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Twistorial versus spacetime formulations: Unification of various string models

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We introduce the $D = 4$ twistorial tensionfull bosonic string by considering the canonical twistorial 2-form in two-twistor space. We demonstrate its equivalence to two bosonic string models: due to Siegel (with covariant world-sheet vectorial string momenta $P^m_\mu(\tau, \sigma)$) and the one with tensorial string momenta $P_{[\mu\nu]}(\tau, \sigma)$. We show how to obtain in mixed spacetime-twistor formulation the Soroka-Sorokin-Tkach-Volkov (SSTV) string model and subsequently by harmonic gauge fixing the Bandos-Zheltukhin (BZ) model, with constrained spinorial coordinates.

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I. INTRODUCTION

Twistors and supertwistors (see e. g. [1–3]) have been recently widely used [4–8] for the description of (super) particles and (super) strings, as an alternative to spacetime approach. We stress also that recently large class of perturbative amplitudes in $N = 4$ $D = 4$ supersymmetric Yang-Mills theory [9–11] and conformal supergravity (see e.g. [12]) were described in a simple way by using strings moving in supertwistor space. Such a deep connection between supertwistors and non-Abelian supersymmetric gauge fields, from other perspective firstly observed almost 30 years ago, should promote geometric investigations of the links between the spacetime and twistor description of the string model.

In this paper we derive fourlinear twistorial classical string action, with target space described by two-twistor space. Our main aim is to show that the twistorial master action for several string models which all are classical equivalent to $D = 4$ Nambu-Goto string model, can be also described by the fundamental Liouville 2-form in two-twistor space.

Recently also there were described in $D = 4$ two-twistor space $T^{(2)} = T \otimes T$ the models describing free relativistic massive particles with spin [5,13–15]. The corresponding action was derived by suitable choice of the variables from the following free two-twistor oneform

$$\Theta^{(1)} = \Theta^{(1)}_1 + \Theta^{(1)}_2 \tag{1}$$

where ($A = 1, \ldots 4$, $i = 1, 2$; no summation over i):

$$\Theta^{(1)}_i = (\bar{Z}^{Ai} dZ_{Ai} - d\bar{Z}^{Ai} Z_{Ai}) \tag{2}$$

with imposed suitable constraints.

In this paper we shall study the following canonical Liouville twoform in two-twistor space $T^{(2)}$

$$\Theta^{(2)} = \Theta^{(1)}_1 \wedge \Theta^{(1)}_2 \tag{3}$$

restricted further by suitable constraints. We shall show that from the action which follows from (3) one can derive various formulations of $D = 4$ bosonic free string theory.

We start our considerations from the first order formulation of the tensionfull Nambu-Goto string in flat Minkowski space which is due to Siegel [16,17]

$$S = \int d^2\xi \left[P^m_\mu \partial_m X^\mu + \frac{1}{2T}(-h)^{-1/2} h_{mn} P^m_\mu P^{\mu n} \right]. \tag{4}$$

The kinetic part of the action (4) is described equivalently by the twoform

$$\tilde{\Theta}^{(2)} = P_\mu \wedge dX^\mu \tag{5}$$

where $P_\mu = P^m_\mu \epsilon_{mn} d\xi^n$, $dX^\mu = d\xi^m \partial_m X^\mu$ i. e. in Siegel formulation the pair (P^0_μ, P^1_μ) of generalized string momenta are represented by a oneform.

If we apply to (4) the string generalization of the Cartan-Penrose formula on curved world sheet [18]

$$P^m_{\alpha\dot{\alpha}} = e \tilde{\lambda}_{\dot{\alpha}} \rho^m \lambda_\alpha = e e^m_a \tilde{\lambda}^i_{\dot{\alpha}} (\rho^a)_i^{\ j} \lambda_{\alpha j}. \tag{6}$$

we shall obtain the SSTV bosonic string model [19]

$$S = \int d^2\xi e \left[\tilde{\lambda}_{\dot{\alpha}} \rho^m \lambda_\alpha \partial_m X^{\dot{\alpha}\alpha} + \frac{1}{2T}(\lambda^{\alpha i}\lambda_{\alpha i})(\tilde{\lambda}^j_{\dot{\alpha}} \tilde{\lambda}^{\dot{\alpha}}_j) \right] \tag{7}$$

where $\sqrt{-h} = e = \det(e^a_m) = -\frac{1}{2}\epsilon^{mn}\epsilon_{ab} e^a_m e^b_n$. Further we shall discuss the local gauge freedom in the spinorial sector of (7) and consider the suitable gauge fixing. We shall show that by suitable constraints in spinorial space we obtain the BZ formulation [20] which interprets the $D = 4$ spinors $\lambda_{\alpha i}$, $\bar{\lambda}^i_{\dot{\alpha}}$ as the spinorial Lorentz harmonics. Finally we shall derive the second-order action for twistorial string model described by the twoform (3).

Further we shall consider the bosonic string model with tensorial momenta obtained from the Liouville twoform [21,22]

$$\tilde{\tilde{\Theta}}^{(2)} = P_{\mu\nu} dX^\mu \wedge dX^\nu. \tag{8}$$

Such a model is directly related with the interpretation of strings as dynamical world sheets with the surface ele-

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ments

$$dS^{\mu\nu} = dX^\mu \wedge dX^\nu = \partial_m X^\mu \partial_n X^\nu \epsilon^{mn} d^2\xi. \tag{9}$$

If we introduce the composite formula for $P_{\alpha\beta} = P_{\mu\nu}\sigma^{\mu\nu}_{\alpha\beta}$, $\bar{P}_{\dot\alpha\dot\beta} = -P_{\mu\nu}\sigma^{\mu\nu}_{\dot\alpha\dot\beta}$ in terms of spinors (see also [22]) by passing to the first order action we obtain the mixed spinor-spacetime SSTV and BZ string formulations. We see therefore that both bosonic string models, based on (5) and (8), lead via SSTV to the purely twistorial bosonic string with the null twistor constraints and the constraint determining the string tension T

$$P^{\mu\nu}P_{\mu\nu} = -\frac{T^2}{4} \leftrightarrow |\lambda_{\alpha 1}\lambda_2^\alpha|^2 = \frac{T^2}{4}. \tag{10}$$

If we wish to obtain the BZ formulation one should introduce in place of (10) two constraints

$$\lambda_{\alpha 1}\lambda_2^\alpha = \frac{T}{2}, \qquad \bar\lambda^1_{\dot\alpha}\bar\lambda^{\dot\alpha 2} = \frac{T}{2} \tag{11}$$

providing the particular solution of the constraint (10).

II. SIEGEL BOSONIC STRING

Equations of motion following from the action (4) are

$$\partial_m P^m_\mu = 0, \tag{12}$$

$$P^m_\mu = -T(-h)^{1/2} h^{mn}\partial_n X_\mu, \tag{13}$$

$$P^m_\mu P^{n\mu} - \frac{1}{2}h^{mn}h_{kl}P^k_\mu P^{l\mu} = 0. \tag{14}$$

If we solve half of the equations of motion (13) without time derivatives

$$P^1_\mu = -\rho P_\mu - \lambda T X'_\mu \tag{15}$$

where $P^0_\mu = P_\mu$ denotes the string momentum and $\lambda = \frac{\sqrt{-h}}{h_{11}}$, $\rho = \frac{h_{01}}{h_{11}}$, the action (4) takes the form

$$S = \int d^2\xi \left[P_\mu \dot X^\mu - \lambda\frac{1}{2}(T^{-1}P^2_\mu + TX'^2_\mu) - \rho P_\mu X'^\mu \right]. \tag{16}$$

It is easy to see that (16) describes the phase space formulation of the tensionfull Nambu-Goto string

$$S = -T\int d^2\xi\sqrt{-g^{(2)}} \tag{17}$$

where $g^{(2)}$ is the determinant of the induced $D = 2$ metric

$$g_{mn} = \partial_m X^\mu \partial_n X_\mu, \tag{18}$$

T is the string tension, and the string Hamiltonian (see (16)) is described by a sum of first class constraints generating Virasoro algebra.

By substitution of equations of motion (13) into the Siegel action (4) one obtains the Polyakov action

$$S = -\frac{T}{2}\int d^2\xi(-h)^{1/2}h^{mn}\partial_m X^\mu \partial_n X_\mu. \tag{19}$$

Note that the Eqs. (14) describe the Virasoro first class constraints.

III. SSTV STRING MODEL AND ITS RESTRICTION TO BZ MODEL

In order to obtain from the action (4) the mixed spinor-spacetime action (7) we should eliminate the fourmomenta P^m_μ by means of the formula (6). We obtain that the second term in string action (4) takes the form

$$\frac{1}{2T}(-h)^{-1/2}h_{mn}P^m_\mu P^{n\mu} = \frac{1}{2T}e(\lambda^{\alpha i}\lambda_{\alpha i})(\tilde\lambda^j_{\dot\alpha}\tilde\lambda^{\dot\alpha}_j) \tag{20}$$

where we used $\mathrm{Tr}(\rho^m\rho^n) = 2h^{mn}$. Note that $\tilde\lambda^i_{\dot\alpha}\tilde\lambda^{\dot\alpha}_i = \bar\lambda^i_{\dot\alpha}\bar\lambda^{\dot\alpha}_i$.

Putting (6) and (20) in the action (4) we obtain the SSTV string action (7) which provides the mixed spacetime-twistor formulation of bosonic string. We stress that in SSTV formulation the twistor spinors $\lambda_{\alpha i}$ are not constrained. Further, the algebraic field Eq. (14) after substitution (6) is satisfied as an identity.

Calculating from the action (7) the momenta $\pi^{\alpha i}$, $\bar\pi^{\dot\alpha}_i$, $p^{(e)m}_a$ conjugate to the variables $\lambda_{\alpha i}$, $\bar\lambda^i_{\dot\alpha}$, e^a_m one can introduce the following two first class constraints

$$F = \lambda_{\alpha i}\pi^{\alpha i} + \bar\lambda^i_{\dot\alpha}\bar\pi^{\dot\alpha}_i - 2e^a_m p^{(e)m}_a \approx 0, \tag{21}$$

$$G = i(\lambda_{\alpha i}\pi^{\alpha i} - \bar\lambda^i_{\dot\alpha}\bar\pi^{\dot\alpha}_i) \approx 0 \tag{22}$$

generating the following local transformations:

$$\lambda'_{\alpha i} = e^{i(b+ic)}\lambda_{\alpha i}, \qquad \bar\lambda'^i_{\dot\alpha} = e^{-i(b-ic)}\bar\lambda^i_{\dot\alpha},$$
$$e'^a_m = e^{2c}e^a_m.$$

In particular one can fix the real parameters b, c in such a way that we obtain the constraints (11). The relations (11) can be rewritten in $SU(2)$-covariant way as follows (we recall that T is real)

$$A = \lambda^{\alpha i}\lambda_{\alpha i} - T = 0, \qquad \bar A = \bar\lambda^i_{\dot\alpha}\bar\lambda^{\dot\alpha}_i - T = 0. \tag{23}$$

If we introduce the variables $v_{\alpha i} = \sqrt{\frac{2}{T}}\lambda_{\alpha i}$, $\bar v^i_{\dot\alpha} = \sqrt{\frac{2}{T}}\bar\lambda^i_{\dot\alpha}$ we get the orthonormality relations for the spinorial Lorentz harmonics [20].

If we impose the constraints (11) the model (7) can be rewritten in the following way

$$S = \int d^2\xi\left[ee^m_a\tilde\lambda^i_{\dot\alpha}(\rho^a)_i{}^j\lambda_{\alpha j}\partial_m X^{\dot\alpha\alpha} + \frac{T}{2}e + \Lambda A + \bar\Lambda\bar A\right] \tag{24}$$

where the spinors λ, $\bar\lambda$ are constrained by the relations (23), which are imposed additionally in (24) by the Lagrange multipliers. It is easy to see that introducing the light cone notations on the world sheet for the *zweibein* $e^{++}_m = e^0_m + e^1_m$, $e^{--}_m = e^0_m - e^1_m$ and following Weyl representation for

Dirac matrices in two dimensions we obtain string action in the form used by Bandos and Zheltukhin (BZ model) [20,23,24].

IV. PURELY TWISTORIAL FORMULATION

Let us introduce second half of twistor coordinates $\mu_i^{\dot\alpha}$, $\bar\mu^{\alpha i}$ by employing Penrose incidence relations generalized for string

$$\mu_i^{\dot\alpha} = X^{\dot\alpha\alpha}\lambda_{\alpha i}, \qquad \bar\mu^{\alpha i} = \bar\lambda_{\dot\alpha}^i X^{\dot\alpha\alpha}. \tag{25}$$

Incidence relations (25) with real spacetime string position $X^{\dot\alpha\alpha}$ imply that the twistor variables satisfy the constraints

$$V_i^j \equiv \lambda_{\alpha i}\bar\mu^{\alpha j} - \mu_i^{\dot\alpha}\bar\lambda_{\dot\alpha}^j \approx 0 \tag{26}$$

which are antiHermitian ($(\bar V_i^j) = -V_j^i$).

Let us insert the relations (25) into (24). Using

$$P^m_{\alpha\dot\alpha}\partial_m X^{\dot\alpha\alpha} = \frac{1}{2}ee_a^m[\tilde\lambda_{\dot\alpha}\rho^a\partial_m\mu^{\dot\alpha} - \tilde\mu^\alpha\rho^a\partial_m\lambda_\alpha] + \text{c.c.}$$

we obtain the first order string action in twistor formulation

$$S = \int d^2\xi\Big\{\frac{1}{2}ee_a^m[\tilde\lambda_{\dot\alpha}\rho^a\partial_m\mu^{\dot\alpha} - \tilde\mu^\alpha\rho^a\partial_m\lambda_\alpha + \text{c.c.}] + \frac{T}{2}e + \Lambda A + \bar\Lambda\bar A + \Lambda_j{}^i(\lambda_{\alpha i}\bar\mu^{\alpha j} - \mu_i^{\dot\alpha}\bar\lambda_{\dot\alpha}^j)\Big\} \tag{27}$$

where Λ, $\bar\Lambda$, $\Lambda_j{}^i$ are the Lagrange multipliers ($(\bar\Lambda_j{}^i) = -\Lambda_i{}^j$).

The variation with respect to *zweibein* e_m^a of the action (27) gives the equations (we use that $ee_a^m = -\epsilon_{ab}\epsilon^{mn}e_n^b$)

$$e_m^a = -\frac{1}{T}(\tilde\lambda_{\dot\alpha}\rho^a\partial_m\mu^{\dot\alpha} - \tilde\mu^\alpha\rho^a\partial_m\lambda_\alpha) + \text{c.c.}. \tag{28}$$

For compact notation we introduce the string twistors

$$Z_{Ai} = (\lambda_{\alpha i}, \mu_i^{\dot\alpha}), \qquad \bar Z^{Ai} = (\bar\mu^{\alpha i}, -\bar\lambda_{\dot\alpha}^i), \qquad \tilde Z^{Ai} = \bar Z^{Aj}(\rho^0)_j^i.$$

Then

$$e_m^a = -\frac{1}{T}[\partial_m\tilde Z^{Ai}(\rho^a)_i{}^j Z_{Aj} - \tilde Z^{Ai}(\rho^a)_i{}^j\partial_m Z_{Aj}] \tag{29}$$

and the constraints (26) can be rewritten as

$$V_i^j = Z_{Ai}\bar Z^{Aj} \approx 0. \tag{30}$$

Substituting (29) and (30) in the action (27) we obtain our basic twistorial string action:

$$S = \int d^2\xi\Big\{\frac{1}{4T}\epsilon^{mn}\epsilon_{ab}[\partial_m\tilde Z^{Ai}(\rho^a)_i{}^j Z_{Aj} - \tilde Z^{Ai}(\rho^a)_i{}^j\partial_m Z_{Aj}][\partial_n\tilde Z^{Bi}(\rho^b)_i{}^j Z_{Bj} - \tilde Z^{Bi}(\rho^b)_i{}^j\partial_n Z_{Bj}] + \Lambda A + \bar\Lambda\bar A + \Lambda_j{}^i V_i{}^j\Big\}. \tag{31}$$

Using explicit form of $D = 2$ Dirac matrices we can see that the first term in the action (31) equals to

$$\frac{1}{T}\epsilon^{mn}[\partial_m\bar Z^{A1}Z_{A1} - \bar Z^{A1}\partial_m Z_{A1}][\partial_n\bar Z^{B2}Z_{B2} - \bar Z^{B2}\partial_n Z_{B2}]$$

i.e. the action (31) is induced on the world-sheet by the canonical 2-form (3) with supplemented constraints (23) and (26).

V. FROM SSTV ACTION TO TENSORIAL MOMENTUM FORMULATION

The *zweibein* e_m^a can be expressed from the action (7) as follows ($(\lambda\lambda) \equiv \lambda^{\alpha i}\lambda_{\alpha i}$, $(\bar\lambda\,\bar\lambda) \equiv \bar\lambda_{\dot\alpha}^i\bar\lambda_i^{\dot\alpha}$)

$$e_m^a = \frac{2T}{(\lambda\lambda)(\bar\lambda\,\bar\lambda)}\tilde\lambda_{\dot\alpha}^i(\rho^a)_i^j\lambda_{\alpha j}\partial_m X^{\dot\alpha\alpha} \tag{32}$$

Substitution of the relation (32) in the action (7) provides the following string action

$$S = T\int d^2\xi[(\lambda\lambda)(\bar\lambda\,\bar\lambda)]^{-1}\epsilon_{ab}(\tilde\lambda_{\dot\alpha}\rho^a\lambda_\alpha) \times (\tilde\lambda_{\dot\beta}\rho^b\lambda_\beta)\epsilon^{mn}\partial_m X^{\dot\alpha\alpha}\partial_n X^{\dot\beta\beta} \tag{33}$$

Using identities for $D = 2$ Dirac matrices and the relation

$$\epsilon^{mn}\partial_m X_\alpha^{\dot\alpha}\partial_n X_\beta^{\dot\beta} = \epsilon^{mn}\partial_m X_{(\alpha}^{[\dot\alpha}\partial_n X_{\beta)}^{\dot\beta]} + \epsilon^{mn}\partial_m X_{[\alpha}^{(\dot\alpha}\partial_n X_{\beta]}^{\dot\beta)}$$

after contractions of spinorial indices we obtain the action

$$S = \sqrt{2}\int d^2\xi\epsilon^{mn}(P_{\alpha\beta}\partial_m X^{\dot\gamma\alpha}\partial_n X_{\dot\gamma}^\beta + \bar P_{\dot\alpha\dot\beta}\partial_m X^{\dot\alpha\gamma}\partial_n X_\gamma^{\dot\beta}) \tag{34}$$

where the composite second rank spinors

$$P_{\alpha\beta} = \frac{\sqrt{2}T}{(\lambda\lambda)}\lambda_{(\alpha}^1\lambda_{\beta)}^2, \qquad \bar P_{\dot\alpha\dot\beta} = \frac{\sqrt{2}T}{(\bar\lambda\,\bar\lambda)}\bar\lambda_{(\dot\alpha}^1\bar\lambda_{\dot\beta)}^2. \tag{35}$$

satisfy the constraints

$$P^{\alpha\beta}P_{\alpha\beta} = -\frac{T^2}{4}, \qquad \bar P^{\dot\alpha\dot\beta}\bar P_{\dot\alpha\dot\beta} = -\frac{T^2}{4}. \tag{36}$$

Using fourvector notation the relations (36) take the form

$$P^{\mu\nu}P_{\mu\nu} = -\frac{T^2}{4}, \qquad P^{\mu\nu}\tilde P_{\mu\nu} = 0 \tag{37}$$

where $\tilde P_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\lambda\rho}P^{\lambda\rho}$.

The action (34) is the Ferber-Shirafuji form of the string action with tensorial momenta

$$S = \sqrt{2}\int d^2\xi\Big[P_{\mu\nu}\partial_m X^\mu\partial_n X^\nu\epsilon^{mn} - \Lambda\Big(P^{\mu\nu}P_{\mu\nu} + \frac{T^2}{4}\Big)\Big]. \tag{38}$$

Expressing $P_{\mu\nu}$ by its equation of motion, we get

$$P^{\mu\nu} = \frac{1}{2\Lambda}\Pi^{\mu\nu}, \qquad \Pi^{\mu\nu} \equiv \epsilon^{mn}\partial_m X^\mu \partial_n X^\nu. \tag{39}$$

After substituting (39) in the action (38) we obtain the second-order action (see e.g. [25])

$$S = \frac{1}{2\sqrt{2}} \int d^2\xi [\Lambda^{-1}\Pi^{\mu\nu}\Pi_{\mu\nu} - \Lambda T^2]. \tag{40}$$

Eliminating further Λ and using that (see also (18))

$$\Pi^{\mu\nu}\Pi_{\mu\nu} = 2\det(g_{mn}) \tag{41}$$

we obtain the Nambu-Goto string action (17).

It is important to notice that the solution (39) satisfies the constraint $P^{\mu\nu}\tilde{P}_{\mu\nu} = 0$ as an identity. We see therefore that in the action (38) it is sufficient to impose by the Lagrange multiplier only the first constraint (37).

VI. CONCLUSIONS

We have shown the equivalence of five formulations of $D = 4$ tensionfull bosonic string:

(i) two spacetime formulations, with vectorial string momenta (see (4)) and tensorial ones (see (38));

(ii) two mixed twistor-spacetime SSTV (see (7)) and BZ (see (24)) models;

(iii) the generic pure twistorial formulation with the action given by the formula (31).

Following the massive relativistic particle case (see [13–15]) the main tools in the equivalence proof are the string generalizations of Cartan-Penrose string momenta (see (6) and (35)) and the incidence relations (25). The action (27) in conformal gauge $e^a_m = \delta^a_m$ is the commonly used bilinear action for twistorial string.

We would like to stress that the model (31) is substantially different from the one proposed by Witten *et al.* [9–11]. In Witten twistor string model described by $CP(3|4)$ ($N = 4$ supertwistor) σ-model the targed space is described by a single supertwistor, and the Penrose incidence relation, introducing spacetime coordinates appears only after quantization, as the step permitting the spacetime interpretation of holomorphic twistorial fields. In our approach composite spacetime variables enter already into the formulation of classical string model, in a way enforcing the complete equivalence of classical twistorial string and Nambu-Goto action provided that we treat the spacetime target coordinates as 2-twistor composites.

In this paper we restricted the presentation to the case of $D = 4$ bosonic string. The generalization to $D = 6$ is rather straightforward; the extension to $D = 10$ requires clarification how to introduce the $D = 10$ conformal spinors, i.e. $D = 10$ twistors. Other possible generalizations are the following:

(i) If we quantize canonically the model (27) one can show that the PB of the constraints V_i^j satisfy the internal $U(2)$ algebra (see [26]). One can introduce, contrary to (26), nonvanishing V_i^j. The degrees of freedom described by V_i^j can be interpreted (see also [2,14,15]) as introducing on the string the local density of covariantly described spin components and electric charge;

(ii) We presented here the links between various bosonic string models. Introducing two-supertwistor space and following known supersymmetrization techniques (see [23,24]) one can extend the presented equivalence proofs to the relations between different superstring formulations with manifest world-sheet supersymmetry which involved the twistor variables (see e.g. [18,27–29]).

(iii) Particularly interesting would be the twistorial formulation of $D = 4$ $N = 4$ Green-Schwarz superstring, which should be derivable by dimensional reduction from $D = 10$, $N = 1$ Green-Schwarz superstring. Such twistorial $D = 4$, $N = 4$ supersting model could be in our formulation the counterpart of twistorial $N = 4$ superstring considered in [9–11].

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[17] The indices m, $n = 0, 1$ are vector world-sheet indices, $h_{mn} = e^a_m e_{na}$ is a world-sheet metric, e^a_m is the *zweibein*, $e^a_m e^m_b = \delta^a_b$. The indices $a, b = 0, 1$ are $d = 2$ flat indices. The indices $i, j = 1, 2$ are $d = 2$ Dirac spinor indices. We use bar for complex conjugate quantities, $\bar{\lambda}^i_{\dot{\alpha}} = (\bar{\lambda}_{\alpha i})$, and tilde for Dirac conjugated $d = 2$ spinors, $\tilde{\bar{\lambda}}^i_{\dot{\alpha}} = \bar{\lambda}^j_{\dot{\alpha}}(\rho^0)_j{}^i$. $X^\mu(\xi)$, $\mu, \nu = 0, 1, 2, 3$, is a spacetime vector and world-sheet scalar, $P^m_\mu(\xi)$ is a spacetime vector and a world-sheet vector density.
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