Quantization and spectrum of an open 2-brane

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Starting from the free Polyakov action of an open 2-brane, we obtain the solutions to the Euler-Lagrange equation. By relating the normal modes to the raising and lowering operators and defining the vacuum state, we achieve the spectrum of the open 2-brane at different levels. Interestingly, there are two kinds of tachyon states appearing as scalar states and vector states, respectively. Besides, the graviton fields, Kalb-Ramond fields, dilaton, and photon states appear at the same level in the open 2-brane model.

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I. INTRODUCTION

String theory has emerged as the most promising candidate for a quantum theory of all known interactions [1–4]. The dynamical properties of strings are described by Nambu-Goto action, given in terms of the area of string world sheet. It can also be replaced by a classically equivalent action, called Polyakov action, involving an auxiliary world sheet metric and local conformal symmetry. In contrast with the nonpolynomial Nambu-Goto action the new action is quadratic in the derivatives of the coordinates. Some authors investigated the connections between the two kinds of actions by introducing the interpolating actions [5–7].By resolving the Euler-Lagrange equation derived from the Polyakov action and using variation with respect to spacetime coordinates, one can get the solution in terms of harmonic oscillators or normal modes, with which one can construct raising and lowering operators. After defining the vacuum state, one can obtain all the excited states by acting the raising operators on the vacuum state [3].

In addition to fundamental strings, in terms of which string theory is usually formulated, the theory contains other extended p-dimensional objects. The underlying extended objects that motivated recent progress in string theory are p-branes. They are extended structures embedded in a higher-dimensional spacetime from which it inherits an induced metric [8–11]. Over the last decade string theory has been gradually replaced by M-theory as the natural candidate for a fundamental description of nature. While a complete definition of M-theory is yet to be given, it is believed that the five perturbatively consistent string theories are different phases of this theory. Indeed it is known that membrane and five-brane occur naturally in 11-dimensional supergravity, which is argued to be the low-energy limit of M-theory. Also, string theory is effectively described by the low-energy dynamics of a system of branes. For instance, the membrane of M-theory may be "wrapped" around the compact direction of radius R to become the fundamental string of type-IIA string theory, in the limit of vanishing radius [12–18].

In this paper, we mainly work in 26-dimensional Minkowski spacetime, the arrangement is: in Sec. II, we obtain the solutions which satisfy the Neumann boundary conditions to the Euler-Lagrange equation from the free Polyakov action; in Sec. III, we give the Hamiltonian of the open 2-brane and deduce its representation in terms of normal modes; in Sec. IV, we define the vacuum state and get a series of excited states by acting the raising operators on it and discuss different levels. The last section is summary and conclusion.

II. SOLUTION TO EULER-LAGRANGE EQUATION OF 2-BRANE

An open 2-brane is a two dimensional object which sweeps out a three dimensional world-volume parametrized by τ, σ^1, and σ^2. These parameters are collectively referred to as $\xi_i (i = 0, 1, 2)$. Polyakov action for the 2-brane is then given by [5]

$$S_P = -\frac{1}{4\pi\alpha'}\int d^3\xi\sqrt{-h}[h^{\alpha\beta}\partial_\alpha X^\mu\partial_\beta X_\mu - 1], \quad (1)$$

Because $h = h_{ab}\Delta_{ab}$, $h^{ab} = \Delta_{ba}/h$, we have [1,4]

$$\delta h = -hh_{ab}\delta h^{ab}, \quad (2)$$

$$\delta\sqrt{-h} = -\tfrac{1}{2}\sqrt{-h}h_{ab}\delta h^{ab}, \quad (3)$$

and

$$\delta(\sqrt{-h}h^{cd}) = \sqrt{-h}(\delta h^{cd} - \tfrac{1}{2}h_{ab}h^{cd}\delta h^{ab}). \quad (4)$$

Let $\gamma_{ab} = \partial_a X^\mu\partial_b X_\mu$, the Euler-Lagrange equation for h^{ab}

$$\frac{\delta S}{\delta h^{ab}} - \partial_c\frac{\delta S}{\delta(\partial_c h^{ab})} = 0 \quad (5)$$

gives

$$\frac{\delta S}{\delta h^{ab}} = -\frac{\sqrt{-h}}{2\pi\alpha'}\left\{\gamma_{ab} - \frac{1}{2}h_{ab}h^{cd}\gamma_{cd} + \frac{1}{2}h_{ab}\right\} = 0. \quad (6)$$

Define the energy-momentum tensor as

$$T_{ab} = \frac{-2\pi\alpha'}{\sqrt{-h}}\frac{\delta S}{\delta h^{ab}} = \gamma_{ab} - \frac{1}{2}h_{ab}h^{cd}\gamma_{cd} + \frac{1}{2}h_{ab} = 0. \tag{7}$$

Choose the metric h_{ab} as $(-, +, +)$, then

$$T_{ab} = \gamma_{ab} - \tfrac{1}{2}h_{ab}(-\gamma_{00} + \gamma_{11} + \gamma_{22}) + \tfrac{1}{2}h_{ab} \tag{8}$$

$$\begin{aligned} T_{00} &= \gamma_{00} + \tfrac{1}{2}(-\gamma_{00} + \gamma_{11} + \gamma_{22}) - \tfrac{1}{2} \\ &= \tfrac{1}{2}(\gamma_{00} + \gamma_{11} + \gamma_{22} - 1) = 0, \end{aligned} \tag{9}$$

$$\begin{aligned} T_{11} &= \gamma_{11} - \tfrac{1}{2}(-\gamma_{00} + \gamma_{11} + \gamma_{22}) + \tfrac{1}{2} \\ &= \tfrac{1}{2}(\gamma_{00} + \gamma_{11} - \gamma_{22} + 1) = 0, \end{aligned} \tag{10}$$

$$\begin{aligned} T_{22} &= \gamma_{22} - \tfrac{1}{2}(-\gamma_{00} + \gamma_{11} + \gamma_{22}) + \tfrac{1}{2} \\ &= \tfrac{1}{2}(\gamma_{00} - \gamma_{11} + \gamma_{22} + 1) = 0. \end{aligned} \tag{11}$$

Equation (9) indicates that the Hamiltonian of a 2-brane system vanishes, which we will discuss later.

The Euler-Lagrange equation for X^μ can be derived from the variation with respect with X^μ

$$\delta S_P = 0 \tag{12}$$

directly as follows:

$$(\partial_\tau^2 - \partial_1^2 - \partial_2^2)X^\mu(\tau, \sigma^1, \sigma^2) = 0 \tag{13}$$

with the Neumann boundary conditions

$$\partial_{\sigma^1}X^\mu(\tau, 0, \sigma^2) = \partial_{\sigma^1}X^\mu(\tau, \pi, \sigma^2) = 0 \tag{14}$$

$$\partial_{\sigma^2}X^\mu(\tau, \sigma^1, 0) = \partial_{\sigma^2}X^\mu(\tau, \sigma^1, \pi) = 0. \tag{15}$$

On the other hand, because general physical processes should satisfy quantitative causal relation [19,20], e.g., Ref. [21] uses the no-loss-no-gain homeomorphic map transformation satisfying the quantitative causal relation to gain exact strain tensor formulas in Weitzenbök manifold. In fact, some changes (cause) of some quantities in Eq. (12) must result in the relative some changes (result) of the other quantities in Eq. (12) so that Eq. (12)'s right side keeps no-loss-no-gain, i.e., zero, namely, Eq. (12) also satisfies the quantitative causal relation, which just makes X^μ relative to $(\tau, \sigma^1, \sigma^2)$ to form a coupling physical system of different variables.

The canonical momenta for canonical variables X_μ are defined as

$$P^\mu = \frac{\partial \mathcal{L}}{\partial \dot{X}_\mu} = \frac{1}{2\pi\alpha'}\dot{X}^\mu. \tag{16}$$

Therefore, we can find out the solutions satisfying the boundary conditions to the Euler-Lagrange equation as follows:

$$X^0 = \frac{x^0}{\sqrt{\pi}} + \frac{2\alpha' p^0}{\sqrt{\pi}}, \qquad X^1 = \frac{x^1}{\sqrt{\pi}} + \frac{2\alpha' p^1}{\sqrt{\pi}} \tag{17}$$

$$\begin{aligned} X^I(\tau, \sigma^1, \sigma^2) &= \frac{x^I}{\sqrt{\pi}} + \frac{2\alpha' p^I}{\sqrt{\pi}}\tau \\ &\quad + i\sqrt{2\alpha'}\sum_{n,m=0}^{+\infty}(n^2 + m^2)^{-(1/4)} \\ &\quad \times (X^I_{nm}e^{i\tau\sqrt{n^2+m^2}} - (X^I_{nm})^\dagger e^{-i\tau\sqrt{n^2+m^2}}) \\ &\quad \times \cos n\sigma^1 \cos m\sigma^2 \end{aligned} \tag{18}$$

$$\begin{aligned} P^I(\tau, \sigma^1, \sigma^2) &= \frac{1}{\pi}\bigg[\frac{P^I}{\sqrt{\pi}} + \sqrt{\frac{2}{\alpha'}}\sum_{n,m=0}^{+\infty}(n^2 + m^2)^{1/4} \\ &\quad \times ((P^I_{nm})^\dagger e^{i\tau\sqrt{n^2+m^2}} + P^I_{nm}e^{-i\tau\sqrt{n^2+m^2}}) \\ &\quad \times \cos n\sigma^1 \cos m\sigma^2\bigg], \end{aligned} \tag{19}$$

where Eqs. (10) and (11) are two extra constraints which allow us to make some kind of special choices as Eq. (17) to the solutions. We have introduced $(X^I_{nm})^\dagger$ and $(P^I_{nm})^\dagger$ as Hermitian operators for X^I_{nm} and P^I_{nm}, respectively, in order to guarantee the Hermiticity of $X^I(\tau, \sigma^1, \sigma^2)$ and $P^I(\tau, \sigma^1, \sigma^2)$, and n, m cannot be zero simultaneously. Using (16), (18), and (19), we can set up the following relations

$$X^I_{nm} = -2(P^I_{nm})^\dagger; \qquad (X^I_{nm})^\dagger = -2P^I_{nm}. \tag{20}$$

It is easy to see that P^I_{nm} and X^I_{nm} themselves are not Hermitian operators.

In order to determine the commutative relations, we must calculate the commutative relations of $X^\mu(\tau, \sigma^1, \sigma^2)$ and $P^\mu(\tau, \sigma^1, \sigma^2)$ based on the Delta function as [2]

$$\delta(\sigma - \sigma') = \frac{1}{\pi}\bigg(1 + 2\sum_{n=1}^{+\infty}\cos n\sigma \cos n\sigma'\bigg). \tag{21}$$

Therefore, we have

$$
\begin{aligned}
[X^I(\tau,\sigma^1,\sigma^2), P^J(\tau,\sigma'^1,\sigma'^2)] &= \eta^{IJ}\delta(\sigma^1-\sigma'^1)\delta(\sigma^2-\sigma'^2) \\
&= \bigg[\frac{x^I}{\sqrt{\pi}} + \frac{2\alpha' p^I}{\sqrt{\pi}}\tau + i\sqrt{2\alpha'}\sum_{n_1,m_1=0}^{+\infty}(n_1^2+m_1^2)^{-(1/4)}(X^I_{n_1m_1}e^{i\tau\sqrt{n_1^2+m_1^2}} \\
&\quad - (X^I_{n_1m_1})^\dagger e^{-i\tau\sqrt{n_1^2+m_1^2}})\cos n_1\sigma^1\cos m_1\sigma^2, \frac{1}{\pi}\bigg\{\frac{P^J}{\sqrt{\pi}} + \sqrt{\frac{2}{\alpha'}}\sum_{n_2,m_2=0}^{+\infty}(n_2^2+m_2^2)^{1/4} \\
&\quad \times ((P^J_{n_2m_2})^\dagger e^{i\tau\sqrt{n_2^2+m_2^2}} + P^J_{n_2m_2}e^{-i\tau\sqrt{n_2^2+m_2^2}})\cos n_2\sigma^1\cos m_2\sigma^2\bigg\}\bigg]
\end{aligned}
\tag{22}
$$

which requires us to adopt the commutative relations between the normal modes as the form

$$
[X^I_{n_1m_1}, P^J_{n_2m_2}] = \frac{1}{\pi}\bigg[\eta^{IJ}\delta_{n_1n_2}\delta_{m_1m_2} - \frac{1}{2}\eta^{IJ}\delta_{-n_1,n_2}\delta_{m_1m_2} - \frac{1}{2}\eta^{IJ}\delta_{n_1n_2}\delta_{-m_1,m_2}\bigg], \tag{23}
$$

where n_i, $m_i \geq 0$, and n_i, m_i cannot be zero simultaneously. For n_i, $m_i > 0$, Eq. (23) can be divided into the following forms:

$$
[X^I_{n_1m_1}, P^J_{n_2m_2}] = \frac{1}{\pi}\eta^{IJ}\delta_{n_1n_2}\delta_{m_1m_2}, \tag{24}
$$

$$
[X^I_{n_10}, P^J_{n_20}] = \frac{1}{2\pi}\eta^{IJ}\delta_{n_1n_2}, \tag{25}
$$

$$
[X^I_{0m_1}, P^J_{0m_2}] = \frac{1}{2\pi}\eta^{\mu\nu}\delta_{m_1m_2}. \tag{26}
$$

Now we can construct the raising and lowering operators in the state space as $\phi^I_{n_1}\otimes\phi^I_{m_1}$, $\phi^{J\dagger}_{n_2}\otimes\phi^{J\dagger}_{m_2}$, and obtain the fundamental commutative relations

$$
[\phi^I_{n_1}\otimes\phi^I_{m_1}, \phi^{J\dagger}_{n_2}\otimes\phi^{J\dagger}_{m_2}] = \eta^{IJ}\delta_{n_1n_2}\otimes\delta_{m_1m_2}. \tag{27}
$$

It is natural to relate these raising and lowering operators to the modes X^I_{nm} and P^I_{nm} (or X^I_{nm} and $(X^I_{nm})^\dagger$).

In this section, we have quantized the open 2-brane. The quantization of the canonical variables requires a kind of commutative relations between the normal modes, which is important to construct the spectrum of the open 2-brane.

III. HAMILTANIAN OF THE OPEN 2-BRANE AND ITS REPRESENTATION

In order to obtain the mass-squared operator, we must calculate the Hamiltonian in terms of the raising and lowering operators. The Hamiltonian of the 2-brane can be derived from the Polyakov action (1) under the flat world-volume metrics as

$$
\begin{aligned}
H &= \int_0^\pi d\sigma^1\int_0^\pi d\sigma^2(P_\mu\dot{X}^\mu - \mathcal{L}) \\
&= \pi\alpha'\int_0^\pi d\sigma^1\int_0^\pi d\sigma^2\bigg(\frac{(\partial_0X_0)^2}{(2\pi\alpha')^2} + \frac{(\partial_0X_1)^2}{(2\pi\alpha')^2} + P_I^2 \\
&\quad + \frac{(\partial_1X_I)^2}{(2\pi\alpha')^2} + \frac{(\partial_2X_I)^2}{(2\pi\alpha')^2} - \frac{1}{(2\pi\alpha')^2}\bigg).
\end{aligned}
\tag{28}
$$

Here, different from the string case, there is an extra cosmological constant term, which is proportional to $\frac{1}{\alpha'}$. In the zero slope limit, this term will be very large, because of $\alpha' \sim \ell^2 \sim 10^{-66}$ cm, and then we can omit this term as the contributions of background fields.

Substituting solutions (17) and (18) into the Hamiltonian, we obtain the total Hamiltonian for the 2-brane in terms of normal modes as follows:

$$
\begin{aligned}
4\pi\alpha' H &= 2\alpha'\pi^2\eta_{IJ}\sum_{n_1=1}^{+\infty}n_1[(X^I_{n_10})^\dagger X^J_{n_10} + X^I_{n_10}(X^J_{n_10})^\dagger] \\
&\quad + 2\alpha'\pi^2\eta_{IJ}\sum_{m_1=1}^{+\infty}m_1[(X^I_{0m_1})^\dagger X^J_{0m_1} + X^I_{0m_1}(X^J_{0m_1})^\dagger] \\
&\quad + \alpha'\pi^2\eta_{IJ}\sum_{n_1,m_1=1}^{+\infty}(n_1^2+m_1^2)^{1/2}[(X^I_{n_1m_1})^\dagger X^J_{n_1m_1} \\
&\quad + X^I_{n_1m_1}(X^J_{n_1m_1})^\dagger] + 4\pi\alpha'^2p^2.
\end{aligned}
\tag{29}
$$

We can use Eqs. (24)–(26) to construct the raising and lowering operators

$$
\begin{cases} X^I_{n_1m_1} = \sqrt{\frac{2}{\pi}}\phi^{I\dagger}_{n_1}\otimes\phi^{I\dagger}_{m_1} \\ (X^I_{n_1m_1})^\dagger = \sqrt{\frac{2}{\pi}}\phi^I_{n_1}\otimes\phi^I_{m_1} \end{cases}, \tag{30}
$$

$$
\begin{cases} X^I_{n_10} = \sqrt{\frac{1}{\pi}}\phi^{I\dagger}_{n_1}\otimes\phi^{I\dagger}_{0} \\ (X^I_{n_10})^\dagger = \sqrt{\frac{1}{\pi}}\phi^I_{n_1}\otimes\phi^I_{0} \end{cases}, \tag{31}
$$

$$
\begin{cases} X^I_{0m_1} = \sqrt{\frac{1}{\pi}}\phi^{I\dagger}_{0}\otimes\phi^{I\dagger}_{m_1} \\ (X^I_{0m_1})^\dagger = \sqrt{\frac{1}{\pi}}\phi^I_{0}\otimes\phi^I_{m_1} \end{cases}, \tag{32}
$$

where we have used Eq. (20).

Using the on-shell conditions: $p^2 = -M^2$, we have

$$H = \eta_{IJ}\sum_{n_1=1}^{+\infty} n_1\left[[\phi_{n_1}^{I\dagger}\otimes\phi_0^{I\dagger}][\phi_{n_1}^{J}\otimes\phi_0^{J}] + \frac{1}{2}\eta^{IJ}\right] + \eta_{IJ}\sum_{m_1=1}^{+\infty} m_1\left[[\phi_0^{I\dagger}\otimes\phi_{m_1}^{I\dagger}][\phi_0^{J}\otimes\phi_{m_1}^{J}] + \frac{1}{2}\eta^{IJ}\right] + \eta_{IJ}\sum_{n_1,m_1=1}^{+\infty}(n_1^2+m_1^2)^{1/2}\left[[\phi_{n_1}^{I\dagger}\otimes\phi_{m_1}^{I\dagger}][\phi_{n_1}^{J}\otimes\phi_{m_1}^{J}] + \frac{1}{2}\eta^{IJ}\right] + \alpha' p^2 = N_1 + N_2 + N_{12} + a + b - \alpha' M^2, \tag{33}$$

where

$$N_1 = \eta_{IJ}\sum_{n=1}^{+\infty} n[\phi_n^{I+}\otimes\phi_0^{I+}][\phi_n^{J}\otimes\phi_0^{J}], \tag{34}$$

$$N_2 = \eta_{IJ}\sum_{m=1}^{+\infty} m[\phi_0^{I+}\otimes\phi_m^{I+}][\phi_0^{J}\otimes\phi_m^{J}], \tag{35}$$

$$N_{12} = \eta_{IJ}\sum_{n,m=1}^{+\infty}(n^2+m^2)^{1/2}[\phi_n^{I+}\otimes\phi_m^{I+}][\phi_n^{J}\otimes\phi_m^{J}], \tag{36}$$

$$a = \eta_I^I\sum_{n=1}^{+\infty} n = (D-2)\sum_{n=1}^{+\infty} n, \tag{37}$$

$$b = \frac{1}{2}\eta_I^I\sum_{n,m=1}^{+\infty}\sqrt{n^2+m^2} = \frac{(D-2)}{2}\sum_{n,m=1}^{+\infty}\sqrt{n^2+m^2}. \tag{38}$$

Without involving Fermi fields, we can choose the number of the dimension of spacetime 26, and use the Riemann zeta function

$$\zeta(-1) = \sum_n n = -\frac{1}{12}, \tag{39}$$

$$a = (D-2)\sum_{n=1}^{+\infty} n = -\frac{(D-2)}{12} = -2. \tag{40}$$

However, the number b is still infinity, and we have to remove it by viewing it as the effects of background fields or the vacuum zero point energies.

Thus, we give both Hamiltonian of the open 2-brane and its representation in terms of normal modes.

IV. SPECTRUM OF THE OPEN 2-BRANE

Reference [5] investigated the connections between Nambu-Goto action and Polyakov action by introducing the interpolating actions. Because of the internal constraints in phase space, one can find that the canonical Hamiltonian vanishes. The total Hamiltonian would be made up of internal constraints [5]. Equation (9) also shows this result

$$T_{00} = \tfrac{1}{2}(\gamma_{00}+\gamma_{11}+\gamma_{22}-1) = \tfrac{1}{2}[(\partial_0 X^\mu)^2 + (\partial_1 X^\mu)^2 + (\partial_2 X^\mu)^2 - 1] = 0. \tag{41}$$

Then the mass-squared operator is

$$\alpha' M^2 = N_1 + N_2 + N_{12} + a = N_1 + N_2 + N_{12} - 2. \tag{42}$$

The vacuum state $|0,0\rangle \equiv |0\rangle\otimes|0\rangle$ is defined to be annihilated by the lowering operators for n, $m \geq 0$, but n, m cannot be zero simultaneously:

$$\phi_n\otimes\phi_m|0,0\rangle = 0. \tag{43}$$

In general, a basis for the Fock space states can be taken of the form with the raising operators

$$|\lambda,\kappa\rangle = \prod_{n_i,m_i,I}\{\phi_{n_i}^{I\dagger}\otimes\phi_{m_i}^{I\dagger}\}|0,0\rangle, \tag{44}$$

where $\sum n = \lambda$, $\sum m = \kappa$. Because the mass-squared operator is symmetrical with respect to n and m in Eq. (42), the states $|\lambda,\kappa\rangle$ and $|\kappa,\lambda\rangle$ must have the same mass-square. So we can obtain two sets of states with the same masses except for $\lambda = \kappa$.

Now, let us see how this works for the first some levels of the open 2-brane in flat spacetime by removing the contributions of background fields.

At the lowest mass level, the only state is $|0,0\rangle$, which means $N_1 = N_2 = N_{12} = 0$. Then the mass-squared operator can be calculated:

$$\alpha' M^2 = N_1 + N_2 + N_{12} + a = 0 + 0 + 0 + a = -2. \tag{45}$$

It is a tachyon state, and the mass-square is

$$M^2 = -\frac{2}{\alpha'}. \tag{46}$$

At the next level there are two kinds of excited states corresponding to $n = 1$, $m = 0$ and $n = 0$, $m = 1$, respectively:

$$|1,0\rangle = k_I(\phi_1^{I\dagger}\otimes\phi_0^{I\dagger})|0,0\rangle, \tag{47}$$

$$|0,1\rangle = l_I(\phi_0^{I\dagger}\otimes\phi_1^{I\dagger})|0,0\rangle. \tag{48}$$

The mass-squared operators can be calculated as

$$\alpha' M_k^2 = N_1 + N_2 + N_{12} + a = 1 + 0 + 0 + (-2) = -1, \tag{49}$$

$$\alpha' M_l^2 = N_1 + N_2 + N_{12} + a = 0 + 1 + 0 + (-2) = -1. \tag{50}$$

Obviously, they are still the tachyon states with the same mass-square:

$$M_k^2 = M_l^2 = -\frac{1}{\alpha'}. \tag{51}$$

So far, we have obtained more tachyon states than string cases including the usual ground state, and the first excited states which associate with two types of vector states. In string theory, a series of developments starting in 1999 have essentially elucidated the role of the open string tachyon. The presence of tachyon indicates instability of open string theory. More precisely, there is some instability in the theory of open string on the background of a space-filling D25-brane. For quite a few years, superstring theories, i.e., the kind of string theories that also include fermions, seemed blessedly devoid of tachyons. Later studies, however, showed that tachyons can appear when we construct realistic models based on superstrings. In our 2-brane model, there are tachyons not only as scalar states, but vector states, appearing.

The next level is the second excited states, and there are four kinds of state corresponding to

$$\begin{cases} n = 0 \\ m = 2, \end{cases} \qquad \begin{cases} n = 2 \\ m = 0 \end{cases} \quad \begin{cases} n_1 = n_2 = 1 \\ m_1 = m_2 = 0 \end{cases} \quad \begin{cases} n_1 = n_2 = 0 \\ m_1 = m_2 = 1, \end{cases} \tag{52}$$

respectively:

$$|2, 0\rangle_1 = k_{IJ}(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger})(\phi_1^{J\dagger} \otimes \phi_0^{J\dagger})|0, 0\rangle, \tag{53}$$

$$|2, 0\rangle_2 = k_I(\phi_2^{I\dagger} \otimes \phi_0^{I\dagger})|0, 0\rangle, \tag{54}$$

$$|0, 2\rangle_1 = l_{IJ}(\phi_0^{I\dagger} \otimes \phi_1^{I\dagger})(\phi_0^{J\dagger} \otimes \phi_1^{J\dagger})|0, 0\rangle, \tag{55}$$

$$|0, 2\rangle_2 = l_I(\phi_0^{I\dagger} \otimes \phi_I^{\dagger})|0, 0\rangle. \tag{56}$$

Now we have $N_1 = 2$, $N_2 = N_{12} = 0$ or $N_2 = 2$, $N_1 = N_{12} = 0$, so the mass-squared operator is

$$\alpha' M^2 = 0. \tag{57}$$

Here we also find that more massless states than string cases, especially 2-rank tensor states, vector states, and scalar states, all have been produced at the same level. Let us discuss these massless states. We will take the states $|2, 0\rangle_1$ and $|2, 0\rangle_2$ for example. Here k_{IJ} can be viewed as the elements of an arbitrary square matrix of size $D - 2$. Any square matrix can be decomposed into its traceless symmetric part, its antisymmetric part and a multiple of the unit matrix:

$$k_{IJ} = \hat{S}_{IJ} + A_{IJ} + \delta_{IJ} S', \tag{58}$$

where $\hat{S}_{IJ}$ denotes the traceless symmetric part of $k_{\mu\nu}$, and A_{IJ} denotes the antisymmetric part of k_{IJ}, and S' the trace of k_{IJ}. Then the states can be decomposed into the following forms:

$$\hat{S}_{IJ}(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger})(\phi_1^{J\dagger} \otimes \phi_0^{J\dagger})|0, 0\rangle, \tag{59}$$

$$A_{IJ}(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger})(\phi_1^{J\dagger} \otimes \phi_0^{J\dagger})|0, 0\rangle, \tag{60}$$

$$S'(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger})(\phi_{1I}^{\dagger} \otimes \phi_{0I}^{\dagger})|0, 0\rangle. \tag{61}$$

In closed bosonic string theory, Eq. (59) represents one-particle graviton states; Eq. (60) corresponds to the one-particle states of Kalb-Ramond fields; Eq. (61) has no free indices, so it represents one scalar state called the dilaton state. While the states $|2, 0\rangle_2$ simply represent the massless vector states, which can be related to the photon states. The same procedure can be practiced to the states $|0, 2\rangle_1$ and $|0, 2\rangle_2$. So we have two sets of spectrum of massless states resulted from the symmetric directions of σ^1 and σ^2.

Then we will consider the first massive level, which has two kinds of excited states corresponding to $n = 0$, $m = 3$ and $n = 3$, $m = 0$

$$|3, 0\rangle_1 = k_I(\phi_3^{I\dagger} \otimes \phi_0^{I\dagger})|0, 0\rangle, \tag{62}$$

$$|3, 0\rangle_2 = k_{IJ}(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger})(\phi_2^{J\dagger} \otimes \phi_0^{J\dagger})|0, 0\rangle, \tag{63}$$

$$|3, 0\rangle_3 = k_{IJK}(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger})(\phi_1^{J\dagger} \otimes \phi_0^{J\dagger})(\phi_1^{K\dagger} \otimes \phi_0^{K\dagger})|0, 0\rangle, \tag{64}$$

$$|0, 3\rangle_1 = l_I(\phi_0^{I\dagger} \otimes \phi_3^{I\dagger})|0, 0\rangle, \tag{65}$$

$$|0, 3\rangle_2 = l_{IJ}(\phi_0^{I\dagger} \otimes \phi_1^{I\dagger})(\phi_0^{J\dagger} \otimes \phi_2^{J\dagger})|0, 0\rangle, \tag{66}$$

$$|0, 3\rangle_3 = l_{IJK}(\phi_0^{I\dagger} \otimes \phi_1^{I\dagger})(\phi_0^{J\dagger} \otimes \phi_1^{J\dagger})(\phi_0^{K\dagger} \otimes \phi_1^{K\dagger})|0, 0\rangle. \tag{67}$$

Now with the numbers $N_2 = N_{12} = 0$, $N_1 = 3$ and $N_1 = N_{12} = 0$, $N_2 = 3$, we have the mass-squared operator as

$$\alpha' M_k^2 = N_1 + N_2 + N_{12} + a = 3 + 0 + 0 + (-2) = 1, \tag{68}$$

$$\alpha' M_l^2 = N_1 + N_2 + N_{12} + a = 0 + 3 + 0 + (-2) = 1. \tag{69}$$

In this level, the 3-rank tensor field appears.

The following massive level will be the $n = m = 1$ case, while $N_1 = N_2 = 1$ and $N_{12} = \sqrt{1^2 + 1^2} = \sqrt{2}$.

$$|1, 1\rangle_1 = k_I(\phi_1^{I\dagger} \otimes \phi_1^{I\dagger})|0, 0\rangle, \tag{70}$$

$$|1, 1\rangle_2 = k_{IJ}(\phi_1^{I\dagger} \otimes \phi_0^{I\dagger})(\phi_0^{J\dagger} \otimes \phi_1^{J\dagger})|0, 0\rangle. \tag{71}$$

Then the mass-squared operator is

$$\alpha' M^2 = N_1 + N_2 + N_{12} + a = 1 + \sqrt{2} + 1 + (-2) = \sqrt{2}. \tag{72}$$

They belong to the second massive level, including vector states, 2-rank traceless tensor states and scalar states. The mass-square is an irrational number which differs from the string theory.

Now we have obtained different levels of Fock states, the higher levels can analogously be discussed, which do not have more new physical laws to give, and due to length limit of the paper, we do not do further. In these levels, tachyon states, massless states, and massive states all appeared, and we find that there are more fruitful contents than the string case.

V. SUMMARY AND CONCLUSION

In this paper, we have investigated the quantization and the spectrum of the open 2-brane from the free Polyakov action. According to the Euler-Lagrange equation and the Neumann boundary conditions, we have gained the solution to the equation by analogy with the string case. At the first glance, this solution is a bit strange from the string's. However, it is just a suitable solution to guarantee the natural quantization of the canonical variables. Besides, we have derived the commutative relations between the normal modes of the solution.

To investigate the spectrum of the 2-brane, it is natural and necessary to relate the commutative relations between the normal modes to the raising and lowering operators and to define the vacuum states which must be annihilated by the lowering operators. And the basis for the Fock space states can be taken of the form with the raising operators acting on the vacuum state. Benefitting from the work of Ref. [5–7], we can choose the vanishing of the Hamiltonian. On the construction of the spectrum, we have removed the infinite contribution of the background.

In the fourth section, we have discussed the spectrum of the open 2-brane explicitly. Because of the appearance of one more spacial direction, from Eq. (33) we conclude that the mass-square αM^2 has been lowered (e.g., $a = -1$ for string while -2 for open 2-brane), which makes the first two levels relative to tachyon states and the third level relative to the massless states. Because of the exchange symmetry between the two spacial directions, we find two copies of states at each level except the case $n = m$. Besides, for the first two levels, there are two classes of tachyon states appearing as scalar states and vector states, respectively, which are different from the string theory. In string theory, a tachyon state has the mass-square $-1/\alpha'$ and it represents an excitation of the D-brane with open strings attached, which can lower the energy of the D-brane. So the existence of the tachyon is telling us that the D-brane is unstable and will decay. In the 2-brane model, we find from Eq. (46) that the scalar tachyon state has lower mass-square than that of the string case, while the tachyon state as a vector will also contribute to the lowering of the energy of the D-brane with open 2-branes attached. As a result, both the contributions of the two types of tachyon states must be incorporated into the tachyon potential, then the D-brane will be more unstable and decay more easily.

More interestingly, some massless states, such as the graviton states, Kalb-Ramond fields, dilaton states, and photon states, are all produced at the same level in the open 2-brane model. In string theory, however, graviton fields, Kalb-Ramond fields, and dilaton fields only appear in closed bosonic string theory and photon states in the open string theory. Whereafter, we also give two massive levels, including Eqs. (62)–(67), (70), and (71).

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