Stringent phenomenological investigation into heterotic string optical unification

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For the weakly coupled heterotic string (WCHS) there is a well-known factor of 20 conflict between the minimum string coupling unification scale, $\Lambda_H \sim 5 \times 10^{17}$ GeV, and the projected minimal supersymmetric standard model (MSSM) gauge coupling unification scale, $\Lambda_U \sim 2.5 \times 10^{16}$ GeV, assuming an intermediate scale desert (ISD). From a bottom-up approach, renormalization effects of intermediate scale MSSM-charged exotics (ISME), which are endemic to quasirealistic string models, can resolve this issue by pushing the MSSM scale up to the string scale. However, for a generic string model, this implies that the projected Λ_U unification under the ISD assumption is accidental. If the true unification scale is $\Lambda_H \gtrsim 5.0 \times 10^{17}$ GeV, is it possible that an illusionary unification at $\Lambda_U = 2.5 \times 10^{17}$ GeV in the ISD scenario is not accidental? (This is an issue recently raised again by Binétruy *et al.*.) *Optical unification* suggests that Λ_U might not be accidental. Through its ISME constraints, optical unification offers a mechanism whereby a generic MSSM scale $\Lambda_U < \Lambda_H$ is guaranteed. A WCHS model was recently constructed that could yield optical unification, depending on the availability of anomaly-cancelling D- and F-flat directions that meet optical unification ISME requirements. We report the results of a systematic investigation of the optical unification properties of a subset of flat directions of this model that are *stringently* flat. Stringent flat directions do not require significant fine-tuning and can be easily guaranteed to be F-flat to all finite order (or to at least a given finite order consistent with electroweak scale supersymmetry breaking). They are the likely roots of more complicated (and arguably, more finely tuned) flat directions. To realize optical unification, a flat direction must keep all exotic triplets and doublets massless down to an intermediate mass scale, except for three extra pairs of Higgs which must acquire MSSM (or higher) scale mass. Additionally, six out of seven pairs of exotic hypercharged non-Abelian singlets must acquire MSSM (or higher) scale mass, while the remaining pair remains massless down to the intermediate scale. Our investigation revealed that the best stringent directions could induce MSSM scale or higher mass to at most three of the six pairs of exotic singlets, and to only two out of three pairs of exotic Higgs. Each of these stringent flat directions keeps all of the exotic triplets and remaining exotic doublets massless down to an intermediate scale. Thus, some fine-tuning away from stringent flat directions is necessary, if it is possible for an additional three pairs of exotic hypercharged singlets and one more pair of extra Higgs to become MSSM scale massive. Future research may indicate if such flat directions exist. This paper is a product of the 2003-2004 NSF REU program at Baylor University.

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I. REVIEW OF OPTICAL UNIFICATION

The lower limit to string coupling unification in a weakly coupled heterotic string (WCHS) was shown by Kaplunovsky in 1992 to be around $\Lambda_H \sim 5 \times 10^{17}$ GeV [1]. In contrast, under the scenario of an intermediate scale desert (ISD), the runnings of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ ([321]) couplings in the minimal supersymmetric standard model (MSSM) predict a unification scale $\Lambda_U \sim 2.5 \times 10^{16}$ GeV [2]. The issue of this factor-of-twenty difference was raised again in the third of the *Twenty-Five Questions for String Theorists* by Binétruy *et al.* [3].

One resolution to the factor-of-twenty difference between these two scales is a grand unified theory (GUT) between the MSSM and string scales. However, with the exception of flipped $SU(5)$ [4] (or partial GUTs such as the Pati-Salam $SU(4)_C \times SU(2)_L \times SU(2)_R$ [5,6]), string GUTs cannot be generated by level-one Kač-Moody algebras (since they lack the required adjoint Higgs and/or higher dimensional scalar representations) and models based on higher level Kač-Moody algebras vastly prefer

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even numbers of generations [7–9]. Alternately, strong coupling effects of M theory can lower Λ_H down to Λ_U [10]. On the other hand, intermediate scale MSSM-charged exotics (ISME) at $\Lambda_I < \Lambda_U$ could shift the MSSM unification scale upward to the string scale [11]. The near ubiquitous appearance of ISME in (quasi)realistic heterotic string semi-GUT [6,12], (near)-MSSM [13–18], and GUT [4] models adds weight to the third proposal. However, most intermediate scale MSSM-charged exotic solutions might be viewed as accidental.

The string unification scale is, of course, independent of the masses of the exotics. In contrast, the prediction of an apparent MSSM unification scale when MSSM-charged exotics are ignored should generally be unstable under mass shifts of these exotics. However, a set of ISME satisfying optical unification constraints can provide a robust method for stabilizing an apparent MSSM unification scale under such shifts [19]. In optical unification, ISME affect running couplings like a diverging lens, always producing a "virtual" image of the string unification point between the string scale and the exotic particle mass scale. That is, a shift of the intermediate scale Λ_I simply produces a shift in Λ_U, rather than the disappearance of Λ_U. Thus, a string model with optical unification offers a resolution to question three of [3].

Successful optical unification has three general requirements [19]. First, the effective level of the hypercharge generator must be the standard

$$k_Y = \frac{5}{3}. \tag{1.1}$$

Equation (1.1) is a strong constraint on string-derived [321] models, for the vast majority have nonstandard hypercharge levels. Only select classes of models, such as the free fermionic [20] Nanopoulos, Antoniadis, Ellis, and Hagelin (NAHE)-based [21] class, can yield $k_Y = \frac{5}{3}$.

Second, optical unification imposes the relationship

$$\delta b_2 = \frac{7}{12}\delta b_3 + \frac{1}{4}\delta b_Y \tag{1.2}$$

between the exotic particle contributions δb_3, δb_2, and δb_1 to the [321] beta-function coefficients. Each $SU(3)_C$ exotic triplet or antitriplet contributes $\frac{1}{2}$ to δb_3; each $SU(2)_C$ exotic doublet contributes $\frac{1}{2}$ to δb_2. With the hypercharge of a MSSM quark doublet normalized to $\frac{1}{6}$, the contribution to δb_Y from an individual particle with hypercharge Q_Y is Q_Y^2. $\delta b_3 > \delta b_2$ is required to keep the virtual unification scale below the string scale. In combination with (1.2), this imposes

$$\delta b_3 > \delta b_2 \geq \frac{7}{12}\delta b_3, \tag{1.3}$$

since $\delta b_Y \geq 0$.

To acquire intermediate scale mass, the exotic triplets and antitriplets must be equal in number. Similarly, an even number of exotic doublets is required. Hence, δb_3 and δb_2 must be integer. The simplest solution to (1.2) and (1.3) is a set of three exotic triplet/antitriplet pairs ($\delta b_3 = 3$) and two pairs of doublets ($\delta b_2 = 2$). One pair of doublets can carry $Q_Y = \pm\frac{1}{2}$, while the remaining exotics carry no hypercharge [19]. Alternately, if the doublets carry too little hypercharge, some exotic $SU(3)_C \times SU(2)_L$ singlets could make up the hypercharge deficit. The next simplest solution requires four triplet/antitriplet pairs ($\delta b_3 = 4$) and three pairs of doublets ($\delta b_2 = 3$) that yield $\delta b_Y = 2\frac{2}{3}$, either as a set, or with the assistance of additional non-Abelian singlets. For models containing more than four triplet/antitriplet pairs, (1.2) and (1.3) allow varying numbers of pairs of doublets.

In Sec. II we review the particle content of the optical unification model. In Sec. III we review flatness constraints of the WCHS and discuss the properties that optical unification flat directions must possess. In Sec. IV we present the findings of our investigation of stringent flat directions for optical unification. These results are then summarized in Sec. V.

II. HETEROTIC STRING MODEL WITH OPTICAL UNIFICATION POTENTIAL

A search for free fermionic WCHS models with the potential for optical unification was recently conducted [22]. One such model presented in [22] (see Tables A.I, A.II, A.III, and A.IV for the particle content) was discovered by altering Gliozzi-Scherk-Olive (GSO) projection coefficients of a model in [14]. The new model from altered GSO projections contains a set of 4 $SU(3)_C$ exotic triplet/antitriplet pairs, 3 $SU(2)_L$ exotic doublets, and a pair of non-Abelian singlets (chosen from a set of seven such pairs) that together satisfy optical unification requirements [22,23] (see Table A.II). Three pairs of exotic triplet/antitriplets carry hypercharge $Q_Y = \pm\frac{1}{3}$, while one pair carries $Q_Y = \pm\frac{1}{6}$. All three pairs of exotic doublets carry $Q_Y = 0$ while the pairs of non-Abelian singlets carry $Q_Y = \pm\frac{1}{2}$. The only additional exotic MSSM (besides the above mentioned six extra pairs of singlets) are the three extra pairs of MSSM Higgs doublets endemic to (quasi)realistic heterotic models (see Table A.II).

The optical unification constraints ((1.2) and (1.3)) require that, together, the four triplet/antitriplet pairs, the three exotic doublet pairs, and exactly one of the pairs of hypercharged exotic singlets form the set of ISME at $\Lambda_I < \Lambda_U$. Thus, the remaining six pairs of exotic hypercharged singlets and the three extra Higgs must take on Λ_U scale (or higher) masses, induced by vacuum expectation values of MSSM singlet scalars contributing to D- and F-flat directions.

Like most quasirealistic heterotic string models the possible optical unification model contains an anomalous $U(1)_A$ (i.e., for which $\mathrm{Tr}Q^{(A)} \neq 0$) [24]. For this model

$$\mathrm{Tr}\, Q^{(A)} = +72, \tag{2.1}$$

with a net contribution of +24 from the standard MSSM three generations, of +48 from the hidden-sector non-Abelian matter states, and no net contribution from the exotic non-Abelian singlets.

The set of MSSM-uncharged matter states is composed of 27 non-Abelian singlet fields, henceforth denoted "singlets" (see Table A.III), and 16 hidden-sector non-Abelian fields (see Table A.IV). The singlet fields are $\Phi_{i=1,2,3}$ (the three totally uncharged moduli), Φ_{12}, Φ_{23}, Φ_{31} and complex conjugate fields $\bar{\Phi}_{12}$, $\bar{\Phi}_{23}$, $\bar{\Phi}_{31}$, and $S_{j=1\,\mathrm{to}\,9}$ and complex conjugate fields $\bar{S}_j$. Except for the three uncharged moduli, all singlets form vectorlike pairs, $(\Phi_{ij}, \bar{\Phi}_{ij})$ and $(S, \bar{S})_k$. Of these, only S_7, S_8, and S_9 (and $\bar{S}_7$, $\bar{S}_8$, and $\bar{S}_9$) carry an anomalous charge, which is positive for S_7, S_8, and $\bar{S}_9$ and negative for their vector partners.

The set of hidden-sector non-Abelian states is composed of (i) four $SU(5)_H$ **5** reps, $F_{1,2,3,4}$, and four $\bar{\mathbf{5}}$ reps, $\bar{F}_1$, $\bar{F}'_{2,3,4}$, and (ii) four $SU(3)_H$ **3** reps, $K_{1,2,3,4}$, and four $\bar{\mathbf{3}}$ reps, $\bar{K}'_{1,2,3}$, $\bar{K}_4$. $(F_1, \bar{F}_1)$ and $(K_4, \bar{K}_4)$ form vectorlike pairs of states, while $'$ indicates F_n and $\bar{F}'_n$ and K_n and $\bar{K}'_n$ do not form vectorlike pairs, but, instead, have some matching charges.

III. HETEROTIC STRING FLAT DIRECTIONS

For heterotic strings, the Green-Schwarz-Dine-Seiberg-Wittten mechanism [25] breaks the anomalous $U(1)_A$, and in the process generates a Fayet-Iliopoulos (FI) term,

$$\epsilon \equiv \frac{g_s^2 M_P^2}{192\pi^2}\,\mathrm{Tr}Q^{(A)}, \tag{3.1}$$

in the associated D-term. The FI term breaks supersymmetry near the Planck scale and destabilizes the string vacuum, unless it is cancelled by scalar vacuum expectation values (VEVs),

$$\langle D_A\rangle = \sum_m Q_m^{(A)}|\langle\phi_m\rangle|^2 + \epsilon = 0. \tag{3.2}$$

Thus, an anomalous $U(1)_A$ induces a nonperturbatively chosen flat direction of VEVs. Since the fields taking on the VEVs typically carry additional nonanomalous charges, a nontrivial set of constraints is imposed on the VEVs. The VEVs must maintain D-flatness for each non-anomalous gauge symmetry. For Abelian gauges,

$$\langle D_i\rangle = \sum_m Q_m^{(i)}|\langle\phi_m\rangle|^2 = 0, \tag{3.3}$$

while for non-Abelian gauges,

$$\langle D_a^\alpha\rangle = \sum_m \langle\phi_m^\dagger T_a^\alpha \phi_m\rangle = 0, \tag{3.4}$$

where T_a^α is the αth matrix generator for scalar state ϕ_m in the representation R of the gauge group $\mathcal{G}_a$. Since the states with anomalous charge often carry an additional, nonanomalous charge, their VEVs will in general break some, or all, of the nonanomalous gauge symmetries spontaneously.

If matrix generators are T_a^α for states in the representation R, then the matrix generators are

$$\bar{T}_a^{\,\alpha} = -T_a^{\alpha *} \tag{3.5}$$

for states in the representation $\bar{R}$. Thus, for $SU(n)$ groups, the non-Abelian D-term contributions for vectorlike pairs of non-Abelian states can cancel out.

To insure a supersymmetric vacuum, F-flatness,

$$\langle F_m\rangle \equiv \left\langle\frac{\partial W}{\partial\Phi_m}\right\rangle = 0, \tag{3.6}$$

must also be maintained for each superfield Φ_m (containing a scalar field ϕ_m and chiral spin-$\frac{1}{2}$ superpartner ψ_m) appearing in the superpotential W (for which flatness is also required, i.e. $\langle W\rangle = 0$).[1]

Optical unification places strong constraints on viable flat directions for this model. A good optical unification flat direction must, as discussed,

(i) keep all of the MSSM exotic triplets (the D's) and doublets (the X's) massless above the intermediate scale Λ_I, where the optical-unification-producing diverging lense effect occurs,

(ii) generate Λ_U or greater scale mass for six out of seven pairs of the exotic singlets (the A's), while keeping one pair of exotic singlets massless above the Λ_I scale, and

(iii) generate Λ_U or greater masses for three out of four pairs of the MSSM Higgs.

Possible mass terms in the superpotential are given in Appendix C. Those for the exotic triplet are given to sixth order in Appendix C.1, for exotic doublets to sixth order in Appendix C.2, for exotic singlets to seventh order in Appendix C.3, and for MSSM Higgs to sixth order in Appendix C.4. These tables contain all possible relevant gauge invariant mass terms that additionally satisfy the world sheet symmetry selection rules for free fermionic heterotic models. Concise expression of these selection rules for any superpotential term is given in [18].[2]

In WCHSs, FI-term cancellation generically imposes scalar VEVs, $\langle\phi\rangle$, of order .01 $M_{\mathrm{Pl}} \sim 1.2\times 10^{17}$ Gev, which is approximately $0.3\Lambda_H$. For this model, the average D-flat direction that is also F-flat to at least 6th order has an anomalous charge of around 12. This corresponds to an average FI VEV scale of

$$|\langle\phi\rangle|^2 \sim \epsilon/Q_\phi^{(A)} \equiv \frac{g_s^2 M_P^2}{192\pi^2}(\mathrm{Tr}Q^{(A)}/Q_\phi^{(A)}), \tag{3.7}$$

[1]The first analysis of D- and F-flat directions in the presence of an anomalous $U(1)$ in quasirealistic heterotic models were performed in [26].

[2]World sheet selection rules for orbifolds, into which the free fermionic form can be recast, were summarized in [27].

$$= (0.75 \times 1.2 \times 10^{19}\ \text{Gev})^2/(192\pi^2)(72/12)$$
$$= 5 \times 10^{17} \sim \Lambda_H \tag{3.8}$$

(taking $g_s \sim 0.75$). For free fermionic WCHSs, world sheet charge constraints limit dimensionless couplings, λ_3, in the third order superpotential to discrete values of $1/(\sqrt{2})^i$, for $i \in \{0, 1, 2\}$. Thus, masses from third order terms,

$$\lambda_n\langle\phi\rangle\bar{\Phi}\Phi, \tag{3.9}$$

are on the order of the string scale Λ_H. They are, thus, greater than the MSSM unification scale, Λ_U, by a factor of 20 or more.

On the other hand, nonrenormalizable terms of order $n > 3$,

$$\lambda_n\bar{\Phi}\Phi\langle\phi\rangle\left(\frac{\langle\phi\rangle}{M_{\text{Pl}}}\right)^{n-3}, \tag{3.10}$$

produce mass suppression. Factors of order $(1/100)^{n-3}$ are generically acquired from $(\langle\phi\rangle/M_{\text{Pl}})^{n-3}$. However, these are partially counteracted by the world sheet phase space factor.

In the orbifold approach, the numeric value of the nth-order nonrenormalizable (i.e., $n > 3$) dimensionless coupling constant, λ_n, is determined by the geometry of the compactified space [28]. In free fermionic language, coupling constants λ_n can be expressed in terms of an n-point string amplitude A_n, which is proportional to a world sheet integral I_{n-3} of the correlators of the n vertex operators V_i, $i = 1$ to 3 for the fields in the superpotential term [29],

$$A_n = \frac{g}{\sqrt{2}}(\sqrt{8/\pi})^{n-3}C_{n-3}I_{n-3}/(M_{\text{str}})^{n-3}. \tag{3.11}$$

The integral has the form

$$I_{n-3} = \int d^2z_3 \cdots d^2z_{n-1}\langle V_1^f(\infty)V_2^f(1)V_3^b(z_3)\cdots \times V_{n-1}^b(z_{n-1})V_n^b(0)\rangle \tag{3.12}$$

$$= \int d^2z_3 \cdots d^2z_{n-1}f_{n-3}(z_1 = \infty, z_2 = 1, z_3, \cdots, z_{n-1}, z_n = 0), \tag{3.13}$$

where z_i is the world sheet coordinate of the fermion (boson) vertex operator V_i^f (V_i^b) of the ith string state. C_{n-3} is an $\mathcal{O}(1)$ coefficient that includes renormalization factors in the operator product expansion of the string vertex operators and target space gauge group Clebsch-Gordon coefficients. $SL(2, C)$ invariance is used to fix the location of three of the vertex operators at $z = \infty$, 1, 0. When n_v of the fields $\prod_{i=1}^{l}\mathbf{V}_i$ take on VEVs, $\langle\prod_{i=1}^{l}\mathbf{V}_i\rangle$, then the coupling constant for the effective $n_e = (n - n_v)$th order term becomes $A'_{n_e} \equiv A_n\langle\prod_{i=1}^{l}\mathbf{V}_i\rangle$ [16].

An n-point string function trivially vanishes when the correlator $\langle\prod_i V_i\rangle$ itself vanishes, resulting from nonconservation of at least one or more gauged or global (including "Ising") world sheet charges. When all charges are conserved, one must compute I_{n-3} to determine the numeric value of A_n. It might actually be possible for an n-point function to vanish upon integration of $\langle\prod_i V_i\rangle$, even when $\langle\prod_i V_i\rangle$ is nonzero (i.e., when all gauge, picture-changed global world sheet, and Ising charges are conserved). Typical nonzero values of I_1 and I_2 integral for 4- and 5-point string amplitudes are around 100 and 340 [30,31] for free fermionic models [16].

The net mass suppression factor is typically around (1/10), per increase in superpotential order. As pointed out in [32], mass suppression actually begins at fifth, rather than fourth, order. At fourth order, the dimensionless coupling λ_4 can take on values as large as 10 to 100, due to integration over world sheet phase space. Thus, mass terms from fourth order superpotential terms need not yield suppression, but can be on par with (or larger than) masses from third order superpotential terms. Hence, factor of (1/10) suppression per order begins at fifth order.

Therefore, masses from third through fifth or sixth order superpotential terms are above (or on par with) the MSSM unification scale, Λ_U, for $\Lambda_H \sim 5 \times 10^{17}$ GeV. For a somewhat higher WCHS scale, seventh order mass terms might also be viable for the six pairs of unwanted hypercharged MSSM exotic singlets and the three extra pairs of MSSM Higgs.

IV. OPTICAL UNIFICATION INVESTIGATION

A typical WCHS model contains a moduli space of perturbative solutions to the D- and F-flatness constraints, which are supersymmetric and degenerate in energy [33]. Study of the phenomenology of superstring models often involves the analysis and classification of these flat directions. Thus, methods for flat-direction analysis have been systematized in recent years [34–36]. Since our optical unification model contains an anomalous $U(1)_A$, some of the scalar fields will necessarily receive FI-scale VEVs to cancel the FI term. (We assume for obvious reasons that the MSSM-charged scalars do not receive a VEV.)

In this section we report on our investigation of D- and F-flat directions for our optical unification model. In general, the systematic analysis of simultaneously D- and F-flat directions is a complicated, very nonlinear process. In WCHS model-building F-flatness of a specific VEV direction in the low energy effective field theory may be proven to a given order (by cancellation of F-term components), only to be lost a mere one order higher.

To systematize the analysis of F-flat direction, the all-order stringent approach was developed [6,35,36]. Rather than allowing cancellation between two or more components in an F-term, stringent F-flatness requires that each possible component in an F-term have zero vacuum expectation value. When only non-Abelian singlet fields acquire VEVs, this implies that two or more singlet fields in a given F-term cannot take on VEVs. This condition can be relaxed when non-Abelian fields acquire VEVs. Self-cancellation of a single component in a given F-term is possible between various VEVs within non-Abelian reps. Self-cancellation was discussed in [17,37] for $SU(2)$ and $SO(2n)$ states. In the optical unification model, self-cancellation is possible through VEVs of $SU(3)$ $\mathbf{3}$ and $\bar{\mathbf{3}}$ reps and also through VEVs of $SU(5)$ $\mathbf{5}$ and $\bar{\mathbf{5}}$ reps. The $SU(n)$ self-cancelling VEV combinations are extensions of the $SU(2)$ examples presented in [17,37].

At least three $\mathbf{3}$ and $\bar{\mathbf{3}}$ fields must receive VEVs for $SU(3)$ self-cancellation. In the minimal case, two VEV fields must be $\mathbf{3}$'s and one VEV field must be a $\bar{\mathbf{3}}$ (or vice versa). In this case, if triplets $\mathbf{3}_i$, $i = 1, 2$ receive respective VEVs

$$\exp\{i\pi\theta_i\}\langle R_i, G_i, 0\rangle, \tag{4.1}$$

where R_i and G_i are respective magnitudes (up to a sign) of hidden red and green charges, and $\exp\{i\pi\theta_i\}$ are respective overall phases, then $SU(3)$ D-flatness is maintained by a $\bar{\mathbf{3}}$ with VEV

$$\exp\{i\pi\bar{\theta}\}\langle \bar{R} = \sqrt{{R_1}^2 + {R_2}^2}, \qquad \bar{G} = \sqrt{{G_1}^2 + {G_2}^2}, 0\rangle. \tag{4.2}$$

Then, clearly in an F-term containing

$$\epsilon_{\alpha\beta\gamma}\mathbf{3}_1^\alpha \cdot \mathbf{3}_2^\beta \tag{4.3}$$

(with α, β, and γ color indices), self-cancellation occurs if

$$R_1 G_2 = G_1 R_2. \tag{4.4}$$

Note that (4.4) implies

$$R_1/G_1 = R_2/G_2 = \bar{R}/\bar{G}. \tag{4.5}$$

The next simplest $SU(3)$ self-cancellation can occur between two triplet VEVs and two antitriplet VEVs.[3] D-flatness is maintained by

$$\langle \mathbf{3}_1 \rangle = \exp\{i\pi\theta_1\}\langle R, 0, 0\rangle, \quad \langle \bar{\mathbf{3}}_1 \rangle = \exp\{i\pi\bar{\theta}_1\}\langle \bar{R} = R, 0, 0\rangle, \tag{4.6}$$

$$\langle \mathbf{3}_2 \rangle = \exp\{i\pi\theta_2\}\langle 0, G, 0\rangle, \quad \langle \bar{\mathbf{3}}_2 \rangle = \exp\{i\pi\bar{\theta}_2\}\langle 0, \bar{G} = G, 0\rangle. \tag{4.7}$$

Self-cancellation then occurs in F-terms containing either

$$\mathbf{3}_1 \cdot \bar{\mathbf{3}}_2 \quad \text{or} \quad \mathbf{3}_2 \cdot \bar{\mathbf{3}}_1. \tag{4.8}$$

These two self-cancellation classes can be generalized for more field VEVs.

A. *D*-flat basis directions

Our first step in a systematic search for optical unification producing D- and F-flat directions was to construct a basis of D-flat directions for the set of singlet fields with null hypercharge, and for the set of hidden sector non-Abelian fields. We generated a set of 24 D-flat directions $\{\mathcal{D}_i, \text{ for } i = 1, \text{ to } 24\}$ (see Tables I and II) en mass via the singular value decomposition approach (described in [38] and applied in [35]). Note that by *basis* of D-flat directions, we mean a basis of directions specifically D flat with regard to all *nonanomalous* $U(1)$ gauge symmetries. The D-flat basis elements may carry positive, negative, or zero anomalous charge. For a linear combination of basis directions to be *physical*, its net anomalous charge must be of opposite sign to the FI term. Thus, in this model its net anomalous charge must be negative.

In Tables I and II, the first entry in a given row denotes the D-flat basis element label, the second entry denotes its anomalous charge (normalized to $Q^{(A)\prime} = Q^{(A)}/16$), and the remaining entries denote the ratios of the squares of the norms of its field VEVs. The field corresponding to the first norm-square (that given in the third column) is unique to the given flat direction and can be used to denote it. The VEVs for the first nine basis directions ($\mathcal{D}_{10}$ through $\mathcal{D}_{24}$) are formed solely from non-Abelian singlet fields (henceforth referred to simply as singlets), while the VEVs in the remaining 15 basis directions ($\mathcal{D}_{10}$ through $\mathcal{D}_{24}$) contain several non-Abelian VEVs. Each of the 24 D-flat basis directions contains a unique field VEV not present in any of the other basis directions. Thus each flat direction can be identified by its associated VEV.

The VEVs forming $\mathcal{D}_1$ through $\mathcal{D}_6$ are of singlets from vectorlike pairs. Thus there are also corresponding basis vectors, denoted as $\bar{\mathcal{D}}_i$ for $i = 1, \ldots, 6$, formed from vector-partner VEVs. Since the corresponding charges in $\mathcal{D}_i$ and $\bar{\mathcal{D}}_i$ are of opposite sign, we can express the $\bar{\mathcal{D}}_i$ as $-\mathcal{D}_i$ (effectively allowing the $\mathcal{D}_i$ norm-squared components to be negative). The combination of $\mathcal{D}_i$, $\bar{\mathcal{D}}_1$, $i = 1, \ldots, 6$, and the trivial uncharged moduli field directions $\mathcal{D}_{6+l} = \langle \Phi_l \rangle$, for $l \in \{1, 2, 3\}$, form a complete set of singlet D-flat VEVs.

By definition, physical D-flat directions are not allowed negative norm-squares of VEVs for nonvectorlike fields, while they are allowed to have negative norm VEVs for

TABLE I. *D*-flat singlet VEV basis elements.

Dir.	$Q^{(A)\prime}$	Identifying field	S_8	$\bar{\Phi}_{31}$	S_2	S_4	S_5	S_6
$\mathcal{D}_1$	-1	$S_9 = 3$	-3	2	0	0	-1	-1
$\mathcal{D}_2$	0	$S_7 = 1$	-1	1	0	0	-1	-1
$\mathcal{D}_3$	0	$\bar{\Phi}_{23} = 1$	0	1	0	0	-1	-1
$\mathcal{D}_4$	0	$S_3 = 1$	0	0	0	1	-1	-1
$\mathcal{D}_5$	0	$S_1 = 1$	0	0	1	0	-1	-1
$\mathcal{D}_6$	0	$\bar{\Phi}_{12} = 1$	0	0	0	0	-1	-1
$\mathcal{D}_7$	0	$\Phi_1 = 1$	0	0	0	0	0	0
$\mathcal{D}_8$	0	$\Phi_2 = 1$	0	0	0	0	0	0
$\mathcal{D}_9$	0	$\Phi_3 = 1$	0	0	0	0	0	0

[3] From gauge invariance arguments, it can be shown that triplets from a D-flat antisymmetrized $\epsilon_{\alpha\beta\gamma}{\mathbf{3}_1}^\alpha{\mathbf{3}_2}^\beta{\mathbf{3}_3}^\gamma$ combination cannot produce self-cancellation.

TABLE II. D-flat non-Abelian VEV basis elements.

Dir.	$Q^{(A)\prime}$	Identifying field	K_4 $\bar{K}_4$	$\bar{K}'_3$	S_8	$\bar{\Phi}_{31}$	S_2	S_4	S_5	S_6
$\mathcal{D}_{10}$	2	$F_1/\bar{F}_1 = 15$	9	18	−6	−4	0	0	−10	−1
$\mathcal{D}_{11}$	2	$F_2 = 15$	−6	3	−6	−4	0	0	5	−1
$\mathcal{D}_{12}$	4	$F_3 = 30$	−12	6	−12	7	−15	0	−5	−2
$\mathcal{D}_{13}$	4	$F_4 = 30$	−12	6	−12	7	0	15	−5	−2
$\mathcal{D}_{14}$	1	$\bar{F}'_2 = 5$	2	−1	2	−2	0	0	0	2
$\mathcal{D}_{15}$	2	$\bar{F}'_3 = 10$	4	−2	4	1	5	0	−5	−6
$\mathcal{D}_{16}$	2	$\bar{F}'_4 = 10$	4	−2	4	1	0	−5	5	4
$\mathcal{D}_{17}$	2	$K_1 = 6$	0	6	0	−1	0	3	−1	2
$\mathcal{D}_{18}$	2	$K_2 = 6$	0	6	0	−1	−3	0	−1	2
$\mathcal{D}_{19}$	1	$K_3 = 3$	0	3	0	−2	0	0	1	1
$\mathcal{D}_{20}$	1	$\bar{K}'_1 = 2$	0	−2	0	1	0	−1	1	0
$\mathcal{D}_{21}$	1	$\bar{K}'_2 = 2$	0	−2	0	1	1	0	−1	−2
$\mathcal{D}_{22}$	0	$N_1 = 2$	−2	−2	2	−1	0	1	1	0
$\mathcal{D}_{22}$	0	$N_2 = 2$	−2	−2	2	−1	−1	0	3	2
$\mathcal{D}_{24}$	0	$N_3 = 1$	−1	−1	1	0	0	0	1	0

vectorlike components. Since vector pairs have opposite signed charges, a negative norm-squared implies that the vector-partner field acquires the VEV, rather than the field.

Tables I and II reveal one property that all physical D-flat directions possess: $\mathcal{D}_1$ always appears (with positive coefficient), since only $\mathcal{D}_1$ carries a negative anomalous charge, necessary to cancel the positive FI term. This is obvious when only singlet flat directions are allowed since $\mathcal{D}_2$ to $\mathcal{D}_9$ carry no anomalous charge. Thus the field S_9 acquires a VEV in all D-flat directions.

Inclusion of non-Abelian D-flat directions does not change this conclusion. For proof of this, first note that all 15 non-Abelian flat directions, i.e., $\mathcal{D}_{10}$ to $\mathcal{D}_{24}$ carry positive anomalous charge. Further, the fields unique to $\mathcal{D}_{11}$ to $\mathcal{D}_{24}$ are nonvectorlike. Thus, $\bar{\mathcal{D}}_{11}$ to $\bar{\mathcal{D}}_{24}$ cannot appear in physical D-flat directions with negative coefficients. Hence their D-term contributions are of the same sign as the FI term. Next, note that while $\mathcal{D}_{10}$'s unique field VEV, F_1, does have a vector partner, $\bar{F}_1$, $\mathcal{D}_{10}$ also contains the VEV of the nonvectorlike field $\bar{K}'_3$. In a physical direction the norm-square of the $\bar{K}'_3$ VEV must be positive. While the net norm-square $\bar{K}'_3$ VEV can be made positive by adding to $\mathcal{D}_{10}$ a linear combination of basis directions $\mathcal{D}_{11}$, $\mathcal{D}_{12}$, $\mathcal{D}_{13}$, $\mathcal{D}_{17}$, $\mathcal{D}_{18}$, or $\mathcal{D}_{19}$, Table II shows that the net anomalous charge from such a combination turns positive again. Hence a sufficiently large negative anomalous charge contribution from $\mathcal{D}_1$ is again required. Thus, $\mathcal{D}_1$ must be present in all valid D-flat directions, independent of non-Abelian field VEVs. In the following pages, the phenomenological effect of a nonzero $\langle S_9 \rangle$ will often be discussed.

B. Stringent *F*-flat directions

Linear combinations of the D-flat basis generators were systematically examined for stringent F-flatness in two steps: First the D-flat linear combinations were tested for stringent F-flatness through sixth order.[4] (The optical unification model's superpotential is given up to sixth order in Appendix B.) Then, directions passing the first test were examined for either all-order (or at least 17th) order stringent F-flatness. For those not F-flat to all (17th) order, the exact order at which F-flatness breaking occurs was determined. Singlet flat directions were analyzed first, then flat directions containing non-Abelian VEVs.

We searched among our stringent F-flat directions for those that induce FI-scale masses (see Appenedix C) to the MSSM-charged exotics that do not participate in the optical unification "lensing" effect. These unwanted MSSM exotics are (i) the six extra pairs of exotic $Q_Y = \pm\frac{1}{2}$ singlets and (ii) the three extra pairs of Higgs. For optical unification these states must acquire λ_U or higher masses, while, simultaneously, four exotic triplet/antitriplet pairs, three exotic doublet pairs, and one singlet pair remain massless until an intermediate scale λ_I.

1. *Singlet flat directions*

Initially we allowed only non-Abelian singlet fields to take on VEVs. That is, we used only $\mathcal{D}_i$, $i = 1, \ldots, 9$, and $\bar{\mathcal{D}}_j = -\mathcal{D}_j$, $j = 2, \ldots, 6$, as our initial basis set. The range of coefficients was from 0 to n for $\mathcal{D}_1$ and from $-n$ to n for $\mathcal{D}_{2\le i\le 6}$, where $n = 99$ for directions containing up to four basis directions, $n = 31$ for directions containing five or six basis vectors, and $n = 21$ for directions containing seven or more basis directions. The coefficients for $\mathcal{D}_{7,8,9}$ were either 0 or 1.

Note that $\mathcal{D}_4$ has two unique field VEVs, $\langle S_1 \rangle$ and $\langle S_2 \rangle$. Similarly, only $\mathcal{D}_5$ contains $\langle S_3 \rangle$ and $\langle S_4 \rangle$. The superpotential contains terms $S_1 S_2 \Phi_{12}$ and $S_3 S_4 \Phi_{12}$, which thus prohibit $\mathcal{D}_4$ or $\mathcal{D}_5$ from contributing to any stringently F-flat direction. $\bar{\mathcal{D}}_4$ and $\bar{\mathcal{D}}_5$ are similarly prohibited due to superpotential terms $\bar{S}_1 \bar{S}_2 \bar{\Phi}_{12}$ and $\bar{S}_3 \bar{S}_4 \bar{\Phi}_{12}$. Linearly combinations of

$$n_1 \mathcal{D}_1 + n_2 \mathcal{D}_2 + n_3 \mathcal{D}_3 + n_6 \mathcal{D}_6 \tag{4.9}$$

are also constrained by the requirement that $\langle S_5 S_6 \rangle = \langle \bar{S}_5 \bar{S}_6 \rangle = 0$. This requires that

$$n_1 + n_2 + n_3 + n_6 = 0. \tag{4.10}$$

Ultimately, we found that the demand of stringent flatness through sixth order allowed only one class of singlet D-flat directions that is stringently F-flat to all order. All other D-flat directions generated were found to break F-flatness below seventh order in the superpotential.

[4]The number of superpotential terms increases drastically per order after sixth order in the superpotential, so this first test was used to limit the number of directions tested beyond sixth order.

The nonzero field VEVs for this all-order flat class are:

$$\langle S_9\bar{S}_7\bar{S}_8\rangle + \text{ one or more of } \langle\bar{\Phi}_{12}\rangle, \langle\bar{\Phi}_{31}\rangle, \langle\Phi_2\rangle, \langle\Phi_3\rangle. \tag{4.11}$$

The specific set of all-order flat directions is given in Table III: v_2 and v_3 are real, positive, and of FI scale, but otherwise unconstrained.

2. Singlet flat-direction class phenomenology

For the singlet class of flat directions defined by (4.11), no (rather than a hoped for six) linear combinations of the seven pairs of singlets with $Q_Y = \pm\frac{1}{2}$ receive mass (see Appendix C). From (C1) we also find that the VEVs of Φ_2 and Φ_3 should be kept at zero to prevent third and fifth order mass terms

$$\begin{array}{ll} \frac{1}{2}\langle\Phi_3\rangle D_1\bar{D}_1, & \frac{1}{2}\langle\Phi_3\rangle D_2\bar{D}_2, \\ \frac{1}{2}\langle\Phi_2\rangle D_4\bar{D}_4, & \langle S_9\bar{S}_8\Phi_2\rangle D_1\bar{D}_2 \end{array} \tag{4.12}$$

from appearing for the exotic D triplets. The mass terms in (4.12) should be zero, for it is extremely unlikely that the third order terms could be cancelled by the possible sixth order terms in respective m'_{11}, m'_{22}, and m'_{44}, since sixth order terms have order (1/100) suppression. Further, there is a similar suppression ratio between the fifth order term in (4.12) and possible seventh order contributions.

Third order stringent F-flatness constraints also forbid both fields of a given vector pair from simultaneously acquiring VEVs. That is

$$\langle F_1\bar{F}_1\rangle = \langle K_4\bar{K}_4\rangle = \langle S_i\bar{S}_i\rangle = 0. \tag{4.13}$$

Thus, (4.11) implies that

$$\langle S_7\rangle = \langle S_8\rangle = \langle\bar{S}_9\rangle = 0. \tag{4.14}$$

(Third order constraints also require that

$$\langle S_7\bar{S}_8\rangle = \langle S_8\bar{S}_7\rangle = 0, \tag{4.15}$$

but (4.15) is automatically satisfied when (4.13) is combined with (4.11).) Note also that (4.14) removes the third and higher order $D\bar{D}$ mass terms containing S_8 or $\bar{S}_9$. Hence, for stringent flat directions, all $D_2\bar{D}_1$ terms vanish (when combined with $\langle\Phi_1\rangle = \langle\Phi_2\rangle = 0$), as do third order and several fifth order $D_2\bar{D}_4$ terms, and the sixth order $D_3\bar{D}_3$ terms. Therefore, the only $D\bar{D}$ that need to be investigated for non-Abelian stringent directions are those in m'_{11}, m'_{22}, the last half of m'_{24} and m'_{41} terms, and m'_{44}.

TABLE III. Norm-squared components of D-flat singlet VEVs with all-order stringent F-flatness.

$Q^{(A)\prime}$	S_9	$\bar{S}_7$	$\bar{S}_8$	$\bar{\Phi}_{12}$	$\bar{\Phi}_{31}$	Φ_2	Φ_3
−1	3	2	1	1	0	v_2	v_3
−1	3	1	2	0	1	v_2	v_3
−2	6	3	3	1	1	v_2	v_3

The singlet class flat directions do not give unwanted mass to the exotic X_i and $\bar{X}_i$ doublets through at least sixth order. Further, by demanding

$$\langle\Phi_2\rangle = 0 \tag{4.16}$$

because of (4.12), the unwanted possible fifth order $X_1\bar{X}_1$ mass term from Appendix C.2 is also eliminated, independent of $\langle S_1\bar{S}_6 + S_1\bar{S}_6\rangle$, which may not be zero for generic non-Abelian directions. Unfortunately, Appendix C.3 also reveals that (4.16) eliminates the desirable mass terms (of fifth order) for $A_4\bar{A}_4$ and $A_7\bar{A}_7$ and for $A_7\bar{A}_1$ (of sixth order).

Note that Φ_1 was not allowed a VEV in any singlet flat direction because of the third order term, $S_9\bar{S}_9\Phi_1$ (and also because of $S_7\bar{S}_7\Phi_1$ and $S_8\bar{S}_8\Phi_1$). Since $\langle S_9\rangle \neq 0$ also applies to all non-Abelian flat directions, $\langle\Phi_1\rangle = 0$ is also true for all flat directions. $\langle\Phi_1\rangle = 0$ (favorably) prevents a possible $X_1\bar{X}_1$ mass term from appearing for the latter directions. However, as Appendix C.3 indicates, this also prevents the desirable fifth order mass terms for exotic singlets $A_2\bar{A}_2$ and $A_5\bar{A}_5$, which means that, at most, one independent $A\bar{A}$ mass term can be expected from sixth order or lower non-Abelian flat directions,

$$(\langle F_1\bar{F}'_2N_3S_9\rangle A_1 + \langle N_2\rangle A_5)\bar{A}_1. \tag{4.17}$$

One possible difficulty with this is both mass components require left-handed antineutrino singlet VEVs, which might result in unacceptable lepton number violation.

This analysis implies that successful optical unification clearly requires seventh order mass terms for $A\bar{A}$ and possibly for some AA (see Appendix C.4). Since these masses must be at or above the λ_U scale, the WCHS unification scale λ_H for this model must be above the lower bound of 5×10^{17} GeV and on the order of 10^{18} GeV $\times$ order(1). In addition, several seventh order mass terms may also be required for a given $A_i\bar{A}_j$ or A_iA_j, in order to sufficiently counter seventh order mass suppression. (Whether this occurs or not will be studied in the next section.)

We complete our discussion of singlet direction phenomenology with an analysis of MSSM mass matrices: From the Higgs mass matrix given in Appendix C.5, we find that $\langle S_9\rangle$ produces the term

$$\langle S_9\rangle h_1\bar{h}_4. \tag{4.18}$$

Additionally, the nonzero optional singlet VEVs would yield a second (linearly independent) combination

$$h_1(\langle\bar{\Phi}_{12}\rangle\bar{h}_2 + \langle\bar{\Phi}_{31}\rangle\bar{h}_3), \tag{4.19}$$

leaving but one more desirable Higgs mass term to be generated.

Third order diagonal up-quark mass terms of the form

$$Q_iu_i^c\bar{h}_i, \tag{4.20}$$

generically appear in NAHE-based models [14]. These can naturally produce a generational mass hierarchy if each $\bar{h}_i$ appears in the physical massless Higgs combination with coefficients differing by orders of magnitude. Mass hierarchy between two generations is often obtained this way in NAHE-based models, but generally not for all three generations. Rather, additional suppressed quark mass terms are needed. Depending on the optional VEVs acquired, the singlet VEV class can provide suppressed mass terms

$$Q_2 u_2^c \bar{h}_4 \langle S_9 \bar{\Phi}_{12} \rangle \quad \text{and} \quad Q_3 u_3^c \bar{h}_4 \langle S_9 \bar{\Phi}_{31} \rangle \tag{4.21}$$

(see Appendix C.6). Note that the three suppressed sixth order $Q_1 d_1^c h_4$ mass terms and a similar set for $Q_2 d_2^c h_4$ are prohibited by stringent third order F-flatness.

Natural mass suppression between up and down quarks becomes evident with Appendix C.7, which does not contain comparable $Q_i d_i^c h_i$ terms, which follows the pattern discussed in [13,14]. Based on GSO projection choices, third order mass terms for either up quarks or down quarks (but not both) can appear. The singlet flat-direction class does provide one fourth order down-quark mass term

$$Q_2 d_2^c h_4 \langle \bar{S}_8 \rangle, \tag{4.22}$$

and possibly one suppressed sixth order term

$$Q_2 d_2^c h_3 \langle S_9 \bar{S}_7 \bar{\Phi}_{12} \rangle. \tag{4.23}$$

As with the up quarks, the three suppressed sixth order $Q_1 d_1^c h_4$ mass terms are prohibited by stringent third order F-flatness. Further, for upcoming non-Abelian directions, the fifth order $Q_2 d_2^c h_2$ terms are similarly eliminated. Finally, note that (C42) is also the electron mass matrix, providing $m_{e_i} = m_{d_i}$ at the string scale.

As already discussed, other than the singlet D-flat direction class (4.11), all singlet D-flat directions lose F-flatness below sixth order. Thus, the insufficiency of the (4.11) singlet flat-direction class, i.e., its lack of mass terms for the six extra pairs of singlets, led us to investigate the phenomenologies of non-Abelian stringent flat directions, containing VEVs of hidden-sector 5 and $\bar{5}$ fields of $SU(5)_H$ and/or the 3 and $\bar{3}$ fields of $SU(3)_H$.

3. Non-Abelian flat directions

We systematically generated non-Abelian flat directions using the basis directions from both Tables I and II, with at least one direction always from Table II. Under reasonable constraints for the range of basis vector coefficients with regard to program running time, the complete collection of non-Abelian stringent flat directions for our model was generated and analyzed. Linear combinations of up to seven of the basis directions were examined. For linear combinations of three or four basis directions, an integer range of coefficients from −99 to 99 was chosen, whereas for five or six basis directions, a reduced coefficient range from −31 to 31 was used, and for seven basis directions, a further reduced coefficient range from −21 to 21 was applied. The maximum number of basis vectors considered so far was seven because of two factors: the projected running time for eight basis vectors is several weeks and no new stringently flat-direction classes (or self-cancellation possibilities) were found for seven basis vectors.

Our investigation revealed five classes of all-order (or at least 17th order) stringently flat directions and one class that can be made stringently flat by self-cancellation of non-Abelian field VEVs (see Table D.II). Classes are denoted by a $n-d$ label, where n is the number of independent pairs of $A_i\bar{A}_j$ exotic doublets that gain mass and d designates different combinations of the n pairs. We found three classes that provide only one independent MSSM exotic singlet mass term, one class that provides two independent mass terms, and two classes that provide three independent mass terms.

The 30 all-order stringent flat directions in class 1-1 generate third order mass for $A_5\bar{A}_1$ and contain anywhere from 8 to 11 field VEVs. (For brevity, only four example flat directions from the complete set of 30 are listed in Table D.II.) The four all-order stringent flat directions in class 1-2 generate seventh order mass for $A_7\bar{A}_6$ and contain either ten or 11 VEVs. The single all-order stringent flat direction forming class 1-3 yields seventh order mass for $A_3\bar{A}_4$. The four stringent flat directions in class 2-1 produce a third order mass term for $A_5\bar{A}_1$ and seventh order mass terms for $A_5\bar{A}_7$ and $A_2\bar{A}_4$, which generate mass for A_5 and a linear combination of $\lambda_3\bar{A}_1 + \lambda_7\bar{A}_7$. The single all-order stringent flat direction in class 3-1 contributes seventh order masses to $A_4\bar{A}_2$, $A_7\bar{A}_5$, and $A_6\bar{A}_1$.

Half of the class 2-1 directions have phenomenological difficulties: flat direction 2-1.2 generates an unwanted seventh order mass for the exotic doublets $\bar{X}_1\bar{X}_1$, while 2-1.4 generates an unwanted sixth order mass term for the exotic triplets $D_4\bar{D}_1$. Additionally, 3-1.1 produces unwanted 7th order mass for $\bar{X}_2\bar{X}_2$. Thus, although stringent flat direction 3-1.1 can produce MSSM scale or higher mass for three pairs of $A/\bar{A}$ fields, the better phenomenological starting point for a flat direction is either flat direction 2-1.1 or 2-1.3, both of which keep all of the exotic $D/\bar{D}$ triplets and the exotic $X/\bar{X}$ doublets massless at the MSSM scale.

Self-cancellation of the class 3-2 directions in Table D.III, providing stringent flatness to at least 17th order, results in a further improved starting point for optical unification. The four directions in class 3-2 provide mass for three exotic doublet pairs, $(a_3A_5 + a_7A_6)\bar{A}_1$, $A_2\bar{A}_4$, and $A_5\bar{A}_7$, while simultaneously keeping all $D/\bar{D}$ and $X/\bar{X}$ fields massless. Note that the mass terms in classes 1-1 and 2-1 are subsets of the class 3-2 set, while the addition of class 1-3 to 3-2 simply rotates an A mass eigenstate. Linear combinations of 1-2 and 3-2, requiring at least 8 basis vectors, generate four independent exotic singlet mass terms (and are discussed further below).

All four flat directions in class 3-2 are threatened by varying numbers of 16th order superpotential terms and several derived F-terms.[5] Nevertheless, the dangerous $\langle W\rangle$ and $\langle F\rangle$ terms can be eliminated for all four directions by non-Abelian self-cancellation. For example, flat direction 3-2.1 (representative of this class) is endangered by W-term

$$S_9{}^3\bar{S}_8^2\bar{\Phi}_{31}\bar{S}_2^2S_6N_2(K_2\cdot\bar{K}_3')^2(K_2\cdot\bar{K}_4) \tag{4.24}$$

and related F-terms. In flat direction 3-2.1, the squares of the norms of the non-Abelian VEVs are

$$\langle K_1\rangle^2=\langle K_2\rangle^2=\langle\bar{K}_1'\rangle^2=3\langle\bar{K}_3'\rangle^2=3/2\langle\bar{K}_4\rangle^2=6. \tag{4.25}$$

One VEV choice for maintaining hidden $SU(3)$ D-term flatness is

$$\langle K_1\rangle=\langle\sqrt{6},0,0\rangle,\qquad\langle\bar{K}_3'\rangle=\langle\sqrt{2},0,0\rangle,$$
$$\langle\bar{K}_4\rangle=\langle 2,0,0\rangle,\qquad\langle K_2\rangle=\langle\sqrt{6},0,0\rangle,\qquad\langle\bar{K}_1'\rangle\langle\sqrt{6},0,0\rangle. \tag{4.26}$$

This provides for

$$(K_2\cdot\bar{K}_3')=0, \tag{4.27}$$

which eliminates all dangerous W- and F-terms.

As with the singlet flat directions, all of the above six classes of non-Abelian directions also generate: (i) third order mass terms for two of the four Higgs doublets, $\langle S_9\rangle h_1\bar{h}_4$ and $\langle\bar{\Phi}_{31}\rangle h_3\bar{h}_1$, (ii) a suppressed fifth order up-quark mass term, $\langle S_9\bar{\Phi}_{31}\rangle\bar{h}_4u_3\bar{u}_3$, and (iii) matching fourth order down-quark and electron mass terms, $\langle\bar{S}_8\rangle h_4(d_2\bar{d}_2+e_2^-e_2^+)$.

As for the linear combinations of classes 1-2 and 3-2, discussed prior (which generate four independent mass terms): all were found to lose flatness by two 14th order terms,

$$S_9{}^3\bar{S}_8^2\bar{\Phi}_{31}\bar{S}_6N_2[(K_2\cdot\bar{K}_2')(K_3\cdot\bar{K}_2')(K_3\cdot\bar{K}_4)+(K_2\cdot\bar{K}_4')(K_3\cdot\bar{K}_2')^2]. \tag{4.28}$$

Self-cancellation of fifth, sixth, and tenth order terms endangering F-flatness of these linear combinations require

$$\langle K_3\cdot\bar{K}_3'\rangle=\langle K_1\cdot\bar{K}_4\rangle=0, \tag{4.29}$$

which can be shown to be consistent only with self-cancellation of the first term in (4.28).

Several directions that generate mass for not just three, but five independent sets of $A\bar{A}$ were also found (see Table D.IV). Four representatives of class 5-1 and four of class 5-2 are given. Unfortunately, all class 5-1 and 5-2 directions are broken by an 11th order superpotential term

$$S_9{}^2\bar{S}_8^2\bar{\Phi}_{31}\bar{S}_2S_5(K_2\cdot\bar{K}_3')^2. \tag{4.30}$$

None of these directions contain two pairs of K and $\bar{K}$, and thus the self-cancellation condition of (4.27) cannot be imposed.

C. General flat-direction investigation

Several stringent flat directions, were found that generate Λ_U scale or higher mass for up to half of the extra pairs of hypercharge-carrying exotic singlets (the A and $\bar{A}$) that cannot contribute to optical unification. Some of these directions achieve F-flat by self-cancellation of 16th order superpotential terms. Many of these stringent flat directions keep all four pairs of the exotic triplets (the D and $\bar{D}$) and all three pairs of the exotic doublets (the X and $\bar{X}$) massless at or below Λ_U. However, no directions stringently flat to at least 17th order have yet been found that can provide Λ_U scale mass to four, five, or all six of the extra exotic singlets. Stringent directions generating five mass pairs were found, but these lose F-flatness at no higher than 11th order.

That stringent flat directions giving mass to four or more of the exotic singlet pairs have not been found strongly suggests that a nonstringent flat direction is likely required for this model to realize optical unification. However, the F-flatness requirements suggest that such a direction will likely have a stringent flat direction embedded within as a root. Thus, future research will focus on a systematic search for nonstringent F-flat variations derived from stringent directions.

V. SUMMARY

In the context of both the weakly coupled heterotic string and the likelihood of intermediate scale MSSM-charged exotics in realistic models, optical unification proffers an explanation for the perhaps apparently accidental unification of the MSSM running couplings at $\Lambda_U\sim 2.5\times10^{16}$ GeV, for the intermediate scale desert scenario, rather than at or above the lower limit of the string coupling unification scale, $\Lambda_H\sim5\times10^{17}$ GeV. For a set of ISME particles meeting optical unification constraints, a virtual MSSM unification scale below the real string unification scale is guaranteed.

A WCHS model of the NAHE class has been found that offers possible realization of optical unification. In this model optical unification requires that six pairs of exotic non-Abelian singlet states with zero hypercharge acquire Λ_U scale or larger masses, along with three out of four pairs of MSSM Higgs. On the other hand, the four pairs of exotic MSSM triplets and three pairs of MSSM doublets must remain massless down to an intermediate scale Λ_I.

The optical unification properties of systematically generated D- and stringent F-flat directions have been investigated for this model. Stringent flat directions do not require significant fine-tuning. In contrast to generic direc-

[5] 16th order is still likely unacceptably low by one order, producing SUSY breaking at an energy scale too high by approximately a factor of 10.

tions, they need not be examined order-by-order for F-flatness. Instead, F-flatness can be guaranteed to all finite order (or to at least 17th order, which is consistent with electroweak scale supersymmetry breaking). Stringent flat directions are the roots of more complicated (and arguably, more finely tuned) flat directions that, generically, must be examined order-by-order.

A subset of stringent flat directions was found that provides Λ_U scale mass for, at most, three of the six unwanted pairs of exotic singlets and two of the four pairs of MSSM Higgs. Thus, a search is underway for non-stringent F-flat directions that could induce additional sought-after mass to three remaining pairs of exotic singlets and to one remaining Higgs pair. Admittedly, discovery of suitable nonstringent flat directions could imply an element of fine-tuning in this particular optical unification model, unless it could be shown that one direction among these is necessarily chosen by nonperturbative dynamics. Too much unexplained fine-tuning might imply that optical unification would, itself, require fortuitous circumstances.

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APPENDIX A: OPTICAL UNIFICATION MODEL FIELDS AND THEIR CHARGES

APPENDIX B: OPTICAL UNIFICATION MODEL SUPERPOTENTIAL

1. Hidden sector and singlet terms to 6th order

Coupling coefficients are given only for third order terms.

$$\begin{aligned}\text{3rd order: } & \tfrac{1}{2}(F_1\bar{F}'_1\Phi_1 + K_4\bar{K}_4\Phi_2 + S_1\bar{S}_1\Phi_3 + S_2\bar{S}_2\Phi_3 \\ & + S_3\bar{S}_3\Phi_3 + S_4\bar{S}_4\Phi_3 + S_5\bar{S}_5\Phi_3 + S_6\bar{S}_6\Phi_3 \\ & + S_7\bar{S}_7\Phi_1 + S_8\bar{S}_8\Phi_1 + S_9\bar{S}_9\Phi_1) + S_1S_2\Phi_{12} \\ & + S_3S_4\Phi_{12} + S_5S_6\Phi_{12} + S_7\bar{S}_8\Phi_{23} + \bar{S}_1\bar{S}_2\bar{\Phi}_{12} \\ & + \bar{S}_3\bar{S}_4\bar{\Phi}_{12} + \bar{S}_5\bar{S}_6\bar{\Phi}_{12} + S_8\bar{S}_7\bar{\Phi}_{23} \\ & + \Phi_{12}\bar{\Phi}_{23}\bar{\Phi}_{31} + \Phi_{23}\Phi_{31}\bar{\Phi}_{12} \end{aligned} \tag{B1}$$

$$\text{4th order: } F_1\bar{F}'_2S_9N_3 + F_3\bar{F}'_3S_9\bar{S}_7 + K_2\bar{K}'_2S_9\bar{S}_7 \tag{B2}$$

TABLE A.I. MSSM 3 generations and Higgs.

F	$(SU(3)_C, SU(2)_L)$	Q_Y	$Q_{Z'}$	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	$(SU(5)_H, SU(3)_H)$	Q_6	Q_7
Q_1	(3, 2)	1/6	1/6	1/2	−1/2	1/2	0	1/2	0	(1, 1)	0	0
u_1	(1, 2)	−2/3	1/3	1/2	−1/2	1/2	0	−1/2	0	(1, 1)	0	0
d_1	(1, 2)	1/3	−2/3	1/2	−1/2	1/2	0	−1/2	0	(1, 1)	0	0
L_1	(1, 2)	−1/2	−1/2	1/2	−1/2	1/2	0	1/2	0	(1, 1)	0	0
e_1	(1, 2)	1	0	1/2	−1/2	1/2	0	−1/2	0	(1, 1)	0	0
N_1	(1, 2)	0	1	1/2	−1/2	1/2	0	−1/2	0	(1, 1)	0	0
Q_2	(3, 2)	1/6	1/6	1/2	1/2	1/2	−1/2	0	0	(1, 1)	0	0
u_2	(1, 2)	−2/3	1/3	1/2	1/2	1/2	1/2	0	0	(1, 1)	0	0
d_2	(1, 2)	1/3	−2/3	1/2	1/2	1/2	1/2	0	0	(1, 1)	0	0
L_2	(1, 2)	−1/2	−1/2	1/2	1/2	1/2	−1/2	0	0	(1, 1)	0	0
e_2	(1, 2)	1	0	1/2	1/2	1/2	1/2	0	0	(1, 1)	0	0
N_2	(1, 2)	0	1	1/2	1/2	1/2	1/2	0	0	(1, 1)	0	0
Q_3	(3, 2)	1/6	1/6	1/2	0	−1	0	0	−1/2	(1, 1)	0	0
u_3	(1, 2)	−2/3	1/3	1/2	0	−1	0	0	1/2	(1, 1)	0	0
d_3	(1, 2)	1/3	−2/3	1/2	0	−1	0	0	1/2	(1, 1)	0	0
L_3	(1, 2)	−1/2	−1/2	1/2	0	−1	0	0	−1/2	(1, 1)	0	0
e_3	(1, 2)	1	0	1/2	0	−1	0	0	1/2	(1, 1)	0	0
N_3	(1, 2)	0	1	1/2	0	−1	0	0	1/2	(1, 1)	0	0
h_1	(1, 2)	−1/2	1/2	1	−1	1	0	0	0	(1, 1)	0	0
h_2	(1, 2)	−1/2	1/2	1	1	1	0	0	0	(1, 1)	0	0
h_3	(1, 2)	−1/2	1/2	1	0	−2	0	0	0	(1, 1)	0	0
h_4	(1, 2)	−1/2	0	−1/4	−1/2	1/2	0	0	0	(1, 1)	2	0
$\bar{h}_1$	(1, 2)	1/2	−1/2	−1	1	−1	0	0	0	(1, 1)	0	0
$\bar{h}_2$	(1, 2)	1/2	−1/2	−1	−1	−1	0	0	0	(1, 1)	0	0
$\bar{h}_3$	(1, 2)	1/2	−1/2	−1	0	2	0	0	0	(1, 1)	0	0
$\bar{h}_4$	(1, 2)	1/2	0	1/4	1/2	−1/2	0	0	0	(1, 1)	−2	0

TABLE A.II. MSSM-charged exotics.

F	$(SU(3)_C, SU(2)_L)$	Q_Y	$Q_{Z'}$	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	$(SU(5)_H, SU(3)_H)$	Q_6	Q_7
D_1	(3, 1)	$-1/3$	$-1/3$	1	0	1	0	0	0	(1, 1)	0	0
D_2	(3, 1)	$-1/3$	$-1/3$	-1	0	-1	0	0	0	(1, 1)	0	0
D_3	(3, 1)	$-1/3$	$1/6$	$1/4$	$-1/2$	$-1/2$	0	0	0	(1, 1)	-2	0
D_4	(3, 1)	$1/6$	$1/6$	0	0	0	$1/2$	$1/2$	$1/2$	(1, 1)	$1/2$	$-15/2$
$\bar{D}_1$	$(\bar{3}, 1)$	$1/3$	$1/3$	-1	0	-1	0	0	0	(1, 1)	0	0
$\bar{D}_2$	$(\bar{3}, 1)$	$1/3$	$1/3$	1	0	1	0	0	0	(1, 1)	0	0
$\bar{D}_3$	$(\bar{3}, 1)$	$1/3$	$1/6$	$-1/4$	$1/2$	$1/2$	0	0	0	(1, 1)	2	0
$\bar{D}_4$	$(\bar{3}, 1)$	$-1/6$	$-1/6$	0	0	0	$-1/2$	$-1/2$	$-1/2$	(1, 1)	$-1/2$	$15/2$
X_1	(1, 2)	0	0	$1/2$	$-1/2$	$1/2$	$1/2$	0	$1/2$	(1, 1)	$-1/2$	$15/2$
X_2	(1, 2)	0	0	$1/2$	$1/2$	$1/2$	0	$-1/2$	$1/2$	(1, 1)	$-1/2$	$15/2$
X_3	(1, 2)	0	0	$1/2$	0	-1	$1/2$	$-1/2$	0	(1, 1)	$-1/2$	$15/2$
$\bar{X}_1$	(1, 2)	0	0	$-1/2$	$1/2$	$-1/2$	$1/2$	0	$1/2$	(1, 1)	$1/2$	$-15/2$
$\bar{X}_2$	(1, 2)	0	0	$-1/2$	$-1/2$	$-1/2$	0	$-1/2$	$1/2$	(1, 1)	$1/2$	$-15/2$
$\bar{X}_3$	(1, 2)	0	0	$-1/2$	0	1	$1/2$	$-1/2$	0	(1, 1)	$1/2$	$-15/2$
A_1	(1, 1)	$1/2$	$1/2$	0	0	0	$1/2$	$1/2$	$-1/2$	(1, 1)	$-1/2$	$15/2$
A_2	(1, 1)	$-1/2$	$1/2$	$-1/2$	$-1/2$	$-1/2$	0	$1/2$	$-1/2$	(1, 1)	$-1/2$	$15/2$
A_3	(1, 1)	$-1/2$	$1/2$	$-1/2$	0	1	$-1/2$	$1/2$	0	(1, 1)	$-1/2$	$15/2$
A_4	(1, 1)	$-1/2$	$1/2$	$-1/2$	$1/2$	$-1/2$	$-1/2$	0	$-1/2$	(1, 1)	$-1/2$	$15/2$
A_5	(1, 1)	$1/2$	$-1/2$	$-1/2$	$-1/2$	$-1/2$	0	$1/2$	$-1/2$	(1, 1)	$-1/2$	$15/2$
A_6	(1, 1)	$1/2$	$-1/2$	$-1/2$	0	1	$-1/2$	$1/2$	0	(1, 1)	$-1/2$	$15/2$
A_7	(1, 1)	$1/2$	$-1/2$	$-1/2$	$1/2$	$-1/2$	$-1/2$	0	$-1/2$	(1, 1)	$-1/2$	$15/2$
$\bar{A}_1$	(1, 1)	$-1/2$	$-1/2$	0	0	0	$-1/2$	$-1/2$	$1/2$	(1, 1)	$1/2$	$-15/2$
$\bar{A}_2$	(1, 1)	$1/2$	$-1/2$	$1/2$	$1/2$	$1/2$	0	$1/2$	$-1/2$	(1, 1)	$1/2$	$-15/2$
$\bar{A}_3$	(1, 1)	$1/2$	$-1/2$	$1/2$	0	-1	$-1/2$	$1/2$	0	(1, 1)	$1/2$	$-15/2$
$\bar{A}_4$	(1, 1)	$1/2$	$-1/2$	$1/2$	$-1/2$	$1/2$	$-1/2$	0	$-1/2$	(1, 1)	$1/2$	$-15/2$
$\bar{A}_5$	(1, 1)	$-1/2$	$1/2$	$1/2$	$1/2$	$1/2$	0	$1/2$	$-1/2$	(1, 1)	$1/2$	$-15/2$
$\bar{A}_6$	(1, 1)	$-1/2$	$1/2$	$1/2$	0	-1	$-1/2$	$1/2$	0	(1, 1)	$1/2$	$-15/2$
$\bar{A}_7$	(1, 1)	$-1/2$	$1/2$	$1/2$	$-1/2$	$1/2$	$-1/2$	0	$-1/2$	(1, 1)	$1/2$	$-15/2$

5th order:
$$S_1S_2S_8\bar{S}_7\Phi_{31} + S_3S_4S_8\bar{S}_7\Phi_{31} + S_5S_6S_8\bar{S}_7\Phi_{31} + S_7\bar{S}_1\bar{S}_2\bar{S}_8\bar{\Phi}_{31} + S_7\bar{S}_3\bar{S}_4\bar{S}_8\bar{\Phi}_{31} + S_7\bar{S}_5\bar{S}_6\bar{S}_8\bar{\Phi}_{31} + F_1\bar{F}'_3S_9\Phi_3N_2 + F_2\bar{F}'_2S_9\bar{S}_8\Phi_2 + K_3\bar{K}'_3S_9\bar{S}_8\Phi_2 \tag{B3}$$

6th order:
$$F_1F_1\bar{F}'_1\bar{F}'_2S_9N_3 + F_1F_3\bar{F}'_1\bar{F}'_3S_9\bar{S}_7 + F_1\bar{F}'_1K_2\bar{K}'_2S_9\bar{S}_7 + F_1\bar{F}'_2K_4\bar{K}_4S_9N_3 + F_1\bar{F}'_2S_9N_3\sum_{i=1}^{9}S_i\bar{S}_i + F_1\bar{F}'_2S_9N_3\Phi_{31}\bar{\Phi}_{31} + F_1\bar{F}'_2S_9N_3\Phi_2\Phi_2 + F_3\bar{F}'_3S_9\bar{S}_7K_4\bar{K}_4 + F_3\bar{F}'_3S_9\bar{S}_7\sum_{i=1}^{9}S_i\bar{S}_i + F_3\bar{F}'_3S_9\bar{S}_7\Phi_{12}\bar{\Phi}_{12} + F_3\bar{F}'_3S_9\bar{S}_7\Phi_3\Phi_3 + F_4\bar{F}'_4S_9\bar{S}_7S_1S_2 + F_4\bar{F}'_4S_9\bar{S}_7S_3S_4 + F_4\bar{F}'_4S_9\bar{S}_7S_5S_6 + K_1\bar{K}_1S_9\bar{S}_7S_1S_2 + K_1\bar{K}'_1S_9\bar{S}_7S_3S_4 + K_1\bar{K}_1S_9\bar{S}_7S_5S_6 + K_1\bar{K}_4S_4S_9\Phi_2N_1 + K_2\bar{K}'_2S_9\bar{S}_7K_4\bar{K}_4 + K_2\bar{K}'_2S_9\bar{S}_7\sum_{i=1}^{9}S_i\bar{S}_i + K_2\bar{K}'_2S_9\bar{S}_7\Phi_{12}\bar{\Phi}_{12} + K_2\bar{K}'_2S_9\bar{S}_7\Phi_3\Phi_3 + K_2\bar{K}_4S_1S_9\Phi_1N_2 \tag{B4}$$

TABLE A.III. Singlets with $Q_Y = 0$.

F	$(SU(3)_C, SU(2)_L)$	Q_Y	$Q_{Z'}$	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	$(SU(5)_H, SU(3)_H)$	Q_6	Q_7
Φ_1	(1, 1)	0	0	0	0	0	0	0	0	(1, 1)	0	0
Φ_2	(1, 1)	0	0	0	0	0	0	0	0	(1, 1)	0	0
Φ_3	(1, 1)	0	0	0	0	0	0	0	0	(1, 1)	0	0
Φ_{12}	(1, 1)	0	0	0	−2	0	0	0	0	(1, 1)	0	0
Φ_{23}	(1, 1)	0	0	0	1	−3	0	0	0	(1, 1)	0	0
Φ_{31}	(1, 1)	0	0	0	−1	−3	0	0	0	(1, 1)	0	0
$\bar{\Phi}_{12}$	(1, 1)	0	0	0	2	0	0	0	0	(1, 1)	0	0
$\bar{\Phi}_{23}$	(1, 1)	0	0	0	−1	3	0	0	0	(1, 1)	0	0
$\bar{\Phi}_{31}$	(1, 1)	0	0	0	1	3	0	0	0	(1, 1)	0	0
S_1	(1, 1)	0	0	0	−1	0	−1	0	0	(1, 1)	0	0
S_2	(1, 1)	0	0	0	−1	0	1	0	0	(1, 1)	0	0
S_3	(1, 1)	0	0	0	−1	0	0	−1	0	(1, 1)	0	0
S_4	(1, 1)	0	0	0	−1	0	0	1	0	(1, 1)	0	0
S_5	(1, 1)	0	0	0	−1	0	0	0	−1	(1, 1)	0	0
S_6	(1, 1)	0	0	0	−1	0	0	0	1	(1, 1)	0	0
S_7	(1, 1)	0	1/2	3/4	−1/2	−3/2	0	0	0	(1, 1)	2	0
S_8	(1, 1)	0	1/2	3/4	1/2	3/2	0	0	0	(1, 1)	2	0
S_9	(1, 1)	0	1/2	−5/4	1/2	−1/2	0	0	0	(1, 1)	2	0
$\bar{S}_1$	(1, 1)	0	0	0	1	0	1	0	0	(1, 1)	0	0
$\bar{S}_2$	(1, 1)	0	0	0	1	0	−1	0	0	(1, 1)	0	0
$\bar{S}_3$	(1, 1)	0	0	0	1	0	0	1	0	(1, 1)	0	0
$\bar{S}_4$	(1, 1)	0	0	0	1	0	0	−1	0	(1, 1)	0	0
$\bar{S}_5$	(1, 1)	0	0	0	1	0	0	0	1	(1, 1)	0	0
$\bar{S}_6$	(1, 1)	0	0	0	1	0	0	0	−1	(1, 1)	0	0
$\bar{S}_7$	(1, 1)	0	−1/2	−3/4	1/2	3/2	0	0	0	(1, 1)	−2	0
$\bar{S}_8$	(1, 1)	0	−1/2	−3/4	−1/2	−3/2	0	0	0	(1, 1)	−2	0
$\bar{S}_9$	(1, 1)	0	−1/2	5/4	−1/2	1/2	0	0	0	(1, 1)	−2	0

TABLE A.IV. Hidden-sector non-Abelian fields.

F	$(SU(3)_C, SU(2)_L)$	Q_Y	$Q_{Z'}$	Q_A	Q_1	Q_2	Q_3	Q_4	Q_5	$(SU(5)_H, SU(3)_H)$	Q_6	Q_7
F_1	(1, 1)	−1/4	0	−1/2	−1/2	1/2	0	0	0	(5, 1)	−1	−3
F_2	(1, 1)	1	0	0	0	1	0	0	1/2	(5, 1)	1	−3
F_3	(1, 1)	1	0	0	−1/2	−1/2	1/2	0	0	(5, 1)	1	−3
F_4	(1, 1)	1	0	0	1/2	−1/2	0	−1/2	0	(5, 1)	1	−3
$\bar{F}'_1$	(1, 1)	1/4	0	1/2	1/2	−1/2	0	0	0	($\bar{5}$, 1)	1	3
$\bar{F}'_2$	(1, 1)	1	0	0	0	1	0	0	−1/2	($\bar{5}$, 1)	−1	3
$\bar{F}'_3$	(1, 1)	1	0	0	−1/2	−1/2	−1/2	0	0	($\bar{5}$, 1)	−1	3
$\bar{F}'_4$	(1, 1)	1	0	0	1/2	−1/2	0	1/2	0	($\bar{5}$, 1)	−1	3
K_1	(1, 1)	1	0	0	1/2	−1/2	0	1/2	0	(1, 3)	−1	−5
K_2	(1, 1)	1	0	0	−1/2	−1/2	1/2	0	0	(1, 3)	−1	−5
K_3	(1, 1)	1	0	0	0	1	0	0	1/2	(1, 3)	−1	−5
K_4	(1, 1)	1/4	0	1/2	−1/2	−1/2	0	0	0	(1, 3)	1	−5
$\bar{K}'_1$	(1, 1)	1	0	0	1/2	−1/2	0	−1/2	0	(1, $\bar{3}$)	1	5
$\bar{K}'_2$	(1, 1)	1	0	0	−1/2	−1/2	−1/2	0	0	(1, $\bar{3}$)	1	5
$\bar{K}'_3$	(1, 1)	1	0	0	0	1	0	0	−1/2	(1, $\bar{3}$)	1	5
$\bar{K}_4$	(1, 1)	−1/4	0	−1/2	1/2	1/2	0	0	0	(1, $\bar{3}$)	1	5

APPENDIX C: OPTICAL UNIFICATION MODEL MASS MATRICES

1. Possible exotic triplet $D\bar{D}$ mass matrix to 6th order

$$M_{D_i,\bar{D}_j} = \begin{pmatrix} \frac{1}{2}\Phi_3 + m'_{11} & S_9\bar{S}_8\Phi_2 & - & - \\ \bar{S}_8\bar{S}_9 + m'_{21} & \frac{1}{2}\Phi_3 + m'_{22} & - & \bar{S}_9 + m'_{24} \\ - & - & K_4\bar{K}_4\sum_{i=1}^{6} S_i\bar{S}_i & - \\ S_8 + m'_{41} & - & - & \frac{1}{2}\Phi_2 + m'_{44} \end{pmatrix}, \quad \text{(C1)}$$

where

$$m'_{11} = F_1\bar{F}'_2S_9N_3 + F_3\bar{F}'_3S_9\bar{S}_7 + K_2\bar{K}'_2S_9\bar{S}_7, \quad \text{(C2)}$$

$$m'_{21} = (S_8\bar{S}_7 + \Phi_{23}\Phi_1)(F_3\bar{F}'_3 + K_2\bar{K}'_2) + \bar{\Phi}_{31}\Phi_2F_4\bar{F}'_4 + \bar{\Phi}_{31}\Phi_2K_1\bar{K}'_1, \quad \text{(C3)}$$

$$m'_{22} = S_9F_1\bar{F}'_2N_3 + S_9\bar{S}_7(F_3\bar{F}'_3 + K_2\bar{K}'_2), \quad \text{(C4)}$$

$$m'_{24} = \bar{S}_9F_1\bar{F}_1 + \bar{S}_9K_4\bar{K}_4 + \bar{S}_9\sum_{i=1}^{9} S_i\bar{S}_i + F_1\bar{F}'_2N_3\Phi_1 + \bar{S}_7\Phi_1(F_3\bar{F}'_3 + K_2\bar{K}'_2) + \bar{S}_7\Phi_{12}(F_4\bar{F}'_4 + K_2\bar{K}'_2) + \bar{S}_8\Phi_{23}(F_3\bar{F}'_3 + K_2\bar{K}'_2) + \bar{S}_8\Phi_{31}(F_4\bar{F}'_4 + K_1\bar{K}'_1) + S_1N_2K_2\bar{K}_4 + S_4N_1K_1\bar{K}_4 \quad \text{(C5)}$$

$$m'_{41} = S_8F_1\bar{F}_1 + S_8K_4\bar{K}_4 + S_8\sum_{i=7}^{9} S_i\bar{S}_i + S_9\Phi_{23}(F_3\bar{F}'_3 + K_2\bar{K}'_2) + S_9\bar{\Phi}_{31}(F_4\bar{F}'_4 + K_1\bar{K}'_1), \quad \text{(C6)}$$

$$m'_{44} = S_9N_3F_1\bar{F}'_2 + S_9\bar{S}_7(F_3\bar{F}'_3 + K_2\bar{K}'_2). \quad \text{(C7)}$$

2. Possible exotic doublet $X\bar{X}$ mass matrix to 6th order

$$M_{X_i,\bar{X}_j} = \begin{pmatrix} (S_1\bar{S}_6 + S_5\bar{S}_2)\Phi_2 & - & - \\ - & (S_4\bar{S}_6 + S_5\bar{S}_3)\Phi_1 & - \\ - & - & - \end{pmatrix}. \quad \text{(C8)}$$

3. Possible exotic hypercharged singlet $A\bar{A}$ mass matrix to 6th order

$$M_{A_i,\bar{A}_j} = \begin{pmatrix} M_{11} & - & - & - & - & - & - \\ - & M_{22} & - & - & - & - & - \\ - & - & - & - & - & - & - \\ - & - & - & M_{44} & - & - & - \\ N_2 & - & - & - & M_{55} & - & - \\ - & - & - & - & - & - & - \\ M_{71} & - & - & - & - & - & M_{77} \end{pmatrix}, \quad \text{(C9)}$$

where

$$M_{11} = F_1\bar{F}'_2S_9N_3, \quad \text{(C10)}$$

$$M_{22} = M_{55} = (S_3\bar{S}_5 + S_6\bar{S}_4)\Phi_1, \quad \text{(C11)}$$

$$M_{44} = M_{77} = (S_2\bar{S}_5 + S_6\bar{S}_1)\Phi_2, \quad \text{(C12)}$$

$$M_{51} = N_2, \quad \text{(C13)}$$

$$M_{71} = (S_2\bar{S}_3 + S_4\bar{S}_1)N_1\Phi_2. \quad \text{(C14)}$$

4. Possible seventh order AA and $A\bar{A}$ mass terms

$$A_3A_5\colon\ K_2K_3K_4S_9\bar{S}_4 \quad \text{(C15)}$$

$$A_3A_6\colon\ K_1K_1K_4S_2S_9 + K_2K_2K_4S_9\bar{S}_4 \quad \text{(C16)}$$

$$A_4A_6\colon\ K_1K_3K_4S_2S_9 \quad \text{(C17)}$$

$$A_1\bar{A}_1\colon\ (F_2\bar{F}'_2 + K_3\bar{K}'_3)S_9\bar{S}_8\Phi_2 + (S_1S_2 + S_3S_4 + S_5S_6)S_8\bar{S}_7\Phi_{31} + (\bar{S}_1\bar{S}_2 + \bar{S}_3\bar{S}_4 + \bar{S}_5\bar{S}_6)S_7\bar{S}_8\bar{\Phi}_{31} + \quad \text{(C18)}$$

$$A_1\bar{A}_5\colon\ F_1\bar{F}'_3(S_3\bar{S}_5 + S_6\bar{S}_4)S_9 \quad \text{(C19)}$$

$$A_1\bar{A}_6\colon\ F_1\bar{F}'_2(S_3\bar{S}_5 + S_6\bar{S}_4)S_9 \quad \text{(C20)}$$

$$A_1\bar{A}_7\colon\ K_1\bar{K}_4S_6S_9\Phi_2 \quad \text{(C21)}$$

$$A_2\bar{A}_2,\ A_5\bar{A}_5\colon\ F_1\bar{F}_1(S_3S_6\Phi_{12} + \bar{S}_4\bar{S}_5\bar{\Phi}_{12}) + K_3\bar{K}_4N_2S_3S_9 + S_1S_2\bar{S}_4\bar{S}_5\Phi_1 + S_3S_6\bar{S}_1\bar{S}_2\Phi_1 + S_3S_6\sum_{i=7}^{9} S_i\bar{S}_i\Phi_{12} + S_3S_6S_8\bar{S}_7\Phi_{31} + \bar{S}_4\bar{S}_5S_7\bar{S}_8\bar{\Phi}_{31} + \bar{S}_4\bar{S}_5\sum_{i=7}^{9} S_i\bar{S}_i\bar{\Phi}_{12} \quad \text{(C22)}$$

$$A_2\bar{A}_3\colon\ K_3\bar{K}_4N_2S_3S_9 \quad \text{(C23)}$$

$$A_2\bar{A}_4,\ A_5\bar{A}_7\colon\ K_1\bar{K}_4N_2S_6S_9 + K_2\bar{K}_4N_1S_9\bar{S}_5 \quad \text{(C24)}$$

$$A_3\bar{A}_3, A_6\bar{A}_6\colon F_1\bar{F}_1(S_2\bar{S}_4 + S_3\bar{S}_1)\Phi_1 + K_4\bar{K}_4(S_2\bar{S}_4 + S_3\bar{S}_1)\Phi_2 + S_2S_7\bar{S}_4S_8\Phi_{23} + S_3S_7\bar{S}_1S_8\Phi_{23} + S_2S_8\bar{S}_4S_7\bar{\Phi}_{23} + S_3S_8\bar{S}_1S_7\bar{\Phi}_{23} + S_3\bar{S}_1\sum_{i=7}^{9} S_i\bar{S}_i\Phi_1 \tag{C25}$$

$$A_3\bar{A}_4\colon K_1\bar{K}_4N_3S_2S_9 \tag{C26}$$

$$A_4\bar{A}_2, A_7\bar{A}_5\colon K_2\bar{K}_4N_1S_6S_9 \tag{C27}$$

$$A_4\bar{A}_4, A_7\bar{A}_7\colon K_3\bar{K}_4N_3S_2S_9 + K_4\bar{K}_4S_2S_6\Phi_{12} + K_4\bar{K}_4\bar{S}_1\bar{S}_5\bar{\Phi}_{12} + S_2S_6\bar{S}_3\bar{S}_4\Phi_2 + S_3S_4\bar{S}_1\bar{S}_5\Phi_2 \tag{C28}$$

$$A_5\bar{A}_1\colon (F_3\bar{F}'_2)N_3S_9\bar{S}_7 \tag{C29}$$

$$A_6\bar{A}_1\colon K_2\bar{K}'_3N_2S_9\bar{S}_8 \tag{C30}$$

$$A_6\bar{A}_5\colon K_2\bar{K}_4N_3S_3S_9 \tag{C31}$$

$$A_7\bar{A}_6\colon K_3\bar{K}_4N_1S_2S_9 \tag{C32}$$

5. Possible Higgs mass matrix to 6th order

$$M_{h_i,\bar{h}j} = \begin{pmatrix} - & \Phi_{12} & \Phi_{31} & S_9 \\ \bar{\Phi}_{12} & m_{22} & \bar{\Phi}_{23} & m_{24} \\ \bar{\Phi}_{31} & \Phi_{23} & m_{33} & - \\ \bar{S}_9 + m_{41} & m_{42} & m_{43} & \Phi_1 + m_{44} \end{pmatrix}, \tag{C33}$$

where

$$m_{22} = F_1\bar{F}'_2N_3S_9, \tag{C34}$$

$$m_{24} = (S_1S_2 + S_3S_4 + S_5S_6)S_9, \tag{C35}$$

$$m_{33} = (F_3\bar{F}'_3 + K_2\bar{K}'_2)S_9\bar{S}_7, \tag{C36}$$

$$m_{41} = K_2\bar{K}_4N_2S_1, \tag{C37}$$

$$m_{42} = (\bar{S}_1\bar{S}_2 + \bar{S}_3\bar{S}_4 + \bar{S}_5\bar{S}_6)\bar{S}_9, \tag{C38}$$

$$m_{43} = (F_3\bar{F}'_3 + K_2\bar{K}'_2)\bar{S}_8\Phi_{12}, \tag{C39}$$

$$m_{44} = F_1\bar{F}'_2N_3S_9 + (F_3\bar{F}'_3 + K_2\bar{K}'_2)S_9\bar{S}_7. \tag{C40}$$

6. Possible up-quark mass matrix to 6th order

$$M_{Q_i,u_j^c} = \begin{pmatrix} \bar{h}_1 + \bar{h}_2(S_1S_2 + S_3S_4 + S_5S_6) + & - & - \\ - & \bar{h}_2 + \bar{h}_1(S_1S_2 + S_3S_4 + S_5S_6) + \bar{h}_3S_7\bar{S}_8 + \bar{h}_4S_9\bar{\Phi}_{12} & - \\ - & - & \bar{h}_3 + \bar{h}_2S_8\bar{S}_7 + \bar{h}_4S_9\bar{\Phi}_{31} \end{pmatrix}. \tag{C41}$$

7. Possible down-quark and electron mass matrix to 6th order

$$M_{Q_i,d_j^c} = M_{L_i,e_j^c} = \begin{pmatrix} h_4\bar{S}_8(\bar{S}_1\bar{S}_2 + \bar{S}_3\bar{S}_4 + \bar{S}_5\bar{S}_6) & - & - \\ - & h_4\bar{S}_8 + h_3S_9\bar{S}_7\bar{\Phi}_{12} \; h_4(F_1\bar{F}_1 + K_4\bar{K}_4 + \sum_{i=1}^{9} S_i\bar{S}_i + \Phi_{12}\bar{\Phi}_{12} + \Phi_3\bar{\Phi}_3) & - \\ - & - & h_4\bar{S}_7\Phi_2 \end{pmatrix}. \tag{C42}$$

APPENDIX D: *D*- AND STRINGENT *F*-FLAT DIRECTIONS TOWARDS OPTICAL UNIFICATION

In Tables D.II, D.III, and D.IV, the first column specifies the VEV class, with the first component of the class designation indicating the number of independent pairs of massive exotic doublets (AA or AA) produced, the second component distinguishing the mass combinations, and the third component in Tables D.II and D.III identifying a given flat direction. The second column specifies the number of field VEVs, the third column specifies the order at which F-flatness is broken unless self-cancellation via non-Abelian VEVs is induced (∞ indicates F-flatness to all finite orders), the third column specifies the normalized

TABLE D.I. Classes of D- and F-flat directions and related massive exotic doublets.

Class	Massive pairs	Orders of mass terms
1-1	$A_5\bar{A}_1$	3
1-2	$A_7\bar{A}_6$	7
1-3	$A_3\bar{A}_4$	7
2-1	$A_5(a_3\bar{A}_1 + a_7\bar{A}_7)$, $A_2\bar{A}_4$	3, 7
3-1	$A_4\bar{A}_2$, $A_7\bar{A}_5$, $A_6\bar{A}_1$	7, 7, 7
3-2	$(a_3A_5 + a_7A_6)\bar{A}_1$, $A_2\bar{A}_4$, $A_5\bar{A}_7$	3, 7, 7, 7
5-1	$A_2\bar{A}_4$, $A_5\bar{A}_7$, $A_4\bar{A}_2$, $A_7\bar{A}_5$, $A_6\bar{A}_1$	7, 7, 7, 7, 7
5-2	$(a_3A_5 + a_7A_6)\bar{A}_1$, $A_2\bar{A}_4$, $A_5\bar{A}_7$, $A_4\bar{A}_2$, $A_7\bar{A}_5$	3, 7, 7, 7, 7

TABLE D.II. $A_i\bar{A}_j$ or A_iA_j mass-generating stringent flat directions (to at least 17th order).

Class	$\#v$	F-flat	$Q^{(A)\prime}$	VEVs S_9 F_2	 S_7 $\bar{F}'_3$	 S_8 K_4	 $\bar{\Phi}_{12}$ K_1	 $\bar{\Phi}_{23}$ K_2	 $\bar{\Phi}_{31}$ K_3	 S_1 $\bar{K}'_1$	 S_2 $\bar{K}'_2$	 S_3 $\bar{K}'_3$	 S_4 N_1	 S_5 N_2	 S_6 N_3
1-1	8	∞	−6	24	0	−18	0	0	12	0	−3	0	3	0	0
				0	0	−6	6	0	0	0	0	0	0	6	0
	9		−7	27	0	−21	0	0	11	0	−3	0	0	2	−1
				0	0	−6	0	0	6	0	0	0	0	6	0
	10		−7	27	0	−21	0	0	11	0	−2	0	1	0	−3
				0	0	−6	0	0	6	0	0	0	2	4	0
	11		−13	51	0	−39	0	0	23	0	−6	0	3	2	−1
				0	0	−12	6	0	6	0	0	0	0	12	0
1-2	10		−1	9	0	−5	0	0	1	0	1	0	2	0	−3
				0	0	−4	0	0	6	0	2	0	4	0	0
	10		−1	15	0	−7	0	0	0	0	2	0	4	1	−5
				0	0	−8	0	0	12	0	4	0	8	0	0
	11		−3	75	0	−39	0	0	3	0	15	0	18	0	−33
				30	30	−36	0	0	36	0	0	0	36	0	0
	11		−3	81	0	−39	0	0	0	0	15	0	21	3	−33
				30	30	−42	0	0	42	0	0	0	42	0	0
1-3	10		−3	21	0	−11	0	0	13	0	1	0	6	0	−5
				0	0	−10	12	0	0	0	2	0	0	0	10
2-1.1	10	∞	−3	15	0	−11	0	0	7	0	−2	0	3	0	1
				0	0	−4	6	0	0	0	0	2	0	4	0
.2	10		−1	9	0	−5	0	0	3	2	0	0	3	0	1
				0	0	−4	6	0	0	0	0	2	0	4	0
.3	11		−22	72	−22	−48	0	0	24	0	−1	0	3	0	2
				0	0	−2	6	0	0	0	0	4	0	2	0
.4	11		−1	9	0	−7	0	0	5	0	−1	0	2	0	1
				0	0	−2	6	0	0	2	0	2	0	2	0
3-1.1	10		−1	9	0	−5	0	0	3	0	−3	−2	0	0	1
				0	0	−4	0	6	0	0	0	2	4	0	0

TABLE D.III. $A_i\bar{A}_j$ or A_iA_j mass-generating stringent F-flat directions (to at least 17th order) through non-Abelian self-cancellation via $\langle K_2 \cdot \bar{K}_3' \rangle = 0$.

Class & ID #	$\#v$	F-flat	$Q^{(A)\prime}$	VEVs S_9 F_2	 S_7 $\bar{F}_3'$	 S_8 K_4	 $\bar{\Phi}_{12}$ K_1	 $\bar{\Phi}_{23}$ K_2	 $\bar{\Phi}_{31}$ K_3	 S_1 $\bar{K}_1'$	 S_2 $\bar{K}_2'$	 S_3 $\bar{K}_3'$	 S_4 N_1	 S_5 N_2	 S_6 N_3
3-2.1	11	>17(16)	−3	21	0	−17	0	0	13	0	−5	0	0	0	1
				0	0	−4	6	6	0	6	0	2	0	4	0
.2	11	>17(16)	−2	18	0	−16	0	0	12	0	0	−2	0	0	1
				0	0	−2	6	6	0	6	0	4	0	2	0
.3	11	>17(16)	−3	21	0	−17	0	0	13	0	0	0	1	0	1
				0	0	−4	6	6	0	6	0	2	0	4	0
.4	12	>17(16)	−1	15	0	−13	0	0	10	0	0	−1	1	0	1
				0	0	−2	6	6	0	6	0	4	0	2	0

TABLE D.IV. Directions generating 5 pairs of $A_i\bar{A}_j$ or A_iA_j masses, but with F-flatness breaking below 17th order (some of these directions have F-term self-cancellation below 11th order).

Class	$\#v$	F-flat	$Q^{(A)\prime}$	VEVs S_9 F_1	 S_7 $\bar{F}_4'$	 S_8 K_4	 $\bar{\Phi}_{12}$ K_1	 $\bar{\Phi}_{23}$ K_2	 $\bar{\Phi}_{31}$ K_3	 S_1 $\bar{K}_1'$	 S_2 $\bar{K}_2'$	 S_3 $\bar{K}_3'$	 S_4 N_1	 S_5 N_2	 S_6 N_3
5-1	11	11	−1	15	0	−11	0	0	6	0	−6	−2	0	−3	1
				0	0	−4	0	12	0	0	0	8	4	0	0
	11	12	−1	15	0	−9	0	0	5	0	−6	−2	1	−2	1
				0	0	−6	0	12	0	0	0	6	6	0	0
	11	13	−1	45	0	−11	0	0	6	0	−6	−2	0	−3	1
				30	30	−4	0	12	0	0	0	8	34	0	0
	11	14	−1	45	0	−9	0	0	5	0	−6	−2	1	−2	1
				30	30	−6	0	12	0	0	0	6	36	0	0
5-2	11	12	−1	15	0	−11	0	0	6	0	−7	0	1	−3	1
				0	0	−4	0	12	0	0	0	8	2	2	0
	11	14	−20	114	0	−82	0	0	51	0	−27	0	−14	−1	10
				30	30	−2	0	24	0	0	0	22	2	30	0
	11	13	−1	39	0	−7	0	0	4	0	−4	0	0	−1	1
				30	30	−2	0	6	0	0	0	4	30	2	0
	11	14	−1	45	0	−11	0	0	6	0	−7	0	1	−3	1
				30	30	−4	0	12	0	0	0	8	32	2	0

anomalous charge, and the remaining columns specify the ratios of the norm-squared components of the field VEVs. (a_3 and a_7 denote varying normalized coefficients of mass eigenstate components.) Note that none of these directions contain hidden sector $SU(5)$-charged fields.

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