lead to accelerated expansion at different stages of the evolution of the universe. A specific proposal for such a theory is presented and we conclude by discussing a series of open problems.

II. THE THEORY

A general action for a vector field, **A** coupled to gravity can be written in the form

$$S = \int d^4x \sqrt{-g} \left[\frac{R}{16\pi G_N} + \mathcal{L}(g, A) \right] + S_M \quad (1)$$

where **g** is the metric, R the Ricci scalar of that metric, S_M the matter action and $\mathcal{L}$ is constructed to by generally covariant and local. S_M only couples to the metric, **g** and *not* to **A**. We shall use the metric signature (-,+,+,+) throughout.

For most of this paper we will restrict ourselves consider a Lagrangian that only depends on covariant derivatives of A and we will consider a **A** that is timelike. Such a theory can be written in the form

$$\mathcal{L}(A, g) = \frac{M^2}{16\pi G_N} \mathcal{F}(\mathcal{K}) + \frac{1}{16\pi G_N} \lambda(A^\alpha A_\alpha + 1)$$

$$\mathcal{K} = M^{-2} \mathcal{K}^{\alpha\beta}{}_{\gamma\sigma} \nabla_\alpha A^\gamma \nabla_\beta A^\sigma \quad (2)$$

$$\mathcal{K}^{\alpha\beta}{}_{\gamma\delta} = c_1 g^{\alpha\beta} g_{\gamma\sigma} + c_2 \delta^\alpha_\gamma \delta^\beta_\sigma + c_3 \delta^\alpha_\sigma \delta^\beta_\gamma$$

where c_i are dimensionless constants and M has the dimension of mass. λ is a nondynamical Lagrange-multiplier

field with dimensions of mass-squared. Note that it is possible to construct a more complicated $\mathcal{K}$ by including different powers in **A** and its derivatives. Indeed it is possible to show that Bekenstein's theory of modified gravity [5] is formally equivalent to a theory with such an extended $\mathcal{K}$ (though with a more exotic method of achieving a nonvanishing vacuum-expectation value for **A**). We allow for these different possibilities by deriving a general form for the field equations below. We will comment on these models in the discussion.

The gravitational field equations for this theory, obtained by varying $g^{\alpha\beta}$ (see [11] but also [18]) are

$$G_{\alpha\beta} = \tilde{T}_{\alpha\beta} + 8\pi G T^{\text{matter}}_{\alpha\beta} \quad (3)$$

where the stress-energy tensor for the vector field is given by

$$\tilde{T}_{\alpha\beta} = \frac{1}{2} \nabla_\sigma (\mathcal{F}'(J_{(\alpha}{}^\sigma A_{\beta)} - J^\sigma{}_{(\alpha} A_{\beta)} - J_{(\alpha\beta)} A^\sigma))$$

$$- \mathcal{F}' Y_{(\alpha\beta)} + \frac{1}{2} g_{\alpha\beta} M^2 \mathcal{F} + \lambda A_\alpha A_\beta \quad (4)$$

$$\mathcal{F}' = \frac{d\mathcal{F}}{d\mathcal{K}} \qquad J^\alpha{}_\sigma = (\mathcal{K}^{\alpha\beta}{}_{\sigma\gamma} + \mathcal{K}^{\beta\alpha}{}_{\gamma\sigma}) \nabla_\beta \mathcal{A}^\gamma$$

Brackets around indices denote symmetrization and $Y_{\alpha\beta}$ is the functional derivative

$$Y_{\alpha\beta} = \nabla_\sigma A^\eta \nabla_\gamma A^\xi \frac{\delta(\mathcal{K}^{\sigma\gamma}{}_{\eta\xi})}{\delta g^{\alpha\beta}}$$

The equations of motion for the vector field, obtained by varying A^β are

$$\nabla_\alpha(\mathcal{F}' J^\alpha_\beta) + \mathcal{F}' y_\beta = 2\lambda A_\beta \quad (5)$$

where once again we define the functional derivative

$$y_\beta = \nabla_\sigma A^\eta \nabla_\gamma A^\xi \frac{\delta(\mathcal{K}^{\sigma\gamma}{}_{\eta\xi})}{\delta A^\beta}$$

Variations of λ will fix $A^\mu A_\mu = -1$. By inspection, transvecting both sides of (5) with A^β leads to a solution for λ in terms of the vector field and its covariant derivatives.

These equations allow us to study a general theory of the form presented in Eq. (1) with a timelike vector field. For our particular, restricted choice of $\mathcal{K}$ we have $Y_{\alpha\beta} = -c_1[(\nabla_\nu A_\alpha)(\nabla^\nu A_\beta) - (\nabla_\alpha A_\nu)(\nabla_\beta A^\nu)]$ and $y_\beta = 0$.

III. THE NON-RELATIVISTIC REGIME: NEWTONIAN AND MONDIAN LIMIT

Having established our general theory, we can now explore its properties in various different regimes. We start in the static, weak field, nonrelativistic limit. We must expand both the metric and vector field around a fixed, Minkowski space background. The constraint equation very clearly fixes the zeroth order contribution of the vector field to be $A^{\alpha(0)} = \delta^\alpha_0$. Hence we have

$$g_{\alpha\beta} = \eta_{\alpha\beta} + \epsilon h_{\alpha\beta} \qquad A^\alpha = \delta_0^\alpha + \epsilon B^\alpha$$

We will only consider terms which are linear in our parameter ϵ and restrict ourselves to the Poisson gauge (so $h_{00} = -2\Phi, h_{ij} = -2\Psi\delta_{ij}$). We have that

$$\nabla\mathbf{A} = \epsilon\Sigma + \mathbf{O}(\epsilon^2) \qquad (6)$$

where $\Sigma_{ij} = 0$, $\Sigma_{0\mu} = \partial_\mu\Phi$ and $\Sigma_{i0} = \partial_i(\Phi + B^0)$ and (i) indexes the three coordinates, (x^1, x^2, x^3). Terms in $\nabla A\nabla A$ and $\vec{B}$ may be consistently regarded as being of $O(\epsilon^2)$ or greater and so can be ignored in what follows. An immediate consequence of this is that, to first order, the ijth component of the Aether's stress-energy tensor disappears. The ijth component of the Einstein equations then yields $\Phi = \Psi$. The equality of scalar potentials is true of General Relativity in the absence of anisotropic stress and simplifies the equations dramatically.

The modified Newton-Poisson and vector equations then are

$$2\nabla^2\Phi - (c_1 - c_3)\nabla.(\mathcal{F}'\nabla B^0) - \lambda = 8\pi G\rho$$
$$2c_3\nabla.(\mathcal{F}'\nabla\Phi) + 2c_1\nabla.(\mathcal{F}'\nabla(\Phi + B^0)) = -2\lambda \qquad (7)$$

The constraint fixes $\mathcal{K}$ to be

$$\mathcal{K} = -c_1\frac{(\nabla\Phi)^2}{M^2} \qquad (8)$$

Taking $c_1 < 0$ ensures that $\mathcal{K}$ is positive. We can manipulate the system of Eqs. (7) so as to obtain the field equation for Φ:

$$\nabla.((2 + c_1\mathcal{F}')\nabla\Phi) = 8\pi G\rho \qquad (9)$$

If such a theory is to have $\nabla.(|\nabla\Phi|\nabla\Phi) \propto \rho$ in the limit of small $\nabla\Phi$ we require that:

$$\lim_{|\nabla\Phi|\ll M}[2 + c_1\mathcal{F}'] \propto \mathcal{K}^{1/2} \qquad (10)$$

Integrating then we have that $\mathcal{F} = \alpha\mathcal{K} + \beta\mathcal{K}^{3/2}$ where α and β are constants. Hence we can construct a theory with "MONDian" limit on galactic scales, this holding as long as we identify M with something of the order of a_0 [4].

The limit as $\mathcal{K} \to 0$ is worth considering in more detail. Considering a test particle a distance r away from an isolated source, the Modified Poisson Equation dictates that for $|\nabla\Phi| \ll M$ the gravitational force will vary as $1/r$. This combined with the geodesic equation suggests that in this regime the metric component g_{00} grows approximately as $+\ln(r)$ and therefore becomes ill defined in the limit of $r \to \infty$. This has been a generic feature of past metric theories of MOND. It was noted in [19] that the value of this growing term will typically vary by around only 10^{-6} from the radius of onset of MOND in a system to the present Hubble radius ($> 10^{27}$ cm). Therefore, the weak-field approximation is unlikely to be seen to break down. The situation becomes more complicated if the

theory is such that MOND only arises in a cosmological background (for instance if A^2 is not fixed and the "scale" M is contingent on its value at the particular cosmological era). It would only be consistent then to formulate the weak-field limit in a FRW background rather than Minkowski space-time and the asymptotic form of the metric may be expected to differ.

The Solar System can supply us with stringent constraints on the weak-field limit of these theories. Accelerations are typically substantially larger than M (again assumed to be $\sim a_0$) and so requiring concordance with observations could constrain the possible form of $\mathcal{F}$ for large $\mathcal{K}$. Poisson's equation $\nabla^2\Phi = 4\pi G\rho$ is an excellent approximation in the Solar System and thus we shall expect the contribution of $c_1\mathcal{F}'$ in (9) to be small. We may expect then that $\mathcal{F}'$ in this limit can be expanded as a power series in inverse powers of $\mathcal{K}^{1/2}$. That is:

$$\lim_{|\nabla\Phi|\ll M}[\mathcal{F}'(\mathcal{K})] = \sum_{i=1}^{\infty}\frac{\eta_i}{\mathcal{K}^{i/2}} \qquad (11)$$

where η_i are constants.

Consider the leading term $\eta_1/\mathcal{K}^{1/2}$. We would expect such a term in spherical symmetry to result in a constant anomalous acceleration equal to $\frac{\sqrt{-c_1\eta_1^2}M}{2}$. The existence of such a term is particularly constrained by observed bounds on the variation of Kepler's constant $GM_\odot$. For instance, observations between Earth and Mars restrict the acceleration to be less than approximately 10^{-9} ms^{-1} [20]. We shall see that η and c values are typically of order unity so the above is rather restrictive. Even under the assumption of spherical symmetry, the field equations for the theory are enormously complicated. The inherent nonlinearity in the weak-field limit provided by $\mathcal{F}'$ presents a considerable challenge [21]. We note however that the fixed norm constraint on $\mathbf{A}$ will, in the weak-field limit, force terms of the form $\nabla A\nabla A$ and $\nabla\nabla A$ to be at most of the order of terms comprising the components of the Einstein tensor $\mathbf{G}$. For the limiting form of $\mathcal{F}'$ given in (11) we expect terms in the Aether stress-energy tensor to be schematically of the form $\frac{\alpha_i}{\mathcal{K}^{i/2}}c_j[\nabla\nabla A$ and $\nabla A\nabla A]$ and higher order derivative terms suppressed relatively by factors of $1/\mathcal{K}^{i/2}$.

At Mercury, the ratio $|\nabla\Phi|/M$ is of order 10^8. Provisionally neglecting terms in $\mathcal{F}'$ of $\mathcal{K}^{-(1/2)}$ (see above), we then expect corrections to terms in $\mathbf{G}$ in the inner solar system to be of order $10^{-16}c_j\alpha_i/c_k$. It is tempting to conclude that two "Parameterized Post Newtonian" (PPN) parameters measurable by inner solar system effects, β and γ, would generally be expected to deviate from the predictions of General Relativity by a similar order. The complete set of PPN coefficients have been obtained for the case $\mathcal{F}(\mathcal{K}) \propto \mathcal{K}$ (Ref. [10], part 3). In particular the coefficients describing "preferred frame" effects were found to be only consistent with experimental bounds for specific combinations of the c_i; they are ex-

pected to be a particularly strong test of more general forms of $\mathcal{F}$.

Additionally we note that the asymptotic behavior of $\mathcal{F}$ is consistent with our assumption that the term $M^2 \frac{\mathcal{F}}{2} \mathbf{g}$ is second order in perturbations. We see that for $\mathcal{K} \ll 1$, $M^2 \mathcal{F} \to |\nabla \Phi|^2$, and for $\mathcal{K} \gg 1$, $M^2 \mathcal{F}$ shall be at most $\alpha_1 M |\nabla \Phi|$.

Finally we briefly consider the effect of allowing for first-order spatial components of $\mathbf{A}$, A^i. It has been found [22] that in Bekenstein's theory of modified gravity the growth of large scale structure is necessarily accompanied by growth in A^i so it is not unreasonable to expect that such fields shall be present in the model considered here. It may be readily checked that in the *static* weak-field limit that an A^i of order ϵ will only contribute to $\tilde{T}_{00}$ and $\tilde{T}_{ii}$ at order two in ϵ and above. However, as the numerator in $\mathcal{K}$ is a second order quantity, it shall generally be affected. If A^i has only a radial component (i.e. $A^r \neq 0$), it may be checked that $\mathcal{K}$ is modified as:

$$\mathcal{K} = \frac{-c_1(\nabla \Phi)^2 + c_2(\nabla_r A^r)^2}{M^2} \quad (12)$$

where the covariant derivative $\nabla_r A^r$ is a of order two or greater in ϵ. We shall see later that c_2 is preferably the same sign as c_1. The effect of gradients of the radial component of the vector field then is to decrease $\mathcal{K}$ for a given $|\nabla \Phi|$. Recall that the onset of MONDian behavior coincides with $\mathcal{K} \ll 1$. Therefore an $A^r(r)$ in this model will generally hasten the onset of this limit, the effect being to further increase the gravitational field for a given ρ (see (9)). It requires further work to see whether this can appreciably counter MOND's problems on the scale of clusters of galaxies.

IV. FURTHER CONSTRAINTS

Recall that $\mathcal{F}$ tends to $\alpha \mathcal{K} + \beta \mathcal{K}^{3/2}$ for small $\mathcal{K}$ (i.e. far from a source). Therefore, in general, when considering classical perturbations one is effectively considering a theory with $\mathcal{F} \sim \mathcal{K}$, as in the Einstein-Aether theory with minimal couplings. This can be used to study the consistency of these theories in the perturbative regime. Lim [23] has considered the dominant term in the limit where metric and vector field perturbations decouple. The vector field propagates in flat space-time and allows for a decomposition of perturbations into spin-0 and spin-1 components. The requirement that the perturbations can be consistently quantized, that the spin-0 component propagates subluminally and nontachyonically when quantized, and that metric perturbations do not propagate superluminally place the following restrictions on the c_i:

$$c_1 < 0 \quad (13)$$

$$c_2 \leq 0 \quad (14)$$

$$c_1 + c_2 + c_3 \leq 0. \quad (15)$$

Additional constraints can be obtained via astroparticle physics. The observation of ultrahigh energy cosmic rays implies a lack of energy loss via gravitational Cherenkov radiation. This radiation is expected when gravitational waves propagate subluminally. With the Aether, the usual transverse-traceless modes exist along with three coupled Aether-metric modes. The speeds of propagation of each mode are functions of the c_i and have been calculated in [10] for perturbations in Minkowski space-time. The squared speed s_{tt}^2 of the transverse-traceless mode is:

$$s_{tt}^2 = 1/(1 + (c_1 + c_3))$$

and the squared speeds of the transverse Aether and trace Aether-metric modes (s_{tA}^2 and s_{tr}^2 respectively) are given by:

$$s_{tA}^2 = \frac{(c_1 + \frac{c_1^2}{2} - \frac{c_3^2}{2})}{(c_1(1 + c_{13}))}$$

$$s_{tr}^2 = \frac{(c_{123}/c_1)(2 + c_1)}{[2(1 - c_2)^2 + c_{123}(1 - c_2 - c_{123})]}$$

where $c_{ijk..} = c_i + c_j + c_k + ..$ and c_i are the opposite sign to those considered in [10].

Computing the expected degree of gravitational Cherenkov radiation for the above modes and comparing this with observational bounds, it has been found [24] that for the case $\mathcal{F}(\mathcal{K}) = \mathcal{K}$ that extremely stringent constraints could be placed on the c_i. For instance, they may be satisfied when the c_i are mutually related such that the metric and Aether-metric modes propagate at precisely the speed of light. If this is not the case then the magnitudes of the c_i must be severely diminished. It is expected that such an analysis could powerfully constrain more general combinations of $\mathcal{F}, c_i$, though any nonlinearity is expected to complicate the results when propagation is considered against a curved background space-time.

Note that such an analysis is by no means complete. It has been shown that a very restricted class of Einstein-Aether theories do not have positive Hamiltonians and therefore are inherently unstable at both the classical and quantum levels [25]. Furthermore we are considering non-linear functions of $\mathcal{K}$ and hence instabilities may arise in non trivial backgrounds. A more detailed analysis of individual cases for $\mathcal{F}$ is needed yet first indications are that these theories are healthy.

V. COSMOLOGY

This class of modified theories are generally covariant. This gives us the possibility of exploring their properties on large scales and, in particular, we can consider the case of a homogeneous and isotropic universe in which the metric is of the form $ds^2 = -dt^2 + a(t)^2 d\mathbf{x}^2$ where t is physical time and $a(t)$ is the scale factor. The vector field must

respect the spatial homogeneity and isotropy of the system and so will only have a nonvanishing 't' component; the constraint fixes $\mathbf{A} = (1, 0, 0, 0)$. The energy-momentum tensor of the matter is of the form $T_{\alpha\beta}^{\text{matter}} = \rho U_\alpha U_\beta + P(g_{\alpha\beta} + U_\alpha U_\beta)$ where ρ is the energy density, P is pressure and we have introduce a four-vector $\mathbf{U}$ which satisfies $g_{ab} U^a U^b = -1$.

The equations of motion simplify dramatically with these symmetries and we find that:

$$\nabla_e A^e = 3H \qquad \mathcal{K} = 3\frac{\alpha H^2}{M^2}$$

where $H \equiv \frac{\dot{a}}{a}$, the dot denotes differentiation with respect to t, and, following [11], we define $\alpha = c_1 + 3c_2 + c_3$.

Note that now $\mathcal{K}$ is negative, unlike the nonrelativistic limit encountered above. This means that the dynamics of static, spherically symmetric systems on the one hand and of relativistic cosmologies on the other probe completely different branches of $\mathcal{F}$.

The modified Einstein's equations now become:

$$[1 - \mathcal{F}'\alpha]H^2 + \frac{1}{6}\mathcal{F}M^2 = \frac{8\pi G}{3}\rho$$

$$-[1 - 2\mathcal{F}'\alpha]H^2 - 2\left[1 - \frac{1}{2}\mathcal{F}'\alpha\right]\frac{\ddot{a}}{a} + \dot{\mathcal{F}}'\alpha H$$

$$-\frac{1}{2}\mathcal{F}M^2 = 8\pi G P$$

These can in fact be rewritten in a more useful form:

$$\left[1 - \alpha\mathcal{K}^{1/2}\frac{d}{d\mathcal{K}}\left(\frac{\mathcal{F}}{\mathcal{K}^{1/2}}\right)\right]H^2 = \frac{8\pi G}{3}\rho \qquad (16)$$

$$\frac{d}{dt}(-2H + \mathcal{F}'\alpha H) = 8\pi G(\rho + P) \qquad (17)$$

Once again these equations are general but we can see in their structure, interesting possibilities. If we take $\mathcal{F} = 0$, we recover the standard cosmology. We can do this by either setting it to 0 or choosing $\mathcal{K}$ to have $c_1 = -c_3$ and $c_2 = 0$ so that it becomes the Maxwell tensor. We then have a theory which will modify gravity on galactic and supergalactic scales but leaves the expansion of the universe unchanged.

It is interesting to explore the possibility that the vector field may affect the late time expansion of the Universe. From Einstein's equations, we can see that for it to behave like a cosmological constant, Λ we must have

$$-\mathcal{K}\mathcal{F}' + \frac{1}{2}\mathcal{F} = \frac{\Lambda}{M^2} \qquad (18)$$

which we can solve to find: $\mathcal{F} = \beta(-\mathcal{K})^{1/2} - \frac{2\Lambda}{M^2}$.

Where β is a constant of integration. Now note that for positive arguments of $\mathcal{F}$ we have been equating the mass scale M with the acceleration scale $a_0 \simeq 10^{-10} \text{ ms}^{-2} \simeq cH_0$ where c is the speed of light and H_0 is the Hubble

constant. If we assume that is also true for negative values, we find that $\mathcal{F}$ is of order unity. I.e. there is a natural relation between the scale of the cosmological constant and the fundamental mass scale in our theory. A choice of such asymptotic behavior close to $\mathcal{K} = 0$ does introduce an undesirable discontinuity in $\mathcal{F}$ at the origin; however, as the behavior in (18) will lead to a constant $\mathcal{K}$ (as $\dot{H} = 0$), this form of $\mathcal{F}$ need only hold in the domain $|\mathcal{K}/3\beta| > \sim H_0$ and does not inform the desirable choice of $\mathcal{F}$ around $\mathcal{K} = 0$.

Accelerate expansion can be reached in a more natural way by positing

$$\mathcal{F} = \gamma(-\mathcal{K})^n$$

for a certain range of $\mathcal{K}$ For arbitrary n Eq. (16) becomes

$$\left[1 + \epsilon\left(\frac{H}{M}\right)^{2(n-1)}\right]H^2 = \frac{8\pi G}{3}\rho$$

where $\epsilon = -(1 - 2n)\gamma(-3\alpha)^n/6$. For an appropriate choice of γ and n we have $\epsilon < 0$ and we find that H tends to an attractor

$$H \rightarrow H_{\text{eq}} = M(-\epsilon)^{1/2(1-n)}$$

This regime is approached asymptotically with

$$H - H_{\text{eq}} \simeq H_{\text{eq}}^{-1}\left(\frac{8\pi G\rho}{3}\right)$$

i.e. as a^{-3} in the matter era. This gives us a particularly elegant, dynamical mechanism for approaching accelerated expansion without invoking a cosmological constant. A small scale is still invoked but it is naturally related to the acceleration scale needed to trigger the onset of the MONDian regime on galactic scales.

VI. DISCUSSION

The discussion throughout this letter has been general. We have neither chosen a specific form of $\mathcal{K}$ or $\mathcal{F}$ although we have constrained the asymptotic form of the latter. It is instructive to pick a simple example. If we choose $c_1 = -1$, $c_2 = 0$ and $c_3 = 1$ we will recover the canonical form $\mathcal{K} \propto F_{\alpha\beta}F^{\alpha\beta}$ where $\mathbf{F}$ is the field-strength of the four-vector $\mathbf{A}$. A possible functional form for $\mathcal{F}$ is then:

$$\mathcal{F}(\mathcal{K}) = 4(\mathcal{K}^{1/2} - \ln(\mathcal{K}^{1/2} + 1))$$

From which we recover the modified Newton-Poisson equation:

$$\nabla.\left(\frac{\frac{|\nabla\Phi|}{M}}{\frac{|\nabla\Phi|}{M} + 1}\nabla\Phi\right) = 4\pi G\rho \qquad (19)$$

With an appropriate choice of the value of M we recover the field equation considered in [26], which was found to give a satisfactory fit to the terminal velocity curve of the

Milky Way. As noted, the choice $c_1 = -c_3$, $c_2 = 0$ as here forces $\mathcal{K}$ to vanish in the case of spatial homogeneity and isotropy. Therefore with this choice of $\mathcal{F}$ the Aether has no influence on cosmological background evolution. The inclusion of a nonzero c_2 modifies the kinetic term of the metric and may have significant effects both in the static, weak-field limit and the cosmological background.

There are a number of theoretical and phenomenological issues that remain to be addressed.

For a start, the presence of the nondynamical Lagrange-multiplier field λ in the Lagrangian (2) is perhaps unappealing. Its sole role is to impose the constraint that A^μ is unit-timelike. The same can be accomplished by replacing the λ-term in the Lagrangian by a potential term $-V(A^\mu)$, by dint of which A^μ at low energy acquires a vacuum-expectation value such that $A^\mu A_\mu = -1$. For example $V(A^\mu) = \frac{1}{2}\mu^4(A^\mu A_\mu + 1)^2$, with μ a constant with dimensions of mass. At energy scales below μ, one expects that indeed $A^2 = -1$, and that "radial" excitation of A (i.e. of $a \equiv \mu A$) will have positive mass-squared $m_a^2 \simeq \mu^2$. So long as μ^2 is sufficiently large, the low-energy phenomenology of this model should be identical to that of (2). A more exotic possibility is to construct more complex $\mathcal{K}$ which have minima for timelike **A**. Indeed the theory proposed in [5] is of this form [27].

Closely related to the previous point is the fundamental origin of such an Aether. We have kept the discussion general in the hope that a more fundamental theory of fields or strings may pin down the form of $\mathcal{K}$ and $\mathcal{F}$. Indeed, the possibility of Lorentz violating vector fields has cropped up in attempts to extend the standard model of particle physics. Most notably it has been argued that such

a field may arise in higher dimensional theories as a low-energy by-product of string theory [28]. It would be interesting to explore the range of current candidates for low-energy string theory to find a possible candidate for such an Aether field. What is clear is that a Lagrangian of the form we require will *not* appear from the standard perturbative approach to constructing effective field theories as in [29]. Nonperturbative effects must come into play.

We have endeavoured to explore some of the observational constraints. Preliminary indications are that these theories are compatible with Solar System constraints. A more detailed analysis is needed with complete spherically symmetric solutions that can then be used to calculate the Post Newtonian Parameters [30]. We have also shown that, as yet there is sufficient freedom in the choice of $\mathcal{F}$ to obtain different cosmological behaviors, from no effect to early or late time acceleration. The next step is to follow in the footsteps of [31] and calculate the evolution of linear perturbations. *A priori* it is unclear whether perturbations in the Aether will have the same effect as dark matter in sustaining perturbations through the Silk damping regime during recombination [32]. Indeed this may be the most stringent test such theories have to pass to be viable alternatives to dark matter.

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