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Path integral quantization corresponding to Faddeev-Jackiw canonical quantization

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We analyze the structure of the Faddeev-Jackiw method and show that the canonical 2-form of the Lagrangian constructed in the last step of the Faddeev-Jackiw method is always nondegenerate. So according to the Darboux theorem, there must exist a coordinate transformation that can transform the Lagrangian into a standard form. We take the coordinates after the transformation as these in a phase space, and use this standard form of the Lagrangian, we achieve its path integral expression over the symplectic space, give the Faddeev-Jackiw canonical quantization of the path integral, and then we further show up the concrete application of the Faddeev-Jackiw canonical quantization of the path integral.

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I. INTRODUCTION

Systems described by singular Lagrangians are called singular systems and this kind of system contains inherent constraints [1,2]. In a lot of physical domains, there extensively exist different singular systems [3–5], such as gauge field theories, gravitational theory, supersymmetric theory, supergravity, superstring theory, and so on. The investigation on inherent constraints has become one basic task of theoretical research in these theories.

The Faddeev-Jackiw method is a kind of quantization method that showed up in the 80's of the last century. This method has some very useful properties of obviating the need to distinguish primary and secondary constraints and the first and the second types of constraints. The method is simpler and does not have such a hypothesis of Dirac's conjecture, thus it has evoked much attention. In the development of the Faddeev-Jackiw method, Floreanini and Jackiw proposed a kind of brackets, defined in the configuration space of a 2-dimensional self-dual field, in order to give the canonical quantization [6]. Subsequently, Faddeev and Jackiw interpreted its reasonability and systematically gave the Faddeev-Jackiw method [7]. In succession, Barcelos-Neto and Wotzasek represented the procedure of dealing with constraints in the Faddeev-Jackiw method [8–10].

However, Faddeev-Jackiw quantization is only one canonical quantization. Up to now, there is still no path integral quantization in Faddeev-Jackiw formalism. So, in this report, we will construct the path integral quantization in the Faddeev-Jackiw formalism by the Darboux theorem, which keeps the property of the Faddeev-Jackiw method, and does not have the need of distinguishing the first and second class constraints and the primary and secondary constraints.

The plan of this letter is: Sec. II investigates the mathematical structure of the Faddeev-Jackiw method, Sec. III proposes the path integral quantization corresponding to Faddeev-Jackiw canonical quantization, Sec. IV gives the application of the path integral quantization to the Schrödinger field, and the last section is a summary and conclusion.

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II. THE MATHEMATICAL STRUCTURE OF THE FADDEEV-JACKIW METHOD

At first, we begin with the Lagrangian $L(q^i, \dot{q}^i)$, $(i = 1, \ldots, N)$ in configuration space. $L(q^i, \dot{q}^i)$ is not always the one-order Lagrangian (a one-order Lagrangian means that there are only one-power terms of general velocities in the Lagrangian [8]). So, if $L(q^i, \dot{q}^i)$ is not the one-order Lagrangian, we must introduce variables of auxiliary fields to transform the Lagrangian into a one-order Lagrangian. Usually, the momenta are chosen as auxiliary fields, but this is not absolute.

Here, after introducing variables of auxiliary fields we obtain the form of the one-order Lagrangian

$$L^{(0)} = a_i(\xi)\dot{\xi}^i - V^{(0)}(\xi), \qquad (i = 1, \ldots, m). \tag{2.1}$$

(2.1) can also be written as

$$L^{(0)}dt = a_i(\xi)d\xi^i - V^{(0)}(\xi)dt. \tag{2.2}$$

The one-form $a_i(\xi)d\xi^i$ can induce the two-form

$$f^{(0)} = d[a_i(\xi)d\xi^i] = \tfrac{1}{2}f^{(0)}_{ij}d\xi^i \wedge d\xi^j, \quad (i, j = 1, \ldots, m), \tag{2.3}$$

where $f^{(0)}_{ij} = \frac{\partial a_j}{\partial \xi^i} - \frac{\partial a_i}{\partial \xi^j}$. It is obvious that $(f^{(0)}_{ij})$ is an antisymmetrical matrix.

Using (2.1) we can deduce Euler-Lagrange equations

$$f^{(0)}_{ij}\dot{\xi}^{(0)j} - \frac{\partial V^{(0)}(\xi^{(0)})}{\partial \xi^{(0)i}} = 0. \tag{2.4}$$

On the other hand, because a lot of general physical processes should satisfy the quantitative causal relation with a no-loss-no-gain character [11–13], e.g., Ref. [14]

uses the no-loss-no-gain homeomorphic map transformation satisfying the quantitative causal relation to gain exact strain tensor formulas in Weitzenböck manifold. In fact, some changes (cause) of some quantities in (2.4) must result in the relative changes (result) of the other quantities in (2.4) so that (2.4)'s right side keeps no-loss-no-gain, i.e., zero, namely, (2.4) also satisfies the quantitative causal relation.

There are two classes for (2.4):

(1) The matrix $(f_{ij}^{(0)})$ is reversible, then (2.4) can be written as

$$\dot{\xi}^{(0)j} = f_{ij}^{(0)-1} \frac{\partial V^{(0)}(\xi^{(0)})}{\partial \xi^{(0)i}}. \tag{2.5}$$

So the general bracket can be defined through the Faddeev-Jackiw method as

$$\{F, G\}^* = \frac{\partial F}{\partial \xi^{(0)i}} f_{ij}^{(0)-1} \frac{\partial G}{\partial \xi^{(0)j}}. \tag{2.6}$$

By (2.6), the quantum commutator can be written as $[\hat{P}, \hat{G}] = i\hbar\{P, G\}^*$, so we complete the quantization. From another angle, we can analyze this Lagrangian.

Two-form $f^{(0)}$ is not a degenerate two-form, so according to the Darboux theorem [15], there exists the coordinate transformation

$$(Q_1(\xi^{(0)}), \ldots, Q_{m/2}(\xi^{(0)}), P_1(\xi^{(0)}), \ldots, P_{m/2}(\xi^{(0)})), \tag{2.7}$$

which transforms (2.1) into

$$L^{(0)} = P_k \dot{Q}_k - V^{(0)}(P, Q), \qquad \left(k = 1, \ldots, \frac{m}{2}\right). \tag{2.8}$$

The corresponding Euler-Lagrange equations are

$$\dot{Q}_k = \frac{\partial V^{(0)}}{\partial P_k}, \qquad \dot{P}_k = -\frac{\partial V^{(0)}}{\partial Q_k}. \tag{2.9}$$

Setting $x_k = Q_k$, $x_{(m/2)+k} = P_k$, (2.9) can be written as $\dot{x}_i = J_{ij}^{-1} \frac{\partial V^{(0)}}{\partial x_j}$, where

$$(J_{ij}^{-1}) = \begin{pmatrix} 0 & I_{m/2} \\ -I_{m/2} & 0 \end{pmatrix},$$

and $I_{m/2}$ is the unit matrix of $(m/2) \times (m/2)$ (m is an even number).

So, according to the Faddeev-Jackiw method, we can construct a general bracket

$$\{F, G\}^* = \frac{\partial F}{\partial x_i} J_{ij}^{-1} \frac{\partial G}{\partial x_j}. \tag{2.10}$$

Then, (2.10) is equivalent to (2.6), i.e., $\{F, G\}^* = \frac{\partial F}{\partial x_i} J_{ij}^{-1} \frac{\partial G}{\partial x_j} = \frac{\partial F}{\partial \xi^i} f_{ij}^{(0)-1} \frac{\partial G}{\partial \xi^j}$, which has merely the difference of the forms under the different formalism of coordinates, and (2.10) is regular and may make the calculation of quantization simple, because J_{ij}^{-1} is regular.

(2) The matrix $(f_{ij}^{(0)})$ is irreversible.

Under this condition, which cannot satisfy the Darboux theorem, there is not a coordinate transformation as (2.7), but we can obtain some inherent constraints according to the Faddeev-Jackiw method.

According to the Faddeev-Jackiw method, $(f_{ij}^{(0)})$ has zero-modes satisfying

$$(\nu_\alpha^{(0)})^T (f_{ij}^{(0)}) = 0. \tag{2.11}$$

We left multiply these zero-modes to the left side of (2.4), and we can obtain

$$\Omega_\alpha^{(0)} = (\nu_\alpha^{(0)})_i^{(0)} \frac{\partial V^{(0)}}{\partial \xi^{i(0)}} = 0, \tag{2.12}$$

which are zero-iterated Faddeev-Jackiw constraints.

Substituting these constraints into the Lagrangian in agreement with the Faddeev-Jackiw method, we can obtain one new Lagrangian

$$L^{(1)} = a_i^{(0)}(\xi^{(0)})\dot{\xi}^{(0)i} + \Omega_\alpha^{(0)} \dot{\lambda}_\alpha^{(0)} - V^{(1)}(\xi^{(0)}), \tag{2.13}$$

where $V^{(1)}(\xi^{(0)}) = V^{(0)}(\xi^{(0)})|_{\Omega^{(0)}=0}$.

We also regard λ_α as symplectic variables, so the symplectic variables become

$$\xi^{(1)} = \{\xi^{(0)}, \lambda^{(0)}\}, \tag{2.14}$$

and $a_{\xi^{(0)i}}^{(1)} = a_i^{(0)}(\xi^{(0)})$, $i = 1, \ldots, n$; $a_{\lambda_\alpha^{(0)}}^{(1)} = \Omega_\alpha^{(0)}(\xi^{(0)})$, $\alpha = 1, \ldots, m$.

Judging whether (2.13) is a singular Lagrangian; if it is, we repeat the above procedures to a new similar Lagrangian, and we can get more constraints and construct a new Lagrangian by using these new constraints, again and again, until we get a nonsingular Lagrangian, i.e., we cannot get more new constraints.

Supposing that after obtaining s constraints after h steps through the Faddeev-Jackiw method, there are no constraints generated, we can construct a h-interated Lagrangian as

$$L^{(h)} = a_i(\xi^{(0)})\dot{\xi}^{(0)i} + \Omega_\alpha \dot{\lambda}_\alpha - V^{(h)}(\xi^{(0)}), \quad (\alpha = 1, \ldots, s), \qquad V^{(h)}(\xi^{(0)}) = V^{(0)}(\xi^{(0)})|_{\Omega=0}. \tag{2.15}$$

In the Faddeev-Jackiw method, the Lagrangian multipliers λ_α are also regarded as new symplectic variables to expand the symplectic variables, and the all new symplectic variables are

$$\xi^{(h)} = (\xi^{(0)1}, \ldots, \xi^{(0)m}, \lambda_1, \ldots, \lambda_s).$$

Thus, (2.15) can be rewritten as

$$L^{(h)} = a^{(h)}_{i'}(\xi^{(0)})\dot{\xi}^{(h)i'} - V^{(h)}(\xi^{(0)}), \quad (i' = 1, \ldots, m + s). \tag{2.16}$$

In the new space of new symplectic variables, the canonical two-form of this Lagrangian is

$$f^{(h)} = d(a^{(h)}_{i'}(\xi^{(0)})d\xi^{(h)i'}) = \tfrac{1}{2}f^{(h)}_{i'j'}d\xi^{(h)i'} \wedge d\xi^{(h)j'}, \quad (i', j' = 1, \ldots, m + s). \tag{2.17}$$

As in (2.4), there are two classes of (2.16).

(i) Two-form $f^{(h)}$ is not degenerate, i.e., $(f^{(h)}_{i'j'})$ is reversible.
Similarly, according to the Darboux theorem, in the new space of new symplectic variables, there exists coordinate transformation $(Q^{(h)}_1(\xi^{(h)}), \ldots, Q^{(h)}_{(m+s)/2}(\xi^{(h)}), P^{(h)}_1(\xi^{(h)}), \ldots, P^{(h)}_{(m+s)/2}(\xi^{(h)}))$, which transforms (2.15) into the following expression

$$L^{(h)} = P^{(h)}_{k'}\dot{Q}^{(h)}_{k'} - V^{(h)}(P^{(h)}, Q^{(h)}), \quad \left(k' = 1, \ldots, \frac{m + s}{2}\right). \tag{2.18}$$

Setting $x^{(h)}_{k'} = Q^{(h)}_{k'}$, $x^{(h)}_{k'+((m+g)/2)} = P^{(h)}_{k'}$ the motion equations of (2.18) can be written as

$$x^{(h)}_{i'} = J^{-1}_{i'j'}\frac{\partial V^{(h)}}{\partial x^{(h)}_{j'}},$$

$$\text{where } (J^{-1}_{i'j'}) = \begin{pmatrix} 0 & I_{(m+s)/2} \\ -I_{(m+s)/2} & 0 \end{pmatrix}, \quad (i', j' = 1, \ldots, m + g). \tag{2.19}$$

So, the general bracket can be rewritten as

$$\{F, G\}^* = \frac{\partial F}{\partial x^{(h)}_{i'}}J^{-1}_{i'j'}\frac{\partial G}{\partial x^{(h)}_{j'}}. \tag{2.20}$$

(ii) Two-form $f^{(h)}$ is degenerate, namely, $(f^{(h)}_{i'j'})$ is irreversible.
So, according to the Faddeev-Jackiw method, there exists gauge symmetry in the system and we must introduce the gauge condition.
In practice, the gauge condition can be regarded as constraints and we construct the new Lagrangian by introducing the gauge condition as constraints, thus, we have

$$L^{(h)} \to \tilde{L}^{(h)} = a^{(0)}_i(\xi^{(0)})\dot{\xi}^{(0)} + \Omega_\alpha\dot{\lambda}_\alpha + \phi_\beta\dot{\lambda}'_\beta - \tilde{V}^{(h)}(\xi^{(0)}), \tag{2.21}$$

where $\tilde{V}^{(h)} = V^{(h)}|_{\phi^\beta_{gf}=0}$, and ϕ_{gf} is the gauge condition.
Consequently, the variables ξ^i, λ_α, λ'_β form a larger symplectic space, and the symplectic variables are

$$\xi''^{(h)} = (\xi^{(0)1}, \ldots, \xi^{(0)m}, \lambda_1, \ldots, \lambda_s, \lambda'_1, \ldots, \lambda'_g). \tag{2.22}$$

Thus, (2.21) can be written as

$$\tilde{L}^{(h)} = a^{(h)}_{i''}(\xi^{(0)})\dot{\xi}''^{(h)i''} - \tilde{V}^{(h)}(\xi^{(0)}), \quad (i'' = 1, \ldots, m + s + g). \tag{2.23}$$

The canonical two-form of (2.23) is nondegenerate, so there exists the transformation $((Q''^{(h)}_1(\xi''^{(h)}), \ldots, Q''^{(h)}_{(m+s+g)/2}(\xi''^{(h)}), P''^{(h)}_1(\xi''^{(h)}), \ldots, P''^{(h)}_{(m+s+g)/2}(\xi''^{(h)}))$ which transforms (2.23) into

$$\tilde{L}^{(h)} = P''^{(h)}_{k''}\dot{Q}''^{(h)}_{k''} - \tilde{V}^{(h)}(P''^{(h)}, Q''^{(h)}). \tag{2.24}$$

So, it is easy to construct the general bracket as the above.

In conclusion, in any case in the Faddeev-Jackiw method, at the last step, we can always obtain the new Lagrangian that has nondegenerate canonical two-form, so according to the Darboux theorem, there must be a special transformation which can transform the new Lagrangian into a standard form as (2.8) and (2.18) or (2.24). From the mathematical view, the key of the Faddeev-Jackiw method is just to construct such a Lagrangian that satisfies the Darboux theorem, and the Faddeev-Jackiw canonical quantization is established on such a form of the Lagrangian. Moreover, in the next section, we can find that it is also the crucial fact in the construction of the path integral quantization corresponding to Faddeev-Jackiw canonical quantization.

III. THE PATH INTEGRAL QUANTIZATION CORRESPONDING TO FADDEEV-JACKIW CANONICAL QUANTIZATION

In this section, such a conclusion is reached, that is, no matter what form is the Lagrangian, in the Faddeev-Jackiw method, at last, we can always obtain a new Lagrangian satisfying the Darboux theorem, which can transform the Lagrangian into a standard form.

So, supposing we have completed all the procedure of the Faddeev-Jackiw method, and obtain a new Lagrangian

$$\tilde{L}^{(h)} = a^{(0)}_{i'}(\xi^{(0)})\dot{\xi}^{(0)i'} + \Omega_\alpha\dot{\lambda}_\alpha + \phi_\beta\dot{\lambda}_\beta - \tilde{V}^{(h)}(\xi^{(0)}), \quad (i' = 1, \ldots, m; \alpha = 1, \ldots, s; \beta = 1, \ldots, g), \tag{3.1}$$

where Ω_α are the constraints and ϕ_β are gauge conditions (if there is no gauge symmetry, gauge conditions vanish).

According to the Darboux theorem, we can transform (3.1) into

$$\tilde{L}^{(h)} = P_i \dot{Q}_i - \tilde{V}^{(h)}(P, Q), \qquad \left(i = 1, \ldots, \frac{m+s+g}{2}\right). \tag{3.2}$$

The motion equations of (3.2) are

$$\dot{P}_i = -\frac{\partial \tilde{V}^{(h)}}{\partial Q_i}, \qquad \dot{Q}_i = \frac{\partial \tilde{V}^{(h)}}{\partial P_i}. \tag{3.3}$$

And the general brackets are

$$\{P_i, Q_j\}^* = \delta_{ij}, \qquad \{P_i, P_j\}^* = \{Q_i, Q_j\}^* = 0. \tag{3.4}$$

Consequently, according to the Faddeev-Jackiw quantization, the quantum commutators are obtained as follows

$$[\hat{Q}_i, \hat{P}_j] = i\hbar\delta_{ij}, \qquad [\hat{P}_i, \hat{P}_j] = [\hat{Q}_i, \hat{Q}_j] = 0. \tag{3.5}$$

From (3.5), we can see that the quantum commutative relation between $\hat{Q}$, $\hat{P}$ is similar to the relation between the canonical coordinate operators and the momentum operators of the nonsingular system. So, being similar to the usual Feynman path integral, it is easy to obtain the quantum transition amplitude of (3.2) as follows

$$Z[0] = \int DQ_i(\tau)DP_i(\tau)\exp\left\{\frac{i}{\hbar}\int_t^{t'}[P_i(\tau)\dot{Q}_i(\tau) - \tilde{V}^{(h)}(Q(\tau), P(\tau))]d\tau\right\}. \tag{3.6}$$

And, (3.6) can be written by the coordinates $(\xi^{(0)i'}, \lambda_\alpha, \lambda_\beta)$ as

$$\begin{aligned} Z[0] = \int & D\xi^{(0)i'}(\tau)D\lambda_\alpha(\tau)D\lambda_\beta(\tau) \\ &\times \prod_{j=0}^{\infty} J(\xi^{(0)i'}(\tau_j), \lambda_\alpha(\tau_j), \lambda_\alpha(\tau_j)) \\ &\times \exp\left\{\frac{i}{\hbar}\int_t^{t'}[a_{i'}^{(0)}(\xi^{(0)})\dot{\xi}^{(0)i'} + \Omega_\alpha\dot{\lambda}_\alpha + \phi_\beta\dot{\lambda}_\beta \right. \\ &\left. - \tilde{V}^{(h)}(\xi^{(0)})]\right\}, \end{aligned} \tag{3.7}$$

where

$$DQ_i(\tau)DP_i(\tau) = D\xi^{(0)i'}(\tau)D\lambda_\alpha(\tau)D\lambda_\beta(\tau) \times \prod_{j=0}^{\infty} J(\xi^{(0)i'}(\tau_j), \lambda_\alpha(\tau_j), \lambda_\alpha(\tau_j)), \tag{3.8}$$

and in which $\prod_{j=0}^{\infty} J(\xi^{(0)i'}(\tau_j), \lambda_\alpha(\tau_j), \lambda_\alpha(\tau_j))$ is the Jacobian determinant of the transformation. τ_j denotes the time point of the time interval $[t, t']$, which is divided into infinitesimal equal parts of length ε and $\tau_i = j\varepsilon + t (j = 1, 2, \ldots, \infty)$. So, in fact, we can rewrite the Jacobian determinant as

$$\begin{aligned} &\prod_{j=0}^{\infty} J(\xi^{(0)i'}(\tau_j), \lambda_\alpha(\tau_j), \lambda_\alpha(\tau_j)) \\ &= \exp\left\{\sum_{j=0}^{\infty} \ln J(\xi^{(0)i'}(\tau_j), \lambda_\alpha(\tau_j), \lambda_\alpha(\tau_j))\right\} \\ &= \exp\left\{\int_t^{t'} d\tau[\ln(J(\xi^{(0)i'}(\tau), \lambda_\alpha(\tau), \lambda_\alpha(\tau))]/\varepsilon\right\}. \end{aligned} \tag{3.9}$$

Then there are two kinds of situations:

(i) When the Jacobian determinant $J = 1$ in (3.9), then $\ln J = 0$, in this time, (3.9) reduces to the unit element. This is the usual discussion situation in general field theories.

(ii) When the Jacobian determinant $J \neq 1$ in (3.9), then (3.7) can be rewritten as

$$\begin{aligned} Z[0] &= \int D\xi^{(0)i'}(\tau)D\lambda_\alpha(\tau)D\lambda_\beta(\tau) \\ &\quad \times \exp\left\{\frac{i}{\hbar}\int_t^{t'}[a_{i'}^{(0)}(\xi^{(0)})\dot{\xi}^{(0)i'} + \Omega_\alpha\dot{\lambda}_\alpha \right. \\ &\quad + \phi_\beta\dot{\lambda}_\beta - \tilde{V}^{(h)}(\xi^{(0)}) \\ &\quad \left. - i\hbar[\ln(J(\xi^{(0)i'}(\tau), \lambda_\alpha(\tau), \lambda_\alpha(\tau))]/\varepsilon]d\tau\right\} \\ &= \int D\xi^{(0)i'}(\tau)D\lambda_\alpha(\tau)D\lambda_\beta(\tau) \\ &\quad \times \exp\left\{\frac{i}{\hbar}\int_t^{t'} L'(\xi^{(0)i'}(\tau), \lambda_\alpha(\tau), \lambda_\beta(\tau))d\tau\right\}, \end{aligned} \tag{3.10}$$

where the Lagrangian $L(\xi^{(0)i'}(\tau), \lambda_\alpha(\tau), \lambda_\beta(\tau))$ is revised as $L'(\xi^{(0)i'}(\tau), \lambda_\alpha(\tau), \lambda_\beta(\tau))$ (called an equivalently extended Lagrangian) by the contribution coming from the Jacobian determinant J, which is just the reason that the Jacobian determinant J is usually taken as a unit element in general field theories. Information about the studies on the case of $J \neq 1$ will be given in another paper.

Therefore, (3.7) is the representation of the quantum transition amplitude with the symplectic variables, and this is one procedure of path integral quantization. Moreover, from the above analysis, we can consider it corresponding to Faddeev-Jackiw canonical quantization of the path integral.

IV. APPLICATION OF THE PATH INTEGRAL QUANTIZATION TO THE SCHRÖDINGER FIELD

The Lagrangian density of the Schrödinger field is [16]

$$\mathcal{L} = \frac{i\hbar}{2}(\phi^*\dot{\phi} - \phi\dot{\phi}^*) - \frac{\hbar^2}{2m}\nabla\phi^*\cdot\nabla\phi - V(r)\phi^*\phi, \tag{4.1}$$

the Euler-Lagrange equation is just the Schrödinger equation

$$i\hbar\dot{\phi} + \frac{\hbar^2}{2m}\nabla^2\phi - V(r)\phi = 0. \tag{4.2}$$

The Dirac primary constraints of (4.1) are

$$\Phi^1 = \pi - \frac{i\hbar}{2}\phi^* \approx 0, \qquad \Phi^2 = \pi^* + \frac{i\hbar}{2}\phi \approx 0. \tag{4.3}$$

The Hamiltonian density is

$$\mathcal{H}_e = \frac{\hbar^2}{2m}\nabla\phi^* \cdot \nabla\phi + V(r)\phi^*\phi, \tag{4.4}$$

and there exist no secondary constraints.

Consequently, according to the Faddeev-Senjanovic path integral quantization, the quantum transition amplitude is obtained as follows

$$Z[0] = \int D\phi D\phi^* D\pi D\pi^* \delta\left(\pi - \frac{i\hbar}{2}\phi^*\right)\delta\left(\pi^* + \frac{i\hbar}{2}\phi\right) \times \exp\left\{\frac{i}{\hbar}\int d^4x\left[\pi\dot{\phi} + \pi^*\dot{\phi}^* - \frac{\hbar^2}{2m}\nabla\phi^*\cdot\nabla\phi + V(r)\phi^*\phi\right]\right\}. \tag{4.5}$$

Calculating the integral of (4.5) over the π, π^*, we obtain

$$Z[0] = \int D\phi D\phi^* \exp\left\{\frac{i}{\hbar}\int d^4x\left[\frac{i\hbar}{2}(\phi^*\dot{\phi} - \phi\dot{\phi}^*) - \frac{\hbar^2}{2m}\nabla\phi^*\cdot\nabla\phi + V(r)\phi^*\phi\right]\right\}. \tag{4.6}$$

When obtaining (4.6), we complete Dirac-Senjanovic quantization, and adding the external source to (4.6), we can obtain the generating functional of the Green function.

Now, we turn to the Faddeev-Jackiw formalism.

Setting $\xi^1 = \phi$, $\xi^2 = \phi^*$, then (4.1) can be written as

$$\mathcal{L} = \frac{i\hbar}{2}(\xi^2\dot{\xi}^1 - \xi^1\dot{\xi}^2) - \frac{\hbar^2}{2m}\nabla\xi^1\cdot\nabla\xi^2 - V(r)\xi^1\xi^2, \tag{4.7}$$

which is just a first-order Lagrangian density, and the symplectic matrix of (4.7) is

$$f_{ij}(r', r'') = i\hbar\begin{pmatrix} 0 & -\delta(r' - r'') \\ \delta(r' - r'') & 0 \end{pmatrix}, \tag{4.8}$$

(4.8) is invertible, the inverse of (4.8) is

$$f_{ij}^{-1}(r', r'') = \frac{1}{i\hbar}\begin{pmatrix} 0 & \delta(r' - r'') \\ -\delta(r' - r'') & 0 \end{pmatrix}. \tag{4.9}$$

So, according to the Darboux theorem, there must be the transformation to transform (4.7) into the standard form.

Then, we can have

$$Q = \frac{\sqrt{2}}{2}(\xi^1 + \xi^2), \qquad P = -\frac{i\hbar\sqrt{2}}{2}(\xi^1 - \xi^2),$$

or we obtain

$$\xi^1 = \frac{\sqrt{2}}{2}Q + \frac{\sqrt{2}i}{2\hbar}P, \qquad \xi^2 = \frac{\sqrt{2}}{2}Q - \frac{\sqrt{2}i}{2\hbar}P, \tag{4.10}$$

which is just the transformation satisfying the Darboux theorem.

Then (4.7) can be rewritten as

$$\mathcal{L} = \frac{1}{2}(P\dot{Q} - Q\dot{P}) - \frac{\hbar^2}{4m}\left[(\nabla Q)^2 + \left(\frac{\nabla P}{\hbar}\right)^2\right] - V\left(Q^2 + \frac{P^2}{\hbar^2}\right). \tag{4.11}$$

So, as the analysis of Sec. III, the quantum transition amplitude can be expressed as

$$Z[0] = \int DQDP\exp\left\{\frac{i}{\hbar}\int d^4x\left[\frac{1}{2}(P\dot{Q} - Q\dot{P}) - \frac{\hbar^2}{4m}(\nabla Q)^2 - \frac{h^2}{4m}\left(\frac{\nabla P}{\hbar}\right)^2 - \frac{V}{2}\left(Q^2 + \frac{P^2}{\hbar^2}\right)\right]\right\}. \tag{4.12}$$

At last, we transform (4.12) into the path integral expressions, respectively, over the symplectic variables ξ^1, ξ^2 and the ϕ, ϕ^* as

$$\begin{aligned} Z[0] &= \int D\xi^1 D\xi^2 \exp\left\{\frac{i}{\hbar}\int dx^4\left[\frac{i\hbar}{2}(\xi^2\dot{\xi}^1 - \xi^1\dot{\xi}^2) - \frac{\hbar^2}{2m}\nabla\xi^1\cdot\nabla\xi^2 - V(r)\xi^1\xi^2\right]\right\} \\ &= \int D\phi D\phi^* \exp\left\{\frac{i}{\hbar}\int d^4x\left[\frac{i\hbar}{2}(\phi^*\dot{\phi} - \phi\dot{\phi}^*) - \frac{\hbar^2}{2m}\nabla\phi^*\cdot\nabla\phi + V(r)\phi^*\phi\right]\right\}, \end{aligned} \tag{4.13}$$

where the Jacobian determinant of (4.10) is independent of the variables, then, the determinant can be eliminated from their transition amplitudes.

From (4.13), we can see that the quantum transition amplitude (4.13) is identical with the transition amplitude (4.6). Therefore, for the Schrödinger field, we concretely proved that the path integral quantization corresponding to the Faddeev-Jackiw canonical quantization is equivalent to the Dirac-Senjanovic path integral quantization.

V. SUMMARY AND CONCLUSION

In any case, in the last step of the Faddeev-Jackiw method, we show that a one-power Lagrangian is able to be transformed into the Lagrangian whose canonical two-form is nondegenerate, so according to the Darboux theorem, there exists a kind of special transformation which transforms the Lagrangian into a standard form, and after this transformation, we can regard the space of new variables as a phase space, thus we achieve the path integral expression in this new space, and, at last, transform the quantum transition amplitude into that over the symplectic variables. Therefore, we obtain the path integral quantization expression in the Faddeev-Jackiw formalism. In succession, we use this method concretely to quantify the Schrödinger field and concretely show that the path integral expression is equivalent to the Faddeev-Senjanovic path integral expression.

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