

**Reconstruction of hessence dark energy and the latest type Ia supernovae gold dataset**

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Recently, many efforts have been made to build dark energy models whose equation-of-state parameter can cross the so-called phantom divide  $w_{\text{de}} = -1$ . One of them is the so-called hessence dark energy model in which the role of dark energy is played by a noncanonical complex scalar field. In this work, we develop a simple method based on Hubble parameter  $H(z)$  to reconstruct the hessence dark energy. As examples, we use two familiar parametrizations for  $H(z)$  and fit them to the latest 182 type Ia supernovae Gold dataset. In the reconstruction, measurement errors are fully considered.

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**I. INTRODUCTION**

Dark energy [1] has been one of the most active fields in modern cosmology since the discovery of accelerated expansion of our universe [2–8]. The simplest candidate of dark energy is a tiny positive cosmological constant. However, as is well-known, it is plagued with the “cosmological constant problem” and “coincidence problem” [1]. In the observational cosmology of dark energy, equation-of-state parameter (EoS)  $w_{\text{de}} \equiv p_{\text{de}}/\rho_{\text{de}}$  plays an important role, where  $p_{\text{de}}$  and  $\rho_{\text{de}}$  are the pressure and energy density of dark energy, respectively. The most important difference between cosmological constant and dynamical scalar fields is that the EoS of the former is always a constant,  $-1$ , while the EoS of the latter can be variable during the evolution of the universe.

Recently, evidence for  $w_{\text{de}}(z) < -1$  at redshift  $z < 0.2 \sim 0.3$  has been found by fitting observational data (see [9–17] for examples). In addition, many best-fits of the present value of  $w_{\text{de}}$  are less than  $-1$  in various data fittings with different parametrizations (see [18] for a recent review). The present data seem to slightly favor an evolving dark energy with  $w_{\text{de}}$  being below  $-1$  around present epoch from  $w_{\text{de}} > -1$  in the near past [10]. Obviously, the EoS cannot cross the so-called phantom divide  $w_{\text{de}} = -1$  for quintessence or phantom alone. Some efforts have been made to build dark energy model whose EoS can cross the phantom divide (see for examples [10,19–32] and references therein).

In [10], Feng, Wang and Zhang proposed a so-called quintom model which is a hybrid of quintessence and phantom (thus the name quintom). Phenomenologically, one may consider a Lagrangian density [10,24,25]

$$\mathcal{L}_{\text{quintom}} = \frac{1}{2}(\partial_{\mu}\phi_1)^2 - \frac{1}{2}(\partial_{\mu}\phi_2)^2 - V(\phi_1, \phi_2), \quad (1)$$

where  $\phi_1$  and  $\phi_2$  are two real scalar fields and play the roles of quintessence and phantom, respectively. Considering a spatially flat Friedmann-Robertson-Walker (FRW) universe and assuming the scalar fields  $\phi_1$  and  $\phi_2$  are homogeneous, one obtains the effective pressure and energy density for the quintom, i.e.

$$\begin{aligned} p_{\text{quintom}} &= \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 - V(\phi_1, \phi_2), \\ \rho_{\text{quintom}} &= \frac{1}{2}\dot{\phi}_1^2 - \frac{1}{2}\dot{\phi}_2^2 + V(\phi_1, \phi_2), \end{aligned} \quad (2)$$

respectively. The corresponding effective EoS is given by

$$w_{\text{quintom}} = \frac{\dot{\phi}_1^2 - \dot{\phi}_2^2 - 2V(\phi_1, \phi_2)}{\dot{\phi}_1^2 - \dot{\phi}_2^2 + 2V(\phi_1, \phi_2)}. \quad (3)$$

It is easy to see that  $w_{\text{quintom}} \geq -1$  when  $\dot{\phi}_1^2 \geq \dot{\phi}_2^2$  while  $w_{\text{quintom}} < -1$  when  $\dot{\phi}_1^2 < \dot{\phi}_2^2$ . The transition occurs when  $\dot{\phi}_1^2 = \dot{\phi}_2^2$ . The cosmological evolution of the quintom dark energy was studied in [24,25]. Perturbations of the quintom dark energy were investigated in [33,34]; and it is found that the quintom model is stable when EoS crosses  $-1$ , in contrast to many dark energy models whose EoS can cross the phantom divide [29].

In [19], by a new view of quintom dark energy, one of us (H.W.) and his collaborators proposed a novel noncanonical complex scalar field, which was named “hessence”, to play the role of quintom. In the hessence model, the phantomlike role is played by the so-called internal motion  $\theta$ , where  $\theta$  is the internal degree of freedom of hessence. The transition from  $w_h > -1$  to  $w_h < -1$  or vice versa is also possible in the hessence model [19]. We will briefly review the main points of hessence model in Sec. II. The cosmological evolution of the hessence dark energy was studied in [20] and then was extended to the more general cases in [21]. The  $w$ - $w'$  analysis of hessence dark energy was performed in [22].

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In this work, we are interested in reconstructing the hessence dark energy. In fact, reconstruction of cosmological models is an important task of modern cosmology. For instance, the inflaton potential reconstruction was extensively studied in [35] and references therein. The parametrizations and reconstruction of quintessence/phantom was considered in [36,37]. The other recent reconstructions of quintessence also include e.g. [38–44]. The reconstruction of k-essence was studied in [45–47]. For the reconstructions of other cosmological models, see [48–52] for examples. We refer to [53] for a recent review on the reconstructions of dark energy. In this paper, after a brief review of the hessence model, we develop a simple method based on the Hubble parameter  $H(z)$  to reconstruct the hessence dark energy. As examples, we use two familiar parametrizations for  $H(z)$  and fit them to the latest 182 type Ia supernovae (SNe Ia) Gold dataset [7]. In the reconstruction, measurement errors are fully considered.

## II. HESSENCE DARK ENERGY

Following [19,20], we consider a noncanonical complex scalar field as the dark energy, namely, hessence,

$$\Phi = \phi_1 + i\phi_2, \quad (4)$$

with a Lagrangian density

$$\begin{aligned} \mathcal{L}_h &= \frac{1}{4}[(\partial_\mu \Phi)^2 + (\partial_\mu \Phi^*)^2] - U(\Phi^2 + \Phi^{*2}) \\ &= \frac{1}{2}[(\partial_\mu \phi)^2 - \phi^2(\partial_\mu \theta)^2] - V(\phi), \end{aligned} \quad (5)$$

where we have introduced two new variables  $(\phi, \theta)$  to describe the hessence, i.e.

$$\phi_1 = \phi \cosh\theta, \quad \phi_2 = \phi \sinh\theta, \quad (6)$$

which are defined by

$$\phi^2 = \phi_1^2 - \phi_2^2, \quad \coth\theta = \frac{\phi_1}{\phi_2}. \quad (7)$$

In fact, it is easy to see that the hessence can be regarded as a special case of quintom dark energy in terms of  $\phi_1$  and  $\phi_2$ . Considering a spatially flat FRW universe with scale factor  $a(t)$  and assuming  $\phi$  and  $\theta$  are homogeneous, from Eq. (5) we obtain the equations of motion for  $\phi$  and  $\theta$ ,

$$\ddot{\phi} + 3H\dot{\phi} + \phi\dot{\theta}^2 + V_{,\phi} = 0, \quad (8)$$

$$\phi^2\ddot{\theta} + (2\phi\dot{\phi} + 3H\phi^2)\dot{\theta} = 0, \quad (9)$$

where  $H \equiv \dot{a}/a$  is the Hubble parameter, a dot and the subscript “ $\phi$ ” denote the derivatives with respect to cosmic time  $t$  and  $\phi$ , respectively. The pressure and energy density of the hessence are

$$\begin{aligned} p_h &= \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) - V(\phi), \\ \rho_h &= \frac{1}{2}(\dot{\phi}^2 - \phi^2\dot{\theta}^2) + V(\phi), \end{aligned} \quad (10)$$

respectively. Equation (9) implies

$$Q = a^3\phi^2\dot{\theta} = \text{const.} \quad (11)$$

which is associated with the total conserved charge within the physical volume due to the internal symmetry [19,20]. It turns out

$$\dot{\theta} = \frac{Q}{a^3\phi^2}. \quad (12)$$

Substituting into Eqs. (8) and (10), they can be rewritten as

$$\ddot{\phi} + 3H\dot{\phi} + \frac{Q^2}{a^6\phi^3} + V_{,\phi} = 0, \quad (13)$$

$$p_h = \frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6\phi^2} - V(\phi), \quad (14)$$

$$\rho_h = \frac{1}{2}\dot{\phi}^2 - \frac{Q^2}{2a^6\phi^2} + V(\phi).$$

It is worth noting that Eq. (13) is equivalent to the energy conservation equation of hessence, namely,  $\dot{\rho}_h + 3H(\rho_h + p_h) = 0$ . The Friedmann equation and Raychaudhuri equation are given by

$$H^2 = \frac{1}{3M_{\text{pl}}^2}(\rho_h + \rho_m), \quad (15)$$

$$\dot{H} = -\frac{1}{2M_{\text{pl}}^2}(\rho_h + \rho_m + p_h), \quad (16)$$

where  $\rho_m$  is the energy density of dust matter;  $M_{\text{pl}} \equiv (8\pi G)^{-1/2}$  is the reduced Planck mass. The EoS of hessence  $w_h \equiv p_h/\rho_h$ . It is easy to see that  $w_h \geq -1$  when  $\dot{\phi}^2 \geq Q^2/(a^6\phi^2)$ , while  $w_h < -1$  when  $\dot{\phi}^2 < Q^2/(a^6\phi^2)$ . The transition occurs when  $\dot{\phi}^2 = Q^2/(a^6\phi^2)$ . We refer to the original papers [19,20] for more details.

## III. RECONSTRUCTION OF HESSENCE DARK ENERGY

Here, we develop a simple reconstruction method based on the Hubble parameter  $H(z)$  for hessence dark energy. From Eqs. (15) and (16), we get

$$V(\phi) = 3M_{\text{pl}}^2 H^2 + M_{\text{pl}}^2 \dot{H} - \frac{1}{2}\rho_m, \quad (17)$$

and

$$\dot{\phi}^2 - \frac{Q^2}{a^6\phi^2} = -2M_{\text{pl}}^2 \dot{H} - \rho_m. \quad (18)$$

Note that

$$\dot{f} = -(1+z)H \frac{df}{dz}$$

for any function  $f$ , where  $z = a^{-1} - 1$  is the redshift (we set  $a_0 = 1$ ; the subscript “0” indicates the present value of the corresponding quantity). We can recast

Eqs. (17) and (18) as

$$V(z) = 3M_{\text{pl}}^2 H^2 - M_{\text{pl}}^2 (1+z) H \frac{dH}{dz} - \frac{1}{2} \rho_{m0} (1+z)^3, \quad (19)$$

$$\left(\frac{d\phi}{dz}\right)^2 - \frac{Q^2}{\phi^2} (1+z)^4 H^{-2} = 2M_{\text{pl}}^2 (1+z)^{-1} H^{-1} \frac{dH}{dz} - \rho_{m0} (1+z) H^{-2}. \quad (20)$$

Introducing the following dimensionless quantities

$$\begin{aligned} \tilde{V} &\equiv \frac{V}{M_{\text{pl}}^2 H_0^2}, & \tilde{\phi} &\equiv \frac{\phi}{M_{\text{pl}}}, \\ \tilde{H} &\equiv \frac{H}{H_0}, & \tilde{Q} &\equiv \frac{Q}{M_{\text{pl}}^2 H_0}, \end{aligned} \quad (21)$$

Equations (19) and (20) can be rewritten as

$$\tilde{V}(z) = 3\tilde{H}^2 - (1+z)\tilde{H} \frac{d\tilde{H}}{dz} - \frac{3}{2} \Omega_{m0} (1+z)^3, \quad (22)$$

$$\begin{aligned} \left(\frac{d\tilde{\phi}}{dz}\right)^2 - \tilde{Q}^2 \tilde{\phi}^{-2} (1+z)^4 \tilde{H}^{-2} \\ = 2(1+z)^{-1} \tilde{H} \frac{d\tilde{H}}{dz} - 3\Omega_{m0} (1+z) \tilde{H}^{-2}, \end{aligned} \quad (23)$$

where  $\Omega_{m0} \equiv \rho_{m0}/(3M_{\text{pl}}^2 H_0^2)$  is the present fractional energy density of dust matter. Once the  $\tilde{H}(z)$ , or the  $H(z)$ , is given, we can reconstruct  $V(z)$  and  $\phi(z)$  by using Eqs. (22) and (23) respectively. Then, the potential  $V(\phi)$  can be reconstructed from  $V(z)$  and  $\phi(z)$  readily. By using Eqs. (18) and (22), we can reconstruct the EoS of hessence

$$w_h(z) \equiv \frac{p_h}{\rho_h} = \frac{-1 + \frac{2}{3}(1+z) \frac{d \ln \tilde{H}}{dz}}{1 - \Omega_{m0} \tilde{H}^{-2} (1+z)^3}. \quad (24)$$

The deceleration parameter

$$q(z) \equiv -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2} = -1 + (1+z)\tilde{H}^{-1} \frac{d\tilde{H}}{dz}. \quad (25)$$

After all, it is of interest to reconstruct the kinetic energy term of hessence,  $K \equiv \dot{\phi}^2/2 - Q^2/(2a^6 \phi^2)$ . From Eq. (18), we have

$$\tilde{K}(z) \equiv \frac{K}{M_{\text{pl}}^2 H_0^2} = (1+z)\tilde{H} \frac{d\tilde{H}}{dz} - \frac{3}{2} \Omega_{m0} (1+z)^3. \quad (26)$$

It is worth noting that the reconstruction method presented here is sufficiently versatile for any  $H(z)$ .

#### IV. EXAMPLES

In this section, as examples, we consider two familiar parametrizations for  $H(z)$  and fit them to the latest 182 SNe Ia Gold dataset [7]. And then, we reconstruct the EoS of

hessence  $w_h(z)$ , deceleration parameter  $q(z)$ , the kinetic energy term of hessence  $K(z)$ , the potential of hessence  $V(z)$ , and the  $\phi(z)$  as functions of the redshift  $z$ . Also, we reconstruct the potential of hessence as function of  $\phi$ , namely  $V(\phi)$ . In our reconstruction, measurement errors are fully considered.

#### A. Parametrizations for $H(z)$ and the latest 182 SNe Ia Gold dataset

The latest 182 SNe Ia Gold dataset compiled in [7] provides the apparent magnitude  $m(z)$  of the supernovae at peak brightness after implementing corrections for galactic extinction, K-correction, and light curve width-luminosity correction. The resulting apparent magnitude  $m(z)$  is related to the luminosity distance  $d_L(z)$  through (see e.g. [54,72])

$$m_{\text{th}}(z) = \bar{M}(M, H_0) + 5 \log_{10} D_L(z), \quad (27)$$

where

$$D_L(z) = (1+z) \int_0^z d\tilde{z} \frac{H_0}{H(\tilde{z}; \text{parameters})} \quad (28)$$

is the Hubble-free luminosity distance  $H_0 d_L/c$  in a spatially flat FRW universe ( $c$  is the speed of light); and

$$\bar{M} = M + 5 \log_{10} \left( \frac{c H_0^{-1}}{\text{Mpc}} \right) + 25 = M - 5 \log_{10} h + 42.38 \quad (29)$$

is the magnitude zero offset ( $h$  is  $H_0$  in units of 100 km/s/Mpc); the absolute magnitude  $M$  is assumed to be constant after the corrections mentioned above. The data points of the latest 182 SNe Ia Gold dataset compiled in [7] are given in terms of the distance modulus

$$\mu_{\text{obs}}(z_i) \equiv m_{\text{obs}}(z_i) - M. \quad (30)$$

On the other hand, the theoretical distance modulus is defined as

$$\mu_{\text{th}}(z_i) \equiv m_{\text{th}}(z_i) - M = 5 \log_{10} D_L(z_i) + \mu_0, \quad (31)$$

where

$$\mu_0 \equiv 42.38 - 5 \log_{10} h. \quad (32)$$

The theoretical model parameters are determined by minimizing

$$\chi^2(\text{parameters}) = \sum_{i=1}^{182} \frac{[\mu_{\text{obs}}(z_i) - \mu_{\text{th}}(z_i)]^2}{\sigma^2(z_i)}, \quad (33)$$

where  $\sigma$  is the corresponding  $1\sigma$  error. The parameter  $\mu_0$  is a nuisance parameter but it is independent of the data points. One can perform an uniform marginalization over  $\mu_0$ . However, there is an alternative way. Following [54–56], the minimization with respect to  $\mu_0$  can be made by expanding the  $\chi^2$  of Eq. (33) with respect to  $\mu_0$  as

$$\chi^2(\text{parameters}) = A - 2\mu_0 B + \mu_0^2 C, \quad (34)$$

where

$$A(\text{parameters}) = \sum_{i=1}^{182} \frac{[m_{\text{obs}}(z_i) - m_{\text{th}}(z_i; \mu_0 = 0, \text{parameters})]^2}{\sigma_{m_{\text{obs}}}^2(z_i)},$$

$$B(\text{parameters}) = \sum_{i=1}^{182} \frac{m_{\text{obs}}(z_i) - m_{\text{th}}(z_i; \mu_0 = 0, \text{parameters})}{\sigma_{m_{\text{obs}}}^2(z_i)}, \quad C = \sum_{i=1}^{182} \frac{1}{\sigma_{m_{\text{obs}}}^2(z_i)}.$$

Equation (34) has a minimum for  $\mu_0 = B/C$  at

$$\tilde{\chi}^2(\text{parameters}) = A(\text{parameters}) - \frac{B(\text{parameters})^2}{C}. \quad (35)$$

Therefore, we can instead minimize  $\tilde{\chi}^2$  which is independent of  $\mu_0$ , since  $\chi_{\text{min}}^2 = \tilde{\chi}_{\text{min}}^2$  obviously.

In this work, we consider two familiar parametrizations for  $H(z)$  and fit them to the latest 182 SNe Ia Gold data [7]. At first, we consider the Ansatz I with

$$H(z) = H_0[\Omega_{m0}(1+z)^3 + A_1(1+z) + A_2(1+z)^2 + (1 - \Omega_{m0} - A_1 - A_2)]^{1/2}, \quad (36)$$

which has been discussed in [13,16,45,50,51,57]. Obviously, it includes  $\Lambda$ CDM and XCDM with particular time-independent EoS of dark energy as special cases. As shown in [45,57], even for the cases where this ansatz is not exact, one can recover the luminosity distance to within 0.5% accuracy using this ansatz in the relevant redshift range for the old 157 SNe Ia Gold dataset [2]. Therefore, this ansatz is trustworthy to some extent. Here, by fitting it to the latest 182 SNe Ia Gold data [7], for the prior  $\Omega_{m0} = 0.30$  [58], we find that the best fit parameters (with  $1\sigma$  errors) are  $A_1 = -3.28 \pm 2.11$  and  $A_2 = 1.36 \pm 0.84$ , while  $\tilde{\chi}_{\text{min}}^2 = 156.53$  for 180 degrees of freedom. The corresponding covariance matrix [59] (see also [57]) is given by

$$\text{Cov}(A_1, A_2) = \begin{pmatrix} 4.469 & -1.774 \\ -1.774 & 0.711 \end{pmatrix}. \quad (37)$$

In Fig. 1, we present the corresponding 68% and 95% confidence level (c.l.) contours in the  $A_1$ - $A_2$  parameter space.

Next, we consider the Ansatz II with

$$H(z) = H_0 \left[ \Omega_{m0}(1+z)^3 + (1 - \Omega_{m0})(1+z)^{3(1+w_0+w_1)} \times \exp\left(-\frac{3w_1 z}{1+z}\right) \right]^{1/2}, \quad (38)$$

which is in fact equivalent to the familiar parametrization  $w_{\text{de}} = w_0 + w_1 z/(1+z)$  [15,16,50,60,61]. By fitting it to the latest 182 SNe Ia Gold dataset [7], for the prior  $\Omega_{m0} = 0.30$  [58], we find that the best fit parameters (with  $1\sigma$  errors) are  $w_0 = -1.43 \pm 0.32$  and  $w_1 = 2.79 \pm 1.55$ ,

while  $\tilde{\chi}_{\text{min}}^2 = 156.56$  for 180 degrees of freedom. The corresponding covariance matrix reads

$$\text{Cov}(w_0, w_1) = \begin{pmatrix} 0.101 & -0.466 \\ -0.466 & 2.407 \end{pmatrix}. \quad (39)$$

In Fig. 2, we present the corresponding 68% and 95% c.l. contours in the  $w_0$ - $w_1$  parameter space.

## B. Reconstruction results

In our reconstruction, measurement errors are fully considered. The well-known error propagation equation for any  $y(x_1, x_2, \dots, x_n)$ ,

$$\sigma^2(y) = \sum_i^n \left( \frac{\partial y}{\partial x_i} \right)_{x=\bar{x}}^2 \text{Cov}(x_i, x_i) + 2 \sum_{i=1}^n \sum_{j=i+1}^n \left( \frac{\partial y}{\partial x_i} \frac{\partial y}{\partial x_j} \right)_{x=\bar{x}} \text{Cov}(x_i, x_j), \quad (40)$$

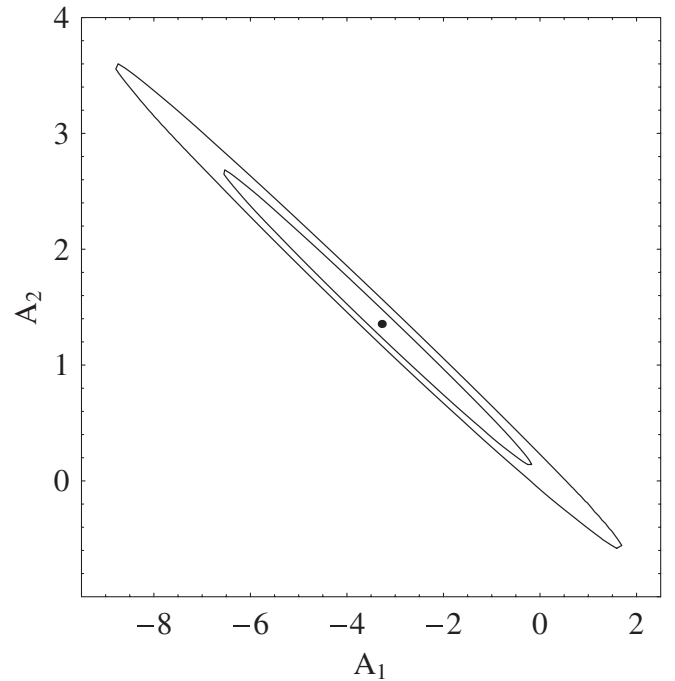


FIG. 1. The 68% and 95% confidence level contours in the  $A_1$ - $A_2$  parameter space for Ansatz I with the prior  $\Omega_{m0} = 0.30$ . The best fit parameters are also indicated by a solid point.

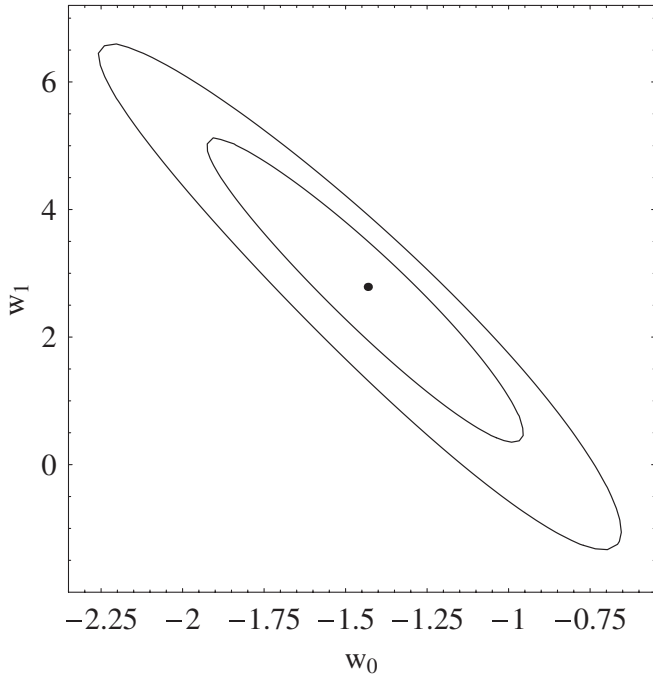


FIG. 2. The 68% and 95% c.i. contours in the  $w_0$ - $w_1$  parameter space for Ansatz II with the prior  $\Omega_{m0} = 0.30$ . The best fit parameters are also indicated by a solid point.

is used extensively (see [59] for instance). For Ansatz I with the prior  $\Omega_{m0} = 0.30$ , by using Eqs. (22), (24)–(26), (37), and (40) and the corresponding best fit values of  $A_1$  and  $A_2$ , we can reconstruct the EoS of hessence  $w_h(z)$ , deceleration parameter  $q(z)$ , the potential of hessence  $V(z)$ , and the kinetic energy term of hessence  $K(z)$  as functions of the redshift  $z$ , with the corresponding  $1\sigma$  errors. We show the results in Fig. 3. It is easy to see that  $w_h$  crossed  $-1$  and the universe transited from deceleration ( $q > 0$ ) to acceleration ( $q < 0$ ); the reconstructed  $w_h(z)$  is well consistent with the three uncorrelated  $\mathcal{W}_{0.25}$ ,  $\mathcal{W}_{0.70}$  and  $\mathcal{W}_{1.35}$  data points with their corresponding  $1\sigma$  error bars for the weak prior [7] which are model-independent.

However, the error propagation equation (40) is invalid when we reconstruct the  $\phi(z)$  and hence the  $V(\phi)$ , since  $\phi(z)$  is obtained from a differential equation, i.e. Eq. (23). To evaluate the error propagations, we use the Monte Carlo method instead. That is, we generate a multivariate Gaussian distribution from the best fit parameters and the corresponding covariance matrix. And then, we randomly sample  $N$  pairs of the parameters  $\{A_1, A_2\}$  from this distribution. For each pair of  $\{A_1, A_2\}$ , we can find the corresponding  $\phi(z)$  and  $V(z)$  from Eqs. (22) and (23) respectively. Hence, the  $V(\phi)$  is in hand. Finally, we can

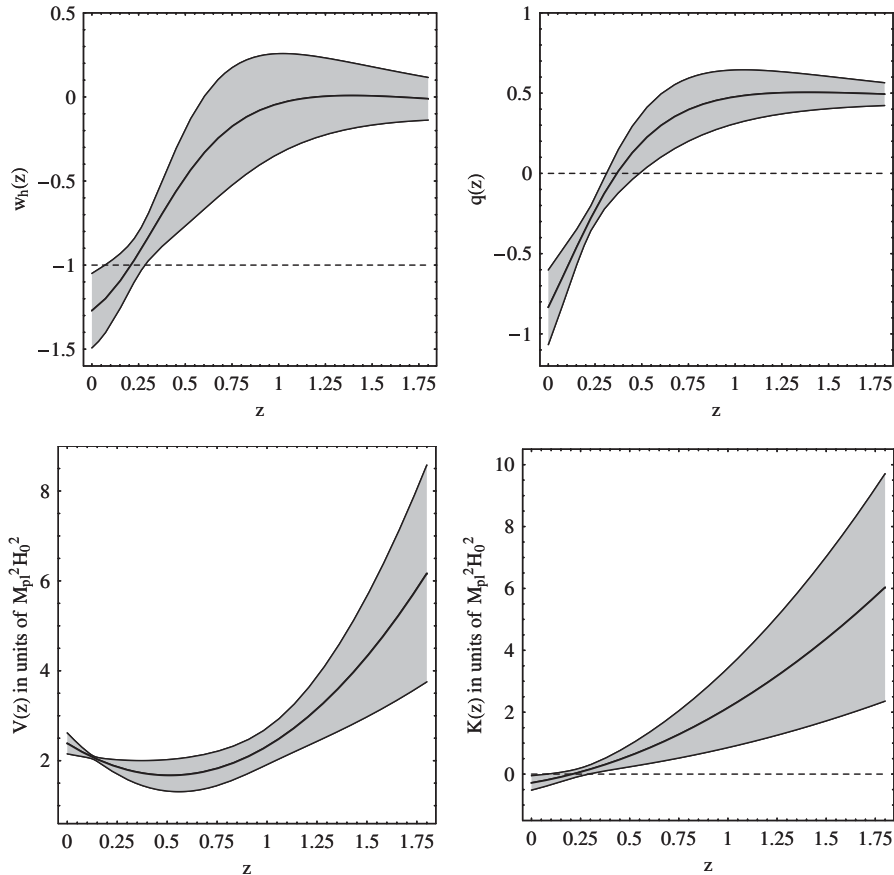


FIG. 3. The reconstructed best fit  $w_h(z)$ ,  $q(z)$ ,  $V(z)$  and  $K(z)$  (thick solid lines) with the corresponding  $1\sigma$  errors (shaded region) for Ansatz I.

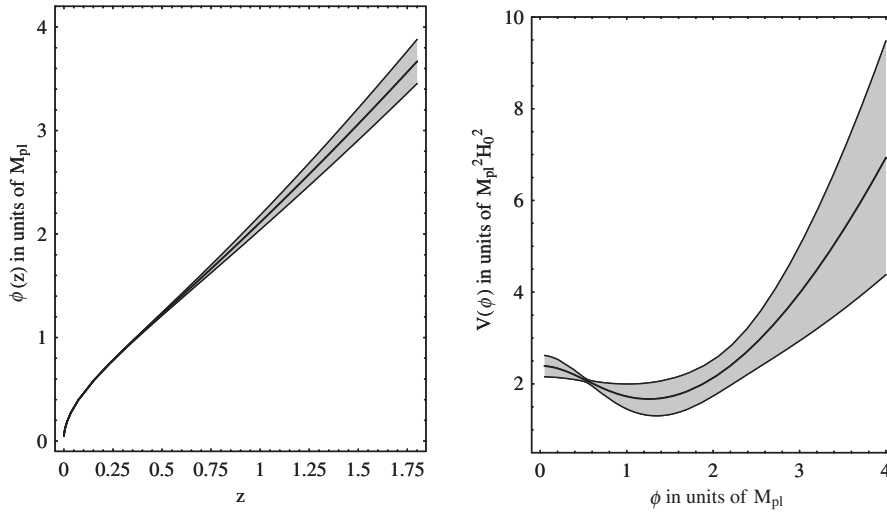


FIG. 4. The reconstructed  $\phi(z)$  and  $V(\phi)$  (thick solid lines) with the corresponding  $1\sigma$  errors (shaded region) for Ansatz I. See text for details.

determine the mean and the corresponding  $1\sigma$  error for the  $\phi(z)$  and  $V(\phi)$  from these  $N$  samples, respectively. In Fig. 4, we show the reconstructed  $\phi(z)$  and  $V(\phi)$  with the corresponding  $1\sigma$  errors for Ansatz I with the prior

$\Omega_{m0} = 0.30$ . In which, we have used the demonstrative initial value  $\tilde{\phi}_0 = 0.05$  at  $z = 0$  and  $\tilde{Q} = 1$ , and have chosen the solution with  $d\tilde{\phi}/dz > 0$  for the reconstructed  $\phi(z)$ ; we have done  $N = 1000$  samplings.

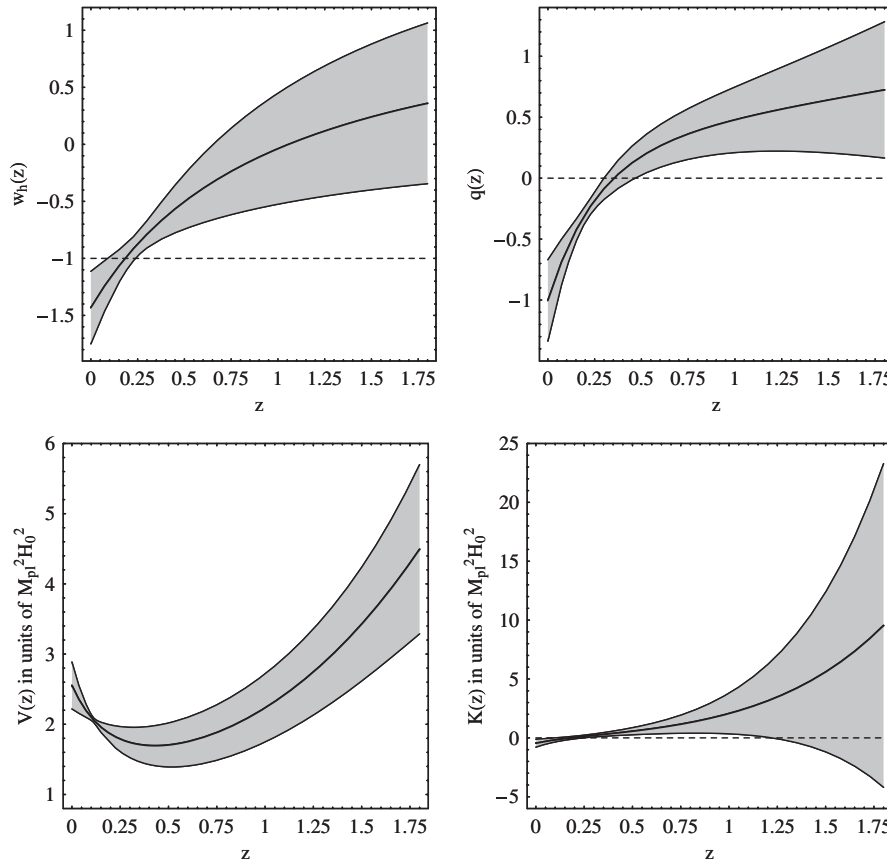


FIG. 5. The reconstructed best fit  $w_h(z)$ ,  $q(z)$ ,  $V(z)$  and  $K(z)$  (thick solid lines) with the corresponding  $1\sigma$  errors (shaded region) for Ansatz II.

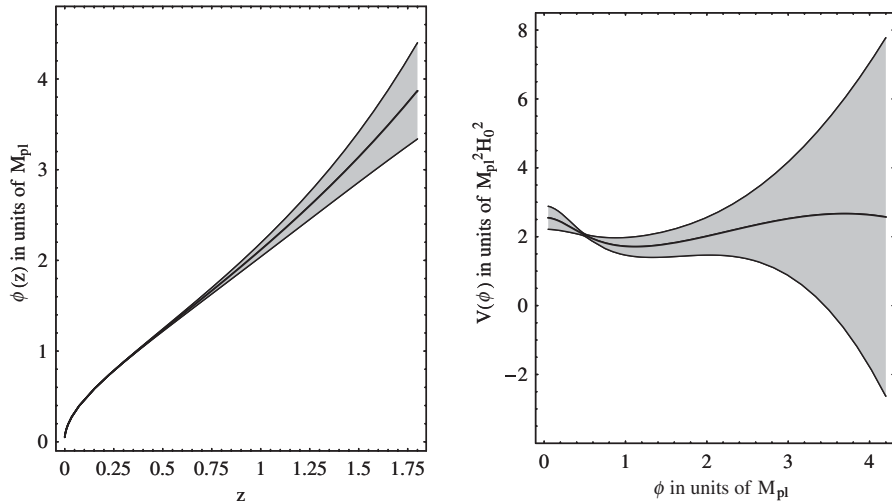


FIG. 6. The reconstructed  $\phi(z)$  and  $V(\phi)$  (thick solid lines) with the corresponding  $1\sigma$  errors (shaded region) for Ansatz II. See text for details.

For Ansatz II, the method to reconstruct the EoS of hessence  $w_h(z)$ , deceleration parameter  $q(z)$ , the kinetic energy term of hessence  $K(z)$ , the potential of hessence  $V(z)$ , the  $\phi(z)$  as functions of the redshift  $z$ , and the potential of hessence as function of  $\phi$ , namely  $V(\phi)$ , is the same for Ansatz I. We present the results in Figs. 5 and 6. Once again, it is easy to see that  $w_h$  crossed  $-1$  and the universe transited from deceleration ( $q > 0$ ) to acceleration ( $q < 0$ ); the reconstructed  $w_h(z)$  is well consistent with the three uncorrelated  $\mathcal{W}_{0.25}$ ,  $\mathcal{W}_{0.70}$  and  $\mathcal{W}_{1.35}$  data points with their corresponding  $1\sigma$  error bars for the weak prior [7] which are model-independent.

## V. DISCUSSION

In this work, we have developed a simple method based on the Hubble parameter  $H(z)$  to reconstruct the hessence dark energy. If the observational  $H(z)$  is obtained, the reconstruction of hessence dark energy is ready. It is worth noting that the reconstruction method presented here is sufficiently versatile for any  $H(z)$ .

As examples, we reconstructed the hessence dark energy with two familiar parametrizations for  $H(z)$ . It is easy to see that this reconstruction method works well. However, these parametrizations for  $H(z)$  are not the direct measure of  $H(z)$  from observational data. We can just say that they are consistent with the recent observational data. Can we be in a more comfortable situation? In fact, some efforts are aiming to a *direct measure* of  $H(z)$  from observational data. Analogous to the estimates of  $w(z)$  [9], a method to obtaining the estimates of  $H(z)$  is proposed in [12] (see also [62]). Actually, in the latest paper [7] by the

Supernova Search Team led by Riess, a rough estimate of  $H(z)$  is obtained by using the new Hubble Space Telescope discoveries of SNe Ia at  $z \geq 1$ . Other new method to determine the Hubble parameter as function of redshift,  $H(z)$ , is also proposed in [63] recently. In a very different way, by using the differential ages of passively evolving galaxies determined from the Gemini Deep Deep Survey (GDDS) [64] and archival data [65], Simon *et al.* determined  $H(z)$  in the range  $0 \leq z \leq 1.8$  [66–71]. However, up to now, all observational  $H(z)$  obtained by various methods are too rough to give a reliable reconstruction of hessence dark energy. A good news from [68] is that a large amount of  $H(z)$  data is expected to become available in the next few years. These include data from the AGN and Galaxy Survey (AGES) and the Atacama Cosmology Telescope (ACT), and by 2009 an order of magnitude increase in  $H(z)$  data is anticipated. Therefore, we are optimistic to the feasibility of the reconstruction method of hessence dark energy proposed in this work.

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