

**New physics and  $CP$  violation in singly Cabibbo suppressed  $D$  decays**

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We analyze various theoretical aspects of  $CP$  violation in singly Cabibbo suppressed (SCS)  $D$  meson decays, such as  $D \rightarrow KK, \pi\pi$ . In particular, we explore the possibility that  $CP$  asymmetries will be measured close to the present level of experimental sensitivity of  $\mathcal{O}(10^{-2})$ . Such measurements would signal new physics. We make the following points: (i) The mechanism at work in neutral  $D$  decays could be indirect or direct  $CP$  violation (or both). (ii) One can experimentally distinguish between these possibilities. (iii) If the dominant  $CP$  violation is indirect, then there are clear predictions for other modes. (iv) Tree-level direct  $CP$  violation in various known models is constrained to be much smaller than  $10^{-2}$ . (v) SCS decays, unlike Cabibbo favored or doubly Cabibbo suppressed decays, are sensitive to new contributions from QCD penguin operators and especially from chromomagnetic dipole operators. This point is illustrated with supersymmetric gluino-squark loops, which can yield direct  $CP$  violating effects of  $\mathcal{O}(10^{-2})$ .

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**I. INTRODUCTION**

$CP$  violation (CPV) in  $D$  meson decays provides a unique probe of new physics. First, the standard model (SM) predicts very small effects, smaller than  $\mathcal{O}(10^{-3})$ , so that a signal at the present level of experimental sensitivity [1–7],  $\mathcal{O}(10^{-2})$ , would clearly signal new physics. Second, the neutral  $D$  system is the only one where the external up-sector quarks are involved. Thus it probes models in which the up sector plays a special role, such as supersymmetric models with alignment [8,9] and, more generally, models in which Cabibbo-Kobayashi-Maskawa (CKM) mixing is generated in the up sector. Third, singly Cabibbo suppressed (SCS) decays are sensitive to new physics contributions to penguin and dipole operators.

Let us elaborate on the first point, that is, the smallness of  $CP$  violation within the SM. The basic argument is that the physics of both  $D^0 - \bar{D}^0$  mixing and SCS  $D$  decays involves, to an excellent approximation, only the first two quark generations and is therefore  $CP$  conserving [10]. In other words, SM  $CP$  violation in these decays is CKM suppressed. As concerns the  $D^0 - \bar{D}^0$  mixing amplitude, SM  $CP$  violation enters at  $\mathcal{O}[(V_{cb}V_{ub})/(V_{cs}V_{us})] \sim 10^{-3}$ . Furthermore, this suppression is relative to the short distance contribution, which is known to lie well below the present experimental sensitivity. (The SM contribution

could saturate the present bounds on  $y$  [11] and  $x$  [12], but this would necessarily be due to the long distance contribution.) The  $CP$  violation contribution to the  $c \rightarrow u\bar{s}s$  and  $c \rightarrow u\bar{d}d$  decays is both CKM- and loop-suppressed and, therefore, entirely negligible. We conclude that  $CP$  violation in SCS  $D$  decays at the percent level signals new physics [13–15].

As concerns the third point, among all hadronic  $D$  decays, the SCS decays are uniquely sensitive to  $CP$  violation in  $c \rightarrow u\bar{q}q$  transitions and, consequently, to new contributions to the  $\Delta C = 1$  QCD penguin and chromomagnetic dipole operators. In particular, such contributions can affect neither the Cabibbo favored ( $c \rightarrow s\bar{d}u$ ) nor the doubly Cabibbo suppressed ( $c \rightarrow d\bar{s}u$ ) decays.

In Secs. II and III we present the formalism of  $CP$  violation in SCS  $D$  decays. For final  $CP$  eigenstates, indirect  $CP$  violation is universal. Thus, for example, equal time-integrated  $CP$  asymmetries in  $D \rightarrow K^+K^-$  and  $D \rightarrow \pi^+\pi^-$  would be a signal for indirect  $CP$  violation. By combining time-dependent and time-integrated measurements it is possible to separate out the universal indirect and generally nonuniversal direct  $CP$  asymmetry contributions. In the case of final non- $CP$  eigenstates, such as  $\rho^\pm\pi^\mp$  or  $K^{*\pm}K^\mp$ , a Dalitz plot analysis allows one to further separate out the indirect  $CP$  asymmetries originating from  $CP$  violation in mixing and from  $CP$  violation in the interference of decays with and without mixing, and to separately determine the neutral  $D$ -meson mass and lifetime differences, up to discrete ambiguities.

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In Secs. IV and V we discuss direct  $CP$  violation. In Sec. IV, we survey models which give rise to direct  $CP$  violation in SCS decays via *tree*-level decay amplitudes, e.g., flavor-changing  $Z$  or  $Z'$  couplings or supersymmetric  $R$ -parity violating couplings. We find that typically these contributions are constrained to lie well below the present experimental sensitivity.

In Sec. V we discuss *loop*-induced effects. Here the situation is different, as direct  $CP$  violation at the level of  $10^{-2}$  is often allowed and, for specific models, even expected. Two specific supersymmetric examples employing up-squark/gluino loops are discussed: contributions to the dipole operators due to flavor-changing “left-right” (LR) squark mixing, and contributions to the QCD penguin and dipole operators due to flavor-changing “left-left” (LL) squark mixing. Remarkably, we find that LR squark mixing can yield direct  $CP$  violation at the current level of sensitivity, while indirect  $CP$  violation remains negligible. The key factor is a strong enhancement of the requisite quark chirality flip in the dipole operators by a factor  $m_{\bar{g}}/m_c$  which is absent in the mixing amplitude. For LL squark mixing, annihilation leads to an order-of-magnitude uncertainty in the QCD penguin operator matrix elements, so that direct  $CP$  asymmetries of  $\mathcal{O}(10^{-2})$  cannot be ruled out. In this case, however, indirect  $CP$  violation is also non-negligible. Implications for  $CP$  violation in supersymmetric flavor models with alignment, which predict the orders of magnitude of the LR and LL squark mixings, are discussed.

In this analysis, some hadronic subtleties are involved. We employ naive factorization to evaluate the impact of new contributions to the QCD penguin operators, and QCD factorization [16] to estimate the contributions of chromomagnetic dipole operators. We argue that there is a large theoretical uncertainty related to annihilation in both (SM) tree and (new physics) penguin contributions: Experimental information as well as hadronic models lead us to think that annihilation could play a prominent role and, in particular, strongly enhance the latter. Details are provided in the appendix. Finally, isospin invariance and, to a lesser extent,  $U$ -spin invariance of the gluonic transitions predict patterns of direct  $CP$  violation among various SCS decay modes. These can be used to test for new contributions to the QCD penguin and dipole operators.

We conclude in Sec. VI with a summary of our results and a brief discussion of additional decay modes which will be useful for learning about the possible intervention of new physics in SCS  $D$  meson decays.

## II. FORMALISM

The SCS decays,  $c \rightarrow u\bar{s}s$  and  $c \rightarrow u\bar{d}d$ , lead to final states that are common to  $D^0$  and  $\bar{D}^0$ . These could be  $CP$  eigenstates (such as  $K^+K^-$ ,  $\pi^+\pi^-$ ,  $\phi\pi^0$ , and  $\rho^0\phi^0$ ), or non- $CP$  eigenstates (such as  $\rho^+\pi^-$ ,  $K^{*+}K^-$ , and  $K^{*0}K_S$ ).

We use the following standard notations:

$$\begin{aligned} \tau &\equiv \Gamma_D t, & \Gamma_D &\equiv \frac{\Gamma_{D_H} + \Gamma_{D_L}}{2}, \\ A_f &\equiv A(D^0 \rightarrow f), & \bar{A}_f &\equiv A(\bar{D}^0 \rightarrow f), \\ A_{\bar{f}} &\equiv A(D^0 \rightarrow \bar{f}), & \bar{A}_{\bar{f}} &\equiv A(\bar{D}^0 \rightarrow \bar{f}), \\ x &\equiv \frac{\Delta m_D}{\Gamma_D} \equiv \frac{m_{D_H} - m_{D_L}}{\Gamma_D}, & y &\equiv \frac{\Delta \Gamma_D}{2\Gamma_D} \equiv \frac{\Gamma_{D_H} - \Gamma_{D_L}}{2\Gamma_D}, \\ \lambda_f &\equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}, & R_m &\equiv \left| \frac{q}{p} \right|, & R_f &\equiv \left| \frac{\bar{A}_f}{A_f} \right|. \end{aligned} \quad (1)$$

Here  $D_H$  and  $D_L$  stand for the heavy and light mass eigenstates, and  $q$  and  $p$  are defined via  $|D_{H,L}\rangle = p|D^0\rangle \mp q|\bar{D}^0\rangle$ .

The time-dependent decay rates into a final state  $f$  can be written as follows (see, for example, [17]):

$$\begin{aligned} \Gamma(D^0(t) \rightarrow f) &= \frac{1}{2} e^{-\tau} |A_f|^2 \{ (1 + |\lambda_f|^2) \cosh(y\tau) \\ &\quad + (1 - |\lambda_f|^2) \cos(x\tau) + 2\mathcal{R}e(\lambda_f) \\ &\quad \times \sinh(y\tau) - 2\mathcal{I}m(\lambda_f) \sin(x\tau) \}, \end{aligned} \quad (2)$$

$$\begin{aligned} \Gamma(\bar{D}^0(t) \rightarrow f) &= \frac{1}{2} e^{-\tau} |\bar{A}_f|^2 \{ (1 + |\lambda_f^{-1}|^2) \cosh(y\tau) \\ &\quad + (1 - |\lambda_f^{-1}|^2) \cos(x\tau) + 2\mathcal{R}e(\lambda_f^{-1}) \\ &\quad \times \sinh(y\tau) - 2\mathcal{I}m(\lambda_f^{-1}) \sin(x\tau) \}. \end{aligned} \quad (3)$$

The time-integrated rates are given by

$$\begin{aligned} \Gamma(D^0 \rightarrow f) &= \int_0^\infty \Gamma(D^0(t) \rightarrow f) dt = \frac{1}{2} |A_f|^2 \left\{ (1 + |\lambda_f|^2) \frac{1}{1 - y^2} + (1 - |\lambda_f|^2) \frac{1}{1 + x^2} \right. \\ &\quad \left. + 2\mathcal{R}e(\lambda_f) \frac{y}{1 - y^2} - 2\mathcal{I}m(\lambda_f) \frac{x}{1 + x^2} \right\}, \end{aligned} \quad (4)$$

$$\begin{aligned} \Gamma(\bar{D}^0 \rightarrow f) &= \int_0^\infty \Gamma(\bar{D}^0(t) \rightarrow f) dt = \frac{1}{2} |\bar{A}_f|^2 \left\{ (1 + |\lambda_f^{-1}|^2) \frac{1}{1 - y^2} + (1 - |\lambda_f^{-1}|^2) \frac{1}{1 + x^2} \right. \\ &\quad \left. + 2\mathcal{R}e(\lambda_f^{-1}) \frac{y}{1 - y^2} - 2\mathcal{I}m(\lambda_f^{-1}) \frac{x}{1 + x^2} \right\}. \end{aligned} \quad (5)$$

The corresponding expressions for decays into  $\bar{f}$  follow via the substitutions  $f \rightarrow \bar{f}$  in the above expressions.

In general the four decay amplitudes can be written as

$$\begin{aligned} A_f &= A_f^T e^{+i\phi_f^T} [1 + r_f e^{i(\delta_f + \phi_f)}], \\ A_{\bar{f}} &= A_{\bar{f}}^T e^{i(\Delta_f + \phi_{\bar{f}}^T)} [1 + r_{\bar{f}} e^{i(\delta_{\bar{f}} + \phi_{\bar{f}})}], \\ \bar{A}_{\bar{f}} &= A_{\bar{f}}^T e^{-i\phi_{\bar{f}}^T} [1 + r_f e^{i(\delta_f - \phi_f)}], \\ \bar{A}_f &= A_{\bar{f}}^T e^{i(\Delta_f - \phi_{\bar{f}}^T)} [1 + r_{\bar{f}} e^{i(\delta_{\bar{f}} - \phi_{\bar{f}})}], \end{aligned} \quad (6)$$

where  $A_f^T e^{\pm i\phi_f^T}$  is the SM tree-level contribution. The phases  $\phi_f^T$ ,  $\phi_{\bar{f}}^T$ ,  $\phi_f$ , and  $\phi_{\bar{f}}$  are weak,  $CP$  violating phases, while  $\Delta_f$  and  $\delta_f$  are strong,  $CP$  conserving phases. Neglecting terms of order  $|(V_{ub}V_{cb})/(V_{us}V_{cs})| \sim 10^{-3}$ ,  $\phi_f^T = \phi_{\bar{f}}^T$  is the same for all final states.

### A. $CP$ eigenstates

We consider final states that are  $CP$  eigenstates. (Note that this analysis also applies to Cabibbo favored (CF)  $CP$  eigenstates, like  $K_S \pi^0$ .) For a similar analysis see [18]. For  $CP$  even (odd) eigenstates,  $\Delta_f = 0$  ( $\pi$ ). We can then write

$$\begin{aligned} A_f &= A_f^T e^{+i\phi_f^T} [1 + r_f e^{i(\delta_f + \phi_f)}], \\ \eta_f^{CP} \bar{A}_f &= A_f^T e^{-i\phi_f^T} [1 + r_f e^{i(\delta_f - \phi_f)}], \end{aligned} \quad (7)$$

where  $\eta_f^{CP} = +(-)$  for  $CP$  even (odd) states. Neglecting  $r_f$  in Eq. (7),  $\lambda_f$  is universal and we can define

$$\lambda_f \equiv -\eta_f^{CP} R_m e^{i\phi}, \quad (8)$$

where  $R_m \equiv |q/p|$  and  $\phi$  is the relative weak phase between the mixing amplitude and the decay amplitude. The time-integrated  $CP$  asymmetry for a final  $CP$  eigenstate  $f$  is defined as follows:

$$a_f \equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow f)}. \quad (9)$$

Given experimental constraints, we take  $x, y, r_f \ll 1$  and expand to leading order in these parameters. Then, we can separate the contributions to  $a_f$  to three parts,

$$a_f = a_f^d + a_f^m + a_f^i, \quad (10)$$

with the following underlying mechanisms:

(i)  $a_f^d$  signals  $CP$  violation in decay:

$$a_f^d = 2r_f \sin\phi_f \sin\delta_f. \quad (11)$$

(ii)  $a_f^m$  signals  $CP$  violation in mixing. With our approximations, it is universal:

$$a_f^m = -\eta_f^{CP} \frac{y}{2} (R_m - R_m^{-1}) \cos\phi. \quad (12)$$

(iii)  $a_f^i$  signals  $CP$  violation in the interference of decays with and without mixing. With our approximations, it is universal:

$$a_f^i = \eta_f^{CP} \frac{x}{2} (R_m + R_m^{-1}) \sin\phi. \quad (13)$$

Consider the time-dependent decay rates in Eqs. (2) and (3). The mixing processes modify the time dependence from a pure exponential. However, given the small values of  $x$  and  $y$ , the time dependences can be recast, to a good approximation, into purely exponential forms,

$$\begin{aligned} \Gamma(D^0(t) \rightarrow f) &\propto \exp[-\hat{\Gamma}_{D^0 \rightarrow f} t], \\ \Gamma(\bar{D}^0(t) \rightarrow f) &\propto \exp[-\hat{\Gamma}_{\bar{D}^0 \rightarrow f} t], \end{aligned} \quad (14)$$

with modified decay rate parameters [15]:

$$\begin{aligned} \hat{\Gamma}_{D^0 \rightarrow f} &= \Gamma_D [1 + \eta_f^{CP} R_m (y \cos\phi - x \sin\phi)], \\ \hat{\Gamma}_{\bar{D}^0 \rightarrow f} &= \Gamma_D [1 + \eta_f^{CP} R_m^{-1} (y \cos\phi + x \sin\phi)]. \end{aligned} \quad (15)$$

One can define the following  $CP$  violating combination of these two observables:

$$\Delta Y_f \equiv \frac{\hat{\Gamma}_{\bar{D}^0 \rightarrow f} - \hat{\Gamma}_{D^0 \rightarrow f}}{2\Gamma_D} = a_f^m + a_f^i. \quad (16)$$

Note that  $a_f^m$  and  $a_f^i$  contribute to  $a_f$  of Eq. (9) and  $\Delta Y_f$  of Eq. (16) in the same way, but  $a_f^d$  contributes only to the former. In particular,  $\Delta Y_f$  is universal while  $a_f$ , in general, is not.

The experimental results from *BABAR* [1],  $\Delta Y = (-0.8 \pm 0.6 \pm 0.2) \times 10^{-2}$ , and from *Belle* [2],  $\Delta Y = (+0.20 \pm 0.63 \pm 0.30) \times 10^{-2}$ , give the following world average:

$$\Delta Y = (-0.35 \pm 0.47) \times 10^{-2}. \quad (17)$$

### B. Non- $CP$ eigenstates

Here we consider final states that are not  $CP$  eigenstates. For each pair of  $CP$  conjugate states  $f$  and  $\bar{f}$ , there are four relevant amplitudes, Eq. (6). Neglecting  $r_f$  and  $r_{\bar{f}}$  we have

$$\begin{aligned} \lambda_f &\equiv \frac{q}{p} \frac{\bar{A}_f}{A_f} = -R_m R_f e^{i(\phi + \Delta_f)}, \\ \lambda_{\bar{f}} &\equiv \frac{q}{p} \frac{\bar{A}_{\bar{f}}}{A_{\bar{f}}} = -R_m R_f^{-1} e^{i(\phi - \Delta_f)}. \end{aligned} \quad (18)$$

Here  $R_m$  and  $\phi$  are the same as in Eq. (8),  $R_f \equiv |\bar{A}_f/A_f|$ , and  $\Delta_f$  is a strong ( $CP$ -conserving) phase. There are two time-integrated  $CP$  asymmetries to consider:

$$\begin{aligned}
a_f &\equiv \frac{\Gamma(D^0 \rightarrow f) - \Gamma(\bar{D}^0 \rightarrow \bar{f})}{\Gamma(D^0 \rightarrow f) + \Gamma(\bar{D}^0 \rightarrow \bar{f})}, \\
a_{\bar{f}} &\equiv \frac{\Gamma(D^0 \rightarrow \bar{f}) - \Gamma(\bar{D}^0 \rightarrow f)}{\Gamma(D^0 \rightarrow \bar{f}) + \Gamma(\bar{D}^0 \rightarrow f)}.
\end{aligned} \tag{19}$$

Again, we take  $x, y, r_f, r_{\bar{f}} \ll 1$  and expand to leading order in these parameters. Then

$$a_f = a_f^d + a_f^m + a_f^i, \quad a_{\bar{f}} = a_{\bar{f}}^d + a_{\bar{f}}^m + a_{\bar{f}}^i, \tag{20}$$

where

$$\begin{aligned}
a_f^d &= 2r_f \sin\phi_f \sin\delta_f, \\
a_f^m &= -R_f \frac{y_f'}{2} (R_m - R_m^{-1}) \cos\phi, \\
a_f^i &= R_f \frac{x_f'}{2} (R_m + R_m^{-1}) \sin\phi,
\end{aligned} \tag{21}$$

(for  $a_{\bar{f}}$  the result is the same with the replacement  $f \rightarrow \bar{f}$ ) and

$$\begin{aligned}
x_f' &= x \cos\Delta_f + y \sin\Delta_f, & y_f' &= y \cos\Delta_f - x \sin\Delta_f, \\
x_{\bar{f}}' &= x \cos\Delta_f - y \sin\Delta_f, & y_{\bar{f}}' &= y \cos\Delta_f + x \sin\Delta_f.
\end{aligned} \tag{22}$$

Since in SCS decays we expect, in general, that  $R_f = \mathcal{O}(1)$ , the decays into final non- $CP$  eigenstates should exhibit  $CP$  asymmetries of the same order of magnitude as for  $CP$  eigenstates.

Several points are in order:

- (1) One can, again, look for  $CP$  violation using the time dependence of the decay, see Eq. (14). The result is similar to Eq. (16):

$$\begin{aligned}
\Delta Y_f &\equiv \frac{\hat{\Gamma}_{\bar{D}^0 \rightarrow \bar{f}} - \hat{\Gamma}_{D^0 \rightarrow f}}{2\Gamma_D} = a_f^m + a_f^i, \\
\Delta Y_{\bar{f}} &\equiv \frac{\hat{\Gamma}_{D^0 \rightarrow \bar{f}} - \hat{\Gamma}_{\bar{D}^0 \rightarrow f}}{2\Gamma_D} = a_{\bar{f}}^m + a_{\bar{f}}^i,
\end{aligned} \tag{23}$$

where  $a_f^m$  and  $a_f^i$  are given in Eq. (21).

- (2) A final state that is a  $CP$  eigenstate is a special case of the non- $CP$  final state, with  $R_f = 1$  and  $\Delta_f = 0$  ( $\pi$ ) for  $CP$  even (odd) final state. Then, Eqs. (21) reduce to Eqs. (11)–(13).
- (3) In analyses of CF and doubly Cabibbo suppressed (DCS) decays, such as  $D \rightarrow K\pi$ , one usually finds expressions that depend on  $x_f'$  and  $y_f'$ , but not on  $x_{\bar{f}}'$  and  $y_{\bar{f}}'$  (see e.g. [19]). The reason is not that the  $CP$  asymmetries are independent of  $x_{\bar{f}}'$  and  $y_{\bar{f}}'$ , but rather that these contributions are relatively suppressed by  $\tan^4\theta_c$ .

### C. Dalitz plot analysis for $D^0 \rightarrow VP$

In practice, all final non- $CP$  eigenstates are resonances. Thus, we can perform a Dalitz plot analysis and sum up several resonances. Such an analysis has several advantages. First, the statistics is increased. Second, information about the strong phases can be obtained. A simple case is to concentrate on a single resonance in the Dalitz plot, for example,  $KK^*$ . Then, from the interference region of  $K^+K^{*-}$  with  $K^-K^{*+}$  the strong phase  $\Delta_{KK^*}$  can be determined [20].

The knowledge of the strong phase can be used to determine  $x$  and  $y$ , and not only  $x_f'$  and  $y_f'$ . (Note that in the standard analysis of DCS decays, only the latter can be determined.) This can be seen by comparing the terms linear in  $\tau$  to the constant ones. We see from Eqs. (2) and (3) that we can measure the following four quantities:

$$\begin{aligned}
yR_m \cos(\phi + \Delta_f), & \quad yR_m^{-1} \cos(\phi - \Delta_f), \\
xR_m \sin(\phi - \Delta_f), & \quad xR_m^{-1} \sin(\phi + \Delta_f).
\end{aligned} \tag{24}$$

Once these four quantities are measured, generally, one can separately determine  $x$ ,  $y$ ,  $R_m$ , and  $\phi$  (up to discrete ambiguities), and thus separately measure the two types of indirect  $CP$  violation,  $a^m$  and  $a^i$ . This cannot be done with a  $CP$  eigenstate.

### III. DIRECT VS INDIRECT $CP$ VIOLATION

New  $CP$  violation could affect  $a_f$  through either a contribution to the mixing amplitude  $M_{12}$ , that is *indirect*  $CP$  violation, or a contribution to the decay amplitudes  $A_f$ , that is *direct*  $CP$  violation, or both. Indirect  $CP$  violation generates  $a_f^m$  and  $a_f^i$ , while direct  $CP$  violation generates  $a_f^d$ . (Contributions to the decay amplitudes affect  $\Gamma_{12}$  but this effect is always very small and can be safely neglected.)

The SM contribution to the mixing is suppressed by three factors: double Cabibbo suppression, flavor SU(3) suppression (which, in the short distance language, is the Glashow-Iliopoulos-Maiani (GIM) suppression) and weak-interaction loop suppression. The long distance contribution avoids the loop factor and can have a much milder SU(3)-breaking suppression. Consequently, it is estimated that the SM gives  $x, y = \mathcal{O}(10^{-3})$ , but with very large uncertainties. In particular, it cannot be excluded that the SM gives values as high as  $x, y = \mathcal{O}(10^{-2})$  [11,12,21].

New physics can avoid some or all of the three suppression factors. Indeed, it is well known that there are many models that can accommodate or even predict  $x$  close to the current experimental limit (for a review see [22,23]). The best known example is that of supersymmetric models with quark-squark alignment [8,9,24]. Here, box diagrams with intermediate squarks and gluinos have a double Cabibbo suppression, but neither SU(3) nor  $\alpha_w^2$ -suppression (but only  $\alpha_s^2$  factor). Furthermore, the gluino couplings carry new  $CP$  violating phases. These, and other models, dem-



onstrate that it is quite possible that indirect  $CP$  violation could account for  $a_f$  of  $\mathcal{O}(10^{-2})$ .

Note that new short distance contributions can enhance  $x$  but not  $y$ . If the SM value of  $y$  is small,  $y \lesssim 10^{-3}$ , then  $a_f^m$  is negligible (in the case of a  $CP$ -eigenstate final state). If  $y$  is large,  $y \sim 10^{-2}$ , then new physics in the mixing amplitude would result in similar contributions from  $a_f^i$  and  $a_f^m$ .

The SM contribution to the decay is through tree-level  $W$ -mediated diagrams. Thus, the amplitude depends on  $G_F \sin\theta_c$ . New physics cannot give competing contributions but, to generate  $a_f^d \sim 10^{-2}$ , it is only required that

$$Im(G_N) \sim 10^{-2} \sin\theta_c G_F, \quad (25)$$

where  $G_N$  denotes the effective four-Fermi coupling from new physics. If, for example, the scale of new physics is  $\Lambda_{\text{NP}} \gtrsim 1 \text{ TeV}$  then the scale suppression of  $G_N$  is  $\mathcal{O}(m_W^2/\Lambda_{\text{NP}}^2) \lesssim 10^{-2}$ . Thus, quite generically, Eq. (25) can only be realized with  $\Lambda_{\text{NP}} \lesssim 1 \text{ TeV}$  and (at least) one of the following conditions satisfied:

- (1) There is neither flavor suppression stronger than  $\sin\theta_c$  nor loop suppression;
- (2) There are enhancement factors related to hadronic factors or chiral enhancement;
- (3)  $\Lambda_{\text{NP}}$  is actually much closer to  $m_W$ .

As we show later, there exist well-motivated models where indeed such conditions apply and consequently (25) can be satisfied. It is thus quite possible that an  $\mathcal{O}(10^{-2})$  effect is generated solely or dominantly from direct  $CP$  violation.

In the absence of direct  $CP$  violation from new physics, the  $CP$  asymmetries in SCS decays into final  $CP$  eigenstates would be *universal*, i.e. independent of the final state. (The SM would give tiny nonuniversal corrections, i.e.  $(a_{KK} - a_{\pi\pi})/(a_{KK} + a_{\pi\pi}) = \mathcal{O}\{\arg[(V_{cd}^* V_{ud})/(V_{cs}^* V_{us})]\} \sim 10^{-3}$ .) We note that this universality would extend to CF decays to final  $CP$  eigenstates, e.g.,  $D \rightarrow K_s \pi^0$ . Let us define the universal, indirect contribution to  $CP$  violation as follows:

$$a^{\text{ind}} = a^m + a^i. \quad (26)$$

As mentioned above,  $a^{\text{ind}}$  is the *only* possible source of  $\Delta Y$  defined in Eq. (16). Thus, Eq. (17) implies

$$a^{\text{ind}} = (-0.35 \pm 0.47) \times 10^{-2}. \quad (27)$$

We note that, if the time-integrated measurements yield a nonzero asymmetry while the time-dependent measurements show no signal then only direct  $CP$  violation must be playing a role. More generally, if a difference between the two classes of measurements is experimentally established, and both are nonzero, then both direct and indirect  $CP$  violation are present, and can be cleanly separated. Such a scenario is quite possible. In fact, supersymmetric models with quark-squark alignment [8,9] provide such an example, as we shall see.

We note that it is also possible to cleanly separate direct and indirect  $CP$  violation in SCS decays only with time-integrated  $CP$  asymmetry measurements. Assuming negligible new  $CP$  violation effects in CF and DCS decays (it is difficult to construct a model in which this is not the case [25]), the time-integrated  $CP$  asymmetry for a CF decay to a final  $CP$  eigenstate would give the universal indirect  $CP$  asymmetry. Subtracting this from the time-integrated  $CP$  asymmetry for a SCS decay to a final  $CP$  eigenstate would give the nonuniversal direct  $CP$  asymmetry for the latter. For example,

$$a_{P^+P^-}^d = a_{P^+P^-} - a_{K_s\pi^0}, \quad P = K, \pi. \quad (28)$$

Finally we mention that charged  $D$  decays are sensitive only to direct  $CP$  violation. If a nonvanishing  $CP$  asymmetry is experimentally established in charged  $D$  decay, that would signal direct  $CP$  violation. If experiments establish time-integrated  $CP$  asymmetries in neutral  $D$  decays but not in charged  $D$  decays, that would be suggestive of indirect  $CP$  violation, but would not prove it. It is possible that the new physics could be such that it induces direct  $CP$  violation only in neutral decays.

#### IV. DIRECT $CP$ VIOLATION AT TREE LEVEL

In this section we examine whether various specific models can generate  $a_f^d \gtrsim 10^{-2}$  via tree-level contributions. For concreteness we focus on  $f = K^+K^-$  and  $\pi^+\pi^-$ . The main purpose is to find, for each model, an upper bound on the  $r_f$  factor of Eq. (7). We assume that the weak phase  $\phi_f$  is of  $\mathcal{O}(1)$ . The strong phase  $\delta_f$  suffers from hadronic uncertainties, but we point out cases where it is formally suppressed by  $1/N_c$ . In practice, however, the strong phase could be of  $\mathcal{O}(1)$  even if it is color suppressed.

##### A. Extra quarks in SM vectorlike representations

In models with nonsequential (“exotic”) quarks, the  $Z$ -boson has flavor-changing couplings, leading to  $Z$ -mediated contributions to the SCS decays. (For a review see, for example, [26].) In models with additional up quarks in the vectorlike representation  $(\mathbf{3}, \mathbf{1}, +2/3) \oplus (\mathbf{3}, \mathbf{1}, -2/3)$ , the flavor-changing  $Z$  couplings have the form

$$-\mathcal{L}_Z = \frac{gU_{ij}^u}{2\cos\theta_W} \bar{u}_{Li}\gamma_\mu u_{Lj}Z^\mu + \text{H.c.} \longrightarrow G_N^Z = G_F U_{cu}^u. \quad (29)$$

The flavor-changing coupling is constrained by  $\Delta m_D$  [25]:

$$|U_{cu}^u| \lesssim 5 \times 10^{-4} \longrightarrow r_f \lesssim 10^{-3}. \quad (30)$$

A somewhat stronger bound (from  $\Delta m_K$ ) applies for the case of vectorlike quark doublets,  $(\mathbf{3}, \mathbf{2}, +1/6) \oplus (\mathbf{3}, \mathbf{2}, -1/6)$ .

We learn that a significant contribution to  $D^0 \rightarrow K^+K^-, \pi^+\pi^-$  from  $Z$ -mediated flavor-changing interac-

tions is ruled out. In fact, this lesson applies to a much broader class of models, that is, all models with a tree-level contribution mediated by a neutral heavy boson. In all of these models, the combination  $Y_{cu}/M$  (with  $Y_{cu}$  the flavor-changing coupling and  $M$  the mass of the heavy boson) is constrained by  $\Delta m_D$ . The contribution to the decay has an extra factor of  $Y_{qq}/M$  ( $q = s$  or  $d$ ) that is maximized for large  $Y_{qq}$  and light  $M$ . Thus, the model discussed here, with  $Y_{qq} = g/(2\cos\theta_W)$  and  $M = m_Z$ , gives a contribution that is near maximal among all models with  $Y_{qq} \lesssim 1$  and  $M \gtrsim m_Z$ .

### B. Supersymmetry without $R$ -parity

We consider supersymmetry without  $R$ -parity models (for a description of the framework, see, for example, [27]). The lepton number violating terms in the superpotential  $\lambda'_{ijk} L_i Q_j d_k^c$  give a slepton-mediated tree-level contribution with an effective coupling

$$G_f' = \frac{\lambda'_{i2k} \lambda'_{i1k}{}^*}{4\sqrt{2}M^2(\tilde{\ell}_{Li})} \quad \text{with} \quad k = \begin{cases} 2 & f = K^+ K^- \\ 1 & f = \pi^+ \pi^- \end{cases} \quad (31)$$

The same combinations of couplings contribute to the  $K^+ \rightarrow \pi^+ \nu \bar{\nu}$  decay. That provides the following bound (see e.g. [27]):

$$|\lambda'_{i2k} \lambda'_{i1k}{}^*| \times \left( \frac{100 \text{ GeV}}{M(\tilde{d}_k^c)} \right)^2 \lesssim 2 \times 10^{-5} \longrightarrow r_f \lesssim 1.5 \times 10^{-4}, \quad (32)$$

where we take all sfermion masses to be of the same order.

The baryon number violating terms  $\lambda''_{ijk} u_i^c d_j^c d_k^c$  give a squark-mediated tree-level contribution with an effective coupling

$$G_f'' = \frac{\lambda''_{2jk} \lambda''_{1jk}{}^*}{4\sqrt{2}M^2(\tilde{d}_k^c)} \quad \text{with} \quad \begin{cases} j = 2, k = 1, 3 & f = K^+ K^- \\ j = 1, k = 2, 3 & f = \pi^+ \pi^- \end{cases} \quad (33)$$

Strong bounds are often quoted from  $n - \bar{n}$  oscillations (see e.g. [27]):

$$|\lambda''_{11k}| \lesssim 10^{-7} \quad (\text{for } M(\tilde{d}_k^c) = 100 \text{ GeV}). \quad (34)$$

(This would rule out any significant contribution to  $G''_{\pi\pi}$ , and a significant contribution to  $G''_{KK}$  from  $k = 1$ .) However, it was shown in [28] that important suppression factors were missed in obtaining these bounds, and that the strongest individual bound on these couplings comes from double nucleon decay,

$$|\lambda''_{112}| < 10^{-15} \left( \frac{m_{\tilde{g}} m_{\tilde{q}}^4}{\Lambda_h} \right)^{5/2}, \quad (35)$$

where  $\Lambda_h$  is some hadronic mass scale. This leaves only the  $k = 3$  contributions to  $G_{\pi\pi}$  and  $G_{KK}$  as potentially significant (the revised bound from  $n - \bar{n}$  oscillations in [28],

$\lambda''_{113} < 0.002(0.1)$  for  $m_{\tilde{q}} = 200(600)$  GeV, allows  $r_{\pi\pi} \sim 10^{-2}$ ). However, the  $K^0 - \bar{K}^0$  system yields the bounds [27,29]

$$\begin{aligned} \text{Im}(\lambda''_{123} \lambda''_{113}{}^*) &< 10^{-5}, \\ \text{Re}(\lambda''_{213} \lambda''_{223}{}^*) &< 3 \times 10^{-4}, \\ \text{Im}(\lambda''_{213} \lambda''_{223}{}^*) &< 3 \times 10^{-6}, \end{aligned} \quad (36)$$

from  $\epsilon'/\epsilon$ ,  $\Delta m_K$ , and  $\epsilon_K$ , respectively, for 100 GeV squark masses. Note that each coupling appearing in these bounds also appears in either  $G_{\pi\pi}$  or  $G_{KK}$ , and vice versa. From this we conclude that it is not possible to simultaneously obtain  $r_{\pi\pi} \sim 10^{-2}$  and  $r_{KK} \sim 10^{-2}$  for  $k = 3$ , as this would require a tuning among the  $\lambda''$  couplings of at least 1 part in  $10^3$ . (Also note that  $\lambda''_{ijk} \gtrsim 10^{-7}$  would, in general, wash out a baryon asymmetry generated before the electroweak phase transition (EWPT).)

In order to obtain a nonvanishing direct  $CP$  asymmetry in  $D \rightarrow K^+ K^-$ , a relative strong phase is required between the SM and NP amplitudes. At the weak scale, the SM Hamiltonian mediating, e.g.,  $D \rightarrow K^+ K^-$ , is of the form  $(\bar{u}_i s_i)_{V-A} (\bar{s}_j c_j)_{V-A}$  ( $i, j$  are color indices), while in the case of  $R$ -parity violation, the relevant Hamiltonian is of the form

$$(\bar{u}_i s_i)_{V+A} (\bar{s}_j c_j)_{V+A} - (\bar{u}_i s_j)_{V+A} (\bar{s}_j c_i)_{V+A}. \quad (37)$$

Since the strong interactions conserve parity, the first term gives the same strong phase as the SM. The second term, however, has a different color structure and thus it can generate a different strong phase. The contribution of the second term, however, is suppressed compared to the first one by  $1/N_c$ . Thus, the resulting strong phase relative to the SM amplitude is color suppressed. As mentioned earlier, while this may mean that the direct  $CP$  violation is further suppressed, an  $O(1)$  relative strong phase cannot be ruled out. The same argument applies to the  $D \rightarrow \pi^+ \pi^-$  amplitude.

### C. Two Higgs doublet models (2HDM)

We consider multi Higgs doublet models with natural flavor conservation (for a review see, for example, [30]). In these models a charged Higgs ( $H^\pm$ ) mediates a tree-level contribution. In the 2HDM the relevant couplings are

$$-\mathcal{L}_{H^\pm} = \frac{ig}{\sqrt{2}m_W} \bar{u}_i [m_{u_i} \cot\beta P_L + m_{d_j} \tan\beta P_R] V_{ij} d_j H^\pm + \text{H.c.} \quad (38)$$

It follows that the charged Higgs mediated contribution is also singly Cabibbo suppressed. Then, for large  $\tan\beta$ , the suppression with respect to the SM contribution is given by

$$r_{KK} \simeq \frac{m_s^2 \tan^2 \beta}{m_{H^\pm}^2}. \quad (39)$$

To obtain the upper bound, we consider the constraint on  $R_\tau \equiv \mathcal{B}(B \rightarrow \tau\nu)/\mathcal{B}^{\text{SM}}(B \rightarrow \tau\nu)$  [31]:

$$R_\tau \simeq \left[ 1 - \left( \frac{m_B}{m_{H^\pm}} \right)^2 \tan^2 \beta \right]^2 \sim 0.7 \pm 0.3. \quad (40)$$

We can write

$$r_{KK} \simeq \frac{m_s^2}{m_b^2} (1 - \sqrt{R_\tau}) \lesssim 4 \times 10^{-4}. \quad (41)$$

The bound on  $r_{\pi\pi}^{H^\pm}$  is stronger by a factor of  $m_d^2/m_s^2$ . For  $\tan\beta \sim 1$  the bound is even stronger,  $r_{KK} \simeq m_s m_c / m_{H^\pm}^2 \lesssim 5 \times 10^{-5}$  (we use [32]  $m_{H^\pm} \geq 80$  GeV). We learn that the charged Higgs contributions to the direct  $CP$  violation are negligible.

The situation is somewhat different in models with more than two Higgs doublets. In particular, when two different doublets couple to the down and charged lepton sectors, the bound from  $B \rightarrow \tau\nu$  does not apply to the SCS  $D$  decays. One can still obtain a bound from charm counting in  $B$  decays. Using  $n_{\text{charm}} = 1.22 \pm 0.04$ , we conclude that in this case  $r_{KK} \lesssim 10^{-2}$  and  $r_{\pi\pi} \lesssim 10^{-4}$ . Thus, direct  $CP$  violation from a charged Higgs contribution in 3HDM can marginally account for  $a_{KK} = \mathcal{O}(10^{-2})$  but is negligible for  $a_{\pi\pi}$ .

## V. DIRECT $CP$ VIOLATION AT ONE LOOP

In the previous section we saw that, in models in which new decay amplitudes are generated at the tree level, the direct  $CP$  asymmetries in SCS decays are typically constrained to lie well below the 1% level. In this section we examine whether one-loop effects due to new contributions to the  $\Delta C = 1$  QCD penguin and chromomagnetic dipole operators can generate  $a_f^d \sim 10^{-2}$ . Again, we consider  $KK$  and  $\pi\pi$  final states, focus on  $r_f$ , and assume that the new weak phase  $\phi_f$  in Eq. (7) is of  $\mathcal{O}(1)$ .

### A. QCD penguin and dipole operators: General considerations

The  $\Delta C = 1$  effective Hamiltonian that is relevant to SCS decays is given by

$$H_{\text{eff}}^{\Delta C=1} = \frac{G_F}{\sqrt{2}} \left[ \sum_{p=d,s} \lambda_p (C_1 Q_1^p + C_2 Q_2^p) + \sum_{i=3}^6 C_i(\mu) Q_i(\mu) + C_{8g} Q_{8g} \right] + \text{H.c.}, \quad (42)$$

where  $\lambda_p = V_{cp}^* V_{up}$  with  $p = d, s$  are CKM factors, and  $\lambda_d + \lambda_s + \lambda_b = 0$  due to the unitarity of the CKM matrix. The operators are given in the appendix in Eq. (A3)).  $Q_{1,2}$  are the current-current operators,  $Q_{3,\dots,6}$  are the QCD penguin operators, and  $Q_{8g}$  is the QCD dipole operator. The dominant contribution to the tree-level coefficients  $C_1$  and  $C_2$  is from the SM. New physics amplitudes contribute to

$C_{3,\dots,6}$ ,  $C_{8g}$ . The standard model contributions to these operators can be neglected, as they enter at  $\mathcal{O}(V_{cb} V_{ub})$  (leading to  $a_f^d \sim (V_{cb} V_{ub} / V_{cs} V_{us}) \alpha_s / \pi \sim 10^{-4}$ ). We have therefore opted to omit the CKM factor in front of the penguin and dipole operators in Eq. (42). We emphasize that for CF decays, as well as DCS decays, only the tree operators contribute. Penguin operators only contribute to SCS decays.

There are also opposite chirality operators  $\tilde{Q}_i$  which are obtained from the  $Q_i$ 's via the substitutions  $L \leftrightarrow R$ . In general their effects are of the same order of magnitude as the operators that we discuss. In particular cases, like in left-right symmetric models, there could be cancellations between the opposite chirality contributions. Here we consider only the general case where such cancellations are not present. Furthermore, for simplicity we do not write down explicitly the contributions of the opposite chirality operators.

In many models the strongest bounds arise from  $D^0 - \bar{D}^0$  mixing. The relevant  $\Delta C = 2$  effective Hamiltonian is given by [33]

$$H_{\text{eff}}^{\Delta C=2} = \sum_{i=1}^5 c_i O_i. \quad (43)$$

Again, we do not write explicitly the opposite chirality operators explicitly. The operators  $O_i$  are given in Eq. (B2) and their matrix elements are estimated in Eq. (B5). Experimental data yield bounds on the relevant operators. In particular, we use [34]

$$|M_{12}^D| < 6.2 \times 10^{-11} \text{ MeV}. \quad (44)$$

In order to obtain rough estimates of the  $D \rightarrow KK$  and  $D \rightarrow \pi\pi$  amplitudes we use the QCD factorization framework [16]. We adapt the original  $B$  decay discussion of [16] to the case of  $D$  decays. We work primarily at leading order in  $1/m_c$ , using naive factorization for  $Q_{1,\dots,6}$ , and QCD factorization for  $Q_{8g}$ . We identify, however, possibly large power corrections associated with the annihilation topology for the current-current and penguin operators, which formally enter at  $\mathcal{O}(1/m_c)$ .

Clearly, the  $1/m_c$  expansion is not expected to work very well for hadronic  $D$  decays. Thus, our analysis only provides order-of-magnitude estimates for the full decay amplitudes, which suffice for our purposes. It should also be noted that the QCD factorization approach is useful for organizing the matrix elements of the various operators in order of importance.

In Appendix A we give the details of our analysis and quantitative estimates. Our conclusions with regard to annihilation amplitudes can however be simply stated:

- (i) For the SM operators, the spectator and the annihilation amplitudes are roughly of the same order (see Eq. (A26) for details).

- (ii) For the penguin operators, the annihilation amplitudes are likely to give the dominant contribution (see Eq. (A27) for details).

### B. Implications of isospin and $SU(3)_F$

Model independently there are no significant bounds on the relevant operators, so we can get  $a_f^d \sim 10^{-2}$ . There are, however, several general results that can be obtained based on symmetries, in particular, isospin and  $U$ -spin.

Very generally isospin predicts

$$A(D^0 \rightarrow \pi^0 \pi^0) + \sqrt{2}A(D^+ \rightarrow \pi^+ \pi^0) - A(D^0 \rightarrow \pi^+ \pi^-) = 0. \quad (45)$$

As for the new penguin amplitudes, the isospin predictions follow from the fact that the  $c \rightarrow ug$  operator is  $\Delta I = 1/2$ . Thus, it cannot generate an  $I = 2$  final state. In particular, it cannot contribute to  $D^+ \rightarrow \pi^+ \pi^0$ . Thus, we expect no direct CPV in this mode,  $a_{\pi^+ \pi^0} = 0$ . In contrast, we can get direct CPV in  $D^0$  decays as well as in  $D^+ \rightarrow K^+ K_S$ . Other isospin-based predictions would need further assumptions. For example, neglecting annihilation diagrams, isospin predicts that  $a_{K^+ K^-}^d = a_{K^+ K_S}^d$ . As we just argued, neglecting annihilation cannot be justified. In principle, it could flip the sign between the two asymmetries.

$U$ -spin predicts that  $a_{K^+ K^-}^d = -a_{\pi^+ \pi^-}^d$  for new  $c \rightarrow ug$  transitions. (This is in contrast to the indirect  $CP$  violation which gives the same sign,  $a_{K^+ K^-}^{\text{ind}} = a_{\pi^+ \pi^-}^{\text{ind}}$ .)  $U$ -spin predicts that the SM amplitudes for the two processes have opposite signs ( $O(\lambda^4)$  effects coming from  $(V_{cs} V_{us}^*)/(V_{cd} V_{ud}^*) \neq 1$  are negligible), whereas penguin amplitudes have the same sign. Further study of  $U$ -spin violation, especially in annihilation, is needed in order to check the resulting prediction of opposite signs for  $a_{K^+ K^-}^d$  and  $a_{\pi^+ \pi^-}^d$ .

Another  $U$ -spin prediction is that in the SM  $A(D \rightarrow K^0 \bar{K}^0)$  vanishes. This is a pure annihilation process with two contributing diagrams: One where  $c\bar{u} \rightarrow d\bar{d}$  ( $\propto V_{cd} V_{ud}^*$ ) and the  $s\bar{s}$  pair pops out of the vacuum, and a second one where  $c\bar{u} \rightarrow s\bar{s}$  ( $\propto V_{cs} V_{us}^*$ ) and the  $d\bar{d}$  pair pops out of the vacuum. Again, due to the sign difference between the two CKM combinations, the total amplitude is proportional to  $d\bar{d} - s\bar{s}$  which vanishes in the  $U$ -spin limit. Thus, the data (A23) shows not only that annihilation is large but also that  $U$ -spin breaking is large for annihilation.

### C. QCD penguin and dipole operators: Examples from SUSY

We study contributions to the QCD penguin and dipole operator Wilson coefficients arising from up-squark-gluino loops. For simplicity, we work in the squark mass-insertion approximation. The common squark mass is denoted by  $\tilde{m}$ . We consider the contributions of the up-squark mass insertions

$$\delta_{LL} \equiv \frac{(\tilde{m}_{LL}^{2u})_{12}}{\tilde{m}^2}, \quad \delta_{LR} \equiv \frac{(\tilde{m}_{LR}^{2u})_{12}}{\tilde{m}^2}. \quad (46)$$

(The opposite chirality mass insertions  $\delta_{RR}$  and  $\delta_{RL}$  are obtained via the substitutions  $L \leftrightarrow R$  above.) The Wilson coefficients are given by

$$C_i = E_i(x)\delta_{LL}, \quad i = 3, \dots, 6, \quad (47)$$

$$C_{8g} = F(x)\delta_{LL} + G(x)\frac{m_{\tilde{g}}}{m_c}\delta_{LR},$$

where  $x = m_{\tilde{g}}^2/\tilde{m}^2$ .  $E_i(x)$ ,  $F(x)$ , and  $G(x)$  contain loop functions, and can be read from Eq. (B1). We learn that  $\delta_{LL}$  contributes to all of the penguin operators, while  $\delta_{LR}$  only contributes to  $C_{8g}$ . Note that the contribution from  $\delta_{LR}$  is enhanced by a large factor of  $m_{\tilde{g}}/m_c$ . In addition, the loop function  $G(x)$  that accompanies  $\delta_{LR}$  gives a further enhancement, which is numerically of order five in the relevant parameter space, relative to  $F(x)$ .

The most severe bounds arise from  $D^0 - \bar{D}^0$  mixing. Note, in particular, that the bounds that arise from  $K^0 - \bar{K}^0$  mixing do not apply to  $\delta_{LL}$  of Eq. (46). In the interaction basis we have  $\tilde{m}_{LL}^{2u} = \tilde{m}_{LL}^{2d}$ . However, such an equality does not apply in the super-CKM basis (where the up- and down-quark mass matrices are diagonal) that we use. The rotation from the first to the latter basis involves the respective diagonalizing matrices,  $V_L^u$  and  $V_L^d$ . For example, in alignment models,  $V_L^d \approx 1$  while  $V_L^u \approx V_{\text{CKM}}$ , leading to sizable  $\tilde{m}_{LL}^{2u}$ , close to the  $\Delta m_D$  bound, and negligible  $\tilde{m}_{LL}^{2d}$ , well below the  $\Delta m_K$  bound.

The full expressions for the Wilson coefficients are given in Eq. (B3). What we find is that all of the mass insertions enter the expressions with similar coefficients. In particular, there is no enhancement for the chirality changing insertions.

We begin with a discussion of the effects of the left-right squark mass insertion,  $\delta_{LR}$ . It generates new contributions to  $D$  meson decays via the dipole operator  $\mathcal{Q}_{8g}$ , and to  $D^0 - \bar{D}^0$  mixing via the operators  $O_2, O_3$ . The crucial point is that the contribution to the decay (but not to the mixing) is enhanced by a large factor,  $m_{\tilde{g}}/m_c$ , and therefore the  $D^0 - \bar{D}^0$  mixing bounds are not restrictive. Consequently,  $O(10^{-2})$  contributions to the  $D \rightarrow KK, \pi\pi$  amplitudes are not excluded.

The situation is illustrated in Fig. 1. The contours in these plots correspond to a fixed ratio,  $r_f = 10^{-2}$ . This ratio is calculated using QCD factorization at leading-power for the dipole operator amplitude and naive factorization for the standard model amplitude, see Eqs. (A9) and (A11). In Fig. 1(a) we plot the values of  $\delta_{LR}$  that yield  $r_f = 10^{-2}$  as a function of the gluino mass,  $m_{\tilde{g}}$ , for several values of  $\tilde{m}$ .  $\delta_{LR}$  is plotted in units of  $\theta_c m_c(\mu_{\text{susy}})/\tilde{m}$ . (For simplicity, we take  $\mu_{\text{susy}} = m_t$  and neglect the small running of  $m_c$  between  $m_t$  and the squark mass scale,



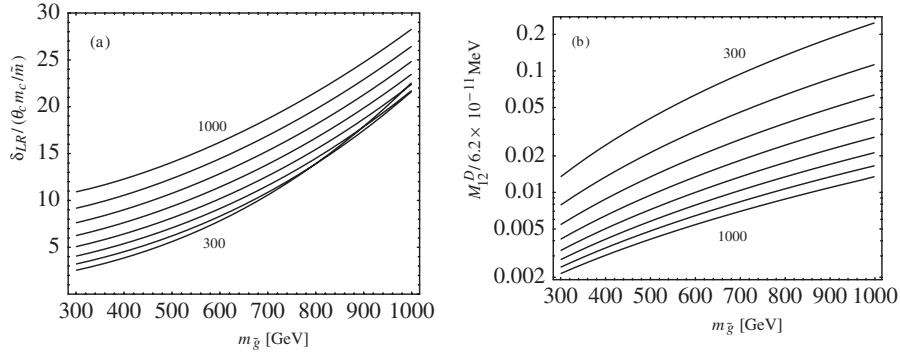


FIG. 1. (a)  $\delta_{LR}$  [in units of  $(\theta_c m_c / \tilde{m})$ ] vs  $m_{\tilde{g}}$ , and (b)  $M_{12}^D$  [in units of  $6.2 \times 10^{-11}$  MeV] vs  $m_{\tilde{g}}$ , for  $r_f = 0.01$  ( $f = K^+ K^-, \pi^+ \pi^-$ ). The lines correspond to  $\tilde{m} = 300$ – $1000$  GeV in increments of 100 GeV.

which yields  $m_c(\mu_{\text{susy}}) = 0.85$  GeV for  $m_c(m_c) = 1.64$  GeV.) This is useful for later comparison to the magnitudes expected for  $\delta_{LR}$  in various supersymmetric models of flavor. In Fig. 1(b) we plot the corresponding contributions to  $|M_{12}^D|$ , normalized to the upper bound of  $6.2 \times 10^{-11}$  MeV, see Eq. (46). We learn that it is possible to obtain  $O(10^{-2})$  contributions to the decay amplitudes, accompanied by new contributions to  $|M_{12}^D|$  lying one to 2 orders of magnitude below the experimental bound. In the standard model the annihilation amplitude could be of the same order as the leading-power tree amplitude with large relative strong phase (this is probably also true for the annihilation vs leading-power dipole operator amplitudes). Therefore, if  $\arg(\delta_{LR})$  is large, then  $a_f^d = O(10^{-2})$  could

be realized with negligible  $a_f^{\text{ind}}$ . A striking feature of this result is the sensitivity of current  $CP$  asymmetry searches to very small values of  $Im(\delta_{LR}) \geq 2 \times 10^{-3}$ .

Next, we discuss the effects of the left-left squark mass insertion,  $\delta_{LL}$ . New contributions to the  $D$  decay amplitudes are generated via the QCD penguin and dipole operators  $Q_{3,\dots,6}$ ,  $Q_{8g}$ . Their magnitudes are restricted by requiring that the supersymmetric contribution to  $|M_{12}^D|$  is smaller than the bound in Eq. (44). Here, unlike in the case of  $\delta_{LR}$ , there is no  $m_{\tilde{g}}/m_c$  enhancement of the contribution to the decay and, consequently, the bound from the mixing is significant. In Fig. 2(a) the resulting upper bound on  $\delta_{LL}$  is plotted as a function of  $m_{\tilde{g}}$  for several values of  $\tilde{m}$ . The corresponding upper bounds on  $r_f$  ( $f = K^+ K^-, \pi^+ \pi^-$ )

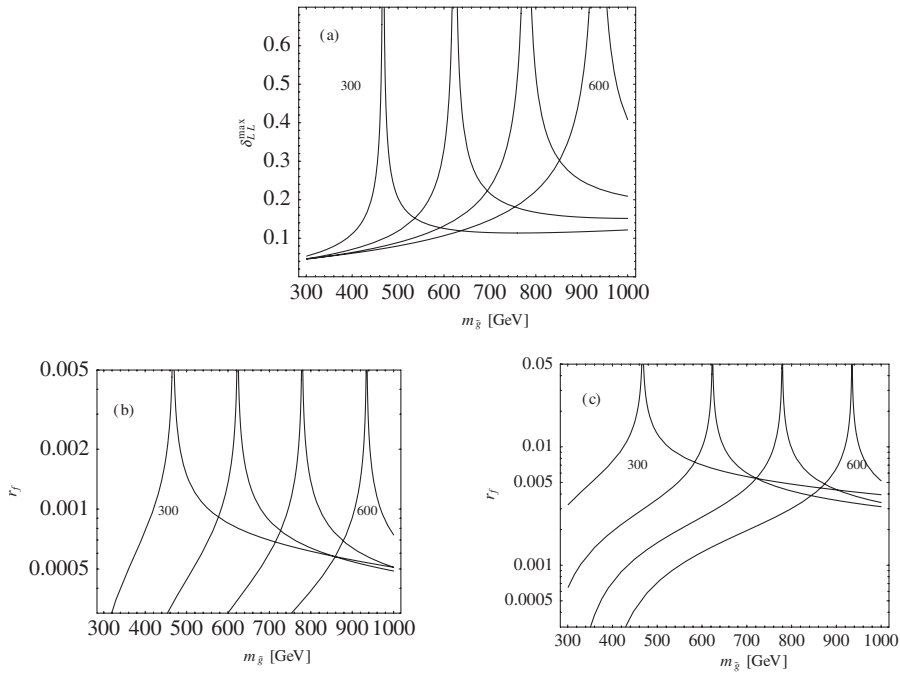


FIG. 2. (a)  $\delta_{LL}^{\text{max}}$  (the upper bound on  $\delta_{LL}$  from  $D^0 - \bar{D}^0$  mixing) vs  $m_{\tilde{g}}$ ; (b) and (c)  $r_f$  ( $f = K^+ K^-, \pi^+ \pi^-$ ) corresponding to  $\delta_{LL}^{\text{max}}$  vs  $m_{\tilde{g}}$  (b) in naive factorization or (c) with annihilation power corrections included in  $A^{\text{NP}}$  (see text). The various lines correspond to  $\tilde{m} = 300, 400, 500, 600$  GeV.

are plotted in Fig. 2(b). (Again, the hadronic matrix elements of the four-quark operators and the dipole operator are estimated in naive factorization and in QCD factorization, respectively.) The supersymmetric contribution to  $M_{12}^D$  in Eq. (B3) vanishes at  $m_{\tilde{g}} \approx 1.56\tilde{m}$  leading to the peaked structures in Fig. 2, also see [24]. In the absence of special tuning of  $m_{\tilde{g}}$  vs  $\tilde{m}$ , we observe that at leading-power  $r_f \lesssim 10^{-3}$ . (We note that the validity of the squark mass-insertion approximation is marginal for  $\delta_{LL} \gtrsim 1/4$ , but it is sufficient for our purposes given the much larger hadronic theoretical uncertainties [35]).

It may well be the case, however, that the  $1/m_c$  expansion fails badly in the evaluation of the QCD penguin contributions. In particular, as argued in Appendix A, annihilation amplitudes could give an order-of-magnitude enhancement. To show how the situation changes if such enhancement is indeed realized, we repeat the calculation with QCD penguin annihilation matrix elements included according to Eqs. (A19), (A20), and (A22). As discussed in Appendix A, we estimate these matrix elements in the one-gluon exchange model of [16,36]. The results are presented in Fig. 2(c). Our conclusion is that, if annihilation enhances the QCD penguin operator contributions, then it is possible that supersymmetric  $\delta_{LL}$  insertions give  $a_f^d \sim 10^{-2}$  without violating the bounds from mixing. In other words, due to hadronic uncertainties, we cannot rule out the possibility of such large direct  $CP$  violation from  $\delta_{LL}$ . In this case, however, we also expect the indirect  $CP$  violation to be of the same order.

#### D. Flavor-changing neutral-currents (FCNCs) in supersymmetric flavor models

Supersymmetric models with minimal flavor violation, such as gauge or anomaly mediation, give no observable  $CP$  violating effects in SCS  $D$  decays. We thus consider supersymmetric models where the SUSY breaking mediation is not flavor blind. In such models there are two main strategies for suppressing FCNCs: (a) quark-squark alignment [8,9,24], (b) squark mass degeneracy, see e.g., [37–44]. Models in each category make specific predictions for the pattern of squark mixing, or for the squark mass-insertions  $\delta_{NM}$ , ( $N, M = L, R$ ). In the following, we compare these predictions with the sensitivity of current direct and indirect  $CP$  asymmetry searches.

The various models are based on approximate horizontal symmetries, and often make predictions in terms of a small symmetry breaking parameter. For concreteness, we use  $\lambda \sim \sin\theta_c \sim 0.2$  as the small parameter.

In models of alignment, Abelian flavor symmetries are responsible for the observed quark mass and mixing hierarchies and lead to a high degree of alignment between the down-quark and down-squark mass eigenstates. Thus, supersymmetric FCNCs in the down sector are highly suppressed. CKM mixing is generated in the up sector, and the up squarks are nondegenerate. The models make

the following order-of-magnitude predictions [24]:

$$\begin{aligned} \delta_{LR} &\sim \frac{\lambda m_c}{\tilde{m}}, & \delta_{LL} &\sim \lambda, \\ \delta_{RR} &\lesssim \lambda^2, & \delta_{RL} &\lesssim \frac{\lambda^2 m_c}{\tilde{m}}. \end{aligned} \quad (48)$$

In addition,  $\mathcal{O}(1)$   $CP$  violating phases are expected.

Comparing the predicted range for  $\delta_{LR}$ , Eq. (48), with the values required to generate  $r_f \sim 0.01$ , Fig. 1(a), one may naively conclude that alignment gives values of  $r_f$  that are a factor of 3–30 too small. It should be kept in mind, however, that the dipole operator matrix elements suffer from large theoretical uncertainties. In particular, we have not taken into account power corrections due to the annihilation topology. Therefore, an enhancement of  $r_f$  by a factor of a few cannot be ruled out. We conclude that for squark and gluino masses at the lower part of the range that we consider,  $\delta_{LR}$  could lead to  $a_f^d \sim 10^{-2}$ . According to Fig. 1(b), the contribution of  $\delta_{LR}$  to indirect  $CP$  violation is bounded to be small.

Comparing the predicted range for  $\delta_{LL}$ , Eq. (48), with the values required to generate  $r_f \sim 0.01$ , Figs. 2(b) and 2(c), we learn that  $\delta_{LL}$  could also lead to  $a_f^d \lesssim 10^{-2}$ , provided that annihilation strongly enhances the penguin operator matrix elements. Finally, Fig. 2(a) confirms that the predicted range for  $\delta_{LL}$  could easily lead to  $a_f^i \sim 10^{-2}$  and, if  $y \sim 10^{-2}$ , also to  $a_f^m \sim 10^{-2}$ . We conclude that models of alignment predict  $a^{\text{ind}} \sim 10^{-2}$  and could also accommodate  $a^d \lesssim 10^{-2}$ .

In models of squark degeneracy, the first two families of quarks constitute a doublet, and the third family a singlet, of a non-Abelian horizontal symmetry. This leads to a high degree of degeneracy between the first and second family squark masses which evades the bounds from  $\Delta m_K$ , and implies  $\delta_{LL}, \delta_{RR} \ll 1$ . Thus, the contributions of  $\delta_{LL}$  and  $\delta_{RR}$  to  $a_f^d$  and  $a_f^{\text{ind}}$  are negligible. The non-Abelian horizontal symmetry is not sufficient for reproducing all features of the quark mass and mixing hierarchies without a large Yukawa coupling hierarchy, and may not lead to a sufficiently high degree of degeneracy between the down and strange squark masses to evade the bounds from  $\epsilon_K$ . Thus, an Abelian flavor symmetry is introduced (it could be a subgroup of a larger non-Abelian symmetry). The resulting predictions for  $\delta_{LR}$  and  $\delta_{RL}$  are model dependent. For example,  $U(2)$  based models, with vanishing (1,1) entries in the quark mass matrices [39–41,43,44], predict

$$\delta_{LR} \sim \delta_{RL} \sim \frac{\sqrt{m_u m_c}}{\tilde{m}} \sim \frac{\lambda^2 m_c}{\tilde{m}}. \quad (51)$$

Therefore, in such models the contributions of  $\delta_{LR}$  and  $\delta_{RL}$  to  $a_f^d$  are well below  $10^{-2}$ . The effect can be larger in models with a discrete non-Abelian  $S_3^3$  horizontal

symmetry [42], which predicts  $\delta_{LR} \sim \lambda m_c / \tilde{m}$  and  $\delta_{RL} \sim \lambda^3 m_c / \tilde{m}$ , quite similar to models of alignment. Therefore,  $a_f^d \sim 10^{-2}$  may again be possible via the dipole operator.

We conclude that  $a_f^d \sim 10^{-2}$  is not generic but could arise in specific models of squark degeneracy via the dipole operator with negligible mixing effects. In models of alignment,  $a_f^d \sim 10^{-2}$  can arise via the dipole operator as well as the penguin operators, the latter being correlated with a large mixing contribution that is likely to yield  $a_f^{\text{ind}} \sim 10^{-2}$ . In both examples a significant dipole operator contribution to  $a_f^d$  is linked to a large contribution to  $\theta_c$  from the up-quark sector.

It is interesting to compare the sensitivity of  $CP$  violation in SCS  $D$  decays and in  $B$  decays to models of flavor. The two sectors provide complementary information. The combination of measurements of  $D$ ,  $B_d$ ,  $B^+$ , and  $B_s$  decays can be used to discriminate between different models of flavor. The details of the comparison are left for a future publication.

## VI. DISCUSSIONS AND CONCLUSIONS

It is well known that  $CP$  violation in  $D$  decays is a clean way to probe new physics. In this paper we study  $CP$  asymmetries in singly Cabibbo suppressed  $D$  decays, focusing, in particular, on the final  $CP$  eigenstates  $K^+K^-$  and  $\pi^+\pi^-$ . The possibility to probe new  $CP$  violation is, however, not limited to these modes. Pseudo-two body  $CP$  eigenstates, such as  $\phi\pi^0$  or  $\phi K_S$ , as well as non- $CP$  eigenstates, for example  $KK^*$  and  $\rho\pi$ , are also worth studying. In particular, we have seen that the formalism for time-integrated  $CP$  asymmetries in decays to non- $CP$  eigenstates allows a separation of indirect  $CP$  violation due to mixing and due to interference of decays with and without mixing. Decays with four (or more) final-state particles, like  $\rho^0\rho^0$ , offer new ways to probe  $CP$  violation via triple product correlations. It is likely that models that lead to large direct  $CP$  asymmetries in two body decays also generate large  $CP$  violating triple products.

To summarize, our main results are as follows:

- (i) The SM cannot account for asymmetries that are significantly larger than  $\mathcal{O}(10^{-4})$ . Thus,  $CP$  violation from new physics must be playing a role if an asymmetry is observed with the present experimental sensitivities [ $\mathcal{O}(0.01)$ ].
- (ii) The underlying mechanism of  $CP$  violation can be any of the three types: in decay ( $a^d$ ), in mixing ( $a^m$ ), and in the interference of decays with and without mixing ( $a^i$ ).
- (iii) In the case of indirect  $CP$  violation ( $a^{\text{ind}} = a^m + a^i$ ) and final  $CP$  eigenstates, the time-integrated  $CP$  asymmetries  $a_f$  and the time-dependent asymmetries  $\Delta Y_f$  are universal (and equal to each other).
- (iv) In contrast, for direct  $CP$  violation, the time-integrated asymmetries  $a_f$  are not expected to be

universal, while the time-dependent asymmetries  $\Delta Y_f$  vanish.

- (v) The pattern of  $CP$  violation can be used to test supersymmetric flavor models. Minimal flavor violation models predict tiny, unobservable, effects. Alignment models predict large  $a^{\text{ind}}$  and possibly also large  $a_f^d$ . Models with squark degeneracy predict small  $a^{\text{ind}}$  but, depending on the model, can accommodate observable  $a_f^d$ .
- (vi) If direct  $CP$  violation is at the 1% level, its likely source is new physics that contributes to the decay via loop diagrams rather than via tree diagrams. The reason is that the experimental bounds on  $D^0 - \bar{D}^0$  mixing are much more effective in constraining models of the latter type.
- (vii) In this regard, SCS  $D$  decays are unique, as they are the only ones that probe gluonic penguin operators. In other words, while we find that direct  $CP$  violation can have observable effects in SCS decays, it is very unlikely to affect CF and DCS decays.

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## APPENDIX A: THE $D \rightarrow KK/\pi\pi$ AMPLITUDES

We use the QCD factorization framework [16] to obtain order-of-magnitude estimates for the  $D \rightarrow KK/\pi\pi$  amplitudes in the presence of new contributions to the QCD penguin and dipole operators. Clearly, the  $1/m_c$  expansion is not expected to work very well for hadronic  $D$  decays. We can therefore ignore  $\mathcal{O}(\alpha_s)$  corrections to the matrix elements, as they are negligible compared to the overall theoretical uncertainties. We work primarily at leading order in  $\Lambda_{\text{QCD}}/m_c$ , using naive factorization for  $Q_{1,\dots,6}$  and QCD factorization for  $Q_{8g}$ . However, we discuss the importance of power corrections, especially annihilation, in the standard model and estimate a large source of

theoretical uncertainty in the QCD penguin operator matrix elements due to annihilation.

Our convention for the flavor wave functions is

$$\begin{aligned}\pi^0 &\sim \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d), & \pi^- &\sim \bar{u}d, & \pi^+ &\sim \bar{d}u, & K^0 &\sim \bar{d}s, \\ K^0 &\sim \bar{s}d, & K^- &\sim \bar{u}s, & K^+ &\sim \bar{s}u.\end{aligned}\quad (\text{A1})$$

### 1. Leading power

The effective  $\Delta C = 1$  Hamiltonian is given in Eq. (42)

$$\begin{aligned}H_{\text{eff}}^{\Delta C=1} &= \frac{G_F}{\sqrt{2}} \left[ \sum_{p=d,s} \lambda_p (C_1 Q_1^p + C_2 Q_2^p) + \sum_{i=3}^6 C_i(\mu) Q_i(\mu) \right. \\ &\quad \left. + C_{8g} Q_{8g} \right] + \text{H.c.}\end{aligned}\quad (\text{A2})$$

The operators are given by:

$$\begin{aligned}Q_1^p &= (\bar{p}c)_{V-A} (\bar{u}p)_{V-A}, \\ Q_2^p &= (\bar{p}_\alpha c_\beta)_{V-A} (\bar{u}_\beta p_\alpha)_{V-A}, \\ Q_3 &= (\bar{u}c)_{V-A} \sum_q (\bar{q}q)_{V-A}, \\ Q_4 &= (\bar{u}_\alpha c_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \\ Q_5 &= (\bar{u}c)_{V-A} \sum_q (\bar{q}q)_{V+A}, \\ Q_6 &= (\bar{u}_\alpha c_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}, \\ Q_{8g} &= -\frac{g_s}{8\pi^2} m_c \bar{u} \sigma_{\mu\nu} (1 + \gamma_5) G^{\mu\nu} c,\end{aligned}\quad (\text{A3})$$

where  $\alpha, \beta$  are color indices and  $q = u, d, s$ . The matrix elements for  $D \rightarrow KK, \pi\pi$  decay can be written in the form [16,36]

$$\langle P_1 P_2 | \mathcal{H}_{\text{eff}} | D \rangle = \langle P_1 P_2 | \mathcal{T}_A + \mathcal{T}_B | D \rangle, \quad (\text{A4})$$

where  $\mathcal{T}_A$  is the transition operator for amplitudes in which the  $D$  spectator quark appears in the final state and  $\mathcal{T}_B$  is the transition operator for annihilation amplitudes which are discussed in subsection A 2. We write  $\mathcal{T}_A$  as

$$\begin{aligned}\mathcal{T}_A &= \sum_{p=d,s} \lambda_p (a_1^p (\bar{p}c)_{V-A} \otimes (\bar{u}p)_{V-A} + a_2^p (\bar{u}c)_{V-A} \\ &\quad \otimes (\bar{p}p)_{V-A}) + a_3^p \sum_q (\bar{u}c)_{V-A} \otimes (\bar{q}q)_{V-A} \\ &\quad + a_4^p \sum_q (\bar{q}c)_{V-A} \otimes (\bar{u}q)_{V-A} + a_5^p \sum_q (\bar{u}c)_{V-A} \\ &\quad \otimes (\bar{q}q)_{V+A} + a_6^p (-2) \sum_q (\bar{q}c)_{S-P} \otimes (\bar{u}q)_{S+P},\end{aligned}\quad (\text{A5})$$

where  $P = K, \pi$  for  $D \rightarrow KK, \pi\pi$  decays, respectively, a summation over  $q = u, d, s$  is implied and  $\lambda_p = V_{cp}^* V_{up}$ . Fierzing of  $Q_5, Q_6$  gives rise to the  $(S - P)(S + P)$  term. The second pair of quarks in each term produces a final-state meson ( $P_2$ ), and the outgoing quark in the first pair combines with the spectator quark to form a final-state meson ( $P_1$ ). The  $\otimes$  indicates that the matrix element of the corresponding operator in  $\mathcal{T}_A$  is to be evaluated in the factorized form:

$$\langle P_1 P_2 | j_1 \otimes j_2 | D \rangle \equiv \langle P_1 | j_1 | \bar{D} \rangle \langle P_2 | j_2 | 0 \rangle = \begin{cases} -ic \mathcal{A}_p, & \text{for } j_1 \otimes j_2 = (V - A) \otimes (V \mp A), \\ -ic r_\chi \mathcal{A}_p, & \text{for } j_1 \otimes j_2 = -2(S - P) \otimes (S + P). \end{cases} \quad (\text{A6})$$

The  $c$  coefficients are products of factors of  $\pm 1, \pm 1/\sqrt{2}$ , which depend on the flavor structures of the mesons, and

$$\mathcal{A}_p = i \frac{G_F}{\sqrt{2}} (m_D^2 - m_p^2) F_0^{D \rightarrow P}(m_p^2) f_p, \quad (\text{A7})$$

where  $F_0^{D \rightarrow P}$  is the  $D \rightarrow P$  transition form factor and  $f_p$  is the decay constant. The factor  $r_\chi$  appearing in the scalar matrix elements is given by

$$\begin{aligned}r_\chi &= \frac{\mu_p}{m_c}, & \mu_p &= \frac{2m_K^2}{m_s + m_q} = \frac{2m_\pi^2}{m_u + m_d}, \\ m_q &= \frac{m_u + m_d}{2}.\end{aligned}\quad (\text{A8})$$

The  $a_i^p$  coefficients in general contain the contributions from naive factorization, penguin contractions, vertex corrections, and hard spectator interactions. We only consider explicitly the naive factorization contributions for  $Q_{1,\dots,6}$ , and the penguin contraction for  $Q_{8g}$  [16]. We therefore

obtain ( $N_c = 3$  and  $P = K, \pi$ )

$$\begin{aligned}a_1^p &= C_1 + \frac{C_2}{N_c}, & a_2^p &= C_2 + \frac{C_1}{N_c}, \\ a_3^p &= C_3 + \frac{C_4}{N_c}, & a_5^p &= C_5 + \frac{C_6}{N_c}, \\ a_4^p &= C_4 + \frac{C_3}{N_c} - \frac{C_F \alpha_s}{2\pi N_c} C_{8g} \int_0^1 \frac{\phi_P(x)}{x} dx, \\ a_6^p &= C_6 + \frac{C_5}{N_c} - \frac{C_F \alpha_s}{2\pi N_c} C_{8g},\end{aligned}\quad (\text{A9})$$

where  $C_F = (N_c^2 - 1)/(2N_c)$ .  $\phi_P(x)$  is the leading-twist light-cone meson distribution amplitude for meson  $P$ . For simplicity we consider asymptotic distribution amplitudes, in which case

$$a_i^K = a_i^\pi \equiv a_i, \quad \int_0^1 \frac{\phi_P(x)}{x} dx = 3. \quad (\text{A10})$$

In that case the only sources of  $SU(3)_F$  breaking are the



form factors and decay constants.  $A_{\text{NF}}$ , the naive factorization amplitudes for  $D \rightarrow KK/\pi\pi$  are then given by

$$\begin{aligned}
A_{\text{NF}}(D \rightarrow K^0 \bar{K}^0) &= 0, \\
A_{\text{NF}}(D^0 \rightarrow K^+ K^-) &= A_{\text{NF}}(D^+ \rightarrow K^+ K^0) \\
&= (\lambda_s a_1 + a_4 + r_\chi a_6) \mathcal{A}_K, \\
A_{\text{NF}}(D^0 \rightarrow \pi^+ \pi^-) &= (\lambda_d a_1 + a_4 + r_\chi a_6) \mathcal{A}_\pi, \\
-\sqrt{2} A_{\text{NF}}(D^+ \rightarrow \pi^+ \pi^0) &= \lambda_d (a_1 + a_2) \mathcal{A}_\pi, \\
A_{\text{NF}}(D^0 \rightarrow \pi^0 \pi^0) &= (-\lambda_d a_2 + a_4 + r_\chi a_6) \mathcal{A}_\pi.
\end{aligned} \tag{A11}$$

The decay  $D \rightarrow K^0 \bar{K}^0$  only proceeds via annihilation and thus vanishes in (A11). The standard model amplitudes are given in terms of  $a_{1,2}$ , and the new physics amplitudes are given in terms of  $a_{3,\dots,6}$ .

Note that there are no strong-phase differences at this point between the SM and NP amplitudes. However, large power corrections (or final-state interactions) could generate them. As we argue below, the measured decay widths point to a large role for such infrared dominated physics, especially annihilation. Thus, the large strong-phase differences that would be necessary to obtain  $a_f^d \sim r_f$  are well motivated.

In our numerical estimates we take  $m_c(m_c) = 1.64$  GeV and  $m_s = 110$  MeV,  $m_u + m_d = 9$  MeV at  $\mu = 2$  GeV. The scale at which the Wilson coefficients and  $r_\chi$  are evaluated is varied within the range  $\mu \approx 1-2$  GeV. At  $\mu \approx m_c$  we obtain  $r_\chi \approx 2.5$ ,  $a_1 \approx 1.05$ , and  $a_2 \approx 0.05$  at next-to-leading order. (For simplicity we ignore the  $m_b$  quark mass threshold, taking  $n_f = 5$  and  $\Lambda_{\text{QCD}} = 225$  MeV). With regards to the form factors, the BES collaboration has measured [45]

$$\begin{aligned}
F_+(0)^{D \rightarrow K} &= 0.78 \pm 0.04 \pm 0.03, \\
F_+(0)^{D \rightarrow \pi} &= 0.73 \pm 0.14 \pm 0.06.
\end{aligned} \tag{A12}$$

(A recent lattice determination obtains  $F_+(0)^{D \rightarrow K} = 0.73 \pm 0.03 \pm 0.07$  [46].) The values of  $F_0^{D \rightarrow K, \pi}(0)$  entering  $\mathcal{A}_{K, \pi}$  follow from the kinematical constraint  $F_0(0) = F_+(0)$ . Small shifts in  $F_0(q^2)$  due to  $q^2 \lesssim m_K^2$  are negligible. Our estimates are obtained by varying the measured values of  $F_0^{D \rightarrow K, \pi}(0)$  in their  $\pm 1\sigma$  ranges quoted in (A12).

The decay rates are given by

$$\begin{aligned}
\Gamma(D \rightarrow PP) &= \frac{|A(D \rightarrow PP)|^2}{16\pi m_D} \sqrt{1 - \frac{4m_P^2}{m_D^2}}, \\
A &= A_{\text{NF}} + A_{\text{ann}}.
\end{aligned} \tag{A13}$$

Taking  $A_{\text{ann}} = 0$  we get the following naive factorization decay widths within the SM,

$$\begin{aligned}
\Gamma(D^0 \rightarrow K^+ K^-) &= \Gamma(D^+ \rightarrow K^+ \bar{K}^0) \\
&= (4.6 - 6.5) \times 10^{-6} \text{ eV}, \\
\Gamma(D^0 \rightarrow \pi^+ \pi^-) &= (2.6 - 6.5) \times 10^{-6} \text{ eV}, \\
\Gamma(D^0 \rightarrow \pi^0 \pi^0) &= (0.1 - 0.3) \times 10^{-6} \text{ eV}, \\
\Gamma(D^+ \rightarrow \pi^+ \pi^0) &= (1.3 - 3.5) \times 10^{-6} \text{ eV}.
\end{aligned} \tag{A14}$$

## 2. Annihilation

Adapting [36] to  $D \rightarrow PP$  decays, the annihilation matrix elements can be organized in terms of flavor operators of the form  $B([\bar{q}_{P_1} q_{P_1}][\bar{q}_{P_2} q_{P_2}][\bar{q}_s c])$ , where  $q_s$  denotes the spectator antiquark in the  $D$  meson. The matrix element of a  $B$  operator is defined as

$$\langle P_1 P_2 | B([\bar{\cdot}][\cdot][\cdot]) | D \rangle = c \mathcal{B}_P \quad \text{with} \quad \mathcal{B}_P = i \frac{G_F}{\sqrt{2}} f_D f_P^2 \tag{A15}$$

whenever the quark flavors of the three brackets match the three mesons, respectively. The notations are as in Eq. (A6). The transition operator for the annihilation contributions of  $Q_{1,\dots,6}$ ,  $Q_{8g}$  in Eq. (A4) can be parametrized in full generality as

$$\begin{aligned}
\mathcal{T}_B &= \sum_{p=d,s} \lambda_p \left( \sum_{q'} b_{1q'}^p B([\bar{p}q'][\bar{q}'p][\bar{u}c]) \right. \\
&\quad \left. + \delta_{pd} \sum_{q'} b_{2q'}^p B([\bar{u}q'][\bar{q}'d][\bar{d}c]) \right) \\
&\quad + \sum_{q,q'} b_{3q}^p B([\bar{u}q']\bar{q}'q][\bar{q}c]) \\
&\quad + \sum_{q,q'} b_{4q}^p B([\bar{q}q']\bar{q}'q][\bar{u}c]),
\end{aligned} \tag{A16}$$

where  $q, q' = u, d, s$ . Here  $q'$  denotes the flavor of the ‘‘popped’’ quark-antiquark pair from gluon splitting,  $g \rightarrow \bar{q}'q'$ . Isospin symmetry implies  $b_{iu}^\pi = b_{id}^\pi \equiv b_i^\pi$ ,  $b_{iu}^K = b_{id}^K$ , and  $U$ -spin symmetry would further imply  $b_{is}^K = b_{id}^K = b_i^\pi$ .  $b_{1,2}^p$  receive contributions from the SM current-current operators, and  $b_{3,4}^p$  from NP via the QCD penguin and dipole operators.

Using isospin all of the  $b_i$  coefficients can be expressed in terms of  $D \rightarrow P^+ P^-$ ,  $K^+ \bar{K}^0$  effective operator annihilation matrix elements. For the SM operators we have

$$\begin{aligned}
\mathcal{B}_P b_{1q'}^p &= C_1 \langle P^+ P^- | (\bar{p}_\alpha p_\beta)_{V-A} \otimes^A (\bar{u}_\beta c_\alpha)_{V-A} | D^0 \rangle \\
&\quad + C_2 \langle P^+ P^- | (\bar{p}p)_{V-A} \otimes^A (\bar{u}c)_{V-A} | D^0 \rangle, \\
\mathcal{B}_K b_{2s}^K &= C_1 \langle K^+ \bar{K}^0 | (\bar{u}d)_{V-A} \otimes^A (\bar{d}c)_{V-A} | D^+ \rangle \\
&\quad + C_2 \langle K^+ \bar{K}^0 | (\bar{u}_\alpha d_\beta)_{V-A} \otimes^A (\bar{d}_\beta c_\alpha)_{V-A} | D^+ \rangle.
\end{aligned} \tag{A17}$$

For the NP operators we have

$$\begin{aligned}
\mathcal{B}_P b_{3q'}^P &= C_3 \langle P^+ P^- | (\bar{u}_\alpha u_\beta)_{V-A} \otimes^A (\bar{u}_\beta c_\alpha)_{V-A} | D^0 \rangle \\
&\quad + C_4 \langle P^+ P^- | (\bar{u}u)_{V-A} \otimes^A (\bar{u}c)_{V-A} | D^0 \rangle \\
&\quad + C_5 \langle P^+ P^- | -2(\bar{u}_\alpha u_\beta)_{S+P} \otimes^A (\bar{u}_\beta c_\alpha)_{S-P} | D^0 \rangle \\
&\quad + C_6 \langle P^+ P^- | -2(\bar{u}u)_{S+P} \otimes^A (\bar{u}c)_{S-P} | D^0 \rangle, \\
\mathcal{B}_P b_{4q'}^P &= C_3 \langle P^+ P^- | (\bar{q}q)_{V-A} \otimes^A (\bar{u}c)_{V-A} | D^0 \rangle \\
&\quad + C_4 \langle P^+ P^- | (\bar{q}_\alpha q_\beta)_{V-A} \otimes^A (\bar{u}_\beta c_\alpha)_{V-A} | D^0 \rangle \\
&\quad + C_5 \langle P^+ P^- | (\bar{q}q)_{V+A} \otimes^A (\bar{u}c)_{V-A} | D^0 \rangle \\
&\quad + C_6 \langle P^+ P^- | (\bar{q}_\alpha q_\beta)_{V+A} \otimes^A (\bar{u}_\beta c_\alpha)_{V-A} | D^0 \rangle.
\end{aligned} \tag{A18}$$

The annihilation product  $j_1 \otimes^A j_2$  means that  $j_2$  destroys the  $D$  meson, and  $j_1$  creates a quark and an antiquark which end up in different mesons. The choices of  $p$  and  $q$  among  $(d, s)$  and  $(u, d, s)$ , respectively, are fixed by the values taken by  $P$  and  $q'$ . In  $b_{4q'}^P$  the  $\langle j_{1V-A} \otimes^A j_{2V-A} \rangle$  and  $\langle j_{1V+A} \otimes^A j_{2V-A} \rangle$  matrix elements are equal because parity implies  $\langle P^+ P^- | (\bar{q}q)_{V-A} | g_1 \dots g_n \rangle = \langle P^+ P^- | (\bar{q}q)_{V+A} | g_1 \dots g_n \rangle$ . Finally, we point out that  $Q_{8g}$  also contributes to  $b_{3q'}^P$  and  $b_{4q'}^P$ . A discussion of the theoretical uncertainty for dipole operator amplitudes due to the annihilation topology is left for future work.

Assuming isospin, the annihilation amplitudes are given by

$$\begin{aligned}
A_{\text{ann}}(D \rightarrow \pi^+ \pi^-) &= \mathcal{B}_\pi (\lambda_d b_1^\pi + b_3^\pi + 2b_4^\pi), \\
A_{\text{ann}}(D \rightarrow K^+ K^-) &= \mathcal{B}_K (\lambda_s b_{1u}^K + b_{3s}^K + b_{4s}^K + b_{4u}^K), \\
A_{\text{ann}}(D \rightarrow \pi^0 \pi^0) &= A_{\text{ann}}(D \rightarrow \pi^+ \pi^-), \\
A_{\text{ann}}(D \rightarrow K^0 \bar{K}^0) &= \mathcal{B}_K (\lambda_s [b_{1d}^K - b_{1s}^K] + b_{4d}^K + b_{4s}^K), \\
A_{\text{ann}}(D \rightarrow \pi^+ \pi^0) &= 0, \\
A_{\text{ann}}(D \rightarrow K^+ \bar{K}^0) &= \mathcal{B}_K (\lambda_s b_{2s}^K + b_{3s}^K).
\end{aligned} \tag{A19}$$

Note that in the  $U$ -spin limit  $A_{\text{ann}}(K^+ K^-) = A_{\text{ann}}(\pi^+ \pi^-)$  and, neglecting the penguin operators,  $A_{\text{ann}}(K^0 \bar{K}^0) = 0$ .

In order to estimate the  $b_i$ 's we make use of the tree-level one-gluon exchange approximation [16,36]. In general, factorizable contributions to  $\langle j_1 \otimes^A j_2 \rangle$ , of the form  $\langle P_1 P_2 | j_1 | 0 \rangle \langle 0 | j_2 | D \rangle$ , vanish for the  $(V \pm A) \otimes^A (V - A)$  matrix elements by the equations of motion. Therefore, many of the matrix elements in Eq. (A17) vanish in the one-gluon approximation. We further simplify our discussion by taking asymptotic meson light-cone distribution amplitudes. Then, the number of independent building blocks appearing in Eq. (A17) reduces to two [16,36],

$$\begin{aligned}
A_1^i &= \langle P^+ P^- | (\bar{q}_\alpha q_\beta)_{V \mp A} \otimes^A (\bar{u}_\beta c_\alpha)_{V-A} | D \rangle / \mathcal{B}_P \\
&= \frac{C_F}{N^2} \pi \alpha_s \left[ 18 \left( X - 4 + \frac{\pi^3}{3} \right) + 2r_\chi^2 X^2 \right], \\
A_3^f &= \langle P^+ P^- | -2(\bar{q}q)_{S+P} \otimes^A (\bar{u}c)_{S-P} | D^0 \rangle / \mathcal{B}_P \\
&= \frac{C_F}{N} 12\pi \alpha_s r_\chi (2X^2 - X).
\end{aligned} \tag{A20}$$

The superscripts  $i(f)$  denote a gluon exchanged from the initial (final-state) quarks in the four-quark operator.  $X$  represents an incalculable infrared logarithmically divergent quantity which signals a breakdown in short/long distance factorization. It is a necessary model-dependent input in the one-gluon approximation. For simplicity, we take  $X$  to be universal. Adopting the model of [16],  $X$  is parametrized as

$$X = \log(m_D / \Lambda_h) (1 + \rho e^{i\phi}). \tag{A21}$$

$\Lambda_h \sim 500$  MeV is a hadronic mass scale corresponding to some physical infrared cutoff,  $\phi$  allows for the presence of an arbitrary strong phase from soft rescattering, and  $\rho$  parametrizes our ignorance of the magnitudes of these amplitudes. With our assumptions, we get

$$\begin{aligned}
b_{1q'}^P &= C_1 A_1^i, & b_{3q'}^P &= C_3 A_1^i + \left( C_6 + \frac{C_5}{N_c} \right) A_3^f, \\
b_{2s}^K &= C_2 A_1^i, & b_{4q'}^P &= (C_4 + C_6) A_1^i.
\end{aligned} \tag{A22}$$

The strong color-suppression  $b_{2s}^K \ll b_{1q'}^K$  may be an artifact of the one-gluon approximation, as beyond it the contribution of the matrix element of  $Q_1$  to  $b_{2s}^K$  does not vanish.

In our numerical evaluation we use  $\alpha_s$  and  $r_\chi$  in Eq. (A20) at a scale  $\mu_h \approx 0.7$  GeV, corresponding to  $\alpha_s \approx 1$  (reflecting the infrared dominance of these matrix elements). The Wilson coefficients are evaluated at a scale  $\mu = m_c(m_c)$ . For  $f_D$  we take the central value of the CLEO-c measurement,  $f_D = 223 \pm 17 \pm 3$  MeV [47].

### 3. Comparison with data

In order to estimate the value of the model parameters we compare the prediction with the measured widths [32]

$$\begin{aligned}
\Gamma(D^0 \rightarrow K^+ K^-) &= (6.16 \pm 0.16) \times 10^{-6} \text{ eV}, \\
\Gamma(D^0 \rightarrow \pi^+ \pi^-) &= (2.19 \pm 0.05) \times 10^{-6} \text{ eV}, \\
\Gamma(D^0 \rightarrow K^0 \bar{K}^0) &= (1.19 \pm 0.22) \times 10^{-6} \text{ eV}, \\
\Gamma(D^0 \rightarrow \pi^0 \pi^0) &= (1.27 \pm 0.13) \times 10^{-6} \text{ eV}, \\
\Gamma(D^+ \rightarrow K^+ \bar{K}^0) &= (3.75 \pm 0.24) \times 10^{-6} \text{ eV}, \\
\Gamma(D^+ \rightarrow \pi^+ \pi^0) &= (0.81 \pm 0.04) \times 10^{-6} \text{ eV}.
\end{aligned} \tag{A23}$$

We always assume that the NP amplitudes are small, so the above measured rates are given by the SM.

To leading order in  $1/m_c$ , only spectator diagrams contribute to the various decay amplitudes. Comparing the

naive factorization predictions, Eq. (A14), with the measured values we see that they are in disagreement. In particular,  $\Gamma(D^0 \rightarrow K^0 \bar{K}^0)$  and  $\Gamma(D^0 \rightarrow \pi^0 \pi^0)$  are considerably larger than the naive factorization predictions. The predicted rates for the  $K^+ K^-$ ,  $K^+ \bar{K}^0$ ,  $\pi^+ \pi^-$ , and  $\pi^+ \pi^0$  modes are of the correct order of magnitude. However, rather than being equal as expected at leading power,  $\Gamma(K^+ K^-)$  is approximately twice  $\Gamma(K^+ K^0)$ .

The disagreement points to a substantial role for annihilation. The magnitudes of the observed amplitudes imply that

$$|A(K^0 \bar{K}^0)| \sim \frac{1}{2}|A(K^+ K^-)|. \quad (\text{A24})$$

$A(K^+ K^-)$  has contributions from both naive factorization and annihilation amplitudes.  $A(K^0 \bar{K}^0)$  on the other hand is pure annihilation and it vanishes in the SM in the  $U$ -spin limit. We therefore expect that

$$|A_{\text{ann}}(K^0 \bar{K}^0)| \lesssim |A_{\text{ann}}(K^+ K^-)|. \quad (\text{A25})$$

Since naive factorization predicts the right orders of magnitude for the  $P^+ P^-$  widths, we expect that the annihilation and naive factorization amplitudes are of the same order for  $K^+ K^-$ . The same should be true for  $\pi^+ \pi^-$  based on any reasonable pattern for  $SU(3)_F$  breaking. We therefore write schematically

$$\frac{A_{\text{ann}}^{\text{SM}}}{A_{\text{NF}}^{\text{SM}}} \sim \frac{\langle P^+ P^- | (\bar{p}_\alpha p_\beta)_{V-A} \otimes^A (\bar{u}_\beta c_\alpha)_{V-A} | D^0 \rangle}{\langle P^+ P^- | (\bar{p}c)_{V-A} \otimes (\bar{u}p)_{V-A} | D^0 \rangle} \sim 1. \quad (\text{A26})$$

Next we try to estimate the size of the NP annihilation amplitudes. We use the one-gluon exchange model discussed above. We see that Eq. (A26) is reproduced with  $X \sim 5$  in Eq. (A20).<sup>1</sup> Using  $X \sim 5$  in Eq. (A20) for  $A_3^f$  we can estimate the size of the NP annihilation amplitude. We find that the chirally enhanced QCD penguin annihilation amplitude is much larger than the corresponding spectator amplitude. They also tend to dominate the total penguin annihilation and total penguin spectator amplitudes, respectively, in NP models. Schematically, we write this as

$$\frac{A_{\text{ann}}^{\text{NP}}}{A_{\text{NF}}^{\text{NP}}} \sim \frac{\langle P^+ P^- | (\bar{u}u)_{S+P} \otimes^A (\bar{u}c)_{S-P} | D^0 \rangle}{\langle P^+ P^- | (\bar{q}c)_{S-P} \otimes (\bar{u}q)_{S-P} | D^0 \rangle} \sim 5. \quad (\text{A27})$$

The large ratio implies that new QCD penguin amplitudes in  $D \rightarrow PP$  decays could receive an order-of-magnitude enhancement from annihilation. This is demonstrated in the numerical example of Fig. 2(c), where the annihilation matrix elements are included as above with  $X \approx 5$  ( $\rho = 3$ ,  $\phi = 0$ ).

<sup>1</sup> $X \approx 5$  arises, e.g., for  $\rho \sim 3$  and  $\phi \sim 0$  in Eq. (A21). It is worth mentioning that similar values of  $\rho$  are required in order to account for the  $e^+ e^- \rightarrow P^+ P^-$  cross sections at  $\sqrt{s} \approx 3.7$  GeV [48].

Given the crude nature of the one-gluon exchange approximation this should only be taken as an indication of the theoretical uncertainty due to QCD penguin operator annihilation. A similar analysis of the theoretical uncertainty for the dipole operator matrix element due to the annihilation topology is left for future work.

## APPENDIX B: QCD PENGUIN AND DIPOLE OPERATORS IN SUSY

We study contributions to the QCD penguin and dipole operator Wilson coefficients arising from up-squark-gluino loops. For simplicity, we work in the squark mass-insertion approximation where to first approximation the squark masses are degenerate with mass  $\tilde{m}$ . In particular, we consider the contributions of the up-squark mass insertions  $\delta_{\text{LL}}$  and  $\delta_{\text{LR}}$  to  $C_{3,\dots,6}$ ,  $C_{8g}$ . (Since in our case  $\delta_{\text{LR}} \ll 1$  and  $\delta_{\text{LL}} \lesssim 1$ , the mass-insertion approximation works very well for  $\delta_{\text{LR}}$  and only provides rough estimates for  $\delta_{\text{LL}}$ .) The expressions for the SUSY Wilson coefficients are given at the scale  $\mu \sim m_{\text{SUSY}}$  by [49]

$$\begin{aligned} C_3 &= -\frac{\alpha_s^2}{2\sqrt{2}G_F\tilde{m}^2} \left( -\frac{1}{9}B_1(x) - \frac{5}{9}B_2(x) - \frac{1}{18}P_1(x) \right. \\ &\quad \left. - \frac{1}{2}P_2(x) \right) \delta_{\text{LL}}, \\ C_4 &= -\frac{\alpha_s^2}{2\sqrt{2}G_F\tilde{m}^2} \left( -\frac{7}{3}B_1(x) + \frac{1}{3}B_2(x) + \frac{1}{6}P_1(x) \right. \\ &\quad \left. + \frac{3}{2}P_2(x) \right) \delta_{\text{LL}}, \\ C_5 &= -\frac{\alpha_s^2}{2\sqrt{2}G_F\tilde{m}^2} \left( \frac{10}{9}B_1(x) + \frac{1}{18}B_2(x) - \frac{1}{18}P_1(x) \right. \\ &\quad \left. - \frac{1}{2}P_2(x) \right) \delta_{\text{LL}}, \\ C_6 &= -\frac{\alpha_s^2}{2\sqrt{2}G_F\tilde{m}^2} \left( -\frac{2}{3}B_1(x) + \frac{7}{6}B_2(x) + \frac{1}{6}P_1(x) \right. \\ &\quad \left. + \frac{3}{2}P_2(x) \right) \delta_{\text{LL}}, \\ C_{8g} &= -\frac{2\pi\alpha_s}{\sqrt{2}G_F\tilde{m}^2} \left[ \delta_{\text{LL}} \left( \frac{3}{2}M_3(x) - \frac{1}{6}M_4(x) \right) \right. \\ &\quad \left. + \delta_{\text{LR}} \left( \frac{m_{\tilde{g}}}{m_c} \right) \frac{1}{6} (4B_1(x) - 9x^{-1}B_2(x)) \right], \quad (\text{B1}) \end{aligned}$$

where  $x \equiv (m_{\tilde{g}}/\tilde{m})^2$ , and the loop functions can be found in Ref. [49]. (The mass insertions  $\delta_{\text{RR}}$  and  $\delta_{\text{RL}}$  generate the opposite chirality operators  $\tilde{Q}_i$ .) For simplicity, we evaluate the above Wilson coefficients at  $\mu = m_t$ , and evolve them to  $\mu = m_c$  at LO.

The  $\Delta C = 2$  effective Hamiltonian,  $H_{\text{eff}}^{\Delta C=2}$ , for supersymmetric up-squark-gluino box graph contributions to  $D - \bar{D}$  mixing is given in Eqs. (43), with

$$\begin{aligned}
O_1 &= \bar{u}_L^\alpha \gamma_\mu c_L^\alpha \bar{u}_L^\beta \gamma^\mu c_L^\beta, & O_2 &= \bar{u}_R^\alpha c_L^\alpha \bar{u}_R^\beta c_L^\beta, \\
O_3 &= \bar{u}_R^\alpha c_L^\beta \bar{u}_R^\beta c_L^\alpha, & O_4 &= \bar{u}_R^\alpha c_L^\alpha \bar{u}_L^\beta c_R^\beta, \\
O_5 &= \bar{u}_R^\alpha c_L^\beta \bar{u}_L^\beta c_R^\alpha.
\end{aligned} \tag{B2}$$

The  $D - \bar{D}$  mixing amplitude is given by  $M_{12}^D = \langle D | H_{\text{eff}}^{\Delta C=2} | \bar{D} \rangle / 2m_D$ , where  $\Delta m_D = 2M_{12}^D = x\Gamma_D$ . In the squark mass-insertion approximation the SUSY Wilson coefficients for the operators  $O_i$  are given by [33],

$$\begin{aligned}
c_1 &= -\frac{\alpha_s^2}{216\tilde{m}^2} (24xf_6(x) + 66\tilde{f}_6(x)) (\delta_{13}^d)_{LL}^2, \\
c_2 &= -\frac{\alpha_s^2}{216\tilde{m}^2} 204xf_6(x) \delta_{RL}^2, \\
c_3 &= \frac{\alpha_s^2}{216\tilde{m}^2} 36xf_6(x) \delta_{RL}^2, \\
c_4 &= -\frac{\alpha_s^2}{216\tilde{m}^2} [(504xf_6(x) - 72\tilde{f}_6(x)) \delta_{LL} \delta_{RR} \\
&\quad - 132\tilde{f}_6(x) \delta_{LR} \delta_{RL}], \\
c_5 &= -\frac{\alpha_s^2}{216\tilde{m}^2} [(24xf_6(x) + 120\tilde{f}_6(x)) \delta_{LL} \delta_{RR} \\
&\quad - 180\tilde{f}_6(x) \delta_{LR} \delta_{RL}].
\end{aligned} \tag{B3}$$

The other Wilson coefficients  $\tilde{c}_{i=1,2,3}$  are obtained from

$c_{i=1,2,3}$  by exchange of  $L \leftrightarrow R$ . The loop functions are given by

$$\begin{aligned}
f_6(x) &= \frac{6(1+3x)\ln x + x^3 - 9x^2 - 9x + 17}{6(x-1)^5}, \\
\tilde{f}_6(x) &= \frac{6x(1+x)\ln x - x^3 - 9x^2 + 9x + 1}{3(x-1)^5}.
\end{aligned} \tag{B4}$$

Again, the Wilson coefficients are evaluated at  $\mu = m_t$  and evolved down to  $\mu \approx m_c$  at LO [50]. For simplicity, we use the vacuum insertion approximation for the operator matrix elements,

$$\begin{aligned}
\langle D | O_1 | \bar{D} \rangle &= \frac{2}{3} m_D^2 f_D^2, \\
\langle D | O_2 | \bar{D} \rangle &= -\frac{5}{12} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D^2 f_D^2, \\
\langle D | O_3 | \bar{D} \rangle &= \frac{1}{12} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D^2 f_D^2, \\
\langle D | O_4 | \bar{D} \rangle &= \frac{1}{2} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D^2 f_D^2, \\
\langle D | O_5 | \bar{D} \rangle &= \frac{1}{6} \left( \frac{m_D}{m_c + m_u} \right)^2 m_D^2 f_D^2.
\end{aligned} \tag{B5}$$

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