

**Radiative decays of the  $D_{s0}(2317)$ ,  $D_{s1}(2460)$  and the related strong coupling constants**

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In this article, we take the point of view that the charmed mesons  $D_{s0}(2317)$  and  $D_{s1}(2460)$  with the spin-parity  $0^+$  and  $1^+$  respectively are the conventional  $c\bar{s}$  states, and calculate the strong coupling constants  $G_S$  (for  $\langle D_s^* \phi | D_{s0} \rangle$ ) and  $G_A$  (for  $\langle D_s \phi | D_{s1} \rangle$ ) in the framework of the light-cone QCD sum rules approach. The strong coupling constants  $G_S$  and  $G_A$  are related to the basic parameter  $\hat{\mu}$  in the heavy quark effective Lagrangian, the numerical value is larger than the existing estimation. With the assumption of the vector meson dominance of the intermediate  $\phi(1020)$ , we study the radiative decays  $D_{s0} \rightarrow D_s^* \gamma$  and  $D_{s1} \rightarrow D_s \gamma$ .

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**I. INTRODUCTION**

The observation of the two charmed resonances  $D_{s0}(2317)$  in the  $D_s \pi^0$  invariant mass distribution and  $D_{s1}(2460)$  in the  $D_s^* \pi^0$  and  $D_s \gamma$  mass distributions has triggered hot debate on their nature, under-structures and whether it is necessary to introduce the exotic states [1,2]. They cannot be comfortably identified as the quark-antiquark bound states in the spectrum of the constituent quark models, their masses are significantly lower than the values of the  $0^+$  and  $1^+$  states, respectively, from the quark models and lattice simulations [3]. The difficulties to identify the  $D_{s0}(2317)$  and  $D_{s1}(2460)$  states with the conventional  $c\bar{s}$  mesons are rather similar to those appearing in the light scalar mesons below 1 GeV. The light scalar mesons are the subject of an intense and continuous controversy in clarifying the hadron spectroscopy [4], the more elusive things are the constituent structures of the  $f_0(980)$  and  $a_0(980)$  mesons with almost the degenerate masses. The mesons  $D_{s0}(2317)$  and  $D_{s1}(2460)$  lie just below the  $DK$  and  $D^*K$  threshold, respectively, which are analogous to the situation that the scalar mesons  $a_0(980)$  and  $f_0(980)$  lie just below the  $K\bar{K}$  threshold and couple strongly to the nearby channels. The mechanism responsible for the low-mass charmed mesons may be the same as the light scalar nonet mesons, the  $f_0(600)$ ,  $f_0(980)$ ,  $a_0(980)$  and  $K_0^*(800)$  [5–9]. There have been a lot of explanations for their nature, for example, the conventional  $c\bar{s}$  states [10], two-meson molecular states [11], four-quark states [12], etc.<sup>1</sup> If we take the scalar mesons  $a_0(980)$  and  $f_0(980)$  as four quark states with the constituents of scalar diquark-antidiquark substructures, the masses of the scalar nonet mesons below 1 GeV can be naturally explained [8,9].

There are other possibilities besides the four-quark state explanations, for example, the scalar mesons  $a_0(980)$ ,  $f_0(980)$ ,  $D_{s0}(2317)$  and the axial-vector meson  $D_{s1}(2460)$  may have bare  $P$ -wave  $q\bar{q}$  and  $c\bar{s}$  kernels with

strong coupling to the nearby thresholds, respectively, the  $S$ -wave virtual intermediate hadronic states (or the virtual mesons loops) play a crucial role in the composition of those bound states (or resonances due to the masses below or above the thresholds). The hadronic dressing mechanism (or unitarized quark models) takes the point of view that the mesons  $f_0(980)$ ,  $a_0(980)$ ,  $D_{s0}(2317)$  and  $D_{s1}(2460)$  have small  $q\bar{q}$  and  $c\bar{s}$  kernels of the typical  $q\bar{q}$  and  $c\bar{s}$  mesons size, respectively. The strong couplings to the virtual intermediate hadronic states (or the virtual mesons loops) may result in smaller masses than the conventional scalar  $q\bar{q}$  and  $c\bar{s}$  mesons in the constituent quark models, enrich the pure  $q\bar{q}$  and  $c\bar{s}$  states with other components [13,14]. Those mesons may spend part (or most part) of their lifetime as virtual  $K\bar{K}$ ,  $DK$  and  $D^*K$  states [5–7,13,14].

The radiative decays can be used to probe the under-structures of the hadrons, and they are suitable to understand the nature of the  $D_{s0}(2317)$ ,  $D_{s1}(2460)$  and distinguish among different interpretations [15–20]. Different under-structures can lead to different decay widths, and the predictions can be compared with the experimental measurements. For example, the value of the strong coupling constant  $g_{D_{s0}DK}$  with the assumption that the  $D_{s0}(2317)$  being a conventional scalar  $c\bar{s}$  state is much larger than (or several times as large as) the corresponding value with the assumption of being a tetraquark state [6,21]. The  $D_{s0}(2317)$  cannot decay to the  $D_s \gamma$  due to the angular momentum and parity conservation, and such a final state has not been observed; the decay  $D_{s0} \rightarrow D_s^* \gamma$  is allowed and no evidence is reported yet of the final state  $D_s \gamma \gamma$  resulting from the decay chain  $D_{s0} \rightarrow D_s^* \gamma \rightarrow D_s \gamma \gamma$ . The radiative decay widths of the  $D_{s0} \rightarrow D_s^* \gamma$  and  $D_{s1} \rightarrow D_s \gamma$  have been calculated with the constituent quark model [17–19], the vector meson dominance (VMD) ansatz [16] and the light-cone QCD sum rules [15], etc.

The amplitudes of the radiative decays  $D_{s0} \rightarrow D_s^* \gamma$  and  $D_{s1} \rightarrow D_s \gamma$  can be written as

$$\begin{aligned} \langle D_s^*(p, \eta) \gamma(k, \epsilon) | D_{s0} \rangle &= ed_S \{ \epsilon^* \cdot \eta^* p \cdot k - \epsilon^* \cdot p \eta^* \cdot k \}, \\ \langle D_s \gamma(k, \epsilon) | D_{s1}(p, \eta) \rangle &= ied_A \{ \epsilon^* \cdot \eta p \cdot k - \epsilon^* \cdot p \eta \cdot k \} \end{aligned} \quad (1)$$

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<sup>1</sup>The literatures listed here are far from complete, for more literatures, one can consult Ref. [2].

due to the Lorentz covariance. The  $p_\mu$  and  $k_\mu$  are the four momenta of the  $D_s^*(D_{s1})$  and  $\gamma$ , respectively; the  $\eta_\mu$  and  $\epsilon_\mu$  are the polarization vectors of the  $D_s^*(D_{s1})$  and  $\gamma$ , respectively. The parameters  $d_s$  and  $d_A$  have the dimension of inverse of the mass, and get contributions from the photon couplings both to the light quark part  $e_s \bar{s} \gamma_\mu s$  and to the heavy quark part  $e_c \bar{c} \gamma_\mu c$  of the electromagnetic current, here the  $e_s$  and  $e_c$  are strange and charm quark charges in units of  $e$ . In order to determine the amplitudes of the  $D_{s0} \rightarrow D_s^* \gamma$  and  $D_{s1} \rightarrow D_s \gamma$ , we follow the VMD ansatz [16,22]. In the heavy quark limit, the matrix elements  $\langle D_s^*(v', \eta) | \bar{c} \gamma_\mu c | D_{s0}(v) \rangle$  and  $\langle D_s(v') | \bar{c} \gamma_\mu c | D_{s1}(v, \eta) \rangle$  vanish for  $v \cdot v' = (M_{D_{s0}}^2 + M_{D_s^*}^2)/2M_{D_{s0}}M_{D_s^*} \approx 1$  and  $v \cdot v' = (M_{D_{s1}}^2 + M_{D_s}^2)/2M_{D_{s1}}M_{D_s} \approx 1$ , here the  $v_\mu, v'_\mu$  are the four-velocities of the heavy mesons. We consider only the contribution of the intermediate  $\phi(1020)$ ,

$$\begin{aligned} \langle D_s^*(p, \eta) \gamma(k, \epsilon) | D_{s0} \rangle &= \langle D_s^*(p, \eta) \phi(q, \xi) | D_{s0} \rangle \frac{i}{q^2 - m_\phi^2} \\ &\quad \times \langle \gamma(k, \epsilon) | \phi(q, \xi) \rangle \\ &= \langle D_s^*(p, \eta) \phi(k, \xi) | D_{s0} \rangle \frac{i}{k^2 - m_\phi^2} \\ &\quad \times f_\phi m_\phi e Q_s (-i) \epsilon^* \cdot \xi \\ &= G_S (\epsilon^* \cdot \eta^* p \cdot k - \epsilon^* \cdot p \eta^* \cdot k) \\ &\quad \times \frac{f_\phi}{m_\phi} e Q_s, \end{aligned} \quad (2)$$

$$\begin{aligned} \langle D_s \gamma(k, \epsilon) | D_{s1}(p, \eta) \rangle &= \langle D_s \phi(q, \xi) | D_{s1}(p, \eta) \rangle \frac{i}{q^2 - m_\phi^2} \\ &\quad \times \langle \gamma(k, \epsilon) | \phi(q, \xi) \rangle \\ &= \langle D_s \phi(k, \xi) | D_{s1}(p, \eta) \rangle \frac{i}{k^2 - m_\phi^2} \\ &\quad \times f_\phi m_\phi e Q_s (-i) \epsilon^* \cdot \xi \\ &= i G_A (\epsilon^* \cdot \eta p \cdot k - \epsilon^* \cdot p \eta \cdot k) \\ &\quad \times \frac{f_\phi}{m_\phi} e Q_s, \end{aligned} \quad (3)$$

where we have use the VMD Lagrangian

$$\mathcal{L} = -eQ \frac{M_\phi^2}{g_\phi} A_\alpha \phi^\alpha = -eQ f_\phi M_\phi A_\alpha \phi^\alpha, \quad (4)$$

and the definitions of the strong coupling constants

$$\begin{aligned} \langle D_s^*(p, \eta) \phi(k, \xi) | D_{s0} \rangle &= G_S (\xi^* \cdot \eta^* p \cdot k - \xi^* \cdot p \eta^* \cdot k), \\ \langle D_s \phi(k, \xi) | D_{s1}(p, \eta) \rangle &= i G_A (\xi^* \cdot \eta p \cdot k - \xi^* \cdot p \eta \cdot k), \\ \langle 0 | \bar{s}(0) \gamma_\mu s(0) | \phi(k, \xi) \rangle &= f_\phi m_\phi \xi_\mu. \end{aligned} \quad (5)$$

The  $G_S$  and  $G_A$  are the strong coupling constants, the  $f_\phi$  is the weak decay constant of the vector meson  $\phi$  and the  $\xi_\mu$

is the polarization vector. The strong coupling constants  $G_S$  and  $G_A$  can be related to the effective coupling constant  $\hat{\mu}$  in the heavy quark effective Lagrangian. The Lagrange density is set up by the hidden gauge symmetry approach with the light vector mesons collected in a  $3 \times 3$  matrix  $\hat{V}_\mu$  [22],

$$\mathcal{L}' = i \hat{\mu} \text{Tr} \{ \bar{S}_a H_b \sigma^{\alpha\beta} V_{\alpha\beta}^{ba} \}, \quad (6)$$

where the effective fields  $H_a$  and  $S_a$  stand for the doublets with  $J^P = (0^-, 1^-)$  and  $(0^+, 1^+)$  respectively,

$$\begin{aligned} H_a &= \frac{1 + \not{v}}{2} [P_{a\mu}^* \gamma^\mu - iP_a \gamma_5], \\ S_a &= \frac{1 + \not{v}}{2} [P_{1a}^{\prime\mu} \gamma_\mu \gamma_5 - P_{0a}], \end{aligned} \quad (7)$$

where the  $v_\mu$  is the four-velocity of the heavy meson and the  $a$  is a light quark flavor index. The  $P_{0a}, P_a, P_a^*$  and  $P'_{1a}$  are the scalar, pseudoscalar, vector and axial-vector mesons, respectively. The  $\bar{H}_a = \gamma^0 H_a^\dagger \gamma^0$ ,  $\bar{S}_a = \gamma^0 S_a^\dagger \gamma^0$ ,  $V_{\alpha\beta} = \partial_\alpha V_\beta - \partial_\beta V_\alpha + [V_\alpha, V_\beta]$  and  $V_\alpha = i \frac{g_V}{\sqrt{2}} \hat{V}_\alpha$ , the  $g_V$  is fixed to be  $g_V = 5.8$  by the KSRF rule [23]. Finally we obtain the relation between the  $G_S$  ( $G_A$ ) and  $\hat{\mu}$

$$G_S = 2\sqrt{2} \hat{\mu} g_V, \quad G_A = 2\sqrt{2} \hat{\mu} g_V. \quad (8)$$

The parameter  $\hat{\mu}$  is a basic parameter in the heavy quark effective Lagrangian, the precise value can lead to more deep understanding of the relevant dynamics. It is interesting to calculate its value with the light-cone QCD sum rules. The  $\hat{\mu}$  is estimated to  $\hat{\mu} = -0.13 \pm 0.05 \text{ GeV}^{-1}$  from the analysis of the  $D \rightarrow K^*$  semileptonic transitions induced by the axial weak current [22].

In this article, we take the point of view that charmed mesons  $D_{s0}(2317)$  and  $D_{s1}(2460)$  are the conventional  $c\bar{s}$  states, calculate the values of the strong coupling constants  $G_S$  and  $G_A$  (and the corresponding  $\hat{\mu}$ ) in the framework of the light-cone QCD sum rules approach, and study the radiative decay widths of the  $D_{s0} \rightarrow D_s^* \gamma$  and  $D_{s1} \rightarrow D_s \gamma$ .<sup>2</sup> The light-cone QCD sum rules approach carries out the operator product expansion near the light-cone  $x^2 \approx 0$  instead of the short distance  $x \approx 0$  while the nonperturbative matrix elements are parametrized by the light-cone distribution amplitudes which classified according to their twists instead of the vacuum condensates [24,25]. The nonperturbative parameters in the light-cone distribution amplitudes are calculated by the conventional QCD sum rules and the values are universal [26].

The article is arranged as follows. In Sec. II, we derive the strong coupling constants  $G_S$  and  $G_A$  within the framework of the light-cone QCD sum rules approach; in Sec. III, the numerical result and discussion; and in Sec. IV, conclusion.

<sup>2</sup>The same approach can be used to study the radiative decay widths of the  $D_{s1} \rightarrow D_s^* \gamma$ ,  $D_{s1} \rightarrow D_{s0} \gamma$ , and explore the structures of the mesons  $D_{s1}(2460)$ ,  $D_{s0}(2317)$ , that may be our next work.

## II. STRONG COUPLING CONSTANTS $G_S$ AND $G_A$ WITH LIGHT-CONE QCD SUM RULES

In the following, we write down the two-point correlation functions  $\Pi_\mu^S(p, q)$  and  $\Pi_\mu^A(p, q)$ ,

$$\Pi_\mu^S(p, q) = i \int d^4x e^{-iq \cdot x} \langle 0 | T \{ J^S(0) J_\mu^V(x) \} | \phi(p) \rangle, \quad (9)$$

$$\Pi_\mu^A(p, q) = i \int d^4x e^{-iq \cdot x} \langle 0 | T \{ J_\mu^A(0) J_5(x) \} | \phi(p) \rangle, \quad (10)$$

$$\begin{aligned} J^S(x) &= \bar{s}(x)c(x), & J_5(x) &= \bar{c}(x)i\gamma_5 s(x), \\ J_\mu^V(x) &= \bar{c}(x)\gamma_\mu s(x), & J_\mu^A(x) &= \bar{s}(x)\gamma_\mu\gamma_5 c(x), \end{aligned} \quad (11)$$

where the currents  $J^S(x)$ ,  $J_5(x)$ ,  $J_\mu^V(x)$  and  $J_\mu^A(x)$  interpolate the mesons  $D_{s0}(2317)$ ,  $D_s$ ,  $D_s^*$  and  $D_{s1}(2460)$ , respectively, the external state  $\phi$  has the four momentum  $p_\mu$  with  $p^2 = m_\phi^2$ . The correlation functions  $\Pi_\mu^S(p, q)$  and  $\Pi_\mu^A(p, q)$  can be decomposed as

$$\begin{aligned} \Pi_\mu^S(p, q) &= \Pi_S \{ \epsilon_\mu q \cdot p - p_\mu \epsilon \cdot q \} + \dots, \\ \Pi_\mu^A(p, q) &= i \Pi_A \{ \epsilon_\mu (q + p) \cdot p - p_\mu \epsilon \cdot (q + p) \} + \dots \end{aligned} \quad (12)$$

due to the Lorentz covariance, here the  $\epsilon_\mu$  is the polarization vector of the  $\phi$  meson.

According to the basic assumption of current-hadron duality in the QCD sum rules approach [26], we can insert a complete series of intermediate states with the same quantum numbers as the current operators  $J^S(x)$ ,  $J_5(x)$ ,  $J_\mu^V(x)$  and  $J_\mu^A(x)$  into the correlation functions  $\Pi_\mu^S(p, q)$  and  $\Pi_\mu^A(p, q)$  to obtain the hadronic representation. After isolating the ground state contributions from the pole terms of the mesons  $D_{s0}(2317)$ ,  $D_s$ ,  $D_s^*$  and  $D_{s1}(2460)$ , we get the following results,

$$\begin{aligned} \Pi_\mu^S(p, q) &= \frac{\langle 0 | J^S(0) | D_{s0}(q+p) \rangle \langle D_{s0} | D_s^* \phi \rangle \langle D_s^*(q) | J_\mu^V(0) | 0 \rangle}{\{M_{D_{s0}}^2 - (q+p)^2\} \{M_{D_s^*}^2 - q^2\}} \\ &+ \dots \\ &= \frac{f_{D_{s0}} f_{D_s^*} M_{D_{s0}} M_{D_s^*} G_S}{\{M_{D_{s0}}^2 - (q+p)^2\} \{M_{D_s^*}^2 - q^2\}} \\ &\times \{ \epsilon_\mu q \cdot p - p_\mu \epsilon \cdot q \} + \dots, \end{aligned} \quad (13)$$

$$\begin{aligned} \Pi_\mu^A(p, q) &= \frac{\langle 0 | J_\mu^A(0) | D_{s1}(q+p) \rangle \langle D_{s1} | D_s \phi \rangle \langle D_s(q) | J_5(0) | 0 \rangle}{\{M_{D_{s1}}^2 - (q+p)^2\} \{M_{D_s}^2 - q^2\}} \\ &+ \dots \\ &= i \frac{f_{D_{s1}} f_{D_s} M_{D_{s1}} M_{D_s} G_A}{(m_c + m_s) \{M_{D_{s1}}^2 - (q+p)^2\} \{M_{D_s}^2 - q^2\}} \\ &\times \{ \epsilon_\mu (q+p) \cdot p - p_\mu \epsilon \cdot (q+p) \} + \dots, \end{aligned} \quad (14)$$

where the following definitions have been used,

$$\begin{aligned} \langle 0 | J^S(0) | D_{s0}(p) \rangle &= f_{D_{s0}} M_{D_{s0}}, \\ \langle 0 | J_5^+(0) | D_s(p) \rangle &= \frac{f_{D_s} M_{D_s}^2}{m_c + m_s}, \\ \langle 0 | J_\mu^{V+}(0) | D_s^*(p, \eta) \rangle &= f_{D_s^*} M_{D_s^*} \eta_\mu, \\ \langle 0 | J_\mu^A(0) | D_{s1}(p, \eta) \rangle &= f_{D_{s1}} M_{D_{s1}} \eta_\mu, \end{aligned}$$

where the  $f_{D_{s0}}$ ,  $f_{D_s}$ ,  $f_{D_s^*}$  and  $f_{D_{s1}}$  are the weak decay constants of the  $D_{s0}(2317)$ ,  $D_s$ ,  $D_s^*$  and  $D_{s1}(2460)$ , respectively. In Eqs. (13) and (14), we have not shown the contributions from the high resonances and continuum states explicitly as they are suppressed due to the double Borel transformation.

In the following, we briefly outline the operator product expansion for the correlation functions  $\Pi_\mu^S(p, q)$  and  $\Pi_\mu^A(p, q)$  in perturbative QCD theory. The calculations are performed at the large spacelike momentum regions  $(q+p)^2 \ll 0$  and  $q^2 \ll 0$ , which correspond to the small light-cone distance  $x^2 \approx 0$  required by the validity of the operator product expansion approach. We write down the propagator of a massive quark in the external gluon field in the Fock-Schwinger gauge firstly [27],

$$\begin{aligned} \langle 0 | T \{ q_i(x_1) \bar{q}_j(x_2) \} | 0 \rangle &= i \int \frac{d^4k}{(2\pi)^4} e^{-ik(x_1-x_2)} \frac{\not{k} + m}{k^2 - m^2} \\ &+ \dots, \end{aligned} \quad (15)$$

here we have neglected the contributions from the gluons  $G_{\mu\nu}$ . The contributions proportional to the  $G_{\mu\nu}$  can give rise to three-particle (and four-particle) meson distribution amplitudes with a gluon (or quark-antiquark pair) in addition to the two valence quarks, their corrections are usually not expected to play any significant roles.<sup>3</sup> Substituting the above  $c$  quark propagator and the corresponding  $\phi$  meson light-cone distribution amplitudes into the correlation functions  $\Pi_\mu^S(p, q)$  and  $\Pi_\mu^A(p, q)$  in Eqs. (9) and (10) and completing the integrals over the variables  $x$  and  $k$ , finally we obtain the results,

<sup>3</sup>For example, in the decay  $B \rightarrow \chi_{c0} K$ , the factorizable contribution is zero and the nonfactorizable contributions from the soft hadronic matrix elements are too small to accommodate the experimental data [28]; the net contributions from the three-valence particle light-cone distribution amplitudes to the strong coupling constant  $g_{D_{s1} D^* K}$  are rather small, about 20% [7]. The contributions of the three-particle (quark-antiquark-gluon) distribution amplitudes of the mesons are always of minor importance comparing with the two-particle (quark-antiquark) distribution amplitudes in the light-cone QCD sum rules. In our previous work, we study the four form-factors  $f_1(Q^2)$ ,  $f_2(Q^2)$ ,  $g_1(Q^2)$  and  $g_2(Q^2)$  of the  $\Sigma \rightarrow n$  in the framework of the light-cone QCD sum rules approach up to twist-6 three-quark light-cone distribution amplitudes and obtain satisfactory results [29]. In the light-cone QCD sum rules, we can neglect the contributions from the valence gluons and make relatively rough estimations.

$$\begin{aligned}
\Pi_S = & -f_\phi^T \int_0^1 du \frac{\phi_\perp(u)}{AA} + \left[ f_\phi^T - f_\phi \frac{2m_s}{m_\phi} \right] m_\phi^2 \int_0^1 duu \frac{h_\parallel^{(s)}(u)}{AA^2} + \frac{f_\phi^T m_\phi^2}{4} \int_0^1 duA(u) \left[ \frac{1}{AA^2} + \frac{2m_c^2}{AA^3} \right] \\
& + 2f_\phi^T m_\phi^2 \int_0^1 du \int_0^u d\tau \int_0^\tau dt \frac{B_\perp(t)}{AA^2} - f_\phi^T m_\phi^2 \int_0^1 duu \int_0^u dt C_\perp(t) \left[ \frac{1}{AA^2} + \frac{2m_c^2}{AA^3} \right] - 2f_\phi m_\phi m_c \int_0^1 duu \frac{g_\perp^{(v)}}{AA^2} \\
& + 2f_\phi m_\phi^3 m_c \int_0^1 duu \int_0^u d\tau \int_0^\tau dt \frac{C(t)}{AA^3}, \tag{16}
\end{aligned}$$

$$\begin{aligned}
\Pi_A = & -f_\phi^T \int_0^1 du \frac{\phi_\perp(u)}{AA} - \left[ f_\phi^T - f_\phi \frac{2m_s}{m_\phi} \right] m_\phi^2 \int_0^1 duu \frac{h_\parallel^{(s)}(u)}{AA^2} + \frac{f_\phi^T m_\phi^2}{4} \int_0^1 duA(u) \left[ \frac{1}{AA^2} + \frac{2m_c^2}{AA^3} \right] \\
& + 2f_\phi^T m_\phi^2 \int_0^1 du \int_0^u d\tau \int_0^\tau dt \frac{B_\perp(t)}{AA^2} - f_\phi^T m_\phi^2 \int_0^1 duu \int_0^u dt C_\perp(t) \left[ \frac{1}{AA^2} + \frac{2m_c^2}{AA^3} \right] - 2f_\phi m_\phi m_c \int_0^1 duu \frac{g_\perp^{(v)}}{AA^2} \\
& + 2f_\phi m_\phi^3 m_c \int_0^1 duu \int_0^u d\tau \int_0^\tau dt \frac{C(t)}{AA^3}, \tag{17}
\end{aligned}$$

where

$$AA = m_c^2 - (q + up)^2.$$

In calculation, the two-particle  $\phi$  meson light-cone distribution amplitudes have been used [30], the explicit expressions are given in the appendix. The parameters in the light-cone distribution amplitudes are scale dependent and can be estimated with the QCD sum rules approach [30]. In this article, the energy scale  $\mu$  is chosen to be  $\mu = 1$  GeV.

Now we perform the double Borel transformation with respect to the variables  $Q_1^2 = -(p + q)^2$  and  $Q_2^2 = -q^2$  for the correlation functions  $\Pi_S$  and  $\Pi_A$  in Eqs. (13) and (14), and obtain the analytical expressions of the invariant functions in the hadronic representation,

$$\begin{aligned}
B_{M_2^2} B_{M_1^2} \Pi_S = & \frac{G_S f_{D_s^*} f_{D_{s0}} M_{D_s^*} M_{D_{s0}}}{M_1^2 M_2^2} \exp \left[ -\frac{M_{D_{s0}}^2}{M_1^2} - \frac{M_{D_s^*}^2}{M_2^2} \right] \\
& + \dots, \tag{18}
\end{aligned}$$

$$\begin{aligned}
B_{M_2^2} B_{M_1^2} \Pi_A = & \frac{G_A f_{D_s} f_{D_{s1}} M_{D_s}^2 M_{D_{s1}}}{M_1^2 M_2^2 (m_c + m_s)} \exp \left[ -\frac{M_{D_{s1}}^2}{M_1^2} - \frac{M_{D_s}^2}{M_2^2} \right] \\
& + \dots, \tag{19}
\end{aligned}$$

here we have not shown the contributions from the high resonances and continuum states explicitly for simplicity. In order to match the duality regions below the thresholds  $s_0$  and  $s'_0$  for the interpolating currents  $J^S(x)$  [or  $J_\mu^A(x)$ ] and  $J_\mu^V(x)$  [or  $J_5(x)$ ], respectively, we can express the correlation functions  $\Pi$  (denote the  $\Pi_S$  and  $\Pi_A$ ) at the level of quark-gluon degrees of freedom into the following form,

$$\Pi = \int ds \int ds' \frac{\rho(s, s')}{\{s - (q + p)^2\} \{s' - q^2\}}, \tag{20}$$

where the  $\rho(s, s')$  are spectral densities, then perform the

double Borel transformation with respect to the variables  $Q_1^2$  and  $Q_2^2$  directly. However, the analytical expressions of the spectral densities  $\rho(s, s')$  are hard to obtain, we have to resort to some approximations. As the contributions from the higher twist terms are suppressed by more powers of  $\frac{1}{m_c^2 - (q + up)^2}$ , the net contributions of the twist-3 and twist-4 terms are of minor importance, less than 20%, the continuum subtractions will not affect the results remarkably. The dominating contribution comes from the two-particle twist-2 term involving the  $\phi_\perp(u)$ . We perform the same trick as Refs. [27,31] and expand the amplitude  $\phi_\perp(u)$  in terms of polynomials of  $1 - u$ ,

$$\phi_\perp(u) = \sum_{k=0}^N b_k (1 - u)^k = \sum_{k=0}^N b_k \left( \frac{s - m_c^2}{s - q^2} \right)^k, \tag{21}$$

then introduce the variable  $s'$  and the spectral density is obtained.

After straightforward calculations, we obtain the final expressions of the double Borel transformed correlation functions  $\Pi(M_1^2, M_2^2)$  at the level of quark-gluon degrees of freedom. The masses of the charmed mesons are  $M_{D_{s1}} = 2.46$  GeV,  $M_{D_{s0}} = 2.317$  GeV,  $M_{D_s} = 1.97$  GeV and  $M_{D_s^*} = 2.112$  GeV,  $\frac{M_{D_s^*}}{M_{D_s^*} + M_{D_{s0}}} \approx 0.48$ ,  $\frac{M_{D_s}}{M_{D_s} + M_{D_{s1}}} \approx 0.45$ , there exists an overlapping working window for the two Borel parameters  $M_1^2$  and  $M_2^2$ , it is convenient to take the value  $M_1^2 = M_2^2$ . We introduce the threshold parameter  $s_0$  and make the simple replacement,

$$e^{-((m_c^2 + u_0(1 - u_0)m_\phi^2)/M^2)} \rightarrow e^{-((m_c^2 + u_0(1 - u_0)m_\phi^2)/M^2)} - e^{-s_0/M^2}$$

to subtract the contributions from the high resonances and continuum states [27], finally we obtain the sum rules for the strong coupling constants  $G_S$  and  $G_A$ ,

$$\begin{aligned}
G_S f_{D_{s0}} f_{D_s^*} M_{D_{s0}} M_{D_s^*} \exp\left\{-\frac{M_{D_{s0}}^2}{M_1^2} - \frac{M_{D_s^*}^2}{M_2^2}\right\} &= -f_\phi^T M^2 \phi_\perp(u_0) \left\{ \exp\left[-\frac{BB}{M^2}\right] - \exp\left[-\frac{s_0}{M^2}\right] \right\} + \exp\left[-\frac{BB}{M^2}\right] \\
&\times \left\{ \left[ f_\phi^T - f_\phi \frac{2m_s}{m_\phi} \right] m_\phi^2 h_\parallel^{(s)}(u_0) u_0 + \frac{f_\phi^T m_\phi^2 A(u_0)}{4} \left[ 1 + \frac{m_c^2}{M^2} \right] \right. \\
&+ 2f_\phi^T m_\phi^2 \int_0^{u_0} d\tau \int_0^\tau dt B_\perp(t) - f_\phi^T m_\phi^2 u_0 \int_0^{u_0} dt C_\perp(t) \left[ 1 + \frac{m_c^2}{M^2} \right] \\
&\left. - 2f_\phi m_\phi m_c u_0 g_\perp^{(v)}(u_0) + f_\phi m_\phi^3 m_c u_0 \int_0^{u_0} d\tau \int_0^\tau \frac{C(t)}{M^2} \right\}, \quad (22)
\end{aligned}$$

$$\begin{aligned}
G_A f_{D_{s1}} f_{D_s} M_{D_{s1}} \frac{M_{D_s}^2}{m_c + m_s} \exp\left\{-\frac{M_{D_{s1}}^2}{M_1^2} - \frac{M_{D_s}^2}{M_2^2}\right\} &= -f_\phi^T M^2 \phi_\perp(u_0) \left\{ \exp\left[-\frac{BB}{M^2}\right] - \exp\left[-\frac{s_0}{M^2}\right] \right\} + \exp\left[-\frac{BB}{M^2}\right] \\
&\times \left\{ -\left[ f_\phi^T - f_\phi \frac{2m_s}{m_\phi} \right] m_\phi^2 h_\parallel^{(s)}(u_0) u_0 + \frac{f_\phi^T m_\phi^2 A(u_0)}{4} \left[ 1 + \frac{m_c^2}{M^2} \right] \right. \\
&+ 2f_\phi^T m_\phi^2 \int_0^{u_0} d\tau \int_0^\tau dt \bar{B}_\perp(t) - f_\phi^T m_\phi^2 u_0 \int_0^{u_0} dt C_\perp(t) \left[ 1 + \frac{m_c^2}{M^2} \right] \\
&\left. - 2f_\phi m_\phi m_c u_0 g_\perp^{(v)}(u_0) + f_\phi m_\phi^3 m_c u_0 \int_0^{u_0} d\tau \int_0^\tau \frac{C(t)}{M^2} \right\}, \quad (23)
\end{aligned}$$

where

$$\begin{aligned}
BB &= m_c^2 + u_0(1 - u_0)m_\phi^2, \quad u_0 = \frac{M_1^2}{M_1^2 + M_2^2}, \\
M^2 &= \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}. \quad (24)
\end{aligned}$$

### III. NUMERICAL RESULT AND DISCUSSION

The parameters are taken as  $m_s = (140 \pm 10)$  MeV,  $m_c = (1.25 \pm 0.10)$  GeV,  $f_\phi = (0.215 \pm 0.005)$  GeV,  $f_\phi^\perp = (0.186 \pm 0.009)$  GeV,  $a_2^\perp = 0.2 \pm 0.2$ ,  $a_2^\parallel = 0.2 \pm 0.1$ ,  $s_3 = 0.032 \pm 0.010$ ,  $s_4 = 0.15 \pm 0.10$ ,  $s_4^T = 0.10 \pm 0.05$ ,  $\tilde{s}_4^T = -0.10 \pm 0.05$ ,  $\omega_3^A = -2.1 \pm 1.0$ ,  $\omega_3^V = 3.8 \pm 1.8$ ,  $\omega_3^T = 7.0 \pm 7.0$ ,  $m_\phi = 1.02$  GeV,  $M_{D_{s0}} = 2.317$  GeV,  $M_{D_{s1}} = 2.46$  GeV,  $M_{D_s^*} = 2.112$  GeV,  $M_{D_s} = 1.97$  GeV,  $f_{D_s^*} = (0.26 \pm 0.02)$  GeV,  $f_{D_s} = (0.26 \pm 0.02)$  GeV,  $f_{D_{s0}} = (0.225 \pm 0.025)$  GeV and  $f_{D_{s1}} = (0.225 \pm 0.025)$  GeV [16,32]. The duality thresholds  $s_0$  in Eqs. (22) and (23) are taken as  $s_0 = (7.0-8.0)$  GeV<sup>2</sup> ( $\sqrt{s_0} = (2.6-2.8)$  GeV) to avoid possible contaminations from the high resonances and continuum states, in this region, the numerical results are not sensitive to the threshold parameters  $s_0$ . The Borel parameters are chosen as  $M_1^2 = M_2^2$  and  $M^2 = (3.5-8)$  GeV<sup>2</sup>, in those regions, the values of the strong coupling constants  $G_S$  and  $G_A$  are rather stable from the sum rules in Eqs. (22) and (23) with the simple subtraction.

The uncertainties of the 11 parameters  $f_\phi$ ,  $f_\phi^\perp$ ,  $a_2^\parallel$ ,  $\omega_3^T$ ,  $\omega_3^V$ ,  $\omega_3^A$ ,  $s_3$ ,  $s_4$ ,  $s_4^T$ ,  $\tilde{s}_4^T$  and  $m_s$  cannot result in large uncertainties for the numerical values. The main uncer-

tainties come from the six parameters  $f_{D_{s0}}$ ,  $f_{D_{s1}}$ ,  $f_{D_s^*}$ ,  $f_{D_s}$ ,  $a_2^\perp$  and  $m_c$ , the variations of those parameters can lead to relatively large changes for the numerical values, while the dominating uncertainty comes from the  $a_2^\perp$ , which are shown in the Fig. 1, refining the parameter  $a_2^\perp$  is of great importance. Taking into account all the uncertainties, finally we obtain the numerical results of the strong coupling constants  $G_S$  and  $G_A$ , which are shown in the Fig. 2,

$$G_S = -(4.4 \pm 2.0) \frac{1}{\text{GeV}}, \quad G_A = -(3.9 \pm 1.6) \frac{1}{\text{GeV}}, \quad (25)$$

$$\begin{aligned}
\hat{\mu}_S &= -(0.27 \pm 0.12) \frac{1}{\text{GeV}}, \\
\hat{\mu}_A &= -(0.24 \pm 0.10) \frac{1}{\text{GeV}}. \quad (26)
\end{aligned}$$

The corresponding values of the parameter  $\hat{\mu}$  are larger than the existing estimation  $\hat{\mu} = -0.13 \pm 0.05$  GeV<sup>-1</sup> from the analysis of the  $D \rightarrow K^*$  semileptonic transitions induced by the axial weak current [22]. From the numerical values of the strong coupling constants  $G_S$  and  $G_A$ , we can obtain the decay widths,

TABLE I. The decay widths of the  $D_{s0} \rightarrow D_s^* \gamma$ ,  $D_{s1} \rightarrow D_s \gamma$ .

KeV	[15]	[16]	[17]	[18]	[19]	[20]	This work
$\Gamma(D_{s0} \rightarrow D_s^* \gamma)$	4-6	1	1.74	1.9	1.1	35	1.3-9.9
$\Gamma(D_{s1} \rightarrow D_s \gamma)$	19-29		5.08	6.2	0.6-2.9		5.5-31.2

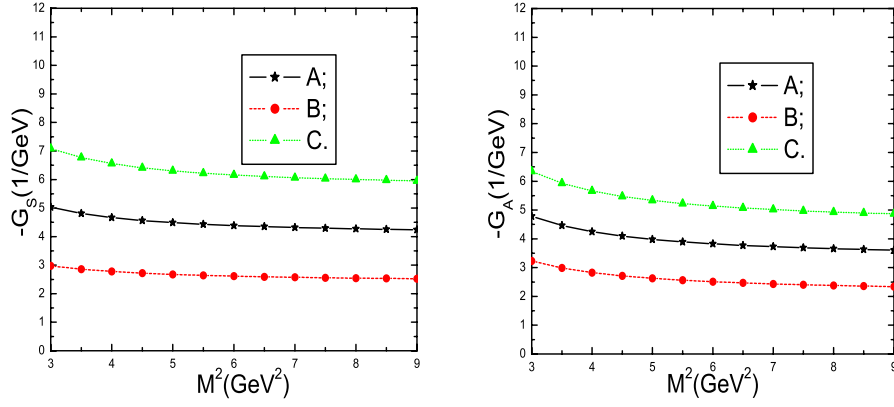


FIG. 1 (color online). The  $G_S$  and  $G_A$  with the Borel parameter  $M^2$  for  $a_2^\perp = 0.2$  (A),  $a_2^\perp = 0.4$  (B) and  $a_2^\perp = 0.0$  (C).

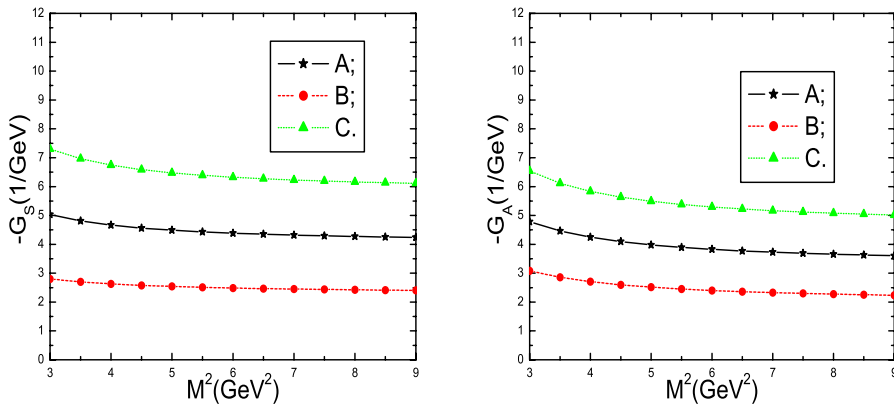


FIG. 2 (color online). The  $G_S$  and  $G_A$  with the Borel parameter  $M^2$ , the A, B and C stand for the central value, down limit and up limit, respectively.

$$\begin{aligned}\Gamma(D_{s0} \rightarrow D_s^* \gamma) &= (1.3-9.9) \text{ KeV}, \\ \Gamma(D_{s1} \rightarrow D_s \gamma) &= (5.5-31.2) \text{ KeV}.\end{aligned}\quad (27)$$

The comparison with the results from other approaches is presented in the Table I.

#### IV. CONCLUSION

In this article, we take the point of view that the charmed mesons  $D_{s0}(2317)$  and  $D_{s1}(2460)$  with the spin-parity  $0^+$  and  $1^+$  respectively are the conventional  $c\bar{s}$  states and calculate the strong coupling constants  $G_S$  and  $G_A$  in the framework of the light-cone QCD sum rules approach. The strong coupling constants  $G_S$  and  $G_A$  are related to the basic parameter  $\hat{\mu}$  in the heavy quark effective Lagrangian, the numerical value of the  $\hat{\mu}$  is larger than the existing estimation. With the assumption of the vector meson dominance of the intermediate  $\phi$ , we study the radiative decays  $D_{s0} \rightarrow D_s^* \gamma$  and  $D_{s1} \rightarrow D_s \gamma$ , the numerical values of the decay widths are compatible with the existing estimations, further experimental data can conform or reject the as-

sumption of the two-quark substructure. Just like the scalar mesons  $f_0(980)$  and  $a_0(980)$ , the scalar meson  $D_{s0}(2317)$  and the axial-vector meson  $D_{s1}(2460)$  may have small  $c\bar{s}$  kernels of typical  $c\bar{s}$  meson size. The strong couplings to virtual intermediate hadronic states (or the virtual mesons loops) can result in smaller masses than the conventional  $0^+$  and  $1^+$  mesons in the constituent quark models, enrich the pure  $c\bar{s}$  states with other components. The  $D_{s0}(2317)$  and  $D_{s1}(2460)$  may spend part (or most part) of their lifetimes as virtual  $DK$  and  $D^*K$  states, respectively.

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#### APPENDIX

The light-cone distribution amplitudes of the  $\phi$  meson are defined by

$$\begin{aligned}
\langle 0|\bar{s}(0)\gamma_\mu s(x)|\phi(p)\rangle &= p_\mu f_\phi m_\phi \frac{\epsilon \cdot x}{p \cdot x} \int_0^1 du e^{-iup \cdot x} \left\{ \phi_{\parallel}(u) + \frac{m_\phi^2 x^2}{16} A(u) \right\} + \left[ \epsilon_\mu - p_\mu \frac{\epsilon \cdot x}{p \cdot x} \right] f_\phi m_\phi \int_0^1 du e^{-iup \cdot x} g_{\perp}^{(v)}(u) \\
&\quad - \frac{1}{2} x_\mu \frac{\epsilon \cdot x}{(p \cdot x)^2} f_\phi m_\phi^3 \int_0^1 du e^{-iup \cdot x} C(u), \\
\langle 0|\bar{s}(0)s(x)|\phi(p)\rangle &= \frac{i}{2} \left[ f_\phi^T - f_\phi \frac{2m_s}{m_\phi} \right] m_\phi^2 \epsilon \cdot x \int_0^1 du e^{-iup \cdot x} h_{\parallel}^{(s)}(u), \\
\langle 0|\bar{s}(0)\sigma_{\mu\nu} s(x)|\phi(p)\rangle &= i[\epsilon_\mu p_\nu - \epsilon_\nu p_\mu] f_\phi^T \int_0^1 du e^{-iup \cdot x} \left\{ \phi_{\perp}(u) + \frac{m_\phi^2 x^2}{16} A_{\perp}(u) \right\} + i[p_\mu x_\nu - p_\nu x_\mu] f_\phi^T m_\phi^2 \frac{\epsilon \cdot x}{(p \cdot x)^2} \\
&\quad \times \int_0^1 du e^{-iup \cdot x} B_{\perp}(u) + i \frac{1}{2} [\epsilon_\mu x_\nu - \epsilon_\nu x_\mu] f_\phi^T m_\phi^2 \frac{1}{p \cdot x} \int_0^1 du e^{-iup \cdot x} C_{\perp}(u). \tag{A1}
\end{aligned}$$

The light-cone distribution amplitudes are parametrized as

$$\begin{aligned}
\phi_{\parallel}(u, \mu) &= 6u(1-u) \left\{ 1 + a_2^{\parallel} \frac{3}{2} (5\xi^2 - 1) \right\}, \\
\phi_{\perp}(u, \mu) &= 6u(1-u) \left\{ 1 + a_2^{\perp} \frac{3}{2} (5\xi^2 - 1) \right\}, \\
g_{\perp}^{(v)}(u, \mu) &= \frac{3}{4} (1 + \xi^2) + \left\{ \frac{3}{7} a_2^{\parallel} + 5s_3 \right\} (3\xi^2 - 1) + \left\{ \frac{9}{112} a_2^{\parallel} + \frac{15}{64} s_3 (3\omega_3^V - \omega_3^A) \right\} (3 - 30\xi^2 + 35\xi^4), \\
g_3(u, \mu) &= 1 + \left\{ -1 - \frac{2}{7} a_2^{\parallel} + \frac{40}{3} s_3 - \frac{20}{3} s_4 \right\} C_2^{1/2}(\xi) + \left\{ -\frac{27}{28} a_2^{\parallel} + \frac{5}{4} s_3 - \frac{15}{16} s_3 (\omega_3^A + 3\omega_3^V) \right\} C_4^{1/2}(\xi), \\
h_3(u, \mu) &= 1 + \left\{ -1 + \frac{3}{7} a_2^{\perp} - 10(s_4^T + \xi_4^T) \right\} C_2^{1/2}(\xi) + \left\{ -\frac{3}{7} a_2^{\perp} - \frac{15}{8} s_3 \omega_3^T \right\} C_4^{1/2}(\xi), \\
h_{\parallel}^{(s)}(u, \mu) &= 6u(1-u) \left\{ 1 + \left( \frac{1}{4} a_2^{\parallel} + \frac{5}{8} s_3 \omega_3^T \right) (5\xi^2 - 1) \right\}, \\
h_{\perp}^{(s)}(u, \mu) &= 3\xi^2 + \frac{3}{2} a_2^{\perp} \xi^2 (5\xi^2 - 3) + \frac{15}{16} s_3 \omega_3^T (3 - 30\xi^2 + 35\xi^4), \\
A_{\perp}(u, \mu) &= 30u^2(1-u)^2 \left\{ \frac{2}{5} + \frac{4}{35} a_2^{\perp} + \frac{4}{3} s_4^T - \frac{8}{3} \xi_4^T \right\}, \\
C(u, \mu) &= g_3(u, \mu) + \phi_{\parallel}(u, \mu) - 2g_{\perp}^{(v)}(u, \mu), \\
B_{\perp}(u, \mu) &= h_{\parallel}^{(s)}(u, \mu) - \frac{1}{2} \phi_{\perp}(u, \mu) - \frac{1}{2} h_3(u, \mu), \\
C_{\perp}(u, \mu) &= h_3(u, \mu) - \phi_{\perp}(u, \mu), \tag{A2}
\end{aligned}$$

where the  $\xi = 2u - 1$ , and the  $C_2^{1/2}$  and  $C_4^{1/2}$  are Gegenbauer polynomials [30].

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