

One-loop renormalization group equations of the neutrino mass matrix in the triplet seesaw model

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Within the framework of the standard model plus one heavy Higgs triplet, we derive a full set of one-loop renormalization group equations of the neutrino mass matrix and Higgs couplings in both full and effective theories. The explicit RGEs of neutrino masses, flavor mixing angles and CP -violating phases are also obtained, and their nontrivial running behaviors around the Higgs triplet mass threshold are numerically illustrated.

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I. INTRODUCTION

To understand the origin of fermion masses and flavor mixing is crucial and essential in modern particle physics. Current neutrino experiments have provided very convincing evidence that neutrinos are massive and lepton flavors are mixed [1–5]. However, it remains unclear why the neutrino masses are quite suppressed compared to the other fermion masses. Among various possible models, the seesaw mechanism [6] should be one of the most favorite mechanisms to explain the unnatural smallness of the neutrino mass scale. In the usual Type-I seesaw mechanism, heavy right-handed Majorana neutrinos are introduced to generate the light neutrino masses. The lepton-flavor-violating and lepton-number-violating processes, which are forbidden in the standard model (SM), can also take place through the exchanges of heavy right-handed neutrinos.

Besides the Type-I seesaw scenario, the triplet seesaw mechanism [7,8], which extends the SM with one Higgs triplet $\xi = (\xi^{++}, \xi^+, \xi^0)^T$ with hypercharge 1, gives another possible solution to the tiny light neutrino masses. Such a Higgs triplet can be introduced in many Grand Unified Theories (GUTs) or $SU(2)_L \times SU(2)_R$ theories. Note that, in general the triplet seesaw model is more predictive than the usual Type-I seesaw model, since there is no unknown right-handed Majorana neutrino mass matrix. The minimal version of the triplet seesaw models contains only one triplet besides the SM particles. That is very different from the Type-I seesaw model, in which at least two heavy right-handed Majorana neutrinos are introduced to give rise to at least two massive left-handed neutrinos [9].

Taking into account that most of the realistic triplet seesaw models are built at some energy scales M_{NP} much higher than the typical electroweak scale $M_Z = 91.2$ GeV [10], it is very meaningful to consider the radiative correc-

tions to the neutrino mixing parameters. In the energy scales below the seesaw scale, the running of the dimension-5 operator has been considered by many authors [11] in the Type-I seesaw framework, and it has been proved that there is no remarkable corrections in the SM or minimal supersymmetric standard model (MSSM) with small $\tan\beta$. However, in the energy scales above the triplet seesaw scale, the renormalization group equations (RGEs) are quite different from those in the low energy effective theory, and the RGE analyses are still lacking in that energy scale. Since the RGE running may bring significant corrections to the physical parameters, it is very important for both model building and phenomenology to study the running effects from the GUT scale ($M_{\text{GUT}} = 10^{16}$ GeV) to the seesaw scale.

In this letter, we derive a full set of one-loop RGEs in the framework of the SM extended with one heavy Higgs triplet. The β -functions of Yukawa couplings are calculated in detail. The RGEs of the couplings between the triplet and doublet Higgs are also obtained. Analytical and numerical analyses based on our formulae are also given for illustration. We show that there may be sizable radiative corrections to the mixing parameters when the triplet Higgs is involved.

The letter is organized as follows: In Sec. II, the basic concepts of the triplet seesaw model is briefly discussed. In Sec. III, we present the β -functions of Yukawa and Higgs couplings in the model. Section IV is dedicated to the numerical analyses of the running effects on mixing parameters. Finally, a summary is given in Sec. V.

II. THE TRIPLET SEESAW MODEL

The full Lagrangian of the triplet seesaw model is given by [8]:

$$\mathcal{L}_{\text{full}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\xi}, \quad (1)$$

where the first part represents the SM Lagrangian, and the second part contains the interactions involving the Higgs triplet. The most general form of \mathcal{L}_{ξ} is

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$$\begin{aligned}
\mathcal{L}_\xi = & (D_\mu \xi)^\dagger (D_\mu \xi) - M_\xi^2 (\xi^\dagger \xi) - \frac{1}{4} \lambda_\xi (\xi^\dagger \xi)^2 \\
& - \lambda_\phi (\xi^\dagger \xi) (\phi^\dagger \phi) - \lambda_C (\xi^T \hat{C} \xi)^\dagger (\xi^T \hat{C} \xi) \\
& - \lambda_T (\xi^\dagger t_i \xi) \left(\phi^\dagger \frac{\tau_i}{2} \phi \right) - \frac{1}{2} [(Y_\xi)_{fg} \bar{l}_L^C \varepsilon \Delta l_L^g \\
& + \lambda_H M_\xi \tilde{\phi}^T \varepsilon \Delta \tilde{\phi} + \text{H.c.}], \tag{2}
\end{aligned}$$

where ϕ is the SM Higgs doublet with $\tilde{\phi} = i\tau_2 \phi^*$ and Δ is a 2×2 representation of the Higgs triplet field [12]:

$$\Delta = \begin{pmatrix} \xi^+/\sqrt{2} & -\xi^{++} \\ \xi^0 & -\xi^+/\sqrt{2} \end{pmatrix}. \tag{3}$$

In Eq. (2), f and g are generation indices, and summation over repeated indices is implied. t_i are the three dimensional representations of the Pauli matrices

$$\begin{aligned}
t_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad t_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \\
t_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \tag{4}
\end{aligned}$$

and \hat{C} is defined as [13]:

$$\hat{C} = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{5}$$

The covariant derivative D_μ reads

$$D_\mu \xi \equiv \partial_\mu \xi + ig_1 Y B_\mu \xi + ig_2 t \cdot W_\mu \xi. \tag{6}$$

As already mentioned in the first section, in order to generate tiny light neutrino masses, the mass scale of the Higgs triplet M_ξ should be much higher than the typical electroweak scale M_Z . In this letter, we take $M_\xi = 10^{10}$ GeV. In the energy scale $\mu \gg M_\xi$, the full Lagrangian is taken into account. In the low energy limit $\mu \ll M_\xi$, we should use the effective theory by integrating out the heavy triplet field. The effective Lagrangian can be defined by [14]:

$$\begin{aligned}
\exp\left\{i \int d^4x \mathcal{L}_{\text{eff}}(x)\right\} & \equiv \int \prod_i^3 \mathcal{D}\xi_i \mathcal{D}\xi_i^\dagger \exp\left\{i \int d^4x \mathcal{L}_{\text{full}}(x)\right\} \\
& = \exp\left\{i \int d^4x \mathcal{L}_{\text{SM}}(x)\right\} \int \prod_i^3 \mathcal{D}\xi_i \mathcal{D}\xi_i^\dagger \exp\left\{i \int d^4x \mathcal{L}_\xi(x)\right\}, \tag{7}
\end{aligned}$$

where $\mathcal{D}\xi_i$ stands for functional integration over ξ_i . Starting from Eq. (7), we can calculate the one-loop level effective Lagrangian by using the steepest-descent method to integrate out the heavy scalar. Keeping only terms of order $\mathcal{O}(1/M_\xi^2)$ through the whole calculation and neglecting all the operators with higher inverse powers of M_ξ , we may get the effective operators:

$$\begin{aligned}
\mathcal{L}_{4\text{-Higgs}} = & -\frac{1}{4} \lambda_H^\dagger \lambda_H (\phi^\dagger \phi)^2; \quad \mathcal{L}_{\nu\text{-mass}} = -\frac{1}{4} \frac{\lambda_H (Y_\xi)_{fg}}{M_\xi} (\bar{l}_L^C \varepsilon \phi) (l_L^g \varepsilon \phi) + \text{H.c.}; \\
\mathcal{L}_{4\text{-Fermi}} = & -\frac{1}{8} \frac{(Y_\xi^\dagger)_{mn} (Y_\xi)_{fg}}{M_\xi^2} [(\bar{l}_L^m \cdot l_L^g) (\bar{l}_L^C \cdot l_L^C) + (\bar{l}_L^C \cdot \bar{l}_L^T) (l_L^g \cdot l_L^C)], \tag{8}
\end{aligned}$$

where m, n, f and g run over 1, 2, 3. $\mathcal{L}_{\text{Higgs}}$ has the same form as the Higgs self-coupling in the SM. The four fermion interaction $\mathcal{L}_{4\text{-Fermi}}$ may contribute to the lepton-flavor-violating processes. However, such processes should be strongly suppressed due to the heavy mass of the Higgs triplet. Thus we will neglect this four fermion coupling in the following calculations. $\mathcal{L}_{\nu\text{-mass}}$ is in proportion to the Majorana neutrino mass matrix. In analogy to the Type-I seesaw model, we can also define the effective dimension-5 operator

$$\mathcal{L}_{\nu\text{-mass}} = -\frac{1}{4} \kappa_{fg} (\bar{l}_L^C \varepsilon \phi) (l_L^g \varepsilon \phi) + \text{H.c.}, \tag{9}$$

where $\kappa = \lambda_H Y_\xi / M_\xi$. After spontaneous symmetry breaking, neutrinos acquire masses and the neutrino mass matrix is given by $M_\nu = Y_\xi \langle \Delta^0 \rangle$ with $\langle \Delta^0 \rangle = \lambda_H v^2 / M_\xi$. Here $v \simeq 174$ GeV denotes the Higgs vacuum expectation value. Since $M_\xi \gg v$, the mass scale of neutrinos is then suppressed, and this is the so-called triplet seesaw mechanism.

III. CALCULATIONS OF THE BETA-FUNCTIONS

In our calculations, we always use the dimensional regularization. For the one-loop wavefunction renormalization constants Z above the seesaw scale, we find that

$$\begin{aligned}\delta Z_{l_L} &= -\frac{1}{16\pi^2} \left(\frac{3}{2} Y_\xi^\dagger Y_\xi + Y_e^\dagger Y_e + \frac{1}{2} g_1^2 \xi_B + \frac{3}{2} g_2^2 \xi_W \right) \frac{1}{\epsilon}, \\ \delta Z_\phi &= -\frac{1}{16\pi^2} \left[2 \text{Tr}(Y_e^\dagger Y_e + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u) + \frac{1}{2} g_1^2 (\xi_B - 3) + \frac{3}{2} g_2^2 (\xi_W - 3) \right] \frac{1}{\epsilon}, \\ \delta Z_\xi &= -\frac{1}{16\pi^2} \left[\text{Tr}(Y_\xi^\dagger Y_\xi) + 2g_1^2 (\xi_B - 3) + 4g_2^2 (\xi_W - 3) \right] \frac{1}{\epsilon}.\end{aligned}\quad (10)$$

For the vertex renormalization constants and the Higgs masses, we obtain

$$\begin{aligned}\delta Z_{Y_\xi} &= \frac{1}{16\pi^2} \left[\frac{1}{2} g_1^2 (3 - 3\xi_B) + \frac{1}{2} g_2^2 (3 - 7\xi_W) \right] \frac{1}{\epsilon}, \quad \delta \lambda_H = -\frac{1}{16\pi^2} \left[\frac{3}{2} g_1^2 \xi_B + \frac{7}{2} g_2^2 \xi_W \right] \frac{1}{\epsilon}, \\ \delta m_\phi^2 &= \frac{1}{16\pi^2} \left[3\lambda m_\phi^2 + 6\lambda_\phi m_\xi^2 + 3\lambda_H^\dagger \lambda_H m_\xi^2 - \frac{1}{2} (g_1^2 \xi_B + 3g_2^2 \xi_W) m_\phi^2 \right] \frac{1}{\epsilon}, \\ \delta m_\xi^2 &= \frac{1}{16\pi^2} \left[4\lambda_\xi m_\xi^2 + 2\lambda_C m_\xi^2 + 4\lambda_\phi m_\phi^2 + \lambda_H^\dagger \lambda_H m_\xi^2 - 2(g_1^2 \xi_B + 2g_2^2 \xi_W) m_\xi^2 \right] \frac{1}{\epsilon}.\end{aligned}\quad (11)$$

The counterterms of the Higgs couplings λ_ξ , λ_C , λ_ϕ and λ_T have also been calculated,

$$\begin{aligned}\delta \lambda_\xi &= \frac{1}{16\pi^2} \{ 6\lambda_\xi^2 + 2\lambda_T^2 + 8\lambda_\phi^2 + 4\lambda_C^2 + 4\lambda_\xi \lambda_C - 4g_1^2 \lambda_\xi - 8g_2^2 \lambda_\xi + 24g_1^4 + 72g_2^4 + 48g_1^2 g_2^2 + \text{Tr}[(Y_\xi^\dagger Y_\xi)^2] \} \frac{1}{\epsilon}, \\ \delta \lambda_C &= \frac{1}{16\pi^2} \{ 3\lambda_C^2 + 6\lambda_\xi \lambda_C - 2\lambda_T^2 - 4g_1^2 \lambda_C - 8g_2^2 \lambda_C - 48g_1^2 g_2^2 - 2 \text{Tr}[(Y_\xi^\dagger Y_\xi)^2] \} \frac{1}{\epsilon}, \\ \delta \lambda_\phi &= \frac{1}{16\pi^2} \left(4\lambda_\phi^2 + 2\lambda_T^2 + 4\lambda_\xi \lambda_\phi + 2\lambda_C \lambda_\phi + 3\lambda_\phi \lambda - \frac{5}{2} g_1^2 \lambda_\phi - \frac{11}{2} g_2^2 \lambda_\phi + 3g_1^4 + 9g_2^4 \right) \frac{1}{\epsilon}, \\ \delta \lambda_T &= \frac{1}{16\pi^2} \left(8\lambda_\phi \lambda_T + \lambda_\xi \lambda_T - 2\lambda_C \lambda_T + \lambda \lambda_T - \frac{5}{2} g_1^2 \lambda_T - \frac{11}{2} g_2^2 \lambda_T + 12g_1^2 g_2^2 \right) \frac{1}{\epsilon}.\end{aligned}\quad (12)$$

By using the counterterms calculated above and the technique described in [11], we obtain the β -functions ($\beta_X = \mu \frac{d}{d\mu} X$) of Yukawa couplings and λ_H :

$$\begin{aligned}16\pi^2 \beta_{Y_\xi} &= Y_\xi \left[\frac{3}{4} (Y_\xi^\dagger Y_\xi) + \frac{1}{2} (Y_e^\dagger Y_e) \right] + \left[\frac{3}{4} (Y_\xi^\dagger Y_\xi)^T + \frac{1}{2} (Y_e^\dagger Y_e)^T \right] Y_\xi + \frac{1}{2} \left[\text{Tr}(Y_\xi^\dagger Y_\xi) - (3g_1^2 + 9g_2^2) \right] Y_\xi, \\ 16\pi^2 \beta_{Y_e} &= Y_e \left[\frac{3}{4} Y_\xi^\dagger Y_\xi + \frac{3}{2} Y_e^\dagger Y_e + \text{Tr}(Y_e^\dagger Y_e + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d) - \frac{15}{4} g_1^2 - \frac{9}{4} g_2^2 \right], \\ 16\pi^2 \beta_{\lambda_H} &= \lambda_H \text{Tr} \left(\frac{1}{2} Y_\xi^\dagger Y_\xi + 2Y_e^\dagger Y_e + 6Y_u^\dagger Y_u + 6Y_d^\dagger Y_d \right) + \frac{1}{2} \lambda_H (-9g_1^2 - 21g_2^2),\end{aligned}\quad (13)$$

and the anomalous dimensions ($\gamma_m = -\frac{1}{m} \mu \frac{dm}{d\mu}$) of the Higgs masses:

$$\begin{aligned}16\pi^2 \gamma_{m_\phi} &= -\left[\frac{3}{2} \lambda + \text{Tr}(Y_e^\dagger Y_e + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d) \right] + \frac{3}{4} g_1^2 + \frac{9}{4} g_2^2 - 3 \left(\lambda_\phi + \frac{1}{2} \lambda_H^\dagger \lambda_H \right) \frac{m_\xi^2}{m_\phi^2}, \\ 16\pi^2 \gamma_{m_\xi} &= -\left[2\lambda_\xi + \lambda_C + \frac{1}{2} \lambda_H^\dagger \lambda_H + \frac{1}{2} \text{Tr}(Y_\xi^\dagger Y_\xi) - 3g_1^2 - 6g_2^2 \right] - 2\lambda_\phi \frac{m_\phi^2}{m_\xi^2}.\end{aligned}\quad (14)$$

It should be noticed that in the triplet seesaw model the mass of the Higgs doublet suffers from the so-called *hierarchy problem*, which can be prevented in some other supersymmetric models. We also calculate the β -functions of Higgs couplings:

$$\begin{aligned}
16\pi^2\beta_\lambda &= 6\lambda^2 + 12\lambda_\phi^2 + 2\lambda_T^2 - 3\lambda(g_1^2 + 3g_2^2) + 3g_2^4 + \frac{3}{2}(g_1^2 + g_2^2)^2 + 4\lambda \text{Tr}(Y_e^\dagger Y_e + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d) \\
&\quad - 8 \text{Tr}[(Y_e^\dagger Y_e)^2 + 3(Y_u^\dagger Y_u)^2 + 3(Y_d^\dagger Y_d)^2] \\
16\pi^2\beta_{\lambda_\xi} &= 2\lambda_T^2 + 8\lambda_\phi^2 + 4\lambda_\xi\lambda_C + 4\lambda_C^2 + 6\lambda_\xi^2 - 12g_1^2\lambda_\xi - 24g_2^2\lambda_\xi + 24g_1^4 + 72g_2^4 + 48g_1^2g_2^2 + 4 \text{Tr}[(Y_\xi^\dagger Y_\xi)^2] \\
&\quad + 2\lambda_\xi \text{Tr}(Y_\xi^\dagger Y_\xi) \\
16\pi^2\beta_{\lambda_C} &= 3\lambda_C^2 + 6\lambda_\xi\lambda_C - 2\lambda_T^2 - 12g_1^2\lambda_C - 24g_2^2\lambda_C - 48g_1^2g_2^2 + 2\lambda_C \text{Tr}(Y_\xi^\dagger Y_\xi) - 2 \text{Tr}[(Y_\xi^\dagger Y_\xi)^2] \\
16\pi^2\beta_{\lambda_\phi} &= 4\lambda_\phi^2 + 2\lambda_T^2 + 4\lambda_\xi\lambda_\phi + 2\lambda_C\lambda_\phi + 3\lambda\lambda_\phi - \frac{15}{2}g_1^2\lambda_\phi - \frac{33}{2}g_2^2\lambda_\phi + 9g_2^4 + 3g_1^4 \\
&\quad + 2\lambda_\phi \text{Tr}\left(\frac{1}{2}Y_\xi^\dagger Y_\xi + Y_e^\dagger Y_e + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right) \\
16\pi^2\beta_{\lambda_T} &= 8\lambda_\phi\lambda_T + \lambda_\xi\lambda_T - 2\lambda_C\lambda_T + \lambda\lambda_T - \frac{15}{2}g_1^2\lambda_T - \frac{33}{2}g_2^2\lambda_T + 12g_1^2g_2^2 \\
&\quad - 2\lambda_T \text{Tr}\left(\frac{1}{2}Y_\xi^\dagger Y_\xi + Y_e^\dagger Y_e + 3Y_d^\dagger Y_d + 3Y_u^\dagger Y_u\right).
\end{aligned} \tag{15}$$

The RGEs for the gauge couplings are changed in this model, and we list the results below:

$$\begin{aligned}
16\pi^2\beta_{g_1} &= \frac{47}{6}g_1^3, & 16\pi^2\beta_{g_2} &= -\frac{5}{2}g_2^3, \\
16\pi^2\beta_{g_3} &= -7g_3^3.
\end{aligned} \tag{16}$$

Since the Higgs triplet field does not couple with quarks, the RGEs of Yukawa couplings for quarks are the same as those in the SM, and the corresponding results can be found in the literature [15].

By calculating the relevant one-loop diagrams, we obtain the β -function of the effective operator which describes the neutrino masses and mixing at the energy scales below the seesaw scale:

$$\begin{aligned}
16\pi^2\beta_\kappa &= -\frac{3}{2}\kappa(Y_e^\dagger Y_e)^T - \frac{3}{2}(Y_e^\dagger Y_e)\kappa + (\lambda + \lambda_H^\dagger\lambda_H)\kappa \\
&\quad - 3g_2^2\kappa + 2 \text{Tr}(Y_e^\dagger Y_e + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d)\kappa.
\end{aligned} \tag{17}$$

It should be mentioned that Eq. (17) has the same form as that in the Type-I seesaw model, only up to a replacement $\lambda \rightarrow \lambda + \lambda_H^\dagger\lambda_H$.

IV. APPLICATIONS

To see the running behaviors of neutrino mixing parameters in the triplet seesaw model, we carry out some numerical analyses by using the β -functions derived above. The lepton flavor mixing matrix, which comes from the mismatch between the diagonalizations of the neutrino mass matrix and the charged lepton mass matrix, is given by $V = V_e^\dagger V_\xi$, where

$$\begin{aligned}
V_e^\dagger \cdot (Y_e^\dagger Y_e) \cdot V_e &= \text{Diag}(y_e^2, y_\mu^2, y_\tau^2), \\
V_\xi^T \cdot Y_\xi \cdot V_\xi &= \text{Diag}(y_1, y_2, y_3),
\end{aligned} \tag{18}$$

with (y_e, y_μ, y_τ) and (y_1, y_2, y_3) being the eigenvalues of Y_e and Y_ξ respectively. In this letter, we adopt the following parameterization [16]:

$$V = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -c_{12}s_{23}s_{13} - s_{12}c_{23}e^{-i\delta} & -s_{12}s_{23}s_{13} + c_{12}c_{23}e^{-i\delta} & s_{23}c_{13} \\ -c_{12}c_{23}s_{13} + s_{12}s_{23}e^{-i\delta} & -s_{12}c_{23}s_{13} - c_{12}s_{23}e^{-i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} e^{i\rho} & 0 & 0 \\ 0 & e^{i\sigma} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{19}$$

where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$ (for $ij = 12, 23$ and 13).

For the typical choice $\langle\Delta^0\rangle \sim \mathcal{O}(0.1 \text{ eV})$, one can estimate that $Y_\xi \sim \mathcal{O}(1)$, which means that the relation $Y_\xi \gg Y_e$ holds, and the tiny Yukawa coupling Y_e in the RGEs of Y_ξ and Y_e can be safely neglected in the hierarchical mass spectrum case.¹ Then we get the approximate equations of Yukawa couplings:

¹When the neutrino mass spectrum is nearly degenerate $m_1 \simeq m_2 \simeq m_3$, such an approximation may not be reasonable. However, in our numerical calculations, we use the exact RGEs and do not make any approximations.

$$\begin{aligned}\mu \frac{dY_\xi}{d\mu} &\simeq \frac{1}{16\pi^2} Y_\xi \left(\frac{3}{2} Y_\xi^\dagger Y_\xi + \alpha_\xi \right), \\ \mu \frac{dY_e}{d\mu} &\simeq \frac{1}{16\pi^2} Y_e \left(\frac{3}{4} Y_\xi^\dagger Y_\xi + \alpha_e \right),\end{aligned}\quad (20)$$

where

$$\begin{aligned}\alpha_\xi &= \frac{1}{2} [\text{Tr}(Y_\xi^\dagger Y_\xi) - (3g_1^2 + 9g_2^2)], \\ \alpha_e &= \text{Tr}(Y_e^\dagger Y_e + 3Y_u^\dagger Y_u + 3Y_d^\dagger Y_d) - \frac{15}{4}g_1^2 - \frac{9}{4}g_2^2.\end{aligned}\quad (21)$$

Using the results above and taking into account the fact that $m_\tau > m_\mu \gg m_e$, we neglect the tiny terms in proportion to y_e^2 and arrive at the approximate analytical results of three mixing angles:

$$\mu \frac{d\theta_{12}}{d\mu} \simeq -\frac{1}{16\pi^2} \cdot \frac{3}{4} s_{12} c_{12} \Delta_{21}, \quad (22)$$

$$\begin{aligned}\mu \frac{d\theta_{23}}{d\mu} &\simeq \frac{1}{16\pi^2} \cdot \frac{3}{4} \{ [2 \cos \delta s_{12} c_{12} c_{23}^2 s_{13} \\ &\quad - s_{23} c_{23} (s_{12}^2 - c_{12}^2 s_{13}^2)] \Delta_{21} - s_{23} c_{23} c_{13}^2 \Delta_{32} \},\end{aligned}\quad (23)$$

$$\mu \frac{d\theta_{13}}{d\mu} \simeq -\frac{1}{16\pi^2} \cdot \frac{3}{4} s_{13} c_{13} (c_{12}^2 \Delta_{21} + \Delta_{32}), \quad (24)$$

where $\Delta_{ij} = y_i^2 - y_j^2$ with $(i, j = 1, 2, 3)$. Note that, in deriving Eqs. (22)–(24), we have adopted the parametrization in Eq. (19). Such instructive expressions allow us to do useful analyses of the running behaviors of mixing angles. Because of the hierarchical charged lepton masses, there is in general no enhanced factor compared with the Type-I seesaw model [11]. However, nontrivial running effects may also be acquired from the sizable Yukawa coupling Y_ξ . From Eqs. (22)–(24), one can immediately conclude that the corrections to θ_{12} and θ_{13} should be milder than that to θ_{23} since the right-hand sides of Eqs. (22) and (24) are in proportion to either Δ_{21} or θ_{13} . In the limit $\theta_{13} \rightarrow 0$ and $\Delta_{21} \rightarrow 0$, we can see from Eq. (23) that $\theta_{23} \propto -\Delta_{32}$. Thus θ_{23} will get negative correction in the normal hierarchy case. For illustration, we only show the evolution of θ_{23} with different $\lambda_H(M_\xi)$ in Fig. 1. We can see that its running is quite sensitive to λ_H and a decrease of several degrees may be acquired from the RGE evolution.

Considering the smallness of θ_{13} , the evolution of the Dirac CP -violating phase δ is the same as those of two Majorana phases (ρ, σ) at the leading order of s_{13}^{-1} ,

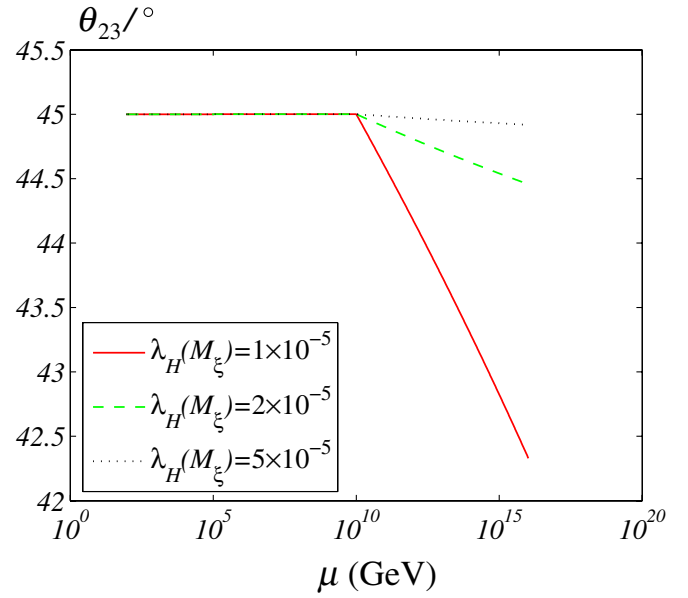


FIG. 1 (color online). The evolution of θ_{23} . We take $\delta = \rho = \sigma = 90^\circ$ and $\theta_{13} = 0.01^\circ$ at the scale $\mu = M_Z$.

$$\begin{aligned}\mu \frac{d\delta}{d\mu} &\simeq \mu \frac{d\rho}{d\mu} \simeq \mu \frac{d\sigma}{d\mu} \\ &\simeq \frac{1}{16\pi^2} \cdot \frac{3y_e^2(y_\tau^2 - y_\mu^2)}{2(y_\tau^2 - y_e^2)(y_\mu^2 - y_e^2)} \frac{\sin \delta s_{12} c_{12} s_{23} c_{23}}{s_{13}} \Delta_{21} \\ &\quad + \mathcal{O}(\theta_{13}).\end{aligned}\quad (25)$$

This is an interesting feature: once three CP -violating

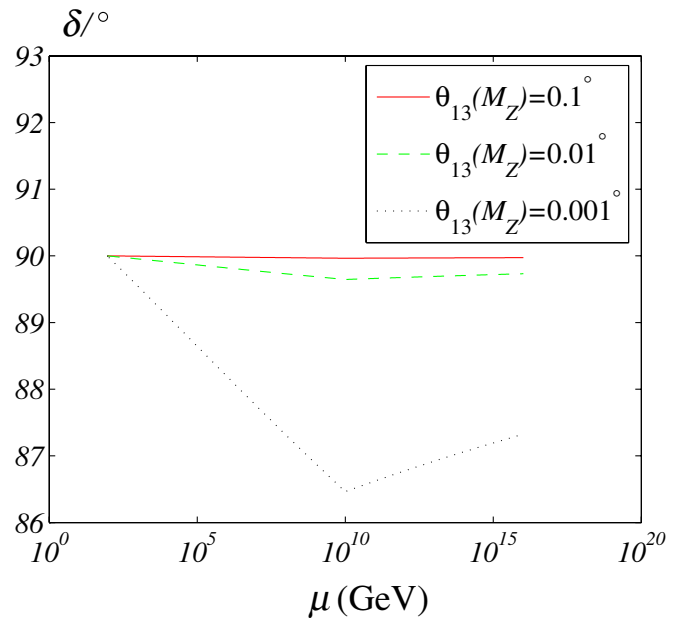


FIG. 2 (color online). Examples of the evolution of the Dirac CP -violating phase δ . We take $\delta = 90^\circ$ and $m_1 = 0.01$ eV at the M_Z scale. We also take $\lambda_H(M_Z) = 5 \times 10^{-5}$. Similar results can be obtained for two Majorana phases ρ and σ .

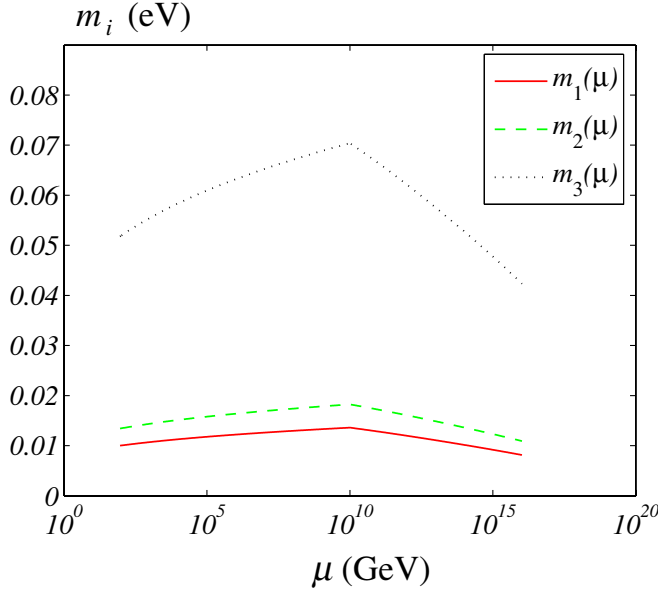


FIG. 3 (color online). The running behaviors of light neutrino masses. Here we choose $m_1(M_Z) = 0.01$ eV and the normal mass hierarchy $m_1 < m_2 < m_3$. We take the same value of $\lambda_H(M_Z)$ as that in Fig. 2 in our calculations.

phases are same at certain energy scale, they will keep this equality against the RGE running. We can also see that the small $\sin\theta_{13}$ in the denominator of Eq. (25) dominates the running of CP phases. That means a fixed point [17] should exist for extremely tiny θ_{13} . As an example, we plot the evolution of δ with different θ_{13} in Fig. 2. Similar results can be obtained for two Majorana phases ρ and σ .

By using Eq. (20), we obtain the RGEs of the eigenvalues of Y_ξ

$$\mu \frac{dy_i}{d\mu} \simeq \frac{1}{16\pi^2} \left(\frac{3}{2} y_i^3 + \alpha_\xi y_i \right), \quad (26)$$

with $i = 1, 2, 3$. Note that, for different signs of α_ξ , the corrections to y_i may be either positive or negative. However, in order to investigate the running of light neutrino masses, one should consider the RGEs of m_ξ and λ_H simultaneously. In Fig. 3, we present the typical evolution of three light neutrino masses with $\lambda_H(M_\xi) = 5 \times 10^{-5}$. We can see that their running effects are appreciable and should not be neglected. A detailed numerical analysis of the triplet seesaw model is worthwhile and the corresponding work will be elaborated elsewhere.

V. SUMMARY

Working in the framework of the SM extended with one heavy Higgs triplet, we have derived a full set of one-loop RGEs for lepton Yukawa and Higgs couplings. Since the triplet seesaw model involves more couplings than the usual Type-I seesaw models, the results are also quite different. Analytical and numerical analyses have been given based on the RGEs we obtained. We find that non-trivial corrections to the mixing parameters can be acquired and they should not be neglected in general. It provides us a possible way to connect the experimental values of lepton flavor mixing parameters with some high energy GUT theories. In conclusion, our formulae are very important for both model building and phenomenological analyses of the triplet seesaw models.

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