Black strings and solitons in five dimensional space-time with positive cosmological constant

Y. Brihaye and T. Delsate*

Faculte des Sciences, Universite de Mons-Hainaut, 7000 Mons, Belgium

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We consider the classical equations of the Einstein-Yang-Mills model in five space-time dimensions and in the presence of a cosmological constant. We assume that the fields do not depend on the extra dimension and that they are spherically symmetric with respect to the three standard space dimensions. The equations are then transformed into a set of ordinary differential equations that we solve numerically. We construct new types of regular (resp. black holes) solutions which, close to the origin (resp. the event horizon) resemble the 4-dimensional gravitating monopole (resp. non-Abelian black hole) but exhibit an unexpected asymptotic behavior.

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I. INTRODUCTION

In recent years, there has been an increasing attention to space-times involving more than four dimensions and particularly to brane-world models [1] to describe space, time, and matter. These assume the standard model fields to be confined on a 3-brane embedded in a higher dimensional manifold. A large number of higher dimensional black holes has been studied in recent years. The first solutions that have been constructed are the hyperspherical generalizations of well-known black holes solutions such as the Schwarzschild and Reissner-Nordström solutions in more than four dimensions [2] as well as the higher dimensional Kerr solutions [3]. In d dimensions, these solutions have horizon topology S^{d-2}.

However, in contrast to 4 dimensions black holes with different horizon topologies should be possible in higher dimensions. An example is a 4-dimensional Schwarzschild black hole extended trivially into one extra dimension, a so-called Schwarzschild black string. These solutions have been discussed extensively especially with view to their stability [4]. A second example, which is important due to its implications for uniqueness conjectures for black holes in higher dimensions, is the black ring solution in 5 dimensions with horizon topology $S^2 \times S^1$ [5].

The largest number of higher dimensional black hole solutions that have been constructed so far are solutions of the vacuum Einstein equations, respectively, Einstein-Maxwell equations.

On the other hand, it is believed that topological defects have occurred and played a role during some phase tansitions in the evolution of the Universe, see e.g. Ref. [6]. In particular, magnetic monopoles [7] must have been produced during the GUT symmetry breaking phase transition. The actual nonobservation of magnetic monopoles leads to constraints which have to be implemented into the models of inflation. On the other hand observational evidence obtained in the last years [8] favors the possibility that space-time has an accelerated expansion which could be related to a positive cosmological constant.

It is, therefore, natural to examine the properties of the various topological defects in presence of a cosmological constant, or said in other words, in asymptotically de Sitter space-time. Recently [9] the magnetic monopole and the sphalerons occuring in an SU(2) gauge theory spontaneously broken by a scalar potential were constructed in an asymptotically de Sitter space-time and it was found that the asymptotic decay of the matter field is not compatible with a finite mass.

The first example of higher dimensional black hole solutions containing non-Abelian gauge fields have been discussed in Ref. [10]. These are non-Abelian black holes solutions of a generalized 5-dimensional Einstein-Yang-Mills system with horizon topology S^3. Using ideas of Refs. [11,12], SU(2)-black strings with $S_2 \times S_1$ topology were constructed in Ref. [13]. Several regular and black hole solutions of an Einstein-Yang-Mills model have been constructed recently with different symmetries [14–16]. These solutions are non-Abelian black hole solutions in 3 + 1-dimensions extended to one extra dimension.

In Refs. [17,18] the Einstein-Yang-Mills model in five dimensions with gauge group SU(2) was considered with a positive cosmological constant. The metric and gauge fields were assumed to be independent of the extra dimension and chosen to be spherically symmetric in the standard three spacelike dimenisons. By adopting a Schwarzschild-dilaton type parametrisation for the metric, it was found that the equations can be integrated only up to a maximal value of the radial coordinate, say $r = r_c$. A coordinate singularity occurs at $r = r_c$. In this paper, we adopt the parametrisation of the metric used in Ref. [19] and we show that the solution of Ref. [18] can be extended up to spatial infinity in these coordinates.

We give the model and the two parametrization of the metric in Sec. II. The relevant reduced action for the gravitating and matter parts are presented in Sec. III together with the boundary conditions. The numerical results corresponding to solutions regular at the origin and solu-

*Electronic address: yves.brihaye@umh.ac.be

tions possessing an event horizon are discussed in Sec. IV. The summary is given in Sec. V.

II. THE MODEL AND THE ANSATZ

The Einstein-Yang-Mills Lagrangian in $d = (4 + 1)$ dimensions is given by

$$S = \int \left(\frac{1}{16\pi G_5} (R - 2\Lambda_5) - \frac{1}{4e^2} F^a_{MN} F^{aMN} \right) \sqrt{g^{(5)}} d^5 x \tag{1}$$

with the SU(2) Yang-Mills field strengths $F^a_{MN} = \partial_M A^a_N - \partial_N A^a_M + \epsilon_{abc} A^b_M A^c_N$, the gauge index $a = 1, 2, 3$ and the space-time index $M = 0, \ldots, 5$. G_5, Λ_5 and e denote, respectively, the 5-dimensional Newton's and cosmological constants and the coupling constant of the gauge field theory. G_5 is related to the Planck mass $M_{\rm pl}$ by $G_5 = M_{\rm pl}^{-3}$ and e^2 has the dimension of [length].

In this paper, we assume that the metric and the matter fields are independent on the extra coordinate y and we will use a spherically symmetric ansatz for the fields.

Our aim is to construct non-Abelian regular and black strings solutions which are spherically symmetric in the four-dimensional space-time and are extended into one extra dimension. The topology of these non-Abelian black strings will thus be $S^2 \times \mathbb{R}$ or $S^2 \times S^1$ if the extra coordinate y is chosen to be periodic.

We will use two different coordinates systems for the metric. On the one hand, the metric can be parametrized according to [11] as follows:

$$g^{(5)}_{MN} dx^M dx^N = e^{-\xi}[-A^2 N dt^2 + N^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d^2\varphi] + e^{2\xi} dy^2\text{: type (1),} \tag{2}$$

where N, A, ξ are function of the coordinate r only. For the gauge fields, we use the spherically symmetric ansatz [7] :

$$\begin{aligned} &A_r{}^a = A_t{}^a = 0, \qquad A_\theta{}^a = (1 - K(r)) e_\varphi{}^a, \\ &A_\varphi{}^a = -(1 - K(r)) \sin\theta e_\theta{}^a, \qquad \Phi^a = v H(r) e_r{}^a, \end{aligned} \tag{3}$$

where v is a mass scale.

The classical equations corresponding to the model above were studied in [17] and more recently in Ref. [18] with the appropriate boundary conditions corresponding to regular solution at the origin $r = 0$ and black string solutions presenting a regular event horizon on a cylinder, i.e. with $N(r_h) = 0$. The equations were solved numerically and it was found that the solutions having the regular behavior at $r = 0$ or $r = r_h$ exist only up to a maximal value $r = r_c$ (with the value r_c depending on both α and Λ). For $r \to r_c$ the fields behave according to

$$\begin{aligned} &N(r) \sim N_c(r_c - r), \qquad \xi(r) \sim \xi_i + \xi_c \sqrt{(r_c - r)}, \\ &A(r) = A_c (r_c - r)^{-a}, \end{aligned} \tag{4}$$

where N_c, ξ_i, ξ_c, A_c, a are constants (with $a > 0$) depending on α and Λ. The behavior of the function ξ clearly indicates an absence of analyticity at $r = r_c$.

In view of these difficulties, we have tried to study the equation by using a different parametrisation of the metric. For instance, we use the length element used e.g. in Ref. [19]. It reads

$$ds^2 = -b(x) dt^2 + \frac{dx^2}{f(x)} + g(x)(d\theta^2 + \sin^2(\theta) d\varphi^2) + a(x) dy^2\text{: type (2),} \tag{5}$$

where the radial variable is named x [to avoid confusion with r used in (2)]. The arbitary redefinition of the coordinate x is left unfixed at this stage but on it will be fixed later according to $g(x) = x^2$. The ansatz for the matter fields is identical to Eq. (3) apart from the fact that the functions K, H now depend on x. Using $g(x) = x^2$ as gauge fixing, the correspondence between the two sets of functions in Eqs. (2) and (5) reads

$$\begin{aligned} &x = r e^{-\xi(r)/2}, \qquad f(x) = N\left(1 - \frac{r}{2}\frac{d\xi}{dr}\right)^2, \\ &a(x) = e^{2\xi(r)}, \qquad b(x) = A(r)^2 N(r) e^{-\xi(r)}. \end{aligned} \tag{6}$$

III. EQUATIONS OF MOTION

Using an appropriate rescaling of the radial variable $evx \to x$ the classical equations associated to the action (1) lead to a set of ordinary differential equations depending on the fundamental coupling $\alpha^2 \equiv 4\pi G_5 v^2$ and on the reduced cosmological constant $\Lambda \equiv 2\alpha^2 \Lambda_5$. In the case of the parametrization (2), the equations are written in details in Ref. [18] together with the appropriate boundary conditions. In the case of the parametrisation (5), the equations are obtained in the standard way. The nonvanishing components of the Einstein tensor are given by

$$G^t_t = \left(\frac{1}{2}\frac{a''}{a} + \frac{g''}{g} - \frac{1}{4}\frac{g'^2}{g^2} + \frac{1}{2}\frac{a'}{a}\frac{g'}{g}\right) f + \frac{1}{2}\left(\frac{1}{2}\frac{a'}{a} + \frac{g'}{g}\right) f' - \frac{1}{g}, \tag{7}$$

$$G^r_r = \left(\frac{1}{4}\frac{a'b'}{ab} + \frac{1}{2}\left(\frac{a'}{a} + \frac{b'}{b}\right)\frac{g'}{g} + \frac{1}{4}\frac{g'^2}{g^2}\right) f - \frac{1}{g}, \tag{8}$$

$$G^\theta_\theta = \left(\frac{1}{2}\left(\frac{a''}{a} + \frac{b''}{b} + \frac{g''}{g}\right) - \frac{1}{4}\left(\frac{a'^2}{a^2} + \frac{b'^2}{b^2} + \frac{g'2}{g^2}\right) + \frac{1}{4}\left(\frac{a'b'}{ab} + \frac{a'g'}{ag} + \frac{b'g'}{bg}\right)\right) f + \frac{1}{4}\left(\frac{a'}{a} + \frac{b'}{b} + \frac{g'}{g}\right) f', \tag{9}$$

$$G^\phi_\phi = \sin^2\theta G^\theta_\theta, \tag{10}$$

$$G_y^y = \left(\left(\frac{1}{2}\frac{b''}{b} + \frac{g''}{g}\right) - \frac{1}{4}\left(\frac{b'^2}{b^2} + \frac{g'^2}{g^2}\right) - \frac{1}{2}\frac{b'g'}{bg}\right)f + \frac{1}{2}\left(\frac{1}{2}\frac{b'}{b} + \frac{g'}{g}\right)f' - \frac{1}{g}, \tag{11}$$

where the prime denote derivative with respect to the rescaled radial variable x; the energy momentum tensor has the following form:

$$T_t^t = \frac{1}{2ag^2}(A + B + C + D), \tag{12}$$

$$T_r^r = \frac{1}{2ag^2}(A - B + C - D), \tag{13}$$

$$T_\theta^\theta = \frac{1}{2ag^2}(D - A), \qquad T_\phi^\phi = \sin^2\theta T_\theta^\theta, \tag{14}$$

$$T_y^y = \frac{1}{2ag^2}(A + B - C - D), \tag{15}$$

where $A \equiv a(K^2 - 1)^2$, $B \equiv 2afgK'^2$, $C \equiv 2gH^2K^2$ and $D \equiv fg^2H'^2$.

In order to treat the Einstein equations numerically, we eliminate the quantity a'' from the (tt) and $(\theta\theta)$-equations and use the remaining equations to obtain a system for a', f', b'', in a quite similar way as in Ref. [19]. The gauge is fixed by $g = x^2$ and the supplementary Einstein equation is finally used as a numerical crosscheck. Finally the 5-dimensional Yang-Mills equations lead to the conditions

$$\frac{1}{\sqrt{-\tilde{g}}}\left(\sqrt{-\tilde{g}}\frac{f}{g}K'\right)' = \frac{1}{g^2}K(K^2 - 1) + \frac{1}{ga}KH^2, \tag{16}$$

$$\frac{1}{\sqrt{-\tilde{g}}}\left(\sqrt{-\tilde{g}}\frac{f}{a}H'\right)' = \frac{2}{ga}HK^2. \tag{17}$$

The Gauss law is trivially fulfilled since the ansatz is static and $A_0 = 0$.

It is interesting to notice that the five relevant radial equations can be obtained directly by the variational principle on the following effective one-dimensional action density

$$S_{\rm red} = \frac{1}{\alpha^2}\sqrt{-\tilde{g}}\frac{1}{\sin\theta}(R - 2\Lambda) + S_{\rm mat}, \qquad \sqrt{-\tilde{g}} = g\sin\theta\sqrt{ab/f}, \tag{18}$$

with the gravitating part proportional to the Ricci scalar

$$\sqrt{-\tilde{g}}\frac{1}{\sin\theta}R = -\left(\sqrt{abf}\left(g\left(\frac{b'}{b} + \frac{a'}{a}\right) + 2g'\right)\right)' + g\sqrt{\frac{ab}{f}}\left(\frac{f}{2}\left(\frac{g'}{g}\right)^2 + f\frac{g'}{g}\left(\frac{a'}{a} + \frac{b'}{b}\right) + \frac{2}{g} + \frac{f}{2}\frac{a'b'}{ab}\right) \tag{19}$$

and

$$S_{\rm mat} = \sqrt{-\tilde{g}}\left(\frac{f}{g}(K')^2 + \frac{1}{2g^2}(1 - K^2)^2 + \frac{f}{2a}(H')^2 + \frac{1}{ga}K^2H^2\right). \tag{20}$$

A. Boundary conditions

It should be noticed that the function a and b can be rescaled arbitrarily. The regularity conditions for a regular solution at the origin read [19]

$$f(0) = 1, \qquad a(0) = 1, \qquad b(0) = 1, \qquad b'(0) = 0, \tag{21}$$

while black strings possessing an horizon at $x = x_h$ should have

$$f(x_h) = 0, \qquad a(x_h) = 1, \qquad b(x_h) = 0, \qquad b'(x_h) = 1 \tag{22}$$

in both cases a natural choice of the normalization of a, b has been supplemented. For the matter fields, the boundary conditions for a regular solution at the origin and the usual asymptotic conditions read

$$K(0) = 1, \qquad H(0) = 0, \qquad K(\infty) = 0, \qquad H(\infty) = 1. \tag{23}$$

In the case of black holes, the regularity at the horizon imposes some peculiar relations between the values $H(x_h)$, $K(x_h)$ and their derivatives. These expressions are cumbersome and will not be presented here.

B. Asymptotic expansion

The study of the classical equations in the vacuum case, i.e. for $K(x) = 1$, $H(x) = 0$ is interesting by itself. A complete analysis of black strings in the case of a negative cosmological constant is reported in Ref. [19]. In that paper, regular solutions at the origin are constructed numerically as well as black holes solutions which present a regular horizon at $x = x_h$. For the two types of solutions, the components of the metric grow asymptotically according to

$$a(x) = \frac{x^2}{\ell^2}, \qquad b(x) = \frac{x^2}{\ell^2},$$
$$f(x) = \frac{x^2}{\ell^2} \quad \text{with} \quad \Lambda \equiv -\frac{6}{\ell^2}. \tag{24}$$

The next terms of these expansions can be found in Ref. [19].

In the case of a positive cosmological constant, a possible asymptotic form of the solutions can be obtained by analytic continuation (i.e. $\ell^2 \to -\ell^2$) of the asymptotic expansion obtained in Ref. [19]; although nothing guarantee that the corresponding solutions will be regular at the origin or admit a regular horizon.

Because it turns out impossible to construct numerically regular solutions approaching $a(x) = b(x) = f(x) \sim -x^2/\ell^2$ asymptotically, we investigated other types of asymptotic behaviours and obtained the following form using the Einstein equations:

$$a = x^{2A}, \quad b = x^{2B},$$
$$f = f_0 x^{2F} \quad \text{with} \quad A = B = -2 - \sqrt{3}, \quad F = 3 + 2\sqrt{3} \tag{25}$$

and where f_0 is a constant. The power-dependence of the metric functions on the radial coordinate and the fact that $A + B + F = -1$ is reminiscent of the Kasner solution in four-dimensional space-time. However (25) is not an exact solution but the leading term of a power expansion:

$$a = b = x^{2A} - \frac{3\Lambda}{2f_0(1+A)(2+A)} x^{4+6A} + \ldots,$$
$$f = f_0 x^{-2-4A} - \frac{2\Lambda}{1+A} x^2 + \frac{1}{1+2A} + O(x^{4+6A}), \tag{26}$$

where $A \equiv -2 - \sqrt{3} \approx -3.732$. The Ricci scalar calculated with this solutions vanishes asymptotically.

C. Known solutions

The system of equations under investigation possesses several known solutions in specific limits of the coupling constants α^2 and Λ. The knowledge of these solutions can be used as a check of our numerical solutions.

(i) In the limit $\alpha^2 = 0$, $\Lambda = 0$ the Einstein equations are trivial (so that $a = b = f = 1$, $g = x^2$) and the matter field equations restrict to the equation of the t'Hooft-Polyakov monopole [7] on the self dual limit.

(ii) In the case $\alpha^2 \neq 0$, $\Lambda = 0$, the equations are solved in Refs. [11–13] with the parametrization (2) of the metric. The solutions can be transformed in the coordinate system (5) by using Eq. (6). It is worth noticing that for $\Lambda \ll 1$ the effect of the cosmological constant appears for large values of x; the profiles of the solutions in Ref. [11–13] in the region of the origin (or of the event horizon) are expected to persist.

(iii) The matter field equations are solved by $W = 1$, $H = 0$. Then the Einsten equations are those studied in Ref. [19] with $\ell^2 \to -\ell^2$. In the case $\Lambda = 0$, the uniform black solution [2–4] is given by $b = f = 1 - 1/x$, $a = 1$ and $g = x^2$. No analytic solutions of these equations is known to our knowledge for $\Lambda \neq 0$. Some numerical solutions will be reported in the next section.

IV. NUMERICAL RESULTS

A. Regular solutions

We solved the equations corresponding to the Lagrangian (1) by numerical methods for several values of α and $\Lambda > 0$. In the system of coordinate (5), our results strongly suggest that the solutions approaching the regular boundary condition at the origin can be extrapolated for $x \to \infty$ and that, in this limit, they approach the asymptotic form (26). This is illustrated on Fig. 1 for $\alpha = 1$, $\Lambda = 0.0005$. Close to the origin, these solutions are similar to the gravitating dilatonic monopole [11,12] but contrary to our expectation they do not extrapolate to a de Sitter space-time for $\Lambda > 0$. It seems that the presence of the cosmological constant and the corresponding Liouville potential leads to an asymptotic space-time obeying the power law (26) asymptotically.

Because this property seems to be related essentially to gravity, we solved the equations in the case $K = 1$, $H = 0$, i.e. when the Yang-Mills field is trivial. The matter Eqs. (16) and (17) are then trivial and the equations correspond to 5-dimensional gravity in the presence of a positive cosmological constant. The profiles of the functions a, b, f are represented in the subplot of Fig. 3. Comparing Figs. 1

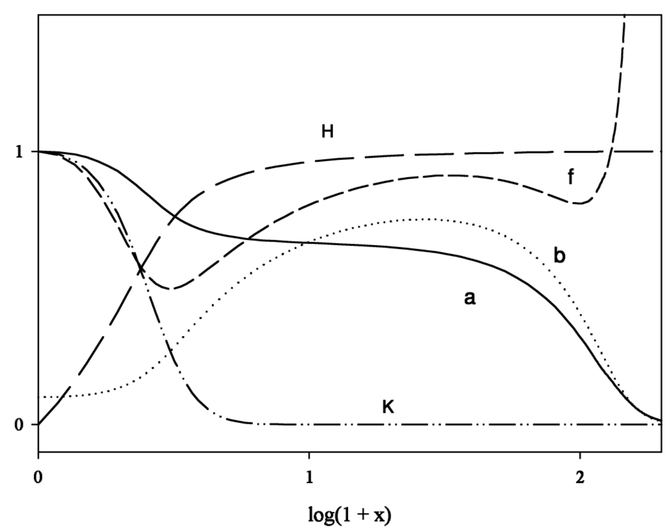

FIG. 1. The profile for a non-Abelian soliton corresponding to $\alpha = 1$, $\Lambda = 0.0005$.

and 3, we can appreciate the significant deformation of the functions a, b, f due to the matter fields in the region $x \sim 0$.

The double logarithmic scale used on Fig. 3 demonstrates the power decay of the metric functions. Our numerical results strongly indicate that $a(x) = b(x)$ in this case. This relation does not hold true for black strings (see the next section) but it suggests that the solution could be expressed in an explicit form (although we failed to find it so far). To finish this section, let us mention that, in the case of a negative cosmological constant, a solution regular at the origin also exists; it was constructed in Ref. [19]. In contrast to the present case, the solution of Ref. [19] asymptotically approaches the solution (24).

B. Black hole solutions

The numerical construction of black hole solutions of Eq. (1) is more difficult with the metric (5) than with the metric (2) parametrization because two functions (for instance f and b) vanish at the event horizon x_h, leading to several singular terms in the equations (with the type-1 parametrization, we have $N(r_h) = 0$ only). Nevertheless, the results of Ref. [18], obtained with the metric (2), suggest that blach holes solutions should exist at least locally. In particular these solutions, once converted with Eq. (6), provide extremely useful starting profiles to solve the equation with Eq. (5). It is worth noticing that, once converted into the system of coordinate (5) system by means of Eq. (6) the functions a, b, f and their derivatives turn out to be smooth in the neighborhood of the maximal value $x_c = r_c e^{-\xi(r_c)/2}$. Solving the equations for the type-2 coordinates confirms indeed the non-Abelian black string of Ref. [18] exist and further shows that these solutions can be extended for $x \in [x_h, \infty]$ with $\Lambda > 0$.

The profile for such a solution is presented on Fig. 2 for $\alpha = 1$, $x_h = 0.3$ and $\Lambda = 0.0005$. Similarly to the case of regular solutions our numerical results strongly suggest that these black hole solution extrapolate between the condition of a regular black string with horizon at $x = x_h$ and the power-type behavior (26) for $x \to \infty$.

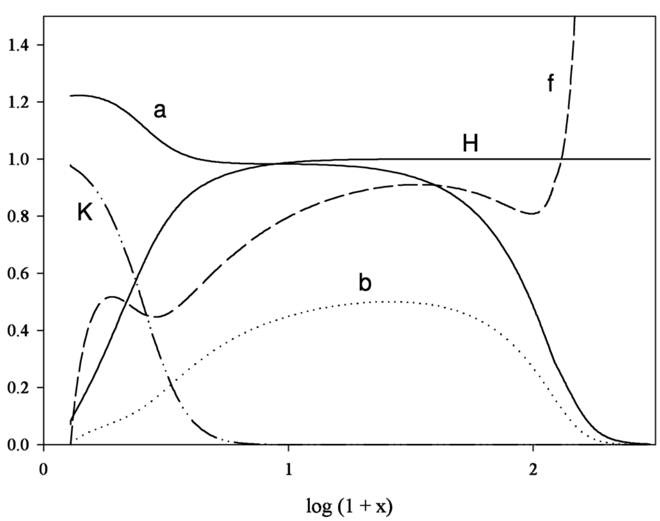

FIG. 2. The profile for a non-Abelian black string corresponding to $\alpha = 1$, $\Lambda = 0.0005$ and $x_h = 0.3$.

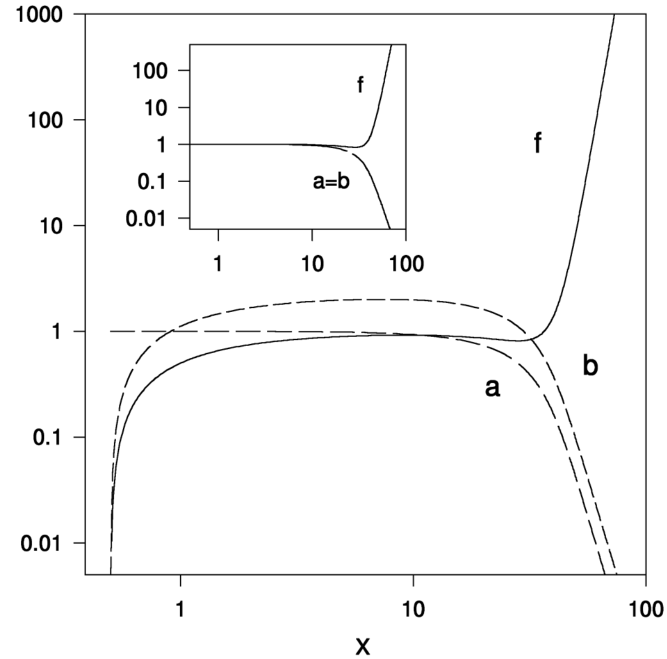

FIG. 3. The profiles for the pure gravity black string and of the regular solution in the window.

We also solved the equations in the pure gravity case (i.e. $\alpha = 0$ and $W = 1$, $H = 0$) and obtained the black string solutions with positive cosmological constant. These are represented on Fig. 3 (main figure), on this figure, the function $b(x)$ has been rescaled by a factor 2 in order to be able to distinguish the profiles of a and b. These solutions are the counterparts for $\Lambda > 0$ of the black string solutions presented in Ref. [19] for $\Lambda < 0$. Let us point out that for $\Lambda < 0$ the black string solutions extrapolate between a regular horizon and ADS space-time. In the neighborhood of the event horizon x_h the two solutions look quit similar but they deviate considerably from each other for $x \gg 1$.

Let us finally mention that, integrating the equations from $x = \infty$ with (24) as initial condition and with $\Lambda > 0$ leads to configurations which become singular for $x \to 0$. A systematic study of black string with $\Lambda > 0$ and $d > 4$ will be presented elsewhere [20].

V. SUMMARY

The construction of solitons and black string solutions for the Einstein-Yang-Mills equations in a five-dimensional space-time and in the presence of a cosmological constant turns out to be numerically difficult. The problem was addressed in Ref. [18] but it appeared that the system of coordinates used was not satisfactory, leading to a coordinate singularity at some maximal value of the radial coordinate. Here we reconsidered the equation with an ansatz of the line element inspired from the literature about black strings (see e.g. Ref. [19]). It turns out that, with the new coordinates, the solutions can be continued up to $x = \infty$ and our numerical results suggest that the metric is of the Kasner-type asymptotically. This feature seems to hold true for pure gravity black strings as well as for the non-Abelian case. In the limit $x_h \to 0$ these two types of black strings approach a nontrivial solution which is regular at the origin.

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