Meson-baryon scattering lengths in heavy baryon chiral perturbation theory

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We calculate the chiral corrections to the s-wave pseudoscalar meson octet-baryon scattering lengths a_{MB} to $\mathcal{O}(p^3)$ in the SU(3) heavy baryon chiral perturbation theory (HBχPT). Hopefully the obtained analytical expressions will be helpful in the chiral extrapolation of these scattering lengths in the future lattice simulation.

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I. INTRODUCTION

Heavy baryon chiral perturbation theory (HBχPT) [1,2] is widely used to study low-energy processes involving chiral fields and ground state baryons. Its Lagrangian is expanded simultaneously with p/Λ_χ and p/M_0, where p represents the meson momentum or its mass or the small residue momentum of baryon in the nonrelativistic limit, $\Lambda_\chi \sim 1$ GeV is the scale of chiral symmetry breaking, and M_0 is the baryon mass in the chiral limit.

Pion-nucleon interactions are widely investigated in the two-flavor HBχPT. For processes involving kaons or hyperons, one has to employ the SU(3) HBχPT, where more unknown low-energy constants (LEC) appear at the same chiral order than in the SU(2) case. Determining these LECs needs more experimental data which are unavailable at present. Broken SU(3) symmetry results in large m_K and m_η. In certain cases, the chiral expansion converges slowly due to $m_K/\Lambda_\chi \sim m_\eta/\Lambda_\chi \sim 1/2$ [3–6].

The scattering length is an important observable which contains important information about low-energy meson-baryon strong interactions. As an effective theory of QCD, HBχPT provides a model-independent approach to calculate meson-baryon scattering lengths. In this paper, we will calculate all s-wave meson-baryon scattering lengths to the third chiral order in the SU(3) HBχPT.

Experimental measurements of πN scattering lengths are relatively easier than those for other processes. Recently, the results of precision X-ray experiments on pionic hydrogen [7] and deuterium [8] were published. These two experiments together constrain the isoscalar and isovector πN scattering lengths [9].

Since the prediction of πN scattering lengths with current algebra, chiral corrections have been calculated to high orders in the two-flavor HBχPT prior [10,11]. There were many theoretical calculations of πN scattering lengths [12]. As we will see below, πN scattering lengths play an important role in determining LECs in the SU(3) HBχPT.

Experimental data is very scarce for kaon-nucleon scattering lengths. The recent DEAR measurements on kaonic hydrogen gave the scattering length a_{K^-p} [13]. However, the direct determination of the two scattering lengths in the $\bar{K}N$ channel needs further experimental measurements. Unlike the pion-nucleon case, there is the possibility that measurements on kaonic deuterium are not enough to give the two $\bar{K}N$ scattering lengths [14].

There had been efforts to extract the scattering lengths from kaon-nucleon scattering data using various models [15–18]. Kaon-nucleon scattering lengths were calculated to order $\mathcal{O}(p^3)$ in the SU(3) HBχPT in Ref. [5]. There were large cancellations at the second and the third chiral order. Recently two lattice simulations also tried to extract the kaon-nucleon scattering lengths [19,20].

ηN scattering is particularly interesting because the attractive ηN interaction may result in η-mesic nuclei which have a long history [21]. The formation of η-mesic hypernuclei is also possible [22]. An early experiment gave a negative result on the search for η-mesic bound states [23]. However, a recent experiment signals the existence of such states [24]. The experimental situation is rather ambiguous.

On the theoretical side, the existence of η-^{3}He quasibound state is also controversial, mostly because it is hard to determine the η-nucleus scattering length. In the optical models, the scattering length can be obtained with ηN scattering length $a_{\eta N}$. However, theoretical predictions of $a_{\eta N}$ are different from various approaches (see overview in [25]). It is worthwhile to calculate ηN scattering length in the SU(3) HBχPT.

There are very few investigations of pion-hyperon, kaon-hyperon and eta-baryon scattering lengths in literature. Experimentally these observables may be studied through the strangeness program at Japan Hadron Facility (JHF) and Lan-Zhou Cooling Storage Ring (CSR). In this paper we will perform an extensive study of these scattering lengths to the third chiral order in the framework of SU(3) HBχPT.

We present the basic notations and definitions in Sec. II. We present the chiral corrections to the threshold T-matrices of meson-baryon interactions in Sec. III. This

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section contains our main results. We discuss useful relations between these threshold T-matrices in Sec. IV. Then we discuss the determination of LECs in Sec. V. The final section is a summary.

II. LAGRANGIAN

For a system involving pions and one octet baryon, the chirally invariant Lagrangian of HBχPT reads

$$\mathcal{L} = \mathcal{L}_{\phi\phi} + \mathcal{L}_{\phi B}, \tag{1}$$

where ϕ represents the pseudoscalar octet mesons, B represents the octet baryons. The purely mesonic part $\mathcal{L}_{\phi\phi}$ incorporates even chiral order terms while the terms in $\mathcal{L}_{\phi B}$ start from $\mathcal{O}(p)$.

$$\mathcal{L}^{(2)}_{\phi\phi} = f^2 \,\mathrm{tr}\left(u_\mu u^\mu + \frac{\chi_+}{4}\right), \tag{2}$$

$$\mathcal{L}^{(1)}_{\phi B} = \mathrm{tr}(\bar{B}(i\partial_0 B + [\Gamma_0, B])) - D\,\mathrm{tr}(\bar{B}\{\vec{\sigma}\cdot\vec{u}, B\}) - F\,\mathrm{tr}(\bar{B}[\vec{\sigma}\cdot\vec{u}, B]), \tag{3}$$

$$\begin{aligned}\mathcal{L}^{(2)}_{\phi B} &= b_D\,\mathrm{tr}(\bar{B}\{\chi_+, B\}) + b_F\,\mathrm{tr}(\bar{B}[\chi_+, B]) + b_0\,\mathrm{tr}(\bar{B}B)\,\mathrm{tr}(\chi_+) + \left(2d_D + \frac{D^2 - 3F^2}{2M_0}\right)\mathrm{tr}(\bar{B}\{u_0^2, B\}) \\ &\quad + \left(2d_F - \frac{DF}{M_0}\right)\mathrm{tr}(\bar{B}[u_0^2, B]) + \left(2d_0 + \frac{F^2 - D^2}{2M_0}\right)\mathrm{tr}(\bar{B}B)\,\mathrm{tr}(u_0^2) + \left(2d_1 + \frac{3F^2 - D^2}{3M_0}\right)\mathrm{tr}(\bar{B}u_0)\,\mathrm{tr}(u_0 B),\end{aligned} \tag{4}$$

where

$$\Gamma_\mu = \frac{i}{2}[\xi^\dagger, \partial_\mu \xi], \qquad u_\mu = \frac{i}{2}\{\xi^\dagger, \partial_\mu \xi\}, \qquad \xi = \exp(i\phi/2f), \tag{5}$$

$$\chi_+ = \xi^\dagger \chi \xi^\dagger + \xi \chi \xi, \qquad \chi = \mathrm{diag}(m_\pi^2, m_\pi^2, 2m_K^2 - m_\pi^2), \tag{6}$$

$$\phi = \sqrt{2}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}, \qquad B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix}. \tag{7}$$

f is the meson decay constant in the chiral limit. Γ_μ is chiral connection which contains even number meson fields. u_μ contains odd number meson fields. $D + F = g_A = 1.26$ where g_A is the axial vector coupling constant. The first three terms in $\mathcal{L}^{(2)}_{\phi B}$ are proportional to SU(3) symmetry breaking. Terms involving M_0 are corrections produced by finite baryon mass in the chiral limit. Others are proportional to LECs d_D, d_F, d_1 and d_0.

We calculate threshold T-matrices for meson-baryon scattering to the third order according to the power counting rule [1,2]. The leading and next leading order contributions can be read from the tree-level Lagrangians $\mathcal{L}^{(1)}_{\phi B}$ and $\mathcal{L}^{(2)}_{\phi B}$ respectively.

At the third order, both loop diagrams and $\mathcal{O}(p^3)$ LECs from $\mathcal{L}^{(3)}_{\phi B}$ contribute. In principle, divergence from the loop integration will be absorbed by these LECs. There are many unknown LECs at $\mathcal{O}(p^3)$ which may reduce the predictive power of HBχPT. The contribution of these $\mathcal{O}(p^3)$ LECs to threshold T-matrix was carefully investigated using the resonance saturation approach in the SU(2) HBχPT in Refs. [10]. Luckily, the counter-term contributions at $\mathcal{O}(p^3)$ are much smaller than the loop corrections and the chiral corrections at this order mainly come from the chiral loop [10]. Therefore, these negligible counter-term contributions were ignored in the calculation of kaon-nucleon scattering lengths in Ref. [5]. We follow the same approach in our crude numerical analysis because of the lack of enough data to fix these small LECs.

III. T-MATRICES FOR MESON-BARYON SCATTERINGS

The threshold T-matrix T_{PB} is related to scattering length a_{PB} through $T_{PB} = 4\pi(1 + \frac{m_P}{M_B})a_{PB}$, where m_P and m_B are the masses of the pseudoscalar meson and baryon, respectively.

There exist many diagrams for a general elastic meson-baryon scattering process. However, the calculation is simpler at threshold due to $\vec{\sigma}\cdot\vec{q} = 0$ where $\vec{q}$ is the three momentum of the meson. The lowest order contribution arises from the chiral connection term in $\mathcal{L}^{(1)}_{\phi B}$. Only meson masses and decay constants appear in the expressions. At the next leading order, the expressions are proportional to the combinations of several LECs in $\mathcal{L}^{(2)}_{\phi B}$. At the third chiral order, the loop diagrams in Fig. 1 have nonvanishing contributions. The vertices come from $\mathcal{L}^{(1)}_{\phi B}$ and $\mathcal{L}^{(2)}_{\phi\phi}$

There are two types of diagrams. The first six contain vertices from the chiral connection, and thus contributions

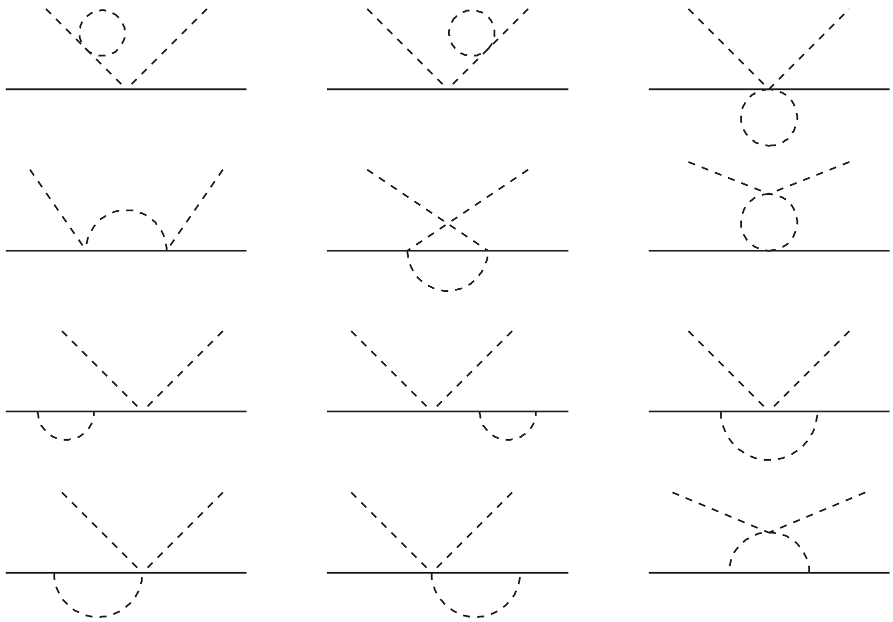

FIG. 1. Nonvanishing loop diagrams in the calculation of meson-baryon scattering lengths to the third chiral order in HBχPT. Dashed lines represent Goldstone bosons while solid lines represent octet baryons. The fourth diagram generates imaginary parts for kaon-baryon and eta-baryon scattering lengths.

from these diagrams do not contain the axial coupling constants D and F. Because the mass of an intermediate meson is always larger than or equal to m_π, no diagrams generate imaginary parts for the pion-baryon scattering lengths. In contrast, the fourth diagram generates imaginary parts for kaon-baryon or eta-baryon threshold T-matrices.

In the following subsections we list the expressions of threshold T-matrices order by order for every channel. This is our main result, which may be helpful to the chiral extrapolations of the scattering lengths on the lattice.

A. Kaon-nucleon scattering

For kaon-nucleon scattering lengths, we reproduce the tree-level expressions of Ref. [5]. At the third chiral order, our expressions are slightly different [26].

At the leading order,

$$T_{KN}^{(1)} = -\frac{m_K}{f_K^2}, \qquad T_{KN}^{(0)} = 0,$$
$$T_{\bar{K}N}^{(1)} = \frac{m_K}{2f_K^2}, \qquad T_{\bar{K}N}^{(0)} = \frac{3m_K}{2f_K^2}, \tag{8}$$

where the superscripts represent total isospin and f_K is the renormalized kaon decay constant.

The second chiral order T-matrices are

$$T_{KN}^{(1)} = \frac{m_K^2}{f_K^2}C_1, \qquad T_{KN}^{(0)} = \frac{m_K^2}{f_K^2}C_0,$$
$$T_{\bar{K}N}^{(1)} = \frac{m_K^2}{2f_K^2}(C_1 + C_0), \qquad T_{\bar{K}N}^{(0)} = \frac{m_K^2}{2f_K^2}(3C_1 - C_0), \tag{9}$$

where $C_{1,0}$ are defined in Ref. [5]

$$C_1 = 2(d_0 - 2b_0) + 2(d_D - 2b_D) + d_1 - \frac{D^2 + 3F^2}{6M_0},$$
$$C_0 = 2(d_0 - 2b_0) - 2(d_F - 2b_F) - d_1 - \frac{D(D - 3F)}{3M_0}. \tag{10}$$

We will express combinations of LECs in the other channels with C_0 and C_1. This can reduce the number of parameters for a given channel.

At the third order, we have

$$T_{KN}^{(1)} = \frac{m_K^2}{16\pi^2 f_K^4}\Big\{m_K\Big(-3 + 2\ln\frac{m_\pi}{\lambda} + \ln\frac{|m_K|}{\lambda} + 3\ln\frac{m_\eta}{\lambda}\Big) + 2\sqrt{m_K^2 - m_\pi^2}\ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}$$
$$- 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{m_K}{m_\eta} - \frac{\pi}{6}(D - 3F)\Big[2(D + F)\frac{m_\pi^2}{m_\eta + m_\pi} + (D + 5F)m_\eta\Big]\Big\}, \tag{11}$$

$$T_{KN}^{(0)} = \frac{3m_K^2}{16\pi^2 f_K^4}\bigg\{m_K\bigg(\ln\frac{m_\pi}{\lambda} - \ln\frac{|m_K|}{\lambda}\bigg) + \sqrt{m_K^2 - m_\pi^2}\ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi} + \frac{\pi}{3}(D-3F)\bigg[(D+F)\frac{m_\pi^2}{m_\eta + m_\pi} + \frac{1}{6}(7D+3F)m_\eta\bigg]\bigg\}, \tag{12}$$

$$T_{\bar{K}N}^{(1)} = \frac{m_K^2}{32\pi^2 f_K^4}\bigg\{m_K\bigg(3 - 5\ln\frac{m_\pi}{\lambda} + 2\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\bigg) + 5\sqrt{m_K^2 - m_\pi^2}\bigg(i\pi - \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\bigg) - 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{-m_K}{m_\eta} + \frac{\pi}{3}(D-3F)\bigg[2(D+F)\frac{m_\pi^2}{m_\eta + m_\pi} + (3D-F)m_\eta\bigg]\bigg\}. \tag{13}$$

$$T_{\bar{K}N}^{(0)} = \frac{3m_K^2}{32\pi^2 f_K^4}\bigg\{m_K\bigg(3 - \ln\frac{m_\pi}{\lambda} - 2\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\bigg) + \sqrt{m_K^2 - m_\pi^2}\bigg(i\pi - \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\bigg) - 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{-m_K}{m_\eta} - \frac{\pi}{3}(D-3F)\bigg[2(D+F)\frac{m_\pi^2}{m_\eta + m_\pi} + \frac{1}{3}(5D+9F)m_\eta\bigg]\bigg\}. \tag{14}$$

In loop calculations, we have used dimensional regularization and minimal subtraction scheme. λ is the cutoff. In deriving the expressions in square brackets, Gell-Mann-Okubo relation was used. For the decay constants in the loop expressions, we have used f_K in kaon processes in the above equations in order to make the expressions more compact. One may also use f_π in π loops, f_K in kaon loops and f_η in η loops. The differences are of high order.

B. Pion-nucleon scattering

With the isospin amplitude, the tree-level expressions are

$$T_{\pi N}^{(3/2)} = -\frac{m_\pi}{2f_\pi^2} \qquad T_{\pi N}^{(1/2)} = \frac{m_\pi}{f_\pi^2}, \tag{15}$$

and

$$T_{\pi N}^{(3/2)} = \frac{m_\pi^2}{2f_\pi^2}(C_1 + C_0 + 4C_\pi)$$
$$T_{\pi N}^{(1/2)} = \frac{m_\pi^2}{2f_\pi^2}(C_1 + C_0 + 4C_\pi), \tag{16}$$

where

$$C_\pi = (d_F - 2b_F) - \frac{DF}{2M_0}. \tag{17}$$

At the third order

$$T_{\pi N}^{(3/2)} = \frac{m_\pi^2}{16\pi^2 f_\pi^4}\bigg\{-m_\pi\bigg(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\bigg) - \sqrt{m_K^2 - m_\pi^2}\bigg(\pi + \arccos\frac{m_\pi}{m_K}\bigg) + \frac{\pi}{4}\bigg[3(D+F)^2 m_\pi - \frac{1}{3}(D-3F)^2 m_\eta\bigg]\bigg\}, \tag{18}$$

$$T_{\pi N}^{(1/2)} = \frac{m_\pi^2}{16\pi^2 f_\pi^4}\bigg\{2m_\pi\bigg(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\bigg) - \sqrt{m_K^2 - m_\pi^2}\bigg(\frac{3}{2}\pi + 2\arcsin\frac{m_\pi}{m_K}\bigg) + \frac{\pi}{4}\bigg[3(D+F)^2 m_\pi - \frac{1}{3}(D-3F)^2 m_\eta\bigg]\bigg\}. \tag{19}$$

The isospin even and isospin odd amplitudes are also widely used in literature.

$$T_{\pi N}^{-} = \frac{m_\pi}{2f_\pi^2} + \frac{m_\pi^2}{16\pi^2 f_\pi^4}\bigg\{m_\pi\bigg(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\bigg) - \sqrt{m_K^2 - m_\pi^2}\arcsin\frac{m_\pi}{m_K}\bigg\}, \tag{20}$$

$$T_{\pi N}^{+} = \frac{m_\pi^2}{f_\pi^2}\bigg\{\frac{1}{2}(C_1 + C_0) + 2C_\pi\bigg\} + \frac{3m_\pi^2}{64\pi f_\pi^4}\bigg\{-2\sqrt{m_K^2 - m_\pi^2} + (D+F)^2 m_\pi - \frac{1}{9}(D-3F)^2 m_\eta\bigg\}. \tag{21}$$

The above expressions agree with those in Ref. [5]. The two conventions are related through the following equations:

$$T_{\pi N}^{(3/2)} = T_{\pi N}^{+} - T_{\pi N}^{-}, \qquad T_{\pi N}^{(1/2)} = T_{\pi N}^{+} + 2T_{\pi N}^{-}. \tag{22}$$

Sometimes a third convention with the isoscalar amplitude $T_{\pi N}^0 = T_{\pi N}^+$ and isovector amplitude $T_{\pi N}^1 = -T_{\pi N}^-$ are used in the literature.

Most of the threshold T-matrices for pion-baryon and kaon-baryon scattering at the second order depend on the above three combinations C_1, C_0 and C_π. If C_1, C_0 and C_π

can be fixed from KN and πN scattering lengths, many predictions can be made.

C. Pion-Sigma scattering

T-matrices at the leading order:

$$T^{(2)}_{\pi\Sigma} = -\frac{m_\pi}{f_\pi^2}, \qquad T^{(1)}_{\pi\Sigma} = \frac{m_\pi}{f_\pi^2}, \qquad T^{(0)}_{\pi\Sigma} = \frac{2m_\pi}{f_\pi^2}. \quad (23)$$

At the next leading order we have

$$T^{(2)}_{\pi\Sigma} = \frac{m_\pi^2}{f_\pi^2}C_1, \qquad T^{(1)}_{\pi\Sigma} = \frac{m_\pi^2}{f_\pi^2}(C_1 - 2C_d), \qquad T^{(0)}_{\pi\Sigma} = \frac{m_\pi^2}{f_\pi^2}(C_1 + 3C_d), \quad (24)$$

where

$$C_d = d_1 - \frac{D^2 - 3F^2}{6M_0}. \quad (25)$$

The extraction of C_d needs additional inputs besides pion-nucleon and kaon-nucleon scattering lengths. Because of the lack of experimental data, one cannot determine it like the determination of $C_{1,0,\pi}$. We will estimate its value when we discuss numerical results.

At the third order, we have

$$T^{(2)}_{\pi\Sigma} = \frac{m_\pi^2}{8\pi^2 f_\pi^4}\left\{-m_\pi\left(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\right) - \sqrt{m_K^2 - m_\pi^2}\arccos\frac{m_\pi}{m_K} + \frac{\pi}{2}\left[3F^2 m_\pi - \frac{1}{3}D^2 m_\eta\right]\right\}, \quad (26)$$

$$T^{(1)}_{\pi\Sigma} = \frac{m_\pi^2}{8\pi^2 f_\pi^4}\left\{m_\pi\left(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\right) - \sqrt{m_K^2 - m_\pi^2}\arccos\frac{-m_\pi}{m_K} + \frac{\pi}{2}\left[(2D^2 - 3F^2)m_\pi - \frac{1}{3}D^2 m_\eta\right]\right\}, \quad (27)$$

$$T^{(0)}_{\pi\Sigma} = \frac{m_\pi^2}{8\pi^2 f_\pi^4}\left\{2m_\pi\left(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\right) - \sqrt{m_K^2 - m_\pi^2}\left(\frac{3}{2}\pi - 2\arccos\frac{m_\pi}{m_K}\right) - \frac{\pi}{2}\left[3(D^2 - 4F^2)m_\pi + \frac{1}{3}D^2 m_\eta\right]\right\}. \quad (28)$$

D. Pion-Cascade scattering

Because both nucleon and cascade are isospin-1/2 baryons, one expects similar expressions. At the leading order, the expressions are the same as those for the πN scattering,

$$T^{(3/2)}_{\pi\Xi} = -\frac{m_\pi}{2f_\pi^2}, \qquad T^{(1/2)}_{\pi\Xi} = \frac{m_\pi}{f_\pi^2}. \quad (29)$$

The next leading order contributions read

$$T^{(3/2)}_{\pi\Xi} = \frac{m_\pi^2}{2f_\pi^2}(C_1 + C_0), \qquad T^{(1/2)}_{\pi\Xi} = \frac{m_\pi^2}{2f_\pi^2}(C_1 + C_0). \quad (30)$$

The expressions involve only combinations of LECs in kaon-nucleon scattering. This is the relic of isopin and U-spin symmetry (see the relations in Sec. IV).

At the third order, we have

$$T^{(3/2)}_{\pi\Xi} = \frac{m_\pi^2}{16\pi^2 f_\pi^4}\left\{-m_\pi\left(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\right) - \sqrt{m_K^2 - m_\pi^2}\left(\pi + \arccos\frac{m_\pi}{m_K}\right) + \frac{\pi}{4}\left[3(D-F)^2 m_\pi - \frac{1}{3}(D+3F)^2 m_\eta\right]\right\}, \quad (31)$$

$$T^{(1/2)}_{\pi\Xi} = \frac{m_\pi^2}{16\pi^2 f_\pi^4}\left\{2m_\pi\left(\frac{3}{2} - 2\ln\frac{m_\pi}{\lambda} - \ln\frac{m_K}{\lambda}\right) - \sqrt{m_K^2 - m_\pi^2}\left(\frac{1}{2}\pi + 2\arccos\frac{-m_\pi}{m_K}\right) + \frac{\pi}{4}\left[3(D-F)^2 m_\pi - \frac{1}{3}(D+3F)^2 m_\eta\right]\right\}. \quad (32)$$

The expressions are similar to πN T-matrices at this order. The difference lies in the factor F. One can get these expressions from those for πN scattering through replacing F by $-F$. In fact, we can also get the second order expressions from those of $T_{\pi N}$ through replacing b_F by $-b_F$, d_F by $-d_F$ and F by $-F$.

E. Pion-Lambda scattering

The leading order contribution vanishes, which can be understood through the crossing symmetry: $T_{\pi^+\Lambda} = T_{\pi^-\Lambda} = [T_{\pi^+\Lambda}]_{m_\pi \to -m_\pi}$. Thus $T_{\pi\Lambda} = -T_{\pi\Lambda}$ at the leading order. This analysis is also available for the following T-matrices, $T_{K\Lambda}$, $T_{\bar{K}\Lambda}$ and $T_{\eta B}$ whose leading order contributions also vanish.

The tree-level contribution appears only at the next leading order

$$T_{\pi\Lambda} = \frac{m_\pi^2}{3f_\pi^2}(C_1 + 2C_0 + 4C_\pi + C_d). \quad (33)$$

At the third order, for a $P\Lambda$ or ηB scattering, only diagram 4, 5, 10, 11 and 12 in Fig. 1 have nonvanishing contributions. The expression for $T_{\pi\Lambda}$ at this order is

$$T_{\pi\Lambda} = \frac{m_\pi^2}{16\pi f_\pi^4}\left\{-3\sqrt{m_K^2 - m_\pi^2} + D^2\left(m_\pi - \frac{1}{3}m_\eta\right)\right\}. \tag{34}$$

The above expression was first derived in Ref. [5].

F. Kaon-Sigma scattering

One expects the similarity between $T_{K\Sigma}$ and $T_{\bar{K}\Sigma}$ because both K and $\bar{K}$ are isospin doublets. The leading order expressions are

$$\begin{aligned} T_{K\Sigma}^{(3/2)} &= -\frac{m_K}{2f_K^2}, \qquad T_{K\Sigma}^{(1/2)} = \frac{m_K}{f_K^2},\\ T_{\bar{K}\Sigma}^{(3/2)} &= -\frac{m_K}{2f_K^2}, \qquad T_{\bar{K}\Sigma}^{(1/2)} = \frac{m_K}{f_K^2}. \end{aligned} \tag{35}$$

At the next leading order, we have

$$\begin{aligned} T_{K\Sigma}^{(3/2)} &= \frac{m_K^2}{2f_K^2}(C_1 + C_0 + 4C_\pi),\\ T_{K\Sigma}^{(1/2)} &= \frac{m_K^2}{2f_K^2}(C_1 + C_0 - 2C_\pi),\\ T_{\bar{K}\Sigma}^{(3/2)} &= \frac{m_K^2}{2f_K^2}(C_1 + C_0),\\ T_{\bar{K}\Sigma}^{(1/2)} &= \frac{m_K^2}{2f_K^2}(C_1 + C_0 + 6C_\pi).. \end{aligned} \tag{36}$$

At the third chiral order

$$\begin{aligned} T_{K\Sigma}^{(3/2)} = \frac{m_K^2}{32\pi^2 f_K^4}\bigg\{&-m_K\left(3 + \ln\frac{m_\pi}{\lambda} - 4\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\right)\\ &+ \sqrt{m_K^2 - m_\pi^2}\left(2i\pi - \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\right)\\ &- 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{m_K}{m_\eta}\\ &+ \frac{4}{3}\pi D\left[2F\frac{m_\pi^2}{m_\eta + m_\pi} + (D + 2F)m_\eta\right]\bigg\}, \end{aligned} \tag{37}$$

$$\begin{aligned} T_{K\Sigma}^{(1/2)} = \frac{m_K^2}{16\pi^2 f_K^4}\bigg\{&m_K\left(3 + \ln\frac{m_\pi}{\lambda} - 4\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\right)\\ &+ \sqrt{m_K^2 - m_\pi^2}\left(\frac{1}{4}i\pi + \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\right)\\ &- 3\sqrt{m_\eta^2 - m_K^2}\left(\frac{3}{4}\pi - \arccos\frac{m_K}{m_\eta}\right)\\ &+ \frac{2}{3}\pi D\left[-4F\frac{m_\pi^2}{m_\eta + m_\pi} + (D - 4F)m_\eta\right]\bigg\}, \end{aligned} \tag{38}$$

$$\begin{aligned} T_{\bar{K}\Sigma}^{(3/2)} = \frac{m_K^2}{32\pi^2 f_K^4}\bigg\{&-m_K\left(3 + \ln\frac{m_\pi}{\lambda} - 4\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\right)\\ &+ \sqrt{m_K^2 - m_\pi^2}\left(2i\pi - \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\right)\\ &- 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{m_K}{m_\eta}\\ &+ \frac{4}{3}\pi D\left[-2F\frac{m_\pi^2}{m_\eta + m_\pi} + (D - 2F)m_\eta\right]\bigg\}, \end{aligned} \tag{39}$$

$$\begin{aligned} T_{\bar{K}\Sigma}^{(1/2)} = \frac{m_K^2}{16\pi^2 f_K^4}\bigg\{&m_K\left(3 + \ln\frac{m_\pi}{\lambda} - 4\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\right)\\ &+ \sqrt{m_K^2 - m_\pi^2}\left(\frac{1}{4}i\pi + \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\right)\\ &- 3\sqrt{m_\eta^2 - m_K^2}\left(\frac{3}{4}\pi - \arccos\frac{m_K}{m_\eta}\right)\\ &+ \frac{2}{3}\pi D\left[4F\frac{m_\pi^2}{m_\eta + m_\pi} + (D + 4F)m_\eta\right]\bigg\}. \end{aligned} \tag{40}$$

$T_{\bar{K}\Sigma}^{(3/2)}$ differs from $T_{K\Sigma}^{(3/2)}$ in the sign in front of F at the third order. The same property holds for $T_{\bar{K}\Sigma}^{(1/2)}$ and $T_{K\Sigma}^{(1/2)}$. One can also testify that differences between $T_{K\Sigma}$ and $T_{\bar{K}\Sigma}$ at the second order are the signs in front of b_F, d_F and F.

G. Kaon-Cascade scattering

The leading order contributions are

$$\begin{aligned} T_{K\Xi}^{(1)} &= \frac{m_K}{2f_K^2}, \qquad T_{K\Xi}^{(0)} = \frac{3m_K}{2f_K^2},\\ T_{\bar{K}\Xi}^{(1)} &= -\frac{m_K}{f_K^2}, \qquad T_{\bar{K}\Xi}^{(0)} = 0. \end{aligned} \tag{41}$$

At the second order, we have

$$\begin{aligned} T_{K\Xi}^{(1)} &= \frac{m_K^2}{2f_K^2}(C_1 + C_0 + 4C_\pi),\\ T_{K\Xi}^{(0)} &= \frac{m_K^2}{2f_K^2}(3C_1 - C_0 - 4C_\pi),\\ T_{\bar{K}\Xi}^{(1)} &= \frac{m_K^2}{f_K^2}C_1,\\ T_{\bar{K}\Xi}^{(0)} &= \frac{m_K^2}{f_K^2}(C_0 + 4C_\pi). \end{aligned} \tag{42}$$

The third order expressions are

$$T^{(1)}_{K\Xi} = \frac{m_K^2}{32\pi^2 f_K^4}\bigg\{m_K\bigg(3 - 5\ln\frac{m_\pi}{\lambda} + 2\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\bigg) + 5\sqrt{m_K^2 - m_\pi^2}\bigg(i\pi - \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\bigg) - 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{-m_K}{m_\eta} + \frac{\pi}{3}(D + 3F)\bigg[2(D - F)\frac{m_\pi^2}{m_\eta + m_\pi} + (3D + F)m_\eta\bigg]\bigg\}, \tag{43}$$

$$T^{(0)}_{K\Xi} = \frac{3m_K^2}{32\pi^2 f_K^4}\bigg\{m_K\bigg(3 - \ln\frac{m_\pi}{\lambda} - 2\ln\frac{|m_K|}{\lambda} - 3\ln\frac{m_\eta}{\lambda}\bigg) + \sqrt{m_K^2 - m_\pi^2}\bigg(i\pi - \ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi}\bigg) - 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{-m_K}{m_\eta} - \frac{\pi}{3}(D + 3F)\bigg[2(D - F)\frac{m_\pi^2}{m_\eta + m_\pi} + \frac{1}{3}(5D - 9F)m_\eta\bigg]\bigg\}, \tag{44}$$

$$T^{(1)}_{\bar{K}\Xi} = \frac{m_K^2}{16\pi^2 f_K^4}\bigg\{m_K\bigg(-3 + 2\ln\frac{m_\pi}{\lambda} + \ln\frac{|m_K|}{\lambda} + 3\ln\frac{m_\eta}{\lambda}\bigg) + 2\sqrt{m_K^2 - m_\pi^2}\ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi} - 3\sqrt{m_\eta^2 - m_K^2}\arccos\frac{m_K}{m_\eta} - \frac{\pi}{6}(D + 3F)\bigg[2(D - F)\frac{m_\pi^2}{m_\eta + m_\pi} + (D - 5F)m_\eta\bigg]\bigg\}, \tag{45}$$

$$T^{(0)}_{\bar{K}\Xi} = \frac{3m_K^2}{16\pi^2 f_K^4}\bigg\{m_K\bigg(\ln\frac{m_\pi}{\lambda} - \ln\frac{|m_K|}{\lambda}\bigg) + \sqrt{m_K^2 - m_\pi^2}\ln\frac{m_K + \sqrt{m_K^2 - m_\pi^2}}{m_\pi} + \frac{\pi}{3}(D + 3F)\bigg[(D - F)\frac{m_\pi^2}{m_\eta + m_\pi} + \frac{1}{6}(7D - 3F)m_\eta\bigg]\bigg\}. \tag{46}$$

H. Kaon-Lambda scattering

Nonvanishing contributions start from the next leading order.

$$T_{\bar{K}\Lambda} = T_{K\Lambda} = \frac{m_K^2}{6f_K^2}[5C_1 + C_0 + 2C_\pi - 4C_d]. \tag{47}$$

At the third chiral order loop corrections are

$$T_{\bar{K}\Lambda} = T_{K\Lambda} = \frac{9m_K^2}{64\pi f_K^4}\bigg\{i\sqrt{m_K^2 - m_\pi^2} - \sqrt{m_\eta^2 - m_K^2} + \frac{8}{27}D^2 m_\eta\bigg\}. \tag{48}$$

Similarly, one may get $T_{\bar{K}\Lambda}$ from $T_{K\Lambda}$ through the replacement $b_F \to -b_F$, $d_F \to -d_F$ and $F \to -F$. However, b_F and d_F disappear in the second order T-matrices and F disappears in the third order T-matrix. Hence $T_{\bar{K}\Lambda} = T_{K\Lambda}$.

I. Eta-baryon scattering

The leading order contribution vanishes for every channel. With the Gell-Mann-Okubo relation the next leading order contributions can be written as:

$$\begin{aligned} T_{\eta N} &= \frac{1}{6f_\eta^2}[(5C_1 + C_0 - 4C_\pi - 4C_d)m_\eta^2 - 4(b_D - 3b_F)(m_\eta^2 - m_\pi^2)], \\ T_{\eta\Sigma} &= \frac{1}{3f_\eta^2}[(C_1 + 2C_0 + 4C_\pi + C_d)m_\eta^2 + 4b_D(m_\eta^2 - m_\pi^2)], \\ T_{\eta\Xi} &= \frac{1}{6f_\eta^2}[(5C_1 + C_0 + 8C_\pi - 4C_d)m_\eta^2 - 4(b_D + 3b_F)(m_\eta^2 - m_\pi^2)], \\ T_{\eta\Lambda} &= \frac{1}{3f_\eta^2}[3(C_1 + C_d)m_\eta^2 - 4b_D(m_\eta^2 - m_\pi^2)]. \end{aligned} \tag{49}$$

These expressions rely on $C_{1,0,\pi,d}$ as well as b_D and b_F. b_D and b_F are determined with the octet-baryon masses.

At the third chiral order, we have

$$T_{\eta N} = \frac{1}{32\pi f_\eta^4}\bigg\{9im_\eta^2\sqrt{m_\eta^2 - m_K^2} - \frac{1}{6}(D - 3F)^2(4m_\eta^2 - m_\pi^2)m_\eta - \frac{3}{2}(D + F)^2 m_\pi^3 + \frac{2}{3}(5D^2 + 9F^2 - 6DF)m_K^3\bigg\}, \tag{50}$$

$$T_{\eta\Sigma} = \frac{1}{16\pi f_\eta^4}\Big\{3im_\eta^2\sqrt{m_\eta^2 - m_K^2} - \frac{1}{3}D^2(4m_\eta^2 - m_\pi^2)m_\eta - \frac{1}{3}(D^2 + 6F^2)m_\pi^3 + 2(D^2 + F^2)m_K^3\Big\}, \tag{51}$$

$$T_{\eta\Xi} = \frac{1}{32\pi f_\eta^4}\Big\{9im_\eta^2\sqrt{m_\eta^2 - m_K^2} - \frac{1}{6}(D + 3F)^2(4m_\eta^2 - m_\pi^2)m_\eta - \frac{3}{2}(D - F)^2 m_\pi^3 + \frac{2}{3}(5D^2 + 9F^2 + 6DF)m_K^3\Big\}, \tag{52}$$

$$T_{\eta\Lambda} = \frac{1}{16\pi f_\eta^4}\Big\{9im_\eta^2\sqrt{m_\eta^2 - m_K^2} - \frac{1}{3}D^2(4m_\eta^2 - m_\pi^2)m_\eta - D^2 m_\pi^3 + \frac{2}{3}(D^2 + 9F^2)m_K^3\Big\}. \tag{53}$$

Again there is the similarity between $T_{\eta N}$ and $T_{\eta\Xi}$.

IV. THRESHOLD T-MATRIX RELATIONS

In this section, we list the relations between kaon-baryon and anti-kaon-baryon threshold T-matrices according to the crossing symmetry. We also present the relations of these T-matrices in the SU(3) symmetry limit. These relations are used to cross-check our calculations.

Crossing symmetry relates KB processes with $\bar{K}B$ processes. According to this symmetry, we have the following relations:

$$\begin{aligned} T_{\bar{K}N}^{(1)} &= \frac{1}{2}[T_{KN}^{(1)} + T_{KN}^{(0)}]_{m_K\to -m_K},\\ T_{\bar{K}N}^{(0)} &= \frac{1}{2}[3T_{KN}^{(1)} - T_{KN}^{(0)}]_{m_K\to -m_K}, \end{aligned} \tag{54}$$

$$\begin{aligned} T_{\bar{K}\Xi}^{(1)} &= \frac{1}{2}[T_{K\Xi}^{(1)} + T_{K\Xi}^{(0)}]_{m_K\to -m_K},\\ T_{\bar{K}\Xi}^{(0)} &= \frac{1}{2}[3T_{K\Xi}^{(1)} - T_{K\Xi}^{(0)}]_{m_K\to -m_K}, \end{aligned} \tag{55}$$

$$\begin{aligned} T_{\bar{K}\Sigma}^{(3/2)} &= \frac{1}{3}[T_{K\Sigma}^{(3/2)} + 2T_{K\Sigma}^{(1/2)}]_{m_K\to -m_K},\\ T_{\bar{K}\Sigma}^{(1/2)} &= \frac{1}{3}[4T_{K\Sigma}^{(3/2)} - T_{K\Sigma}^{(1/2)}]_{m_K\to -m_K}, \end{aligned} \tag{56}$$

and

$$T_{\bar{K}\Lambda} = [T_{K\Lambda}]_{m_K\to -m_K}. \tag{57}$$

In the SU(3) flavor symmetry limit, we derive the following relations using the isospin and U-spin symmetry:

$$\begin{aligned} T_{KN}^{(1)} &= T_{\pi\Sigma}^{(2)} = T_{\bar{K}\Xi}^{(1)},\\ T_{\pi N}^{(3/2)} &= T_{K\Sigma}^{(3/2)} = \frac{1}{2}[T_{\bar{K}\Xi}^{(1)} + T_{\bar{K}\Xi}^{(0)}],\\ T_{\pi\Xi}^{(3/2)} &= T_{\bar{K}\Sigma}^{(3/2)} = \frac{1}{2}[T_{KN}^{(1)} + T_{KN}^{(0)}],\\ T_{K\Xi}^{(1)} &= \frac{1}{3}[T_{\pi N}^{(3/2)} + 2T_{\pi N}^{(1/2)}]\\ &= \frac{1}{3}[T_{\bar{K}\Sigma}^{(3/2)} + 2T_{\bar{K}\Sigma}^{(1/2)}],\\ T_{\bar{K}N}^{(1)} &= \frac{1}{3}[T_{\pi\Xi}^{(3/2)} + 2T_{\pi\Xi}^{(1/2)}]\\ &= \frac{1}{3}[T_{K\Sigma}^{(3/2)} + 2T_{K\Sigma}^{(1/2)}],\\ T_{\bar{K}N}^{(1)} + T_{\bar{K}N}^{(0)} &= T_{K\Xi}^{(1)} + T_{K\Xi}^{(0)}\\ &= \frac{1}{3}[T_{\pi\Sigma}^{(2)} + 3T_{\pi\Sigma}^{(1)} + 2T_{\pi\Sigma}^{(0)}],\\ T_{\eta\Sigma} &= T_{\pi\Lambda},\\ \frac{1}{2}[T_{\pi\Sigma}^{(2)} + T_{\pi\Sigma}^{(1)}] + T_{\pi\Lambda} &= \frac{1}{3}[2T_{K\Sigma}^{(3/2)} + T_{K\Sigma}^{(1/2)}] + T_{K\Lambda}\\ &= \frac{1}{3}[2T_{\bar{K}\Sigma}^{(3/2)} + T_{\bar{K}\Sigma}^{(1/2)}] + T_{\bar{K}\Lambda}\\ &= \frac{1}{3}[2T_{\pi N}^{(3/2)} + T_{\pi N}^{(1/2)}] + T_{\eta N}\\ &= \frac{1}{3}[2T_{\pi\Xi}^{(3/2)} + T_{\pi\Xi}^{(1/2)}] + T_{\eta\Xi}. \end{aligned} \tag{58}$$

V. DETERMINATION OF LOW-ENERGY-CONSTANTS

There are seven unknown LECs in $\mathcal{L}_{\phi B}^{(2)}$, $b_{D,F,0}$, $d_{D,F,0}$, and d_1. The threshold T-matrices depend on four combinations of them: $(d_F - 2b_F)$, $(d_D - 2b_D)$, $(d_0 - 2b_0)$ and d_1. After the regrouping, we defined another four combinations: C_1, C_0, C_π and C_d. The values of C_1, C_0 and C_π can be extracted from the experimental values of T_{KN} and $T_{\pi N}$. With C_1, C_0 and C_π one can predict most of pion-baryon and kaon-baryon scattering lengths.

Threshold T-matrices $T_{K\Sigma}^{(1)}$, $T_{K\Sigma}^{(0)}$, $T_{M\Lambda}$ and $T_{\eta B}$ depend on additional LECs C_d and $b_{D,F}$. The values of b_D and b_F can be extracted from the octet-baryon mass differences. In order to determine $C_d = d_1 - \frac{D^2-3F^2}{6M_0}$, one has to fix both d_1 and M_0.

For the sake of comparison, let us recall the procedure of determining LECs in Ref. [5]. (i) First, Kaiser fixed the energy scale parameter from the loop integration λ to be 0.95 GeV using the experimental value of $T_{\pi N}^-$; (ii) Then he determined the LECs $C_{1,0}$ using the experimental values of $T_{KN}^{(1)}$ and $T_{KN}^{(0)}$ [15]: $C_1 = 2.33\ \text{GeV}^{-1}$, $C_0 = 0.36\ \text{GeV}^{-1}$. Throughout his analysis he used $D = 0.8$ and $F = 0.5$;

(iii) The three LECs $b_D = 0.042\ \text{GeV}^{-1}$, $b_F = -0.557\ \text{GeV}^{-1}$, $b_0 = -0.789\ \text{GeV}^{-1}$ and $M_0 = 918.4$ MeV were obtained by fitting the octet-baryon masses and the πN sigma term $\sigma_{\pi N} = 45$ MeV from Ref. [27]; (iv) Using the experimental value of $T^+_{\pi N}$, the extracted parameters $d_F = -0.968\ \text{GeV}^{-1}$, $2d_0 + d_D = -1.562\ \text{GeV}^{-1}$, $d_1 + d_D = 1.150\ \text{GeV}^{-1}$ and the value of $d_0 \simeq -1.0\ \text{GeV}^{-1}$ determined elsewhere [28], one extracts d_1.

Our procedure is slightly different from Kaiser's. We take $\lambda = 4\pi f_\pi$ as the chiral symmetry breaking scale, which is widely adopted in the chiral perturbation theory. The other steps are somehow similar. The meson masses and decay constants are from PDG [29]: $m_\pi = 139.57$ MeV, $m_K = 493.68$ MeV, $m_\eta = 547.75$ MeV, $f_\pi = 92.4$ MeV, $f_K = 113$ MeV, $f_\eta = 1.2 f_\pi$. $D = 0.8$ and $F = 0.5$.

We first determine the combinations C_1 and C_0. Using the experimental values of $a^{(1)}_{KN} = -0.33$ fm and $a^{(0)}_{KN} = 0.02$ fm [15], we extract

$$C_1 = 1.786\ \text{GeV}^{-1}, \qquad C_0 = 0.413\ \text{GeV}^{-1}. \tag{59}$$

$a^+_{\pi N}$ from one recent experiment is $-0.0001 \pm^{0.0009}_{0.0021} m_\pi^{-1}$ [9], from which $C_\pi = 0.096^{+0.020}_{-0.048}\ \text{GeV}^{-1}$ is extracted. The new value of $a^+_{\pi N}$ indicates a larger $\sigma_{\pi N}$ [9]. Theoretical calculations of $\sigma_{\pi N}$ lie in a large range [30]. For comparison, we use two values $\sigma_{\pi N} = 45$ MeV and 57 MeV for the following calculation. The latter one is compatible with Ref. [9]. We use $\sigma_{\pi N} = 45$ MeV to illustrate the fitting procedure.

We now determine b_D, b_F, b_0 and M_0 with the formulas of the octet-baryon masses and $\sigma_{\pi N}$ given in Ref. [2]. We use $f = f_\pi$ in the π loops, $f = f_K$ in kaon loops and $f = f_\eta$ in η loops in these formulas. By fitting these four parameters to baryon masses $M_N = 938.9 \pm 1.3$ MeV, $M_\Sigma = 1193.4 \pm 8.1$ MeV, $M_\Xi = 1318.1 \pm 6.7$ MeV, $M_\Lambda = 1115.7 \pm 5.4$ MeV and $\sigma_{\pi N} = 45 \pm 8$ MeV [27], we get

$$\begin{aligned} M_0 &= 820.00 \pm 104.24\ \text{MeV}, \\ b_0 &= -0.819 \pm 0.103\ \text{GeV}^{-1}, \\ b_D &= 0.044 \pm 0.008\ \text{GeV}^{-1}, \\ b_F &= -0.491 \pm 0.003\ \text{GeV}^{-1} \end{aligned} \tag{60}$$

with $\chi^2/\text{d.o.f.} \sim 0.98$. Both the up and down quark mass difference and electromagnetic interaction contribute to the baryon mass splitting within an isospin multiplet. The typical electromagnetic correction is roughly around 0.5% of the baryon mass, which is not considered in this work. Therefore we have added some uncertainty to M_Λ. We take the central value of M_N, M_Σ and M_Ξ to be the average of the isospin multiplet. The corresponding error is simply the mass splitting of the isospin multiplet. We determine other LECs and perform numerical evaluations with the above LECs. The values we get differ from those in Ref. [5] because we use a different f_η.

From the above determined $C_{1,0,\pi}$, M_0, $b_{0,D,F}$, we have

$$\begin{aligned} d_F &= -0.642^{+0.038}_{-0.057}\ \text{GeV}^{-1}, \\ 2d_0 + d_D &= -1.722^{+0.413}_{-0.416}\ \text{GeV}^{-1}, \\ d_1 + d_D &= 0.689^{+0.026}_{-0.050}\ \text{GeV}^{-1}. \end{aligned} \tag{61}$$

We derived the errors with the standard error propagation formula. The three LECs d_D, d_0 and d_1 cannot be determined independently in the procedure. Of these LECs, d_0 may have the minimum uncertainty. It is convenient to use it as an input. Up to now, only the second order value is available. With C_1, C_0, C_π, M_0, b_0, b_D and $d_0 = -0.996\ \text{GeV}^{-1}$ [28], we get $d_D = 0.270^{+0.413}_{-0.416}\ \text{GeV}^{-1}$, $d_1 = 0.419^{+0.414}_{-0.423}\ \text{GeV}^{-1}$ and $C_d = 0.441^{+0.414}_{-0.423}\ \text{GeV}^{-1}$.

We denote the parameter set corresponding to $\sigma_{\pi N} = 45(57) \pm 8$ MeV as Set I (II), respectively, which is collected in Table I.

VI. RESULTS AND DISCUSSIONS

The threshold T-matrices with the first and second set of parameters are collected in Tables II, III, IV, V, VI, and VII respectively. The corresponding scattering lengths are listed in the last column of these tables. The scattering lengths do not change significantly with a larger $\sigma_{\pi N}$. The T-matrices at the second order are expressed with C_1, C_0, C_π, C_d, b_D and b_F. The errors of C_1 and C_0 can not been extracted from known experimental sources. Therefore, all the errors in these tables are estimated from C_π, C_d, b_D and b_F with the error propagation formula.

Our $a^-_{\pi N}$ is consistent with the experimental value $0.125^{+0.001}_{-0.003}$ fm [9] while our $a^{(0)}_{\pi\Sigma} = 0.60 \pm 0.04$ fm is smaller than the value 1.10 ± 0.06 fm [31].

TABLE I. Two sets of parameters in the numerical analysis. Units are MeV for $\sigma_{\pi N}$ and M_0 and GeV^{-1} for other quantities. The variation of sigma term has no effect on $b_D = 0.044 \pm 0.008\ \text{GeV}^{-1}$ and $b_F = -0.491 \pm 0.003\ \text{GeV}^{-1}$. d_D, d_1 and C_d are deduced with $d_0 = -0.996\ \text{GeV}^{-1}$ [28]. The error of C_π is from the experimental scattering lengths. The errors of M_0, b_0, b_D and b_F are given by MINUIT. The errors of d_D, d_F, d_1 and C_d are related to those of C_π, b_0, b_D, b_F and M_0.

	$\sigma_{\pi N}$	C_1	C_0	C_π	M_0	b_0	d_D	d_F	d_1	C_d
Set 1	45 ± 8	1.786	0.413	$0.096^{+0.020}_{-0.048}$	820.00 ± 104.24	-0.819 ± 0.103	$0.270^{+0.413}_{-0.416}$	$-0.642^{+0.038}_{-0.057}$	$0.419^{+0.414}_{-0.423}$	$0.441^{+0.414}_{-0.423}$
Set 2	57 ± 8	1.786	0.413	$0.096^{+0.020}_{-0.048}$	663.86 ± 104.26	-0.973 ± 0.102	$-0.282^{+0.415}_{-0.418}$	$-0.585^{+0.052}_{-0.067}$	$0.974^{+0.416}_{-0.425}$	$1.001^{+0.416}_{-0.425}$

TABLE II. Pion-baryon threshold T-matrices order by order with the parameter Set I in unit of fm.

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T^{+}_{\pi N}$	0	$0.58^{+0.02}_{-0.04}$	-0.58	$-0.002^{+0.018}_{-0.043}$	$-0.00014^{+0.00127}_{-0.00297}$ (input)
$T^{-}_{\pi N}$	1.61	0	0.26	1.87	0.13
$T^{(3/2)}_{\pi N}$	-1.61	$0.58^{+0.02}_{-0.04}$	-0.85	$-1.88^{+0.02}_{-0.04}$	$-0.130^{+0.001}_{-0.003}$
$T^{(1/2)}_{\pi N}$	3.23	$0.58^{+0.02}_{-0.04}$	-0.06	$3.75^{+0.02}_{-0.04}$	$0.260^{+0.001}_{-0.003}$
$T^{(2)}_{\pi\Sigma}$	-3.23	0.80	-1.03	-3.46	-0.25
$T^{(1)}_{\pi\Sigma}$	3.23	$0.41^{+0.37}_{-0.38}$	-0.02	$3.61^{+0.37}_{-0.38}$	0.26 ± 0.03
$T^{(0)}_{\pi\Sigma}$	6.45	$1.40^{+0.56}_{-0.57}$	0.59	$8.44^{+0.56}_{-0.57}$	0.60 ± 0.04
$T^{(3/2)}_{\pi\Xi}$	-1.61	0.49	-1.25	-2.37	-0.17
$T^{(1/2)}_{\pi\Xi}$	3.23	0.49	-0.46	3.26	0.23
$T_{\pi\Lambda}$	0	$0.52^{+0.06}_{-0.07}$	-1.52	$-1.00^{+0.06}_{-0.07}$	$-0.071^{+0.004}_{-0.005}$

TABLE III. Kaon-baryon threshold T-matrices order by order with the parameter Set I in unit of fm.

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T^{(1)}_{KN}$	-7.63	6.73	-5.42	-6.33	-0.33 (input)
$T^{(0)}_{KN}$	0	1.56	-1.17	0.38	0.02 (input)
$T^{(1)}_{\bar{K}N}$	3.81	4.14	$-0.32 + 6.95i$	$7.64 + 6.95i$	$0.40 + 0.36i$
$T^{(0)}_{\bar{K}N}$	11.44	9.31	$7.96 + 4.17i$	$28.72 + 4.17i$	$1.50 + 0.22i$
$T^{(3/2)}_{K\Sigma}$	-3.81	$4.86^{+0.15}_{-0.36}$	$-1.00 + 2.78i$	$0.04^{+0.15}_{-0.36} + 2.78i$	$0.0024^{+0.0086}_{-0.0202} + 0.16i$
$T^{(1/2)}_{K\Sigma}$	7.63	$3.78^{+0.08}_{-0.18}$	$2.99 + 0.69i$	$14.40^{+0.08}_{-0.18} + 0.69i$	$(0.81 \pm 0.01) + 0.04i$
$T^{(3/2)}_{\bar{K}\Sigma}$	-3.81	4.14	$-4.61 + 2.78i$	$-4.28 + 2.78i$	$-0.24 + 0.16i$
$T^{(1/2)}_{\bar{K}\Sigma}$	7.63	$5.22^{+0.23}_{-0.54}$	$10.20 + 0.69i$	$23.05^{+0.23}_{-0.54} + 0.69i$	$1.30^{+0.01}_{-0.03} + 0.04i$
$T^{(1)}_{K\Xi}$	3.81	$4.86^{+0.15}_{-0.36}$	$4.06 + 6.95i$	$12.73^{+0.15}_{-0.36} + 6.95i$	$0.74^{+0.01}_{-0.02} + 0.40i$
$T^{(0)}_{K\Xi}$	11.44	$8.59^{+0.15}_{-0.36}$	$5.12 + 4.17i$	$25.16^{+0.15}_{-0.36} + 4.17i$	$1.46^{+0.01}_{-0.02} + 0.24i$
$T^{(1)}_{\bar{K}\Xi}$	-7.63	6.73	-4.66	-5.56	-0.32
$T^{(0)}_{\bar{K}\Xi}$	0	$3.00^{+0.31}_{-0.72}$	6.81	$9.80^{+0.31}_{-0.72}$	$0.57^{+0.02}_{-0.04}$
$T_{K\Lambda}(T_{\bar{K}\Lambda})$	0	$4.88^{+1.04}_{-1.06}$	$-1.76 + 6.25i$	$3.11^{+1.04}_{-1.06} + 6.25i$	$(0.17 \pm 0.06) + 0.34i$

TABLE IV. Eta-baryon threshold T-matrices order by order with the parameter Set I in unit of fm.

	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T_{\eta N}$	$1.22^{+1.33}_{-1.37}$	$2.40 + 8.32i$	$3.63^{+1.33}_{-1.37} + 8.32i$	$(0.18 \pm 0.07) + 0.42i$
$T_{\eta\Sigma}$	$5.78^{+0.68}_{-0.75}$	$1.93 + 5.55i$	$7.70^{+0.68}_{-0.75} + 5.55i$	$(0.42 \pm 0.04) + 0.30i$
$T_{\eta\Xi}$	$10.98^{+1.34}_{-1.39}$	$0.77 + 8.32i$	$11.75^{+1.34}_{-1.39} + 8.32i$	$(0.66 \pm 0.08) + 0.47i$
$T_{\eta\Lambda}$	$10.46^{+2.00}_{-2.04}$	$2.39 + 16.64i$	$12.85^{+2.00}_{-2.04} + 16.64i$	$(0.69 \pm 0.11) + 0.89i$

TABLE V. Pion-baryon threshold T-matrices order by order with the parameter Set II in unit of fm.

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T^{(1)}_{\pi\Sigma}$	3.23	$-0.10^{+0.37}_{-0.38}$	-0.02	$3.11^{+0.37}_{-0.38}$	0.22 ± 0.03
$T^{(0)}_{\pi\Sigma}$	6.45	$2.16^{+0.56}_{-0.57}$	0.59	$9.19^{+0.56}_{-0.57}$	0.66 ± 0.04
$T_{\pi\Lambda}$	0	$0.60^{+0.06}_{-0.07}$	-1.52	$-0.92^{+0.06}_{-0.07}$	$-0.065^{+0.004}_{-0.005}$

TABLE VI. Kaon-baryon threshold T-matrices order by order with the parameter Set II in unit of fm.

	$\mathcal{O}(p)$	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering length
$T_{K\Lambda}(T_{\bar{K}\Lambda})$	0	$3.47^{+1.04}_{-1.07}$	$-1.76 + 6.25i$	$1.71^{+1.04}_{-1.07} + 6.25i$	$(0.09 \pm 0.06) + 0.34i$

TABLE VII. Eta-baryon threshold T-matrices order by order with the parameter Set II in unit of fm.

	$\mathcal{O}(p^2)$	$\mathcal{O}(p^3)$	Total	Scattering lengths
$T_{\eta N}$	$-0.57^{+1.34}_{-1.37}$	$2.40 + 8.32i$	$1.83^{+1.34}_{-1.37} + 8.32i$	$(0.09 \pm 0.07) + 0.42i$
$T_{\eta\Sigma}$	$6.67^{+0.68}_{-0.75}$	$1.93 + 5.55i$	$8.60^{+0.68}_{-0.75} + 5.55i$	$(0.47 \pm 0.04) + 0.30i$
$T_{\eta\Xi}$	$9.19^{+1.34}_{-1.40}$	$0.77 + 8.32i$	$9.95^{+1.34}_{-1.40} + 8.32i$	$(0.56 \pm 0.08) + 0.47i$
$T_{\eta\Lambda}$	$13.16^{+2.00}_{-2.05}$	$2.39 + 16.64i$	$15.55^{+2.00}_{-2.05} + 16.64i$	$(0.83 \pm 0.11) + 0.89i$

Our isovector $\bar{K}N$ scattering length $a^{(1)}_{\bar{K}N} = (0.40 + 0.36i)$ fm is roughly consistent with the empirical value $(0.37 + 0.60i)$ fm [15], although the imaginary part is smaller. The isoscalar $\bar{K}N$ channel is strongly affected by the resonance $\Lambda(1405)$ which lies below $\bar{K}N$ threshold. It is known long ago that it is impossible to get a reasonable description of the isoscalar $\bar{K}N$ scattering length without taking into account $\Lambda(1405)$'s contribution in a nonperturbative way. Especially, $\Lambda(1405)$ affects the real part of $a^{(0)}_{\bar{K}N}$ dramatically. In contrast, the imaginary part of our $a_{K^-p} = 0.95 + 0.29i$ fm is compatible with the recent experimental value, $\mathrm{Im}[a_{K^-p}] = +(0.302 \pm 0.135 \pm 0.036)i$ fm [13].

The real part of our $a_{\eta N} = [(0.18 \pm 0.07) + 0.42i]$ fm is compatible with those in [32,33]. Our imaginary part satisfies the requirement $\mathrm{Im}[a_{\eta N}] \geq (0.28 \pm 0.04)$ fm derived in Ref. [34] and is larger than those in Refs. [22,25]. Again, one should be cautious about our number. Since HBχPT is a perturbative approach, we have completely ignored the contribution from the nearby $N^*(1535)$ resonance.

Our $a_{\eta\Sigma} = [(0.42 \pm 0.04) + 0.30i]$ fm is consistent with the range for $a_{\eta\Sigma} = [(0.10 \sim 1.10) + (0.35 \sim 2.20)i]$ fm in Ref. [22]. Our $a_{\eta\Lambda} = [(0.69 \pm 0.11) + 0.89i]$ fm is also consistent with the value $[(0.64 \pm 0.29) + (0.80 \pm 0.30)i]$ fm in Ref. [22]. The above two imaginary parts are both larger than that of $a_{\eta\Lambda} = [(0.50 \pm 0.05) + (0.27 \pm 0.01)i]$ fm in Ref. [31].

In the SU(3) HBχPT approach, the convergence of the chiral expansion is a serious issue because of the large mass of kaon and eta mesons. One has to investigate case by case to make sure whether the higher order chiral corrections converge or blow up. From Tables II, III, and IV we find the chiral corrections to the threshold T-matrices converge well only in the following few channels: $T^{(1/2)}_{\pi N}$, $T^{(1)}_{\pi\Sigma}$, $T^{(0)}_{\pi\Sigma}$, $T^{(1/2)}_{\pi\Xi}$, $T^{(1/2)}_{K\Sigma}$, $T^{(0)}_{K\Xi}$. HBχPT predictions of scattering lengths in these channels should be reliable. $T^{(1)}_{\pi\Sigma}$ is particular interesting. The chiral corrections converge very fast in this channel. The LEC C_d can be extracted if the scattering length $a^{(1)}_{\pi\Sigma}$ is measured experimentally.

We calculate the scattering lengths at threshold where chiral perturbation theory, in principle, works well. However, some corrections may improve the calculation. First, the complete determination of the LECs in the second and third order Lagrangians may give more accurate predictions in the HBχPT framework. Secondly, due to the complicate convergence in the SU(3) case, higher order corrections may be significant. Future investigations to the fourth order will give us a clearer picture. Thirdly, subthreshold effects of closed channels may give corrections.

One can not consider effects from resonances close to thresholds in the current method. To extend the range for chiral expansion and include resonance effects, unitarized chiral perturbation theory was developed. The unitarity corrections to scattering lengths for πN scattering had been studied in Refs. [35] with such a method. One may also consider coupled channel effects like the treatment in Ref. [36] with this method. To consider the resonance effects from t and u channels for πN scattering, a more complete treatment on the left-cut is possible because there the chiral expansion has been carried out to $\mathcal{O}(p^4)$.

In summary, we have calculated the chiral corrections to the s-wave meson-baryon scattering lengths to the third chiral order in SU(3) HBχPT. Hopefully these explicit expressions of chiral corrections will be helpful to the chiral extrapolations of the scattering lengths in the future lattice simulations. This is the main result of this paper. There is a good possibility of measuring these meson-baryon scattering lengths experimentally from the strangeness program at CSR, Lan-Zhou and JHF in the near future. Therefore we have also done some numerical analysis of these scattering lengths based on the available experimental information. We find that the chiral expansion converges quite well in several channels. Hence, HBχPT predictions of these scattering lengths are reliable, which may be useful to the construction of the meson-baryon interaction models.

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