Branching ratio and *CP* asymmetry of $B_s \to \pi^0\eta^{(\prime)}$ decays in the perturbative QCD approach

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We calculate the branching ratios and *CP*-violating asymmetries for $B_s^0 \to \pi^0\eta^{(\prime)}$ decays in the perturbative QCD (pQCD) factorization approach here. We not only calculate the usual factorizable contributions, but also evaluate the nonfactorizable and annihilation type contributions. The pQCD predictions for the *CP*-averaged branching ratios are $\mathrm{BR}(B_s^0 \to \pi^0\eta) \approx 0.86 \times 10^{-7}$ and $\mathrm{BR}(B_s^0 \to \pi^0\eta') \approx 1.86 \times 10^{-7}$. The pQCD predictions for the *CP*-violating asymmetries are $A_{CP}^{\mathrm{dir}}(\pi^0\eta) \sim -4.5\%$, $A_{CP}^{\mathrm{dir}}(\pi^0\eta') \sim -9.1\%$, $A_{CP}^{\mathrm{mix}}(\pi^0\eta) \sim -0.2\%$, and $A_{CP}^{\mathrm{mix}}(\pi^0\eta') \sim 27\%$ but with large errors. The above pQCD predictions can be tested in the forthcoming LHC-b experiments at CERN.

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I. INTRODUCTION

The experimental measurements and theoretical studies of the two-body charmless hadronic B meson decays play an important role in the precision test of the standard model (SM) and in searching for the new physics beyond the SM [1]. For these charmless B meson decays, the dominant theoretical error comes from the large uncertainty in evaluating the hadronic matrix elements $\langle M_1 M_2 | O_i | B\rangle$ where M_1 and M_2 are light final state mesons. The QCD factorization (QCDF) approach [2] and the perturbative QCD (pQCD) factorization approach [3,4] are the popular methods being used to calculate the hadronic matrix elements.

When the LHC experiment is approaching, the studies about the decays of B_s meson draw much more attentions then ever before. At present, some two-body charmless hadronic B_s meson decays have been calculated, for example, in both the QCDF approach [5] and/or in the pQCD approach [6]. In this paper, we would like to calculate the branching ratios and *CP* asymmetries for $B_s \to \pi^0\eta^{(\prime)}$ decays by employing the low energy effective Hamiltonian [7] and the pQCD factorization approach. Besides the usual factorizable contributions, we here are able to evaluate the nonfactorizable and the annihilation contributions to these decays.

Theoretically, the two $B_s \to \pi^0\eta^{(\prime)}$ decays have been studied in the naive and generalized factorization approach [8,9] or in the QCD factorization approach [10]. On the experimental side, only the poor upper limits for the branching ratios are available now [11]

$$\mathrm{BR}\,(B_s^0 \to \pi^0\eta^{(\prime)}) < 1.0 \times 10^{-3}, \tag{1}$$

Of course, this situation will be improved rapidly when LHC-b starts to run at the year of 2007.

For $B_s \to \pi^0\eta^{(\prime)}$ decays, the light final state mesons are moving very fast in the rest frame of the B_s meson. In this case, the short distance hard process dominates the decay amplitude, while the soft final state interaction (FSI) is not important for such decays. The FSI effect is in nature a subtle and complicated subject. The smallness of FSI effects for B meson decays into two light final state mesons has been put forward by Bjorken [12] based on the color transparency argument [13], and also supported by further renormalization group analysis of soft gluon exchanges among initial and final state mesons [14]. With the Sudakov resummation, we can include the leading double logarithms for all loop diagrams, in association with the soft contribution.

This paper is organized as follows. In Sec. II, we calculate analytically the related Feynman diagrams and present the various decay amplitudes for the studied decay modes. In Sec. III, we show the numerical results for the *CP*-averaged branching ratios and *CP* asymmetries of $B_s \to \pi^0\eta^{(\prime)}$ decays and compare them with the measured values or the theoretical predictions in QCDF approach. The summary and some discussions are included in the final section.

II. PERTURBATIVE CALCULATIONS

For $B_s \to \pi^0\eta^{(\prime)}$ decays, the related weak effective Hamiltonian H_{eff} can be written as [7]

$$\mathcal{H}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}}\Big[V_{ub}V_{us}^*(C_1(\mu)O_1^u(\mu) + C_2(\mu)O_2^u(\mu)) - V_{tb}V_{ts}^*\sum_{i=3}^{10} C_i(\mu)O_i(\mu)\Big]. \tag{2}$$

The explicit expressions of the operators O_i can be found, for example, in Refs. [15,16].

In the pQCD approach, the decay amplitude is conceptually written as the convolution,

$$\mathcal{A}(B_s \to M_1 M_2) \sim \int d^4k_1 d^4k_2 d^4k_3\,\mathrm{Tr}[C(t)\Phi_{B_s}(k_1) \times \Phi_{M_1}(k_2)\Phi_{M_2}(k_3)H(k_1,k_2,k_3,t)], \tag{3}$$

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where k_i's are momenta of light quarks included in each mesons, and Tr denotes the trace over Dirac and color indices. $C(t)$ is the Wilson coefficient which results from the radiative corrections at short distance. The function $H(k_1, k_2, k_3, t)$ describes the four quark operator and the spectator quark connected by a hard gluon whose q^2 is in the order of $\bar{\Lambda} M_{B_s}$, and includes the $\mathcal{O}(\sqrt{\bar{\Lambda} M_{B_s}})$ hard dynamics. Therefore, this hard part H can be perturbatively calculated. The function Φ_M is the wave function which describes hadronization of the quark and antiquark to the meson M. While the function H depends on the processes considered, the wave function Φ_M is independent of the specific processes. Using the wave functions determined from other well-measured processes, one can make quantitative predictions here.

Since the b quark is rather heavy we consider the B_s meson at rest for simplicity. It is convenient to use light-cone coordinate $(p^+, p^-, \mathbf{p}_T)$ to describe the meson's momenta,

$$p^{\pm} = \frac{1}{\sqrt{2}}(p^0 \pm p^3), \quad \text{and} \quad \mathbf{p}_T = (p^1, p^2). \tag{4}$$

Using these coordinates the B_s meson and the two final state meson momenta can be written as

$$P_1 = \frac{M_{B_s}}{\sqrt{2}}(1, 1, \mathbf{0}_T), \qquad P_2 = \frac{M_{B_s}}{\sqrt{2}}(1, 0, \mathbf{0}_T), \qquad P_3 = \frac{M_{B_s}}{\sqrt{2}}(0, 1, \mathbf{0}_T), \tag{5}$$

respectively, here the light meson masses have been neglected. Putting the light (anti) quark momenta in B_s, π^0 and $\eta^{(\prime)}$ mesons as k_1, k_2, and k_3, respectively, we can choose

$$k_1 = (x_1 P_1^+, 0, \mathbf{k}_{1T}), \qquad k_2 = (x_2 P_2^+, 0, \mathbf{k}_{2T}), \qquad k_3 = (0, x_3 P_3^-, \mathbf{k}_{3T}). \tag{6}$$

Then, the integration over k_1^-, k_2^-, and k_3^+ in Eq. (3) will lead to

$$\mathcal{A}(B_s \to \pi^0 \eta^{(\prime)}) \sim \int dx_1 dx_2 dx_3 b_1 db_1 b_2 db_2 b_3 db_3 \times \mathrm{Tr}[C(t) \Phi_{B_s}(x_1, b_1) \Phi_{\pi^0}(x_2, b_2) \times \Phi_{\eta^{(\prime)}}(x_3, b_3) H(x_i, b_i, t) S_t(x_i) e^{-S(t)}], \tag{7}$$

where b_i is the conjugate space coordinate of k_{iT}, and t is the largest energy scale in function $H(x_i, b_i, t)$. The large double logarithms ($\ln^2 x_i$) on the longitudinal direction are summed by the threshold resummation [17], and they lead to $S_t(x_i)$ which smears the endpoint singularities on x_i. The last term, $e^{-S(t)}$, is the Sudakov form factor which suppresses the soft dynamics effectively [14]. In numerical calculations, we use $\alpha_s = 4\pi/[\beta_1 \ln(t^2/\Lambda_{\rm QCD}^{(5)})]$ which is the leading order expression with $\Lambda_{\rm QCD}^{(5)} = 193$ MeV, derived from $\Lambda_{\rm QCD}^{(4)} = 250$ MeV. Here $\beta_1 = (33 - 2n_f)/3$, with the appropriate number of active quarks n_f.

Similar to $B \to \rho \eta^{(\prime)}$ and $B \to \pi \eta^{(\prime)}$ decays, there are 8 type diagrams contributing to the $B_s \to \pi^0 \eta^{(\prime)}$ decays, as illustrated in Fig. 1. We first calculate the usual factorizable diagrams 1(a) and 1(b). Operators O_1, O_2, O_3, O_4, O_9, and O_{10} are $(V-A)(V-A)$ currents, the sum of their amplitudes is given as

$$F_{e\eta} = 4\sqrt{2}\pi G_F C_F f_\pi m_{B_s}^4 \int_0^1 dx_1 dx_3 \int_0^\infty b_1 db_1 b_3 db_3 \phi_{B_s}(x_1, b_1) \{[(1 + x_3)\phi_\eta^A(x_3, b_3) + (1 - 2x_3) r_\eta^s (\phi_\eta^P(x_3, b_3) + \phi_\eta^T(x_3, b_3))] \cdot \alpha_s(t_e^1) h_e(x_1, x_3, b_1, b_3) \exp[-S_{ab}(t_e^1)] + 2 r_\eta^s \phi_\eta^P(x_3, b_3) \alpha_s(t_e^2) h_e(x_3, x_1, b_3, b_1) \exp[-S_{ab}(t_e^2)]\}, \tag{8}$$

where $r_\eta^s = m_0^{\eta_{s\bar{s}}}/m_{B_s}$ [1]; $C_F = 4/3$ is a color factor. The function h_e, the scales t_e^i and the Sudakov factors S_{ab} are displayed in the appendix.

The form factors of B_s to $\eta^{(\prime)}$ decay, $F_{0,1}^{B_s \to \eta^{(\prime)}}(0)$, can thus be extracted from Eq. (8), that is

$$F_{0,1}^{B_s \to \eta^{(\prime)}}(q^2 = 0) = \frac{F_{e\eta^{(\prime)}}}{\sqrt{2} G_F f_\pi M_{B_s}^2}. \tag{9}$$

The operators O_5, O_6, O_7, and O_8 have a structure of $(V-A)(V+A)$. In some decay channels, some of these operators contribute to the decay amplitude in a factorizable way. Since only the axial-vector part of $(V+A)$ current contribute to the pseudoscaler meson production, $\langle \pi | V - A | B \rangle \langle \eta | V + A | 0 \rangle = -\langle \pi | V - A | B \rangle \langle \eta | V - A | 0 \rangle$, that is

$$F_{e\eta}^{P1} = -F_{e\eta}. \tag{10}$$

For the nonfactorizable diagrams 1(c) and 1(d), all three meson wave functions are involved. The integration of b_3 can be performed using δ function $\delta(b_3 - b_2)$, leaving only integration of b_1 and b_2. For the $(V-A)(V-A)$ operators, the result is

[1] The term $m_0^{\eta_{s\bar{s}}}$ is the chiral enhancement factor to be defined lately in Eq. (25).

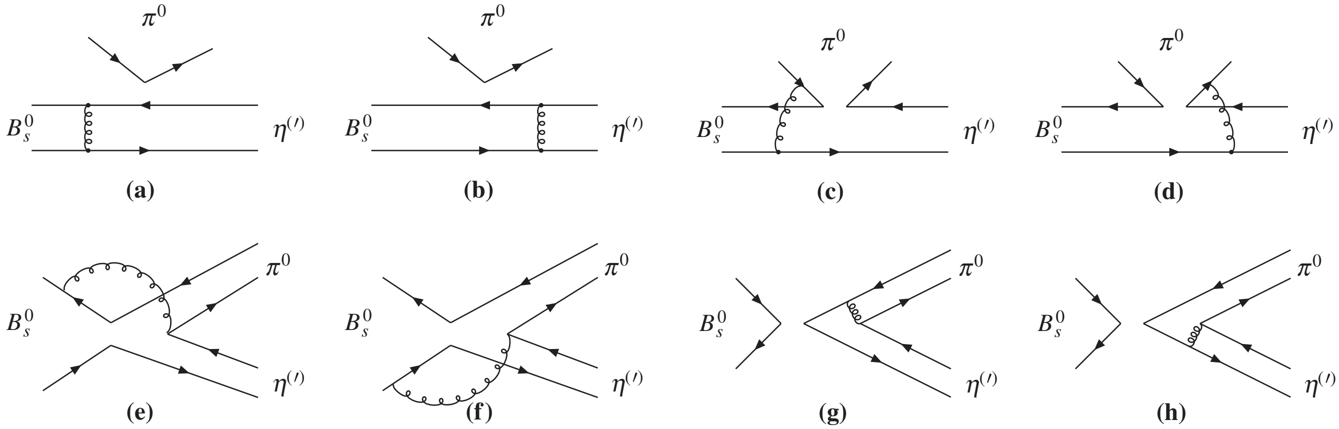

FIG. 1. Diagrams contributing to the $B_s^0 \to \pi^0\eta^{(\prime)}$ decays [diagram (a) and (b) contribute to the $B_s \to \eta^{(\prime)}$ form factor $F_{0,1}^{B_s^0\to\eta^{(\prime)}}$].

$$M_{e\eta} = \frac{16}{\sqrt{3}}\pi G_F C_F m_{B_s}^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \times \phi_{B_s}(x_1, b_1)\phi_\pi^A(x_2, b_2)x_3[\phi_\eta^A(x_3, b_2) - 2r_\eta^s \phi_\eta^T(x_3, b_2)] \cdot \alpha_s(t_f)h_f(x_1, x_2, x_3, b_1, b_2) \times \exp[-S_{cd}(t_f)]. \tag{11}$$

$M_{e\eta}^P$ is for the $(S-P)(S+P)$ type operators, which are from Fierz transformation for $(V-A)(V+A)$ operators:

$$M_{e\eta}^P = -M_{e\eta}. \tag{12}$$

For the nonfactorizable annihilation diagrams 1(e) and 1(f), again all three wave functions are involved. Here we have two kinds of contributions. $M_{a\eta}$ and $M_{a\eta}^P$ describe the contributions from the $(V-A)(V-A)$ and $(S-P)(S+P)$ type operators, respectively,

$$M_{a\eta} = -\frac{16}{\sqrt{3}}\pi G_F C_F m_{B_s}^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1)\{\{x_3\phi_\eta^A(x_3, b_2)\phi_\pi^A(x_2, b_2) + r_\pi r_\eta^{u,d}[x_2(\phi_\pi^P(x_2, b_2) - \phi_\pi^T(x_2, b_2)) \cdot (\phi_\eta^P(x_3, b_2) - \phi_\eta^T(x_3, b_2)) + x_3(\phi_\pi^P(x_2, b_2) + \phi_\pi^T(x_2, b_2)) \cdot (\phi_\eta^P(x_3, b_2) + \phi_\eta^T(x_3, b_2))]\}\alpha_s(t_f^1)h_f^1(x_1, x_2, x_3, b_1, b_2)\exp[-S_{ef}(t_f^1)] - \{x_2\phi_\pi^A(x_3, b_2)\phi_\eta^A(x_2, b_2) + r_\pi r_\eta^{u,d}[((x_2 + x_3 + 2)\phi_\eta^P(x_2, b_2) + (x_2 - x_3)\phi_\eta^T(x_2, b_2))\phi_\pi^P(x_3, b_2) + ((x_2 - x_3)\phi_\eta^P(x_3, b_2) + (x_2 + x_3 - 2)\phi_\eta^T(x_3, b_2))\phi_\pi^T(x_2, b_2)]\}\alpha_s(t_f^2)h_f^2(x_1, x_2, x_3, b_1, b_2)\exp[-S_{ef}(t_f^2)]\}, \tag{13}$$

$$M_{a\eta}^P = \frac{16}{\sqrt{3}}\pi G_F C_F m_{B_s}^4 \int_0^1 dx_1 dx_2 dx_3 \int_0^\infty b_1 db_1 b_2 db_2 \phi_{B_s}(x_1, b_1)\{\{x_2\phi_\eta^A(x_3, b_2)\phi_\pi^A(x_2, b_2) + r_\pi r_\eta^{u,d}[x_3(\phi_\pi^P(x_2, b_2) - \phi_\pi^T(x_2, b_2)) \cdot (\phi_\eta^P(x_3, b_2) - \phi_\eta^T(x_3, b_2)) + x_2(\phi_\pi^P(x_2, b_2) + \phi_\pi^T(x_2, b_2)) \cdot (\phi_\eta^P(x_3, b_2) + \phi_\eta^T(x_3, b_2))]\}\alpha_s(t_f^1)h_f^1(x_1, x_2, x_3, b_1, b_2)\exp[-S_{ef}(t_f^1)] - \{x_2\phi_\pi^A(x_3, b_2)\phi_\eta^A(x_2, b_2) + r_\pi r_\eta^{u,d}[((x_2 + x_3 + 2)\phi_\eta^P(x_2, b_2) + (x_3 - x_2)\phi_\eta^T(x_2, b_2))\phi_\pi^P(x_3, b_2) + ((x_3 - x_2)\phi_\eta^P(x_3, b_2) + (x_2 + x_3 - 2)\phi_\eta^T(x_3, b_2))\phi_\pi^T(x_2, b_2)]\}\alpha_s(t_f^2)h_f^2(x_1, x_2, x_3, b_1, b_2)\exp[-S_{ef}(t_f^2)]\}. \tag{14}$$

where $r_\eta^{u,d} = m_0^{\eta_{u\bar{u},d\bar{d}}}/m_{B_s}$ [for the definition of chiral enhancement factor $m_0^{\eta_{u\bar{u},d\bar{d}}}$, see Eq. (25)].

The factorizable annihilation diagrams 1(g) and 1(h) involve only π^0 and $\eta^{(\prime)}$ wave functions. There are also two kinds of decay amplitudes for these two diagrams. $F_{a\eta}$ is for $(V-A)(V-A)$ type operators, $F_{a\eta}^P$ is for $(V-A)(V+A)$ type operators:

$$F^P_{a\eta} = F_{a\eta} = 4\sqrt{2}\pi G_F C_F f_{B_s} m^4_{B_s} \int_0^1 dx_2 dx_3 \int_0^\infty b_2 db_2 b_3 db_3 \{[x_3 \phi^A_\eta(x_3, b_3)\phi^A_\pi(x_2, b_2) + 2r_\pi r^{u,d}_\eta((x_3+1)\phi^P_\eta(x_3, b_3) + (x_3-1)\phi^T_\eta(x_3, b_3))\phi^P_\pi(x_2, b_2)] \cdot \alpha_s(t^3_e) h_a(x_2, x_3, b_2, b_3) \exp[-S_{gh}(t^3_e)] - [x_2 \phi^A_\eta(x_3, b_3)\phi^A_\pi(x_2, b_2) + 2r_\pi r^{u,d}_\eta \phi^P_\eta(x_3, b_3)((x_2+1)\phi^P_\pi(x_2, b_2) + (x_2-1)\phi^T_\pi(x_2, b_2))] \cdot \alpha_s(t^4_e) h_a(x_3, x_2, b_3, b_2) \exp[-S_{gh}(t^4_e)]\}. \tag{15}$$

If we exchange the π and $\eta^{(\prime)}$ in Fig. 1, the corresponding expressions of amplitudes for new diagrams will be similar with those as given in Eqs. (8)–(15). The expressions of amplitudes for new diagrams can be obtained by the replacements,

$$\phi^A_\pi \leftrightarrow \phi^A_\eta, \qquad \phi^P_\pi \leftrightarrow \phi^P_\eta, \qquad \phi^T_\pi \leftrightarrow \phi^T_\eta, \qquad r_\pi \leftrightarrow r^{u,d}_\eta. \tag{16}$$

For example, we find that

$$F_{a\pi} = -F_{a\eta^{(\prime)}}, \qquad F^P_{a\pi} = -F^P_{a\eta^{(\prime)}}. \tag{17}$$

Now we are able to calculate perturbatively the form factors $F_0^{B_s\to\eta^{(\prime)}}(0)$ and the decay amplitudes for the Feynman diagrams after the integration over x_i and b_i. Since we here calculated the form factors and amplitudes at the leading order (one order of $\alpha_s(t)$), the radiative corrections at the next order would emerge in terms of $\alpha_s(t)\ln(m/t)$, where m's denote some scales, like $m_{B_s}, 1/b_i, \ldots$, in the hard part $H(t)$. We select the largest energy scale among m's appearing in each diagram as the hard scale t's for the purpose of at least killing the large logarithmic corrections partially,

$$\begin{aligned}
t^1_e &= a_t \cdot \max(\sqrt{x_3} m_{B_s}, 1/b_1, 1/b_3),\\
t^2_e &= a_t \cdot \max(\sqrt{x_1} m_{B_s}, 1/b_1, 1/b_3),\\
t^3_e &= a_t \cdot \max(\sqrt{x_3} m_{B_s}, 1/b_2, 1/b_3),\\
t^4_e &= a_t \cdot \max(\sqrt{x_2} m_{B_s}, 1/b_2, 1/b_3),\\
t_f &= a_t \cdot \max(\sqrt{x_1 x_3} m_{B_s}, \sqrt{x_2 x_3} m_{B_s}, 1/b_1, 1/b_2),\\
t^1_f &= a_t \cdot \max(\sqrt{x_2 x_3} m_{B_s}, 1/b_1, 1/b_2),\\
t^2_f &= a_t \cdot \max(\sqrt{x_1 + x_2 + x_3 - x_1 x_3 - x_2 x_3}\\
&\quad \times m_{B_s}, \sqrt{x_2 x_3} m_{B_s}, 1/b_1, 1/b_2),
\end{aligned} \tag{18}$$

where the constant $a_t = 1.0 \pm 0.2$ is introduced in order to estimate the scale dependence of the theoretical predictions for the observables.

In Refs. [15,16], a brief discussion about the $\eta - \eta'$ mixing and the gluonic component of the η' meson have been given. Here we do not show it again.

Combining the contributions from different diagrams, the total decay amplitude for $B^0_s \to \pi^0\eta$ can be written as

$$\begin{aligned}
\sqrt{6}\mathcal{M}(\pi^0\eta) = {} & F_{e\eta}\{\xi_u(C_1 + \tfrac{1}{3}C_2) - \xi_t(-\tfrac{3}{2}C_7 - \tfrac{1}{2}C_8\\
& + \tfrac{3}{2}C_9 + \tfrac{1}{2}C_{10})\}F_2(\theta_p)\\
& + M_{e\eta}\{\xi_u C_2 - \xi_t(-\tfrac{3}{2}C_8 + \tfrac{3}{2}C_{10})\}F_2(\theta_p)\\
& + (M_{a\eta} + M_{a\pi}) \cdot \{\xi_u C_2 - \xi_t \tfrac{3}{2} C_{10}\}F_1(\theta_p)\\
& - \xi_t(M^P_{a\eta} + M^P_{a\pi})\tfrac{3}{2}C_8 F_1(\theta_p).
\end{aligned} \tag{19}$$

The decay amplitudes for $B^0_s \to \pi^0\eta'$ can be obtained easily from Eqs. (19) by the following replacements

$$\begin{aligned}
F_1(\theta_p) &\to F'_1(\theta_p) = \cos\theta_p + \frac{\sin\theta_p}{\sqrt{2}},\\
F_2(\theta_p) &\to F'_2(\theta_p) = \cos\theta_p - \sqrt{2}\sin\theta_p.
\end{aligned} \tag{20}$$

Note that the possible gluonic component of η' meson has been neglected here.

III. NUMERICAL RESULTS AND DISCUSSIONS

A. Input parameters and wave functions

We use the following input parameters in the numerical calculations

$$\begin{aligned}
\Lambda^{(f=4)}_{\overline{\mathrm{MS}}} &= 250 \text{ MeV}, & f_\pi &= 130 \text{ MeV},\\
f_{B_s} &= 230 \text{ MeV}, & m_0^{\eta_{d\bar{d}}} &= 1.4 \text{ GeV},\\
m_0^{\eta_{s\bar{s}}} &= 1.95 \text{ GeV}, & f_K &= 160 \text{ MeV},\\
M_{B_s} &= 5.37 \text{ GeV}, & M_W &= 80.41 \text{ GeV}.
\end{aligned} \tag{21}$$

For the CKM matrix elements, here we adopt the Wolfenstein parametrization for the CKM matrix, and take $\lambda = 0.22$, $A = 0.853$, $\rho = 0.20$ and $\eta = 0.33$ [11].

For the B_s meson wave function, we adopt the model

$$\phi_{B_s}(x, b) = N_{B_s} x^2(1-x)^2 \exp\left[-\frac{M^2_{B_s} x^2}{2\omega^2_{B_s}} - \frac{1}{2}(\omega_{B_s} b)^2\right], \tag{22}$$

where ω_{B_s} is a free parameter and we take $\omega_{B_s} = 0.50 \pm 0.05$ GeV in numerical calculations, and $N_{B_s} = 63.7$ is the normalization factor for $\omega_{B_s} = 0.50$.

For the light meson wave function, we neglect the b dependant part, which is not important in numerical analysis. We use the wave functions of π meson ($\phi^A_\pi(x)$, $\phi^P_\pi(x)$ and $\phi^T_\pi(x)$) as given in Ref. [18]. For η meson's wave function, $\phi^A_{\eta_{d\bar{d}}}$, $\phi^P_{\eta_{d\bar{d}}}$ and $\phi^T_{\eta_{d\bar{d}}}$ represent the axial vector, pseudoscalar and tensor components of the wave function, respectively, for which we utilize the result from the light-cone sum rule [19] including twist-3 contribution. For the explicit expressions of the wave functions and the values of related quantities, one can see Eqs. (50) and (51) of Ref. [15].

We assume that the wave function of $u\bar{u}$ is same as the wave function of $d\bar{d}$. For the wave function of the $s\bar{s}$ components, we also use the same form as $d\bar{d}$ but with $m_0^{s\bar{s}}$ and f_y instead of $m_0^{d\bar{d}}$ and f_x, respectively. For f_x and f_y, we use the values as given in Ref. [20] where isospin symmetry is assumed for f_x and $SU(3)$ breaking effect is included for f_y:

$$f_x = f_\pi, \qquad f_y = \sqrt{2f_K^2 - f_\pi^2}. \tag{23}$$

These values are translated to the values in the two mixing angle method, which is often used in vacuum saturation approach as:

$$\begin{aligned} f_8 &= 169 \text{ MeV}, & f_1 &= 151 \text{ MeV}, \\ \theta_8 &= -25.9°(-18.9°), & \theta_1 &= -7.1°(-0.1°), \end{aligned} \tag{24}$$

where the pseudoscalar mixing angle θ_p is taken as $-17°$ $(-10°)$ [21]. The parameters m_0^i $(i = \eta_{d\bar{d}(u\bar{u})}, \eta_{s\bar{s}})$ are defined as:

$$m_0^{\eta_{d\bar{d}(u\bar{u})}} \equiv m_0^\pi \equiv \frac{m_\pi^2}{(m_u + m_d)}, \qquad m_0^{\eta_{s\bar{s}}} \equiv \frac{2M_K^2 - m_\pi^2}{(2m_s)}. \tag{25}$$

We include full expression of twist-3 wave functions for light mesons. The twist-3 wave functions are also adopted from QCD sum rule calculations [22]. We will see later that this set of parameters will give good results for $B_s \to \pi^0\eta^{(\prime)}$ decays.

B. Branching ratios

For $B_s \to \pi^0\eta^{(\prime)}$ decays, the decay amplitudes in Eqs. (19) can be rewritten as

$$\mathcal{M} = V_{ub}^* V_{us} T - V_{tb}^* V_{ts} P = V_{ub}^* V_{us} T[1 + z e^{i(\gamma+\delta)}], \tag{26}$$

where

$$z = \left| \frac{V_{tb}^* V_{ts}}{V_{ub}^* V_{us}} \right| \left| \frac{P}{T} \right| \tag{27}$$

is the ratio of penguin to tree contributions, $\gamma = \arg[-\frac{V_{ts}V_{tb}^*}{V_{us}V_{ub}^*}]$ is the weak phase (one of the three CKM angles), and δ is the relative strong phase between penguin (P) and tree (T) diagrams. In the pQCD approach, it is easy to calculate the ratio z and the strong phase δ for the decay in study. For $B_s \to \pi^0\eta$ and $\pi^0\eta'$ decays, we find numerically that

$$z(\pi^0\eta) = 38.3, \qquad \delta(\pi^0\eta) = -94°, \tag{28}$$

$$z(\pi^0\eta') = 5.5, \qquad \delta(\pi^0\eta') = -20°. \tag{29}$$

The main error of the ratio z and the strong phase δ is induced by the uncertainty of $\omega_{b_s} = 0.50 \pm 0.05$ GeV.

Using the wave functions and the input parameters as specified in previous sections, it is straightforward to calculate the branching ratios for the four considered decays. The theoretical predictions in the pQCD approach for the CP-averaged branching ratios of the decays under consideration are the following

$$\mathrm{Br}(B_s^0 \to \pi^0\eta) = [0.86^{+0.37}_{-0.24}(\omega_{B_s})^{+0.33}_{-0.21}(m_s)^{+1.00}_{-0.09}(a_t)] \times 10^{-7}, \tag{30}$$

$$\mathrm{Br}(B_s^0 \to \pi^0\eta') = [1.86^{+0.76}_{-0.51}(\omega_{B_s})^{+0.63}_{-0.41}(m_s)^{+1.46}_{-0.21}(a_t)] \times 10^{-7}, \tag{31}$$

for $\theta_p = -17°$, and

$$\mathrm{Br}(B_s^0 \to \pi^0\eta) = [1.18^{+0.50}_{-0.33}(\omega_{B_s})^{+0.45}_{-0.29}(m_s)^{+1.03}_{-0.12}(a_t)] \times 10^{-7}, \tag{32}$$

$$\mathrm{Br}(B_s^0 \to \pi^0\eta') = [1.54^{+0.63}_{-0.42}(\omega_{B_s})^{+0.52}_{-0.34}(m_s)^{+1.19}_{-0.21}(a_t)] \times 10^{-7}. \tag{33}$$

for $\theta_p = -10°$. The main errors are induced by the uncertainties of $a_t = 1.0 \pm 0.2$, $\omega_{B_s} = 0.50 \pm 0.05$ GeV and $m_s = 120 \pm 20$ MeV, respectively.

It is easy to see that (a) the errors of the branching ratios induced by varying a_t in the range of $a_t = [0.8, 1.2]$ can be significant for the penguin-dominated $B_s \to \pi^0\eta^{(\prime)}$ decays; and (b) the variations with respect to the central values are large for the case of $a_t = 0.8$, but very small for the case of $a_t = 1.2$). This feature agrees with general expectations: when the scale t become smaller, the reliability of the perturbative calculation of the form factors in pQCD approach will become weak!

The pQCD predictions of the branching ratios as given in Eqs. (30)–(33) agree well with the theoretical predictions in the QCDF approach, for example, as given in Ref. [5]:

$$\begin{aligned} \mathrm{Br}(B_s^0 \to \pi^0\eta) &= (0.75^{+0.35}_{-0.30}) \times 10^{-7}, \\ \mathrm{Br}(B_s^0 \to \pi^0\eta') &= (1.1^{+0.24}_{-0.24}) \times 10^{-7}, \end{aligned} \tag{34}$$

where the individual errors as given in Ref. [5] have been added in quadrature.

C. CP-violating asymmetries

Now we turn to study the CP-violating asymmetries for $B_s^0 \to \pi^0\eta^{(\prime)}$ decays. For these neutral decay modes, the effects of $B_s^0 - \bar{B}_s^0$ mixing should be considered.

For B_s^0 meson decays, we know that $\Delta\Gamma/\Delta m_s \ll 1$ and $\Delta\Gamma/\Gamma \ll 1$. The CP-violating asymmetry of $B_s^0(\bar{B}_s^0) \to \pi^0\eta^{(\prime)}$ decay is time dependent and can be defined as

$$A_{CP} \equiv \frac{\Gamma(\bar{B}_s^0(\Delta t) \to f_{CP}) - \Gamma(B_s^0(\Delta t) \to f_{CP})}{\Gamma(\bar{B}_s^0(\Delta t) \to f_{CP}) + \Gamma(B_s^0(\Delta t) \to f_{CP})} = A_{CP}^{\rm dir} \cos(\Delta m_s \Delta t) + A_{CP}^{\rm mix} \sin(\Delta m_s \Delta t), \tag{35}$$

where Δm_s is the mass difference between the two B_s^0 mass eigenstates, $\Delta t = t_{CP} - t_{\rm tag}$ is the time difference between the tagged B_s^0 ($\bar{B}_s^0$) and the accompanying $\bar{B}_s^0$ (B_s^0) with opposite b flavor decaying to the final CP-eigenstate f_{CP} at the time t_{CP}. The direct and mixing induced CP-violating asymmetries $A_{CP}^{\rm dir}$ and $A_{CP}^{\rm mix}$ can be written as

$$\mathcal{A}_{CP}^{\rm dir} = \frac{|\lambda_{CP}|^2 - 1}{1 + |\lambda_{CP}|^2}, \qquad A_{CP}^{\rm mix} = \frac{2\,{\rm Im}(\lambda_{CP})}{1 + |\lambda_{CP}|^2}, \tag{36}$$

where the CP-violating parameter λ_{CP} is

$$\lambda_{CP} = \frac{V_{tb}^* V_{ts} \langle \pi^0 \eta^{(\prime)} | H_{\rm eff} | \bar{B}_s^0 \rangle}{V_{tb} V_{ts}^* \langle \pi^0 \eta^{(\prime)} | H_{\rm eff} | B_s^0 \rangle} = e^{2i\gamma} \frac{1 + z e^{i(\delta - \gamma)}}{1 + z e^{i(\delta + \gamma)}}. \tag{37}$$

Here the ratio z and the strong phase δ have been defined previously. In pQCD approach, since both z and δ are calculable, it is easy to find the numerical values of $A_{CP}^{\rm dir}$ and $A_{CP}^{\rm mix}$ for the considered decay processes.

In Figs. 2 and 3, we show the γ dependence of the CP-violating asymmetry $A_{CP}^{\rm dir}$ and $A_{CP}^{\rm mix}$ for $B_s^0 \to \pi^0 \eta$ (solid curve) and $B_s^0 \to \pi^0 \eta'$ (dotted curve) decays for $\theta_p = -17°$.

The pQCD predictions for the direct CP-violating asymmetries of $B_s^0 \to \pi^0 \eta^{(\prime)}$ decays are

$$\mathcal{A}_{CP}^{\rm dir}(B_s^0 \to \pi^0 \eta) = [-4.5^{+1.2}_{-0.6}(\gamma)^{+0.6}_{-0.4}(\omega_{B_s}) \pm 0.6(m_0^\pi)^{+1.7}_{-1.8} \times (m_s)^{+0.7}_{-0.2}(a_t)] \times 10^{-2}, \tag{38}$$

$$\mathcal{A}_{CP}^{\rm dir}(B_s^0 \to \pi^0 \eta') = [-9.1^{+2.8}_{-2.3}(\gamma)^{+0.3}_{-0.6}(\omega_{B_s}) \pm 0.3(m_0^\pi) \pm 1.9(m_s)^{+4.1}_{-1.5}(a_t)] \times 10^{-2}, \tag{39}$$

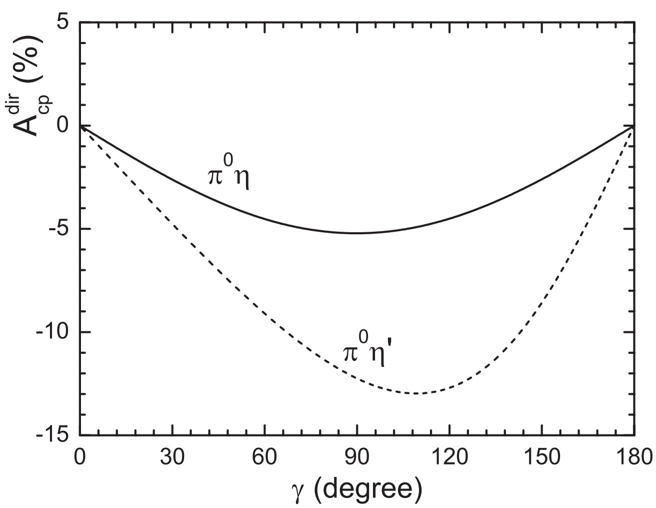

FIG. 2. The direct CP asymmetry $A_{CP}^{\rm dir}$ (in percentage) of $B_s \to \pi^0 \eta$ (solid curve) and $B_s \to \pi^0 \eta'$ (dotted curve) as a function of CKM angle γ for the case of $\theta_p = -17°$.

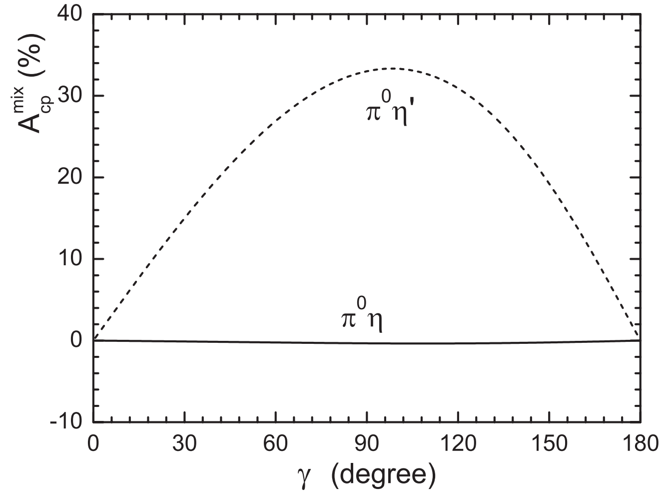

FIG. 3. The mixing induced CP asymmetry $A_{CP}^{\rm mix}$ (in percentage) of $B_s \to \pi^0 \eta$ (solid curve) and $B_s \to \pi^0 \eta'$ (dotted curve) as a function of CKM angle γ for the case of $\theta_p = -17°$.

where the dominant errors come from the variations of $\omega_{B_s} = 0.50 \pm 0.05$ GeV, $m_0^\pi = 1.4 \pm 0.3$ GeV, $a_t = 1.0 \pm 0.2$, $m_s = 120 \pm 20$ MeV and $\gamma = 60° \pm 20°$.

As a comparison, we present the QCDF predictions for $\mathcal{A}_{CP}^{\rm dir}(B_s^0 \to \pi^0 \eta')$ directly quoted from Ref. [5]

$$\mathcal{A}_{CP}^{\rm dir}(B_s^0 \to \pi^0 \eta') = (27.8^{+6.0}_{-7.1}{}^{+9.6}_{-5.7}{}^{+2.0}_{-2.0}{}^{+24.7}_{-27.2}) \times 10^{-2}, \tag{40}$$

where the "default values" of the input parameters have been used in Ref. [5], and the error sources are the same as the first four input parameters in Eqs. (38) and (39). Currently, no relevant experimental measurements for the CP-violating asymmetries of $B_s^0 \to \pi^0 \eta^{(\prime)}$ decays are available.

The pQCD predictions for the mixing induced CP-violating asymmetries of $B_s^0 \to \pi^0 \eta^{(\prime)}$ decays are

$$\mathcal{A}_{CP}^{\rm mix}(B_s^0 \to \pi^0 \eta) = [-0.2 \pm 0.1(\gamma)^{+2.5}_{-2.1}(\omega_{B_s})^{+1.2}_{-1.4}(m_0^\pi)^{+4.4}_{-4.5} \times (m_s)^{+26.3}_{-11.6}(a_t)] \times 10^{-2}, \tag{41}$$

$$\mathcal{A}_{CP}^{\rm mix}(B_s^0 \to \pi^0 \eta') = [27.0^{+4.8}_{-7.5}(\gamma)^{+0.4}_{-0.7}(\omega_{B_s})^{+0.6}_{-0.5}(m_0^\pi) \pm 0.2(m_s)^{+17.1}_{-8.3}(a_t)] \times 10^{-2}, \tag{42}$$

where the dominant errors come from the variations of $\omega_{B_s} = 0.50 \pm 0.05$ GeV, $m_0^\pi = 1.4 \pm 0.3$ GeV, $a_t = 1.0 \pm 0.2$, $m_s = 120 \pm 20$ MeV and $\gamma = 60° \pm 20°$.

If we integrate the time variable t, we will get the total CP asymmetry for $B_s^0 \to \pi^0 \eta^{(\prime)}$ decays,

$$A_{CP} = \frac{1}{1 + x^2} A_{CP}^{\rm dir} + \frac{x}{1 + x^2} A_{CP}^{\rm mix}, \tag{43}$$

where $x = \Delta m_s / \Gamma = 26.5$ for the $B_s^0 - \bar{B}_s^0$ mixing [23]. We found numerically that the magnitude of the total CP asymmetry for ($B_s^0 \to \pi^0 \eta^{(\prime)}$) decays are smaller than 2% in the whole considered parameter space.

D. Effects of possible gluonic component of η'

Now we consider the contributions to the considered $B_s \to \pi\eta^{(\prime)}$ decays induced by the possible gluonic component of $\eta^{(\prime)}$ meson [21,24,25].

Among various mechanisms proposed to account for the distinctive pattern of branching ratios of $B \to K^{(*)}\eta^{(\prime)}$ decays [26–30], the assumption of a nonzero gluonic component of $\eta^{(\prime)}$ meson and its rule in interpreting the anomalously large decay rate $\mathrm{Br}(B \to K\eta')$ have been studied intensively [21,24,25,31,32]

In Ref. [21], Kou examined the contributions to the gluon fusion process $gg \to \eta'$. In his paper [21], the η and η' meson were written as

$$\begin{aligned} |\eta\rangle &= X_\eta|\eta_q\rangle + Y_\eta|\eta_s\rangle, \\ |\eta'\rangle &= X_{\eta'}|\eta_q\rangle + Y_{\eta'}|\eta_s\rangle + Z_{\eta'}|\mathrm{gluonium}\rangle, \end{aligned} \tag{44}$$

where $\eta_q = (u\bar{u} + d\bar{d})/\sqrt{2}$ and $\eta_s = s\bar{s}$. From the experimental data on the radiative light meson decays, such as $\phi \to \eta'\gamma$, $\eta' \to (\phi, \rho, \gamma)\gamma$ and $J/|\psi \to \eta'\gamma$ decays, the author found that the gluonic component in η' should be less than 26%.

By employing the QCD factorization approach, Beneke and Neubert [31] studied the $B \to K^{(*)}\eta^{(\prime)}$ decays by considering systematically the flavor-singlet amplitudes.[2] They estimated the flavor-singlet contributions to $B \to K\eta^{(\prime)}$ branching ratios, including those from the $b \to sgg \to s\eta^{(\prime)}$ transition, from the hard or soft spectator scattering contributions [29–31] (see Fig. 3 of Ref. [31] for the relevant Feynman diagrams), and from the flavor-singlet weak annihilation contribution. They also considered the leading two-gluon contribution to the $B \to \eta^{(\prime)}$ form factors and claimed that such contribution might be significant for the form factor $F_0^{B\to\eta'}$, but small for $F_0^{B\to\eta}$.

In Ref. [32], by employing the pQCD factorization approach, Charng, Kurimoto and Li calculated the flavor-singlet contribution to the $B \to \eta^{(\prime)}$ transition form factors from the gluonic content of the $\eta^{(\prime)}$ meson, induced by the Feynman diagrams with the two gluons emitted from the light quark of the B meson (see Fig. 1 of Ref. [32]). They firstly announced that the enhancement to the form factor $F_{0,1}^{B\to\eta'}$ can reach 10%–40%, but after removing an error in their computer program,[3] they found that the gluonic contributions to both $B \to \eta$ and $B \to \eta'$ form factors are less than 5%.

In order to make a rough numerical estimate of the gluonic effects on the decay modes under study, we here follow the same procedure as being used in Ref. [32] to estimate the gluonic contributions to the $B_s \to \eta^{(\prime)}$ transition form factors $F_{0,1}^{B_s\to\eta^{(\prime)}}$ and in turn to the branching ratios and *CP* violating asymmetries. Using the formulae as given in Ref. [32], we found that the gluonic contributions to the branching ratios are less than 4% for $B_s \to \pi^0\eta$ decay, and around 20% for $B_s \to \pi\eta'$. The central values are now

$$\mathrm{Br}(B_s^0 \to \pi^0\eta) = 0.83 \times 10^{-7}, \tag{45}$$

$$\mathrm{Br}(B_s^0 \to \pi^0\eta') = 2.25 \times 10^{-7}, \tag{46}$$

for $\theta_p = -17°$, and

$$\mathrm{Br}(B_s^0 \to \pi^0\eta) = 1.17 \times 10^{-7}, \tag{47}$$

$$\mathrm{Br}(B_s^0 \to \pi^0\eta') = 1.90 \times 10^{-7} \tag{48}$$

for $\theta_p = -10°$.

As for the *CP*-violating asymmetries of $B_s \to \pi^0\eta^{(\prime)}$ decays, the gluonic corrections are largely canceled in the ratio and therefore negligible: less than 5% and 10% for $B_s \to \pi^0\eta$ and $B_s \to \pi^0\eta'$ decay, respectively.

The smallness of the gluonic corrections to the branching ratios can be understood as follows. First, the gluonic correction to the form factors are small in size: ~2% for $F_0^{B_s\to\eta}$, and around 13% for $F_0^{B_s\to\eta'}$. Second, only the first two diagrams Fig. 1(a) and 1(b) are affected by the gluonic corrections to $B \to \eta'$ form factor, while the contributions from other six diagrams remain unchanged, the total effects are thus not large in size.

Although much progress have been achieved in recent years, but frankly speaking, we currently still do not know how to calculate reliably the contributions of the possible gluonic component of $\eta^{(\prime)}$ meson. From our previous works, as presented in Refs. [15,16] where only the dominant contributions from quark contents of η and η' were taken into account, the pQCD predictions for the branching ratios of $B \to \rho\eta^{(\prime)}$ and $B \to \pi\eta^{(\prime)}$ decays also show a very good agreement with currently available data. It seems that large gluonic contributions are unnecessary for these decay modes. Latest calculations in this paper and in Refs. [32–34] also show that the gluonic contributions to $B \to K\eta^{(\prime)}$, $\eta^{(\prime)}\eta$ and $B_s \to \pi^0\eta^{(\prime)}$ decays are all small.

Of course, more theoretical studies about the gluonic contributions to B meson two-body decays involving $\eta^{(\prime)}$ meson as final state particles are clearly needed, and better experimental measurements for the relevant decay modes are also necessary to clarify this point.

[2]The flavor-singlet amplitude was defined as the one for producing a $q\bar{q}$ pair not containing the spectator quark in the coherent flavor-singlet state $(u\bar{u} + d\bar{d} + s\bar{s})$ or a pair of gluons, where the quark or gluon pairs will hadronize into an η or η' meson.

[3]According to Li's latest talk [33], the analytical formulae as given in Eqs. (32–40) of Ref. [32] are correct, but the numerical results about the gluonic contribution to $B \to \eta'$ form factor as presented in Ref. [32] are not correct because of an error in their computer program.

IV. SUMMARY

In this paper, we calculate the branching ratios and CP-violating asymmetries of $B_s^0 \to \pi^0\eta$, $B_s^0 \to \pi^0\eta'$ decays in the pQCD factorization approach.

Besides the usual factorizable diagrams, the nonfactorizable and annihilation diagrams are also calculated analytically. Although the nonfactorizable and annihilation contributions are subleading for the branching ratios of the considered decays, but they are not negligible. Furthermore these diagrams provide the necessary strong phase required by a nonzero CP-violating asymmetry for the considered decays.

From our calculations and phenomenological analysis, we found the following results:

(i) The pQCD predictions for the form factors are $F_{0,1}^{B_s\to\eta}(0) = -0.276$ and $F_{0,1}^{B_s\to\eta'}(0) = 0.278$, which agree well with those obtained from other methods.

(ii) For the CP-averaged branching ratios of the considered decay modes, the pQCD predictions for $\theta_p = 17°$ are

$$\begin{aligned} \mathrm{Br}(B_s^0 \to \pi^0\eta) &= (0.86^{+1.12}_{-0.33}) \times 10^{-7}, \\ \mathrm{Br}(B_s^0 \to \pi^0\eta') &= (1.86^{+1.76}_{-0.69}) \times 10^{-7}, \end{aligned} \tag{49}$$

here the various errors as specified in Eqs. (30) and (31) have been added in quadrature. The pQCD predictions are also well consistent with the results obtained by employing the QCD factorization approach.

(iii) For the CP-violating asymmetries, the pQCD predictions for $\mathcal{A}_{CP}^{\mathrm{dir}}(B_s \to \pi^0\eta^{(\prime)})$ and $\mathcal{A}_{CP}^{\mathrm{mix}}(B_s \to \pi^0\eta^{(\prime)})$ are generally not very large, while the time-integrated CP asymmetries are less than 2% in magnitude.

(iv) The major theoretical errors of the computed observables are induced by the uncertainties of the hard energy scale t_j's, the parameters ω_{B_s} and m_s, as well as the CKM angle γ for CP asymmetries.

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APPENDIX: RELATED FUNCTIONS

We show here the function h_i's, coming from the Fourier transformations of $H^{(0)}$,

$$\begin{aligned} h_e(x_1, x_3, b_1, b_3) = {} & K_0(\sqrt{x_1x_3}m_B b_1)[\theta(b_1 - b_3) \\ & \times K_0(\sqrt{x_3}m_B b_1)I_0(\sqrt{x_3}m_B b_3) \\ & + \theta(b_3 - b_1)K_0(\sqrt{x_3}m_B b_3) \\ & \times I_0(\sqrt{x_3}m_B b_1)]S_t(x_3), \end{aligned} \tag{A1}$$

$$\begin{aligned} h_a(x_2, x_3, b_2, b_3) = {} & K_0(i\sqrt{x_2x_3}m_B b_3)[\theta(b_3 - b_2) \\ & \times K_0(i\sqrt{x_3}m_B b_3)I_0(i\sqrt{x_3}m_B b_2) \\ & + \theta(b_2 - b_3)K_0(i\sqrt{x_3}m_B b_2) \\ & \times I_0(i\sqrt{x_3}m_B b_3)]S_t(x_3), \end{aligned} \tag{A2}$$

$$\begin{aligned} h_f(x_1, x_2, x_3, b_1, b_2) = {} & \{\theta(b_2 - b_1)I_0(M_B\sqrt{x_1x_3}b_1)K_0(M_B\sqrt{x_1x_3}b_2) \\ & + (b_1 \leftrightarrow b_2)\} \cdot \begin{pmatrix} K_0(M_B F_{(1)} b_1), & \text{for } F^2_{(1)} > 0 \\ \frac{\pi i}{2}\mathrm{H}_0^{(1)}(M_B\sqrt{|F^2_{(1)}|}b_1), & \text{for } F^2_{(1)} < 0 \end{pmatrix}, \end{aligned} \tag{A3}$$

$$\begin{aligned} h_f^3(x_1, x_2, x_3, b_1, b_2) = {} & \{\theta(b_1 - b_2)K_0(i\sqrt{x_2x_3}b_1M_B)I_0(i\sqrt{x_2x_3}b_2M_B) \\ & + (b_1 \leftrightarrow b_2)\} \cdot \frac{\pi i}{2}\mathrm{H}_0^{(1)}(\sqrt{x_1 + x_2 + x_3 - x_1x_3 - x_2x_3}b_1M_B), \end{aligned} \tag{A4}$$

$$\begin{aligned} h_f^4(x_1, x_2, x_3, b_1, b_2) = {} & \{\theta(b_1 - b_2)K_0(i\sqrt{x_2x_3}b_1M_B)I_0(i\sqrt{x_2x_3}b_2M_B) \\ & + (b_1 \leftrightarrow b_2)\} \cdot \begin{pmatrix} K_0(M_B F_{(2)} b_1), & \text{for } F^2_{(2)} > 0 \\ \frac{\pi i}{2}\mathrm{H}_0^{(1)}(M_B\sqrt{|F^2_{(2)}|}b_1), & \text{for } F^2_{(2)} < 0 \end{pmatrix}, \end{aligned} \tag{A5}$$

where J_0 is the Bessel function and K_0, I_0 are modified Bessel functions $K_0(-ix) = -(\pi/2)Y_0(x) + i(\pi/2)J_0(x)$, and $F_{(j)}$'s are defined by

$$F^2_{(1)} = (x_1 - x_2)x_3, \tag{A6}$$

$$F^2_{(2)} = (x_1 - x_2)x_3. \tag{A7}$$

The threshold resummation form factor $S_t(x_i)$ is adopted from Ref. [35]

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{A8}$$

where the parameter $c = 0.3$. This function is normalized to unity.

The Sudakov factors used in the text are defined as

$$\begin{aligned} S_{ab}(t) &= s(x_1 m_B/\sqrt{2}, b_1) + s(x_3 m_B/\sqrt{2}, b_3) \\ &\quad + s((1-x_3)m_B/\sqrt{2}, b_3) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_3\Lambda)}\right], \end{aligned} \tag{A9}$$

$$\begin{aligned} S_{cd}(t) &= s(x_1 m_B/\sqrt{2}, b_1) + s(x_2 m_B/\sqrt{2}, b_2) \\ &\quad + s((1-x_2)m_B/\sqrt{2}, b_2) + s(x_3 m_B/\sqrt{2}, b_2) \\ &\quad + s((1-x_3)m_B/\sqrt{2}, b_2) \\ &\quad - \frac{1}{\beta_1}\left[2\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \end{aligned} \tag{A10}$$

$$\begin{aligned} S_{ef}(t) &= s(x_1 m_B/\sqrt{2}, b_1) + s(x_2 m_B/\sqrt{2}, b_2) \\ &\quad + s((1-x_2)m_B/\sqrt{2}, b_2) + s(x_3 m_B/\sqrt{2}, b_2) \\ &\quad + s((1-x_3)m_B/\sqrt{2}, b_2) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + 2\ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \end{aligned} \tag{A11}$$

$$\begin{aligned} S_{gh}(t) &= s(x_2 m_B/\sqrt{2}, b_2) + s(x_3 m_B/\sqrt{2}, b_3) \\ &\quad + s((1-x_2)m_B/\sqrt{2}, b_2) + s((1-x_3)m_B/\sqrt{2}, b_3) \\ &\quad - \frac{1}{\beta_1}\left[\ln\frac{\ln(t/\Lambda)}{-\ln(b_1\Lambda)} + \ln\frac{\ln(t/\Lambda)}{-\ln(b_2\Lambda)}\right], \end{aligned} \tag{A12}$$

where the function $s(q, b)$ are defined in the Appendix A of Ref. [36].

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