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Mesons and diquarks in neutral color superconducting quark matter with β equilibrium

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The spectrum of meson and diquark excitations in cold color superconducting (2SC) quark matter is investigated under local color and electric neutrality constraints with β equilibrium. A two-flavored Nambu-Jona-Lasinio–type model including baryon μ_B, color μ_8, and electric μ_Q chemical potentials is used. The contribution from free electrons to the free energy is added to take into account the β equilibrium. The sensitivity of the model to the tuning of the interaction constants in the diquark (H) and quark-antiquark (G) channels is examined for two different parametrization schemes by choosing the ratio H/G to be 3/4 and 1, respectively. The gapless neutral color superconductivity is realized at $H = 3G/4$, and the gapped neutral color superconductivity at $H = G$. It is shown that color and electrical neutrality together with β equilibrium lead to a strong mass splitting within the pion isotriplet in the 2SC phase (both gapped and gapless), in contrast with non-neutral matter. The π- and σ-meson masses are evaluated to be $\sim$300 MeV. It is also shown that the properties of the physical $SU(2)_c$-singlet diquark excitation in the 2SC ground state vary for different parametrization schemes. Thus, for $H = 3G/4$ one finds a heavy resonance with mass $\sim$1100 MeV in the non-neutral (gapped) case, whereas, if neutrality is imposed, a stable diquark with mass $\sim|\mu_Q| \sim 200$ MeV appears in the gapless 2SC environment. For a stronger attraction in the diquark channel ($H = G$), there is again a resonance (with the mass $\sim$300 MeV) in the neutral gapped 2SC phase. Hence, the existence of the stable massive $SU(2)_c$-singlet diquark excitation is a new peculiarity of the gapless 2SC. In addition, the behavior of the diquark mass in vacuum, i.e., at $\mu_B = 0$, as a function of H has been investigated.

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I. INTRODUCTION

According to modern theoretical observations made in the framework of perturbative QCD, at asymptotically high baryonic densities and low temperatures the strongly interacting quark matter is expected to undergo a phase transition to the color superconducting state [1,2]. Unfortunately, a perturbative QCD analysis is not applicable at moderate baryon densities (which might exist inside compact stars or in heavy-ion collision experiments) and a study is usually done with the help of effective theories, such as the Nambu-Jona-Lasinio (NJL) model [3–5].

In spite of the lack of quark confinement in the NJL model, it successfully describes low-energy pseudoscalar and vector mesons in the hadronic phase (see, e.g., [6,7]). This success is provided by the fact that many of light meson properties, e.g., masses, are driven by chiral symmetry, rather than by confinement. Moreover, the chiral phase transition at high temperatures and/or density expected in QCD is naturally described by the NJL model [7,8]. The NJL model is also well suited for the consideration of a hot and/or dense medium under the influence of external conditions [9,10] as well as for the investigation of different physical processes in it [11,12].

In the earlier studies of color superconductivity [13–15] for the case of two-flavor quark matter (u and d quarks), only the influence of baryonic density was taken into account. From these investigations, it became evident that the two-flavor color superconducting phase (2SC) might yet be present at rather small values of $\mu_B \sim$ 1 GeV, i.e., at baryon densities only several times larger than the density of ordinary nuclear matter (see reviews [14,16,17]). This is just the density of compact star cores.

The quark matter inside compact stars is considered to be electrically and color neutral in a bulk. Moreover, there must be an equilibrium between the gain and loss in the β decay $d \rightarrow u + e + \bar{\nu}_e$ (here e is the electron and ν_e is the electron neutrino), the so-called β equilibrium. All these physical constraints must be taken into account when studying the equations of state for a compact star. To do this in the NJL model, additional chemical potentials related to the electric charge density of quarks and electrons as well as to color charges must be introduced. Recently, an intensive theoretical study of neutral color superconducting quark matter has been given (see, e.g., [16–26]), which revealed a new possible ground state of the 2SC phase, where some additional number of quasiparticles with a gapless dispersion law appeared [19] (it is the so-called gapless color superconductivity g2SC, an antipode to the usual gapped 2SC).

There is a great interest in the study of different excitations of the 2SC phase, as the application of its results in some related investigations, e.g. in astrophysics and heavy-ion collision experiments, may help to reveal some observable effects evidencing the formation of a quark-gluon plasma. In particular, the bosonic excitations of the 2SC phase ground state, such as π and σ mesons as well as diquarks, are expected to be copiously produced in a dense medium with rather strong correlations between quarks and antiquarks (such is indeed the case at moderate baryonic densities) and affect some scattering and decay processes to a visible effect. Diquarks on their own are also of great interest in hadron physics because of their importance in determining baryon properties [27]. As to the compact stars, one may find an influence of these particles on the equation of state and on the cooling process. Moreover, in dense matter, the deconfinement phase transition might be accompanied by the appearance of Bose-Einstein condensed diquark matter [28], etc.

In our recent papers [29,30], we have studied the masses of mesons and diquarks that are formed in cold ($T = 0$) and dense quark matter in the framework of a two-flavored NJL model with baryon chemical potential μ_B. In particular, it was shown that in the 2SC phase the meson masses lie in the interval $330 \div 500$ MeV, depending on the values of $\mu_B \in (1050, 1200)$ MeV. Since the original $SU(3)_c$ color symmetry of the model is spontaneously broken down in this phase to $SU(2)_c$, one may expect the appearance of five Nambu-Goldstone bosons. However, we have proved that the abnormal number of three, instead of five, massless bosons is allowed for the diquark sector of the model. In addition, there are two light diquarks as well as a heavy diquark resonance that is an $SU(2)_c$ singlet with the mass $\sim$1100 MeV. Qualitatively, the local color neutrality constraint does not affect the masses of mesons or the $SU(2)_c$-singlet diquark. However, in this case only one Nambu-Goldstone boson and four light diquarks are present in the 2SC phase in the NJL model [31] (see also the discussion at the end of the paper). At nonzero temperature some of the properties of mesons and diquarks in a strongly interacting quark matter were discussed in [32,33].

In the present paper, we continue our investigation of mass spectra for mesons and diquarks, imposing the local electrical neutrality and β equilibrium to the cold 2SC medium, in addition to the color neutrality. As in our previous papers, we use a two-flavored NJL, where additional color (μ_8) and electric (μ_Q) chemical potentials are introduced. It will be shown that, in the 2SC phase, the color neutrality constraint supplemented by the electrical neutrality and β equilibrium drastically changes the mass spectrum of the π and σ mesons and $SU(2)_c$ diquark, in comparison with non-neutral quark matter.

The paper is organized as follows. In Sec. II, the thermodynamic potential (TDP) as well as the effective action of the NJL model, extended with baryon (μ_B), color (μ_8), and electric (μ_Q) chemical potentials, are obtained in the one-loop approximation in β equilibrium. Further, in Sec. III, the gap equations and the phase diagram of quark matter are investigated under the local color and electrical neutrality constraints. Here, the behavior of μ_8 and μ_Q vs μ_B are obtained for neutral quark matter with 2SC-type color superconductivity within two different parametrization schemes: $H = 3G/4$ and $H = G$. In the first case, $H = 3G/4$, a gapless 2SC phase revealed itself, whereas for $H = G$ a gapped phase is preferred. In Secs. IV and V, some peculiarities of the mass spectra of the π, σ mesons and scalar diquarks are investigated both in the gapless and gapped neutral 2SC phases. (In addition, the influence of the diquark channel coupling constant on the diquark mass in the vacuum, i.e. at $\mu_B = 0$, is also considered.) Finally, in Appendix B, the expression for the quark propagator in the Nambu-Gorkov representation is obtained.

II. THE MODEL AND THE EFFECTIVE ACTION

Our investigation is based on the NJL-type model with two quark flavors. Its Lagrangian describes the interaction in the quark-antiquark as well as scalar diquark channels:

$$L_q = \bar{q}[\gamma^\nu i\partial_\nu - m]q + G[(\bar{q}q)^2 + (\bar{q}i\gamma^5\vec{\tau}q)^2] + H\sum_{A=2,5,7}[\bar{q}^C i\gamma^5\tau_2\lambda_A q][\bar{q}i\gamma^5\tau_2\lambda_A q^C], \tag{1}$$

where the quark field $q \equiv q_{i\alpha}$ is a flavor doublet ($i = 1, 2$ or $i = u, d$) and color triplet ($\alpha = 1, 2, 3$ or $\alpha = r, g, b$) as well as a four-component Dirac spinor; $q^C = C\bar{q}^t$ and $\bar{q}^C = q^t C$ are charge-conjugated spinors, and $C = i\gamma^2\gamma^0$ is the charge conjugation matrix (the symbol t denotes the transposition operation). It is supposed that up and down quarks have an equal current (bare) mass m. Furthermore, τ_a stands for Pauli matrices and λ_A for Gell-Mann matrices in flavor and color space, respectively. Clearly, the Lagrangian L_q is invariant under transformations from color $SU(3)_c$ as well as baryon $U(1)_B$ groups. In addition, at $m = 0$ this Lagrangian is invariant under the chiral $SU(2)_L \times SU(2)_R$ group. At $m \neq 0$ the chiral symmetry is broken to the diagonal isospin subgroup $SU(2)_I$ with the generators $I_k = \tau_k/2$ ($k = 1, 2, 3$). Moreover, in our system the electric charge is conserved, too, since $Q = I_3 + B/2$, where I_3 is the third generator of the isospin group $SU(2)_I$, Q is the electric charge generator, and B is the baryon charge generator [evidently, these quantities are unit matrices in color space, but in flavor space they are $Q = \mathrm{diag}(2/3, -1/3)$, $I_3 = \mathrm{diag}(1/2, -1/2)$, and $B = \mathrm{diag}(1/3, 1/3)$]. If the Lagrangian (1) is obtained from the QCD one-gluon exchange approximation, then $H = 3G/4$. In addition, we find it interesting to deal with another relation between coupling constants, $H = G$, which results in qualitatively different model properties (see below).

In order to take into account β equilibrium, we include electrons in our consideration, extending the Lagrangian as follows:

$$L_{qe} = L_q + \bar{e}\gamma^\nu i\partial_\nu e. \tag{2}$$

Here e is the electron spinor field (for simplicity, electrons are taken to be massless). Clearly, the Lagrangian (2) is well suited for the description of different processes in the vacuum, i.e. in the empty space. Since the prime object of the present paper is the consideration of dense medium properties, we extend the Lagrangian (2) by including terms with charge densities and chemical potentials, as it is usually done in statistical physics,

$$L = L_{qe} + \mu_B N_B + \mu_Q N_Q + \mu_8 N_8. \tag{3}$$

In (3), N_B, N_Q, N_8 are baryon, electric, and 8th-color charge density expressions, respectively; μ_B, μ_Q, μ_8 are the corresponding chemical potentials. Recall that

$$N_B = \bar{q}B\gamma^0 q, \qquad N_Q = \bar{q}Q\gamma^0 q - \bar{e}\gamma^0 e, \qquad N_8 = \bar{q}T_8\gamma^0 q, \tag{4}$$

where $T_8 \equiv \sqrt{3}\lambda_8 = \mathrm{diag}(1, 1, -2)$ is a matrix in the color space. From Eqs. (4), we have

$$\mu_B N_B + \mu_Q N_Q + \mu_8 N_8 = \mu_e \bar{e}\gamma^0 e + \sum_{i,\alpha} \mu_{i\alpha}\bar{q}_{i\alpha}\gamma^0 q_{i\alpha} \equiv \mu_e \bar{e}\gamma^0 e + \bar{q}\,\hat{\mu}\,\gamma^0 q, \tag{5}$$

where μ_e is the electron number chemical potential, and $\mu_{i\alpha}$ is the chemical potential for the number of quarks with color α and flavor i. Obviously, one has

$$\begin{aligned} \mu_{ur} &= \mu_{ug} = \frac{\mu_B}{3} + \frac{2\mu_Q}{3} + \mu_8, \\ \mu_{dr} &= \mu_{dg} = \frac{\mu_B}{3} - \frac{\mu_Q}{3} + \mu_8, \\ \mu_{ub} &= \frac{\mu_B}{3} + \frac{2\mu_Q}{3} - 2\mu_8, \\ \mu_{db} &= \frac{\mu_B}{3} - \frac{\mu_Q}{3} - 2\mu_8, \\ \mu_e &= -\mu_Q, \end{aligned} \tag{6}$$

$$\hat{\mu} = \mu_B/3 + \mu_Q Q + \mu_8 T_8 = \tilde{\mu} + \delta\mu\tau_3 + \mu_8 T_8, \tag{7}$$

where the last equality in (7) is obtained due to the above-mentioned relation $Q = I_3 + B/2$; moreover, $\tilde{\mu} = \mu_B/3 + \mu_Q/6$, $\delta\mu = \mu_Q/2$. It follows from (6) that $\mu_{d\alpha} = \mu_{u\alpha} + \mu_e$ for each color α. The matrix $\hat{\mu}$ is diagonal in the six-dimensional (color) $\times$ (flavor) space, and its matrix elements are just the quantities $\mu_{i\alpha}$ from (6).

If all chemical potentials in (3) are nonzero and independent quantities, then $SU(3)_c$ and $SU(2)_I$ are not the symmetry groups of this Lagrangian. Instead, due to the μ_8- and μ_Q-terms, it is symmetric under the reduced color $SU(2)_c \times U(1)_{\lambda_8}$ and flavor $U(1)_{I_3}$ groups. With the local neutrality imposed, the chemical potentials in (3) are, however, no longer independent quantities of the model. Equating further $\langle N_Q\rangle$ and $\langle N_8\rangle$ to zero, the chemical potentials are subjected to two constraints, thereby fixing two of them. As a result, μ_Q and μ_8 become dependent on μ_B. It turns out (see below) that there exists a critical value of the baryon chemical potential μ_B^c in the locally neutral matter which separates two phases: if $\mu_B < \mu_B^c$, the normal quark matter phase with $\mu_Q = 0$ and $\mu_8 = 0$ is formed, and the Lagrangian (3) is an $SU(3)_c \times SU(2)_I$ invariant one; if $\mu_B > \mu_B^c$ (2SC phase), the μ_Q and μ_8 are not already equal to zero and have a nontrivial μ_B dependence. It means that at $\mu_B = \mu_B^c$ the $SU(3)_c \times SU(2)_I$ symmetry of the Lagrangian (3) is explicitly (not spontaneously) broken by the chemical potential terms (i.e. no Nambu-Goldstone bosons must appear) to the color $SU(2)_c \times U(1)_{\lambda_8}$ and flavor $U(1)_{I_3}$ groups. (Note that the color $U(1)_{\lambda_8}$ group is broken spontaneously in the 2SC phase; see below.) Because of the above-mentioned flavor symmetry transformation in the critical point μ_B^c, one could expect that all pions would have equal masses for $\mu_B < \mu_B^c$, whereas at larger μ_B ($\mu_B > \mu_B^c$) the pion mass splitting should occur in neutral matter.

To study the phase diagram of the system and the mass spectra of meson and diquark excitations, we need to get the thermodynamic potential as well as an effective action up to second order for the bosonic degrees of freedom. Since electrons and quarks are not mixing, the total thermodynamic potential Ω of the system is the sum of its electronic Ω_e and quark Ω_q parts: $\Omega = \Omega_q + \Omega_e$. It is well known that $\Omega_e = -\mu_e^4/12\pi^2$. To obtain Ω_q, we start from the Lagrangian describing the quark contribution only [see (3)],

$$\mathcal{L} = L_q + \bar{q}\,\hat{\mu}\,\gamma^0 q, \tag{8}$$

where $\hat{\mu}$ is the quark number chemical potential matrix, defined in (7). The linearized version of the Lagrangian (8) that contains auxiliary bosonic fields has the following form:

$$\begin{aligned} \tilde{\mathcal{L}} = {}& \bar{q}[\gamma^\nu i\partial_\nu + \hat{\mu}\gamma^0 - \sigma - m - i\gamma^5\pi_a\tau_a]q \\ & - \frac{1}{4G}[\sigma\sigma + \pi_a\pi_a] - \frac{1}{4H}\Delta_A^*\Delta_A \\ & - \frac{\Delta_A^*}{2}[\bar{q}^C i\gamma^5\tau_2\lambda_A q] - \frac{\Delta_A}{2}[\bar{q}i\gamma^5\tau_2\lambda_A q^C], \end{aligned} \tag{9}$$

where, here and later, a summation over repeated indices $a = 1, 2, 3$ and $A, A' = 2, 5, 7$ is implied. Clearly, the Lagrangians (8) and (9) are equivalent, as can be seen by using the equations of motion for bosonic fields, which take the form

$$\begin{aligned} \sigma(x) &= -2G(\bar{q}q), \\ \pi_a(x) &= -2G(\bar{q}i\gamma^5\tau_a q), \\ \Delta_A(x) &= -2H(\bar{q}^C i\gamma^5\tau_2\lambda_A q), \\ \Delta_A^*(x) &= -2H(\bar{q}i\gamma^5\tau_2\lambda_A q^C). \end{aligned} \tag{10}$$

One can easily see from (10) that the mesonic fields $\sigma(x)$, $\pi_a(x)$ are real quantities, i.e. $(\sigma(x))^\dagger = \sigma(x)$, $(\pi_a(x))^\dagger = \pi_a(x)$ (the superscript symbol $\dagger$ denotes the Hermitian conjugation), but all diquark fields $\Delta_A(x)$ are complex scalars, so $(\Delta_A(x))^\dagger = \Delta_A^*(x)$. Clearly, the real $\sigma(x)$ and $\pi_a(x)$ fields are color singlets, whereas scalar diquarks $\Delta_A(x)$ form a color antitriplet $\bar{3}_c$ of the $SU(3)_c$ group. If some of the scalar diquark fields have a nonzero ground state expectation value, i.e. $\langle\Delta_A(x)\rangle \neq 0$, the color symmetry of the model (8) is spontaneously broken down.

It is more convenient to perform our investigations in terms of the semibosonized Lagrangian (9), since in this case we have a common footing for obtaining both the thermodynamic potential and the effective action of the model. Indeed, in the one-fermion-loop approximation, the effective action $\mathcal{S}_{\rm eff}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*)$ of the model (9) is expressed by means of the path integral over quark fields:

$$\exp(i\mathcal{S}_{\rm eff}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*)) = N' \int [d\bar{q}][dq] \times \exp\left(i\int \tilde{\mathcal{L}} d^4x\right),$$

where

$$\mathcal{S}_{\rm eff}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) = -\int d^4x \left[\frac{\sigma^2 + \pi_a^2}{4G} + \frac{\Delta_A\Delta_A^*}{4H}\right] + \tilde{\mathcal{S}}_{\rm eff}, \tag{11}$$

and N' is a normalization constant. The quark contribution to the effective action, i.e. the term $\tilde{\mathcal{S}}_{\rm eff}$ in (11), is given by

$$\exp(i\tilde{\mathcal{S}}_{\rm eff}) = N' \int [d\bar{q}][dq] \exp\left(\frac{i}{2}\int [\bar{q}D^+q + \bar{q}^C D^- q^C - \bar{q}Kq^C - \bar{q}^C K^* q] d^4x\right). \tag{12}$$

In (12) we have used the following notations:

$$\begin{aligned} D^+ &= i\gamma^\nu\partial_\nu - m + \hat{\mu}\gamma^0 - \Sigma, \\ D^- &= i\gamma^\nu\partial_\nu - m - \hat{\mu}\gamma^0 - \Sigma^t, \\ \Sigma &= \sigma(x) + i\gamma^5\pi_a(x)\tau_a, \\ \Sigma^t &= \sigma(x) + i\gamma^5\pi_a(x)\tau_a^t, \\ K^* &= i\Delta_A^*(x)\gamma^5\tau_2\lambda_A, \\ K &= i\Delta_A(x)\gamma^5\tau_2\lambda_A, \end{aligned} \tag{13}$$

where $D^\pm$ are nontrivial operators in coordinate, spinor, color, and flavor spaces.[1] In the following, it is very convenient to use the Nambu-Gorkov formalism, in which a bispinor Ψ is used instead of quarks, where

$$\Psi = \begin{pmatrix} q \\ q^C \end{pmatrix}, \qquad \Psi^t = (q^t, \bar{q}C^t),$$
$$\bar{\Psi} = (\bar{q}, \bar{q}^C) = (\bar{q}, q^tC) = \Psi^t\begin{pmatrix} 0, & C \\ C, & 0 \end{pmatrix} \equiv \Psi^t Y. \tag{14}$$

Furthermore, by introducing the matrix-valued operator

$$Z = \begin{pmatrix} D^+, & -K \\ -K^*, & D^- \end{pmatrix}, \tag{15}$$

one can rewrite the functional Gaussian integral in (12) in terms of Ψ and Z and then evaluate it as follows (clearly, in this case $[d\bar{q}][dq] = [dq^C][dq] = [d\Psi]$):

$$\begin{aligned} \exp(i\tilde{\mathcal{S}}_{\rm eff}) &= \int [d\Psi]\exp\left\{\frac{i}{2}\int \bar{\Psi}Z\Psi d^4x\right\} \\ &= \int [d\Psi]\exp\left\{\frac{i}{2}\int \Psi^t(YZ)\Psi d^4x\right\} \\ &= \det{}^{1/2}(YZ) = \det{}^{1/2}(Z), \end{aligned}$$

where the last equality is valid due to the evident relation $\det Y = 1$. Then, using the general formula $\det O = \exp \mathrm{Tr}\ln O$, one obtains the expression for the effective action:

$$\mathcal{S}_{\rm eff}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) = -\int d^4x \left[\frac{\sigma^2 + \pi_a^2}{4G} + \frac{\Delta_A\Delta_A^*}{4H}\right] - \frac{i}{2}\mathrm{Tr}_{sfcxNG}\ln Z. \tag{16}$$

As well as being an evident trace over the two-dimensional Nambu-Gorkov (NG) matrix, the trace in (16) is also calculated in spinor (s), flavor (f), color (c), and four-dimensional coordinate (x) spaces, respectively.

Starting from (16), one can define the quark contribution $\Omega_q(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*)$ to the TDP of the model (8). In the mean-field approximation one has

$$\mathcal{S}_{\rm eff}|_{\sigma,\pi_a,\Delta_A,\Delta_{A'}^* = \rm const} = -\Omega_q(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*)\int d^4x. \tag{17}$$

The ground state expectation values (mean values) of the fields, $\langle\sigma_a(x)\rangle \equiv \sigma^o$, $\langle\pi_a(x)\rangle \equiv \pi_a^o$, $\langle\Delta_A(x)\rangle \equiv \Delta_A^o$, $\langle\Delta_{A'}^*(x)\rangle \equiv \Delta_{A'}^{*o}$, are solutions of the gap equations for the TDP Ω_q (in our approach all ground state expectation values do not depend on coordinates x):

$$\frac{\partial\Omega_q}{\partial\pi_a} = 0, \qquad \frac{\partial\Omega_q}{\partial\sigma} = 0, \qquad \frac{\partial\Omega_q}{\partial\Delta_A} = 0, \qquad \frac{\partial\Omega_q}{\partial\Delta_{A'}^*} = 0. \tag{18}$$

Next, let us perform the following shift of bosonic fields in (16): $\sigma(x) \to \sigma(x) + \sigma^o$, $\pi_a(x) \to \pi_a(x) + \pi_a^o$, $\Delta_A^*(x) \to \Delta_A^*(x) + \Delta_A^{*o}$, $\Delta_A(x) \to \Delta_A(x) + \Delta_A^o$. [Obviously, the new shifted bosonic fields $\sigma(x)$, $\pi_a(x)$, $\Delta_A(x)$, $\Delta_A^*(x)$ now denote the small quantum fluctuations around the mean values σ^o, π_a^o, Δ_A^o, Δ_A^{*o} of mesons and

[1] In order to bring the quark sector of the Lagrangian (9) to the expression, given in the square brackets of (12), we use the following well-known relations: $\partial_\nu^t = -\partial_\nu$, $C\gamma^\nu C^{-1} = -(\gamma^\nu)^t$, $C\gamma^5 C^{-1} = (\gamma^5)^t = \gamma^5$, $\tau^2\vec{\tau}\tau^2 = -(\vec{\tau})^t$,

$$\tau^2 = \begin{pmatrix} 0, & -i \\ i, & 0 \end{pmatrix}.$$

diquarks rather than the original fields (10)]. In this case

$$Z = \begin{pmatrix} D_o^+, & -K_o \\ -K_o^*, & D_o^- \end{pmatrix} - \begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix} \equiv S_0^{-1} - \begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix}, \quad (19)$$

where S_0 is the quark propagator matrix in the Nambu-Gorkov representation (its matrix elements S_{ij} are given in Appendix B), and

$$(K_o, K_o^*, D_o^\pm, \Sigma_o, \Sigma_o^t) = (K, K^*, D^\pm, \Sigma, \Sigma^t)|_{\sigma=\sigma^o, \pi_a=\pi_a^o, \ldots}.$$

Then, expanding the obtained expression into a Taylor series up to second order of small bosonic fluctuations, we have

$$\mathcal{S}_{\rm eff}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) = \mathcal{S}_{\rm eff}^{(0)} + \mathcal{S}_{\rm eff}^{(2)}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) + \cdots, \quad (20)$$

where [due to the gap equations, the linear term in meson and diquark fields is absent in (20)]

$$\mathcal{S}_{\rm eff}^{(0)} = -\int d^4x \left[\frac{\sigma^o\sigma^o + \pi_a^o\pi_a^o}{4G} + \frac{\Delta_A^o\Delta_A^{*o}}{4H}\right] - \frac{i}{2}\mathrm{Tr}_{scfxNG}\ln(S_0^{-1}) \equiv -\Omega_q(\sigma^o, \pi_a^o, \Delta_A^o, \Delta_{A'}^{*o})\int d^4x, \quad (21)$$

$$\mathcal{S}_{\rm eff}^{(2)}(\sigma, \pi_a, \Delta_A, \Delta_{A'}^*) = -\int d^4x \left[\frac{\sigma^2 + \pi_a^2}{4G} + \frac{\Delta_A\Delta_A^*}{4H}\right] + \frac{i}{4}\mathrm{Tr}_{scfxNG}\left\{S_0\begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix} \times S_0\begin{pmatrix} \Sigma, & K \\ K^*, & \Sigma^t \end{pmatrix}\right\}. \quad (22)$$

In the following we will study the spectrum of meson/diquark excitations in the color superconducting phase of the NJL model on the basis of the effective action $\mathcal{S}_{\rm eff}^{(2)}$. The effective action (22) can be presented in the explicit form

$$\mathcal{S}_{\rm eff}^{(2)} = \mathcal{S}_{\rm mesons}^{(2)} + \mathcal{S}_{\rm diquarks}^{(2)} + \mathcal{S}_{\rm mixed}^{(2)}, \quad (23)$$

where

$$\mathcal{S}_{\rm mesons}^{(2)} = -\int d^4x \frac{\sigma^2 + \pi_a^2}{4G} + \frac{i}{4}\mathrm{Tr}_{scfx}\{S_{11}\Sigma S_{11}\Sigma + 2S_{12}\Sigma^t S_{21}\Sigma + S_{22}\Sigma^t S_{22}\Sigma^t\}, \quad (24)$$

$$\mathcal{S}_{\rm diquarks}^{(2)} = -\int d^4x \frac{\Delta_A\Delta_A^*}{4H} + \frac{i}{4}\mathrm{Tr}_{scfx}\{S_{12}K^* S_{12}K^* + 2S_{11}KS_{22}K^* + S_{21}KS_{21}K\}, \quad (25)$$

$$\mathcal{S}_{\rm mixed}^{(2)} = \frac{i}{2}\mathrm{Tr}_{scfx}\{S_{11}\Sigma S_{12}K^* + S_{21}\Sigma S_{11}K + S_{12}\Sigma^t S_{22}K^* + S_{21}KS_{22}\Sigma^t\}, \quad (26)$$

and S_{ij} are the matrix elements of the quark propagator matrix S_0 defined in (19) (see also Appendix B). Moreover, some necessary explanations concerning the trace operation over coordinate space in the expressions (24)–(26) are given in Appendix A [see (A4)]. It follows from these formulas that the effective action (24) is a functional of the meson fields $\sigma(x)$, $\pi_i(x)$ only, the effective action (25) is composed from diquark fields only, and the mixing between mesons and diquarks might occur because of the effective action (26).

III. GAP EQUATIONS AND NEUTRALITY CONDITIONS

Earlier (see, e.g., the papers [19–21]), it was shown that for electrically and color neutral cold dense matter, described in the framework of the model (3), only two phases are allowed to exist. In the first one, the symmetric phase that is usually called the normal quark matter phase, only the mean value of the σ field, $\langle\sigma(x)\rangle \equiv \sigma^o$, is nonzero. In the second one, which is just the 2SC phase of dense matter, the mean value of the diquark field $\Delta_2(x)$ is nonzero as well, i.e. $\langle\Delta_2(x)\rangle \equiv \Delta_2^o \neq 0$. Hence, without loss of generality, it is convenient to deal with TDP $\Omega_q(\sigma^o, \pi_a^o, \Delta_A^o, \Delta_{A'}^{*o})$ (note that $A, A' = 2, 5, 7$), in which all arguments, except $\sigma^o \equiv M - m$ (m is a bare quark mass, whereas the parameter M is usually called constituent or dynamical quark mass) and $\Delta_2^o \equiv \Delta$, are identically equal to zero. In this case the calculation of the total TDP Ω of the system (3) (recall that $\Omega = \Omega_e + \Omega_q$) is significantly simplified [19,20], and we have

$$\Omega(\mu_B, \mu_Q, \mu_8; M, \Delta) = -\frac{\mu_Q^4}{12\pi^2} + \frac{(M-m)^2}{4G} + \frac{|\Delta|^2}{4H} - 2\sum_\pm \int \frac{d^3q}{(2\pi)^3}\{|E_\Delta^\pm + \delta\mu| + |E_\Delta^\pm - \delta\mu|\} - \sum_\pm \int \frac{d^3q}{(2\pi)^3}\{|E \pm \mu_{ub}| + |E \pm \mu_{db}|\}, \quad (27)$$

where $E_\Delta^\pm = \sqrt{(E^\pm)^2 + |\Delta|^2}$, $E^\pm = E \pm \bar{\mu}$, $E = \sqrt{\vec{q}^2 + M^2}$, $\bar{\mu} = (\mu_{ur} + \mu_{dg})/2 = (\mu_{ug} + \mu_{dr})/2 = \tilde{\mu} + \mu_8 = \mu_B/3 + \mu_Q/6 + \mu_8$, $\mu_{ub} = \mu_B/3 + 2\mu_Q/3 - 2\mu_8$, $\mu_{db} = \mu_B/3 - \mu_Q/3 - 2\mu_8$, $\delta\mu = \mu_Q/2$ [see also the notations used in (7)]. Note that, apart from the order parameters M and Δ, the TDP Ω really depends on the chemical potentials, which is indicated in (27) in an explicit form. Since the integrals in the right-hand side of (27) are ultraviolet divergent, we regularize them as well as the other three-dimensional divergent integrals below by implementing a cutoff in the integration regions, $|\vec{q}| < \Lambda$. Starting from (27), one can find the gap equations $\partial\Omega/\partial\Delta^* = 0$ and $\partial\Omega/\partial M = 0$, which supply us with the values of M, Δ in the ground state of the system:

$$\frac{\partial\Omega}{\partial\Delta^*} \equiv \frac{\Delta}{4H} - 2\Delta\int\frac{d^3q}{(2\pi)^3}\left[\frac{\theta(E_\Delta^+ - |\delta\mu|)}{E_\Delta^+} + \frac{\theta(E_\Delta^- - |\delta\mu|)}{E_\Delta^-}\right] = 0, \tag{28}$$

$$\frac{\partial\Omega}{\partial M} \equiv \frac{M-m}{2G} - 2M\int\frac{d^3q}{(2\pi)^3E}[\theta(E-|\mu_{ub}|) + \theta(E-|\mu_{db}|)] - 4M\int\frac{d^3p}{(2\pi)^3E}\left[\frac{\theta(E_\Delta^+ - |\delta\mu|)E^+}{E_\Delta^+} + \frac{\theta(E_\Delta^- - |\delta\mu|)E^-}{E_\Delta^-}\right] = 0. \tag{29}$$

Next, let us impose the local color as well as electric charge neutrality requirements on the ground state of the model (3). This means that the quantities μ_8 and μ_Q take such values that the densities of the 8th color charge N_8 and electric charge N_Q are equal to zero in the ground state for arbitrary fixed values of other model parameters, i.e. $\langle N_8\rangle = -\partial\Omega/\partial\mu_8 \equiv 0$, $\langle N_Q\rangle = -\partial\Omega/\partial\mu_Q \equiv 0$. These neutrality constraints look like

$$\langle N_8\rangle \equiv 4\int\frac{d^3q}{(2\pi)^3}\left[\frac{\theta(E_\Delta^+ - |\delta\mu|)E^+}{E_\Delta^+} - \frac{\theta(E_\Delta^- - |\delta\mu|)E^-}{E_\Delta^-}\right] - 4\int\frac{d^3p}{(2\pi)^3}[\mathrm{sign}(\mu_{ub})\theta(|\mu_{ub}| - E) + \mathrm{sign}(\mu_{db})\theta(|\mu_{db}| - E)] = 0, \tag{30}$$

$$\langle N_Q\rangle \equiv \frac{\mu_Q^3}{3\pi^2} + \frac{\langle N_8\rangle}{6} + 2\int\frac{d^3q}{(2\pi)^3}[\mathrm{sign}(\mu_{ub})\theta(|\mu_{ub}| - E) + \mathrm{sign}(\delta\mu)\theta(|\delta\mu| - E_\Delta^+) + \mathrm{sign}(\delta\mu)\theta(|\delta\mu| - E_\Delta^-)] = 0. \tag{31}$$

In the following we suppose, for simplicity, that Δ is a real quantity. Of course, the solution of the common system of gap equations (28) and (29) and neutrality relations (30) and (31) is possible only numerically. In all numerical calculations of the present paper we use the following parameter set:

$$G = 5.86\ \mathrm{GeV}^{-2}, \qquad \Lambda = 618\ \mathrm{MeV}, \qquad m = 5.67\ \mathrm{MeV} \tag{32}$$

which leads, in the framework of the NJL model, to the well-known vacuum phenomenological values of the pion weak-decay constant $F_\pi = 92.4$ MeV, pion mass $M_\pi = 140$ MeV, and chiral quark condensate $\langle\bar{q}q\rangle = -(245\ \mathrm{MeV})^3$. Moreover, we use two different values for the coupling constant H in the diquark channel, $H = 3G/4$ and $H = G$, for which two qualitatively different 2SC phases are realized in the model (see below). The numerical analysis shows that, for the parameter set (32) and both relations $H = 3G/4$ and $H = G$, the system of equations (28)–(31) has only two solutions. As was already discussed after (7), the first one (with $M \neq 0$, $\Delta = 0$, $\mu_8 = 0$, and $\mu_Q = 0$) corresponds to the $SU(3)_c \times SU(2)_I$ symmetry of the model (normal phase), whereas the second one (with $M \neq 0$, $\Delta \neq 0$, $\mu_8 \neq 0$, and $\mu_Q \neq 0$) corresponds to the 2SC phase. As usual, solutions of these equations give local extrema of the thermodynamic potential $\Omega(\mu_B, \mu_Q, \mu_8; M, \Delta)$ (27). Clearly, one should also check in which of them the TDP takes the least value (for each fixed value of μ_B). Only this solution of the system of equations (28)–(31) corresponds to the genuine neutral ground state of the model. In particular, for each fixed μ_B it supplies us with mean values (gaps) M, Δ (see Figs. 1 and 2) as well as with values μ_8 and μ_Q (see Figs. 3 and 4), at which the ground state has zero charges.[2]

It is clear from Figs. 1 and 2 that there is a critical value of the baryon chemical potential μ_B^c (for the case $H = 3G/4$ one has $\mu_B^c = 1.08$ GeV, whereas for the case $H = G$ one gets $\mu_B^c = 1.04$ GeV)[3] such that at $\mu_B < \mu_B^c$ the normal $SU(3)_c \times SU(2)_I$-symmetric phase of the model occurs. However, at $\mu_B > \mu_B^c$ the 2SC phase is realized. Since in this case $\mu_Q \neq 0$ (see Fig. 4), the isospin $SU(2)_I$

[2]In the literature, the zero value of the current quark mass m is of frequent use in the 2SC investigation. In this case the constituent quark mass M is identically equal to zero in the 2SC phase of neutral matter (see, e.g., [19,22]).

[3]The tendency, the greater H the smaller μ_B^c, is indeed supported by earlier investigations of the 2SC phenomenon in the framework of NJL models [34,35]. In particular, it was shown that, at sufficiently large values of the coupling constant H, the 2SC phase may be realized in the model even at zero baryon chemical potential μ_B [35].

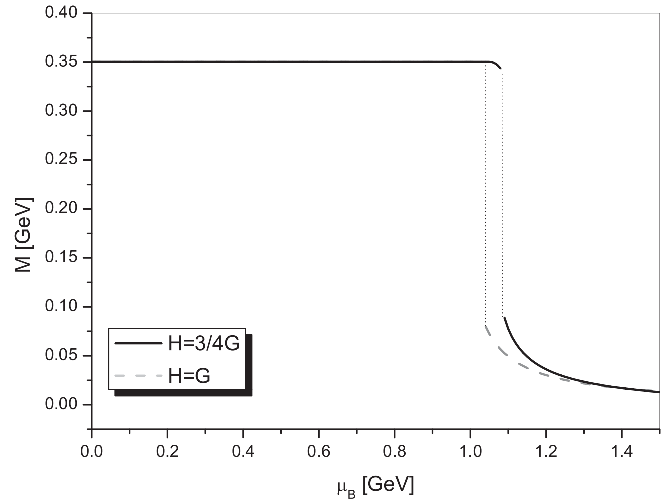

FIG. 1. The behavior of M vs μ_B in neutral matter.

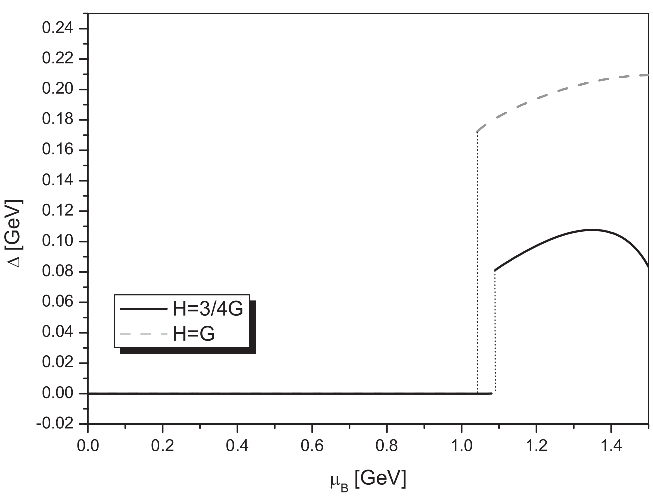

FIG. 2. The behavior of Δ vs μ_B in neutral matter.

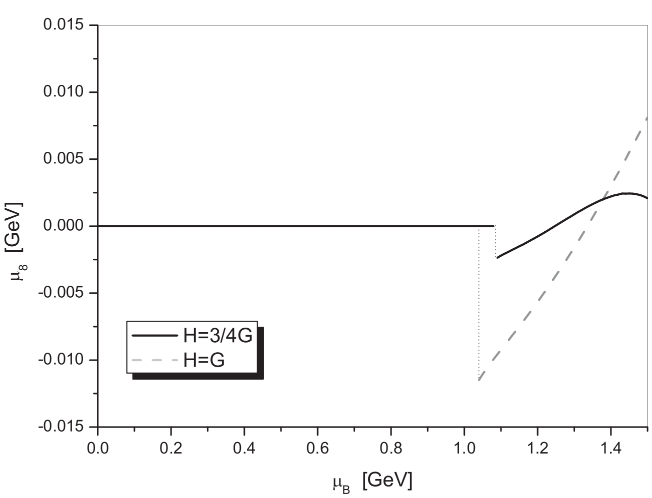

FIG. 3. The behavior of μ_8 vs μ_B in neutral matter.

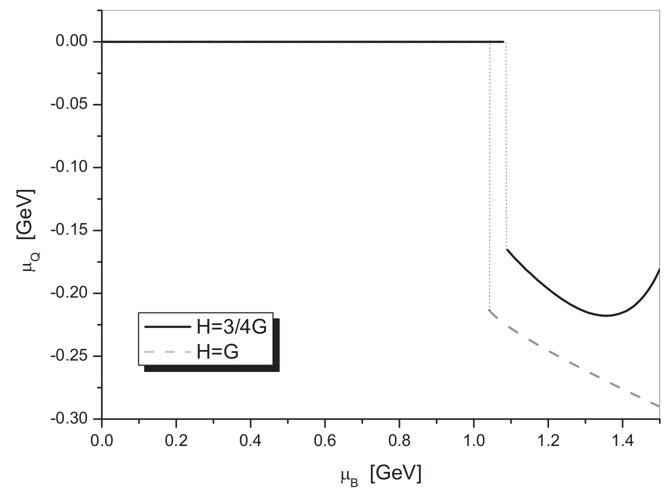

FIG. 4. The behavior of μ_Q vs μ_B in neutral matter.

symmetry of the normal phase is broken in the critical point by hand rather than dynamically, down to the $U(1)_{I_3}$ group. Furthermore, in the 2SC phase we have $\langle\Delta_2(x)\rangle = \Delta \neq 0$, $\langle\Delta_{5,7}(x)\rangle \equiv 0$. So in the critical point μ_B^c the color $SU(3)_c$ symmetry is broken down to the $SU(2)_c$ subgroup. Since at $\mu_B > \mu_B^c$ the μ_8-term of the Lagrangian (3) is nonzero (see Fig. 3), the initial color symmetry of the model is also broken down by hand to the $SU(2)_c \times U(1)_{\lambda_8}$ subgroup, which is further broken down dynamically (spontaneously) to $SU(2)_c$. Hence, one may expect the appearance of only one Nambu-Goldstone boson in the mass spectrum of the 2SC phase (see the discussion below). Finally, we remark that in the critical point μ_B^c the transition between these two phases is of the first order which is characterized by a discontinuity in the behavior of M and Δ vs μ_B (see Figs. 1 and 2).

Up to now we have discussed the properties of the 2SC phase in neutral and β-equilibrated matter which are common for the two particular cases $H = 3G/4$ and $H = G$. However, there exist, at least two, qualitative distinctions between these neutral 2SC phases. (Both of them are based on the fact that in the neutral 2SC matter the relation $\Delta < |\delta\mu|$ is valid for the case of $H = 3G/4$, whereas at $H = G$ the opposite one, $\Delta > |\delta\mu|$, is true.) The first one lies in the diquark mass spectrum and will be discussed below. Now we would like to present the second one which is provided by the quasiparticle dispersion relations, i.e. the momentum dependence of energy. In condensed matter physics quasiparticles are simply the one-fermion excitations of the ground state. In our case the quasiparticle spectrum of the 2SC matter is defined by singularities of the quark propagator S_0 (see Appendix B). Clearly, there are 12 (6 quark and 6 antiquark) quasiparticles in the 2SC matter. Four of them, blue quasiparticles, have the energies $E \pm \mu_{ub}$ and $E \pm \mu_{db}$. The energy spectrum of the other 8, red and green quasiparticles, consists of four values $E_\Delta^\pm \pm |\delta\mu|$, each doubly degenerate. Evidently, for both relations between coupling constants H and G there are momentum values at which the energies of two blue quasiparticles turn into zero, i.e. there are no energy costs to create these quasiparticles (their energies are $E - \mu_{ub}$ and $E - \mu_{db}$). Because of this reason, these excitations are called gapless ones. Similarly, at $H = 3G/4$ there are, in addition, also two gapless red and green fermionic excitations (with energies $E_\Delta^- - |\delta\mu|$) of neutral 2SC matter, since in this case the relation $\Delta < |\delta\mu|$ is true. However, in the neutral 2SC matter of the case $H = G$ there are no additional gapless red and green quasiparticles. Hence, at $H = 3G/4$ the quasiparticle spectrum of the neutral 2SC matter consists of four gapless excitations (it is a so-called gapless color superconductivity). In contrast, at $H = G$ there are only two gapless quasiparticles in the neutral 2SC matter, and color superconductivity is gapped. Thus, our results corroborate the conclusion, made, e.g., in [19,23], that the gapless 2SC may exist for a rather narrow interval of the coupling constant H values.

In the next sections we will calculate the inverse two-point (unnormalized) correlators of meson and diquark fluctuations over the ground state of neutral 2SC matter in the one-loop (mean-field) approximation and find their masses in the two cases, $H = 3G/4$ and $H = G$.

IV. MESON MASSES

After a more detailed study of the $\mathcal{S}^{(2)}_{\text{mixed}}$ part (26) of the effective action, it turns out that it is composed of $\sigma(x)$, $\Delta_2(x)$, and $\Delta_2^*(x)$ fields only, i.e. the π mesons are not mixed with diquarks (this property is justified by the parity conservation both in the normal and 2SC phases of our model). So to find the π-meson masses it is enough to deal with the effective action $\mathcal{S}^{(2)}_{\text{mesons}}$ (24), which is a generating functional of the one-particle irreducible (1PI) Green functions of σ- and π-meson fields. In this case, instead of $\pi_i(x)$ fields (10), we will use the new fields $\pi^0(x) \equiv \pi_3(x)$, $\pi^\pm(x) \equiv (\pi_1(x) \pm \pi_2(x))/\sqrt{2}$, so that the quantities Σ, Σ^t from (13) look like

$$\Sigma = \sigma(x) + i\gamma^5[\pi^0(x)\tau_3 + \pi^-(x)\tau_+ + \pi^+(x)\tau_-],$$
$$\Sigma^t = \sigma(x) + i\gamma^5[\pi^0(x)\tau_3 + \pi^-(x)\tau_- + \pi^+(x)\tau_+], \qquad (33)$$

where $\tau_\pm \equiv (\tau_1 \pm \tau_2)/\sqrt{2}$. Then, the 1PI Green functions of σ- and $\pi^\pm$, π^0-meson fields can be generated through the relation

$$\Gamma_{XY}(x - y) = -\frac{\delta^2 \mathcal{S}^{(2)}_{\text{mesons}}}{\delta Y(y)\delta X(x)}, \qquad (34)$$

where $X, Y = \sigma, \pi^\pm, \pi^0$ [to take the variational derivatives in (34), it is very instructive to refer to the relations (A4) and (A5)]. In momentum space the zeros of the Fourier transformations of these functions are connected with meson masses.

A. The $\pi^\pm$-meson masses

Using the relation (34), it is possible to define the 1PI Green functions of the $\pi^\pm$ fields. In particular,

$$\begin{aligned}\Gamma_{\pi^+\pi^-}(x - y) = \frac{\delta(z)}{2G} &+ 2i\,\mathrm{Tr}_{sc}[2A_{11}(z)\gamma^5 B_{11}(-z)\gamma^5 \\ &+ 2B_{22}(z)\gamma^5 A_{22}(-z)\gamma^5 \\ &+ A_{12}(z)\gamma^5 A_{21}(-z)\gamma^5 \\ &+ B_{21}(z)\gamma^5 B_{12}(-z)\gamma^5] \end{aligned} \qquad (35)$$

(in this expression all traces over flavor indices are calculated, and $z = x - y$), where operators $A_{ij}(z)$ and $B_{kl}(z)$ are defined in Appendix B. Moreover, $\Gamma_{\pi^-\pi^+}(x - y) = \Gamma_{\pi^+\pi^-}(y - x)$ and $\Gamma_{\pi^+\pi^+}(x - y) = \Gamma_{\pi^-\pi^-}(x - y) = 0$. Because of the traces containing an odd number of γ^5, all the mixed Green functions of the form $\Gamma_{\sigma\pi^{0,\pm}}(x - y) = -\{\delta^2\mathcal{S}^{(2)}_{\text{mesons}}/[\delta\pi^{0,\pm}(y)\delta\sigma(x)]\}$ are zero. Moreover, since the traces containing an odd number of $\tau_\pm$ matrices equal zero, all the 1PI Green functions of the form $\Gamma_{\pi^0\pi^\pm}(x - y)$ are also zero. Hence, in the framework of our model there is no mixing between $\pi^\pm$ fields on the one hand, and σ, π^0 fields on the other. After taking the trace over color indices, the Fourier transform of (35) has the following form:

$$\begin{aligned}\bar\Gamma_{\pi^+\pi^-}(p) = \frac{1}{2G} + 4i\,\mathrm{Tr}_s \int \frac{d^4q}{(2\pi)^4}&[2\bar a^{(12)}_{11}(p+q)\gamma^5\bar b^{(12)}_{11}(q)\gamma^5 + \bar a^{(3)}_{11}(p+q)\gamma^5\bar b^{(3)}_{11}(q)\gamma^5 + 2\bar b^{(12)}_{22}(p+q)\gamma^5\bar a^{(12)}_{22}(q)\gamma^5 \\ &+ \bar b^{(3)}_{22}(p+q)\gamma^5\bar a^{(3)}_{22}(q)\gamma^5 + \bar a_{12}(p+q)\gamma^5\bar a_{21}(q)\gamma^5 + \bar b_{21}(p+q)\gamma^5\bar b_{12}(q)\gamma^5],\end{aligned} \qquad (36)$$

where the quantities $\bar a^{(12)}_{11}(q)$, $\bar b^{(12)}_{11}(q)$, etc. are the corresponding Fourier transformations of $a^{(12)}_{11}(z)$, $b^{(12)}_{11}(z)$, etc. presented in Appendix B. Clearly, $\bar\Gamma_{\pi^-\pi^+}(p) = \bar\Gamma_{\pi^+\pi^-}(-p)$. The zeros of these functions determine the $\pi^\pm$-meson dispersion laws, i.e. the relations between their energy and three-momenta. In the present paper, we are mainly interested in the investigation of the modification of meson and diquark masses in dense and cold color and electrically neutral matter. Since in this case a particle mass is defined as the value of its energy in the rest frame, $\vec p = 0$ (see, e.g., [30,31,36]), we put $p = (p_0, 0, 0, 0)$ in the following. As a result, the calculation of 1PI Green functions is significantly simplified. Indeed, in the rest frame one can easily perform all the trace calculations over spinor indices in (36) [see the auxiliary relations (B10)] and get

$$\bar{\Gamma}_{\pi^-\pi^+}(p_0) = \frac{1}{2G} - 16i\int\frac{d^4q}{(2\pi)^4}\frac{(q_0-\delta\mu)(p_0+q_0+\delta\mu) - E^+E^- - |\Delta|^2}{[(q_0-\delta\mu)^2-(E_\Delta^-)^2][(p_0+q_0+\delta\mu)^2-(E_\Delta^+)^2]} - 4i\int\frac{d^4q}{(2\pi)^4}\times\left[\frac{1}{(p_0+q_0+\mu_{ub}+E)(q_0+\mu_{db}-E)} + \frac{1}{(p_0+q_0+\mu_{ub}-E)(q_0+\mu_{db}+E)}\right], \quad (37)$$

where we have used the same notations as in (27). Note also that in (37) q_0 is a shorthand notation for $q_0 + i\varepsilon\cdot\mathrm{sign}(q_0)$ and (p_0+q_0) is a shorthand notation for $(p_0+q_0) + i\varepsilon\cdot\mathrm{sign}(p_0+q_0)$, where $\varepsilon\to 0_+$ [see also [37] and the remark after (B9)]. The q_0 integration in (37) is performed along the real axis in the complex q_0 plane. We will close this contour by an infinite arc in the upper half of the complex q_0 plane. Inside the obtained closed contour, the integrand of the first integral in (37) has four simple poles which are located at the following points:

$$(q_0)_1 = \delta\mu - E_\Delta^- + i\varepsilon\cdot\theta(E_\Delta^- - \delta\mu), \qquad (q_0)_2 = \delta\mu + E_\Delta^- + i\varepsilon\cdot\theta(-E_\Delta^- - \delta\mu),$$
$$(q_0)_3 = E_\Delta^+ - \delta\mu - p_0 + i\varepsilon\cdot\theta(\delta\mu - E_\Delta^+), \qquad (q_0)_4 = -E_\Delta^+ - \delta\mu - p_0 + i\varepsilon\cdot\theta(\delta\mu + E_\Delta^+),$$

whereas the integrand in the second line of (37) has the following four poles in the upper half of the q_0 plane:

$$(\check{q}_0)_1 = E - \mu_{db} + i\varepsilon\cdot\theta(\mu_{db} - E), \qquad (\check{q}_0)_2 = -\mu_{db} - E + i\varepsilon\cdot\theta(\mu_{db} + E),$$
$$(\check{q}_0)_3 = -E - \mu_{ub} - p_0 + i\varepsilon\cdot\theta(E + \mu_{ub}), \qquad (\check{q}_0)_4 = E - \mu_{ub} - p_0 + i\varepsilon\cdot\theta(\mu_{ub} - E).$$

Summing the residues of the integrand function in these poles, we can perform the q_0 integration in (37) and obtain

$$\bar{\Gamma}_{\pi^+\pi^-}(p_0) = \frac{1}{2G} + 8\int\frac{d^3q}{(2\pi)^3}\left[\frac{\theta(-\delta\mu - E_\Delta^-)}{E_\Delta^-}\cdot\frac{(p_0+E_\Delta^-+2\delta\mu)E_\Delta^- - E^+E^- - |\Delta|^2}{(p_0+E_\Delta^-+2\delta\mu)^2-(E_\Delta^+)^2} + \frac{\theta(E_\Delta^- - \delta\mu)}{E_\Delta^-}\cdot\frac{(p_0-E_\Delta^-+2\delta\mu)E_\Delta^- + E^+E^- + |\Delta|^2}{(p_0-E_\Delta^-+2\delta\mu)^2-(E_\Delta^+)^2} - \frac{\theta(\delta\mu - E_\Delta^+)}{E_\Delta^+}\cdot\frac{(p_0-E_\Delta^++2\delta\mu)E_\Delta^+ + E^+E^- + |\Delta|^2}{(p_0-E_\Delta^++2\delta\mu)^2-(E_\Delta^-)^2} - \frac{\theta(\delta\mu + E_\Delta^+)}{E_\Delta^+}\cdot\frac{(p_0+E_\Delta^++2\delta\mu)E_\Delta^+ - E^+E^- - |\Delta|^2}{(p_0+E_\Delta^++2\delta\mu)^2-(E_\Delta^-)^2}\right] + 4\int\frac{d^3q}{(2\pi)^3}\left[\frac{\theta(\mu_{db}-E)}{p_0+2\delta\mu+2E} - \frac{\theta(\mu_{ub}+E)}{p_0+2\delta\mu+2E} + \frac{\theta(E+\mu_{db})}{p_0+2\delta\mu-2E} - \frac{\theta(\mu_{ub}-E)}{p_0+2\delta\mu-2E}\right]. \quad (38)$$

Clearly, the matrix element $\bar{\Gamma}_{\pi^+\pi^-}(p_0)$ depends effectively on the variable $z = (p_0 + 2\delta\mu)$. It follows from our numerical analysis that, for both values of the coupling constant H ($H = G$ and $H = 3G/4$), the expression (38) has only two zeros, $z_1(\mu_B)$ and $z_2(\mu_B)$. Hence, for each fixed value of $\mu_B > \mu_B^c$, i.e. in the 2SC phase, we have $\bar{\Gamma}_{\pi^+\pi^-}(p_0) \sim (p_0 + 2\delta\mu - z_1)(p_0 + 2\delta\mu - z_2)$. Since $\bar{\Gamma}_{\pi^-\pi^+}(p_0) = \bar{\Gamma}_{\pi^+\pi^-}(-p_0)$, the determinant of the inverse propagator matrix of the $\pi^\pm$ mesons has the following form:

$$\bar{\Gamma}_{\pi^+\pi^-}(p_0)\cdot\bar{\Gamma}_{\pi^-\pi^+}(p_0) \sim (p_0^2 - (2\delta\mu - z_1)^2)\times(p_0^2 - (2\delta\mu - z_2)^2). \quad (39)$$

Evidently, in the p_0^2 plane it has two zeros, which are the mass squared of the $\pi^\pm$ mesons. Hence, in the 2SC neutral dense matter the $\pi^\pm$ mesons have different masses:

$$M_{\pi^+}^2 = (2\delta\mu - z_1)^2, \qquad M_{\pi^-}^2 = (2\delta\mu - z_2)^2. \quad (40)$$

The behavior of M_{π^+} and M_{π^-} vs μ_B in the color superconducting and neutral matter is depicted in Figs. 5 and 6 for the cases $H = 3G/4$ and $H = G$, respectively.

B. The π^0, σ-meson masses

As it was already discussed after (35), π^0 mesons are not mixed with other fields in the framework of the NJL model (3), i.e. the 1PI Green functions of the form $\Gamma_{\pi^0 X}(x-y)$, where $X(x) = \sigma(x)$, $\pi^\pm(x)$, $\Delta_A(x)$, $\Delta_{A'}^*(x)$ are equal to zero. In this case the 1PI Green function $\Gamma_{\pi^0\pi^0}(x-y)$ is just the inverse π^0 propagator, which can be found from (34). To get the π^0 mass we need the expression $\bar{\Gamma}_{\pi^0\pi^0}(p)$, i.e. the Fourier transform of $\Gamma_{\pi^0\pi^0}(x-y)$, in the rest frame. Using the technique presented in the previous section, one can obtain, after tedious but straightforward calculations at $p = (p_0, 0, 0, 0)$,

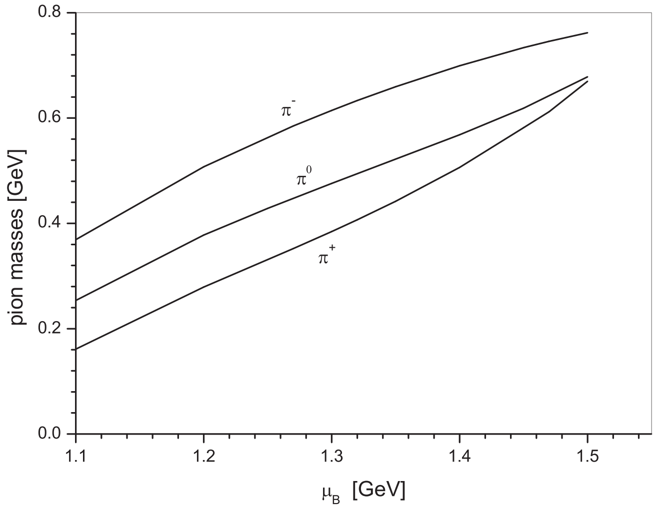

FIG. 5. The behavior of meson masses vs μ_B in the gapless ($H = 3G/4$) color superconducting neutral matter ($\mu_B > \mu_B^c = 1.08$ GeV).

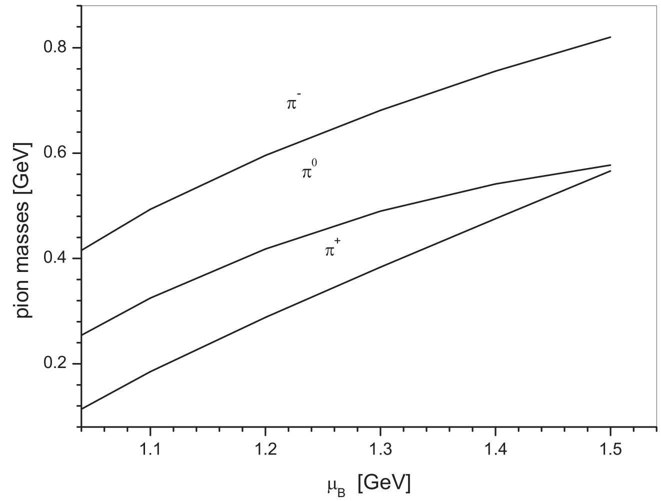

FIG. 6. The behavior of meson masses vs μ_B in the gapped ($H = G$) color superconducting neutral matter ($\mu_B > \mu_B^c = 1.04$ GeV).

$$\begin{aligned}\bar{\Gamma}_{\pi^0\pi^0}(p_0) &= \frac{1}{2G} + 8\int\frac{d^3q}{(2\pi)^3}\frac{E_\Delta^+E_\Delta^- + E^+E^- + \Delta^2}{E_\Delta^+E_\Delta^-}\frac{E_\Delta^+ + E_\Delta^-}{p_0^2 - (E_\Delta^+ + E_\Delta^-)^2}\\
&+ 4\int\frac{d^3q}{(2\pi)^3}\bigg\{\frac{\theta(\delta\mu - E_\Delta^-) + \theta(-\delta\mu - E_\Delta^-)}{E_\Delta^-}\bigg[\frac{(p_0 + E_\Delta^-)E_\Delta^- - E^+E^- - |\Delta|^2}{(p_0 + E_\Delta^-)^2 - (E_\Delta^+)^2}\\
&+ \frac{(E_\Delta^- - p_0)E_\Delta^- - E^+E^- - |\Delta|^2}{(E_\Delta^- - p_0)^2 - (E_\Delta^+)^2}\bigg]\\
&+ \frac{\theta(\delta\mu - E_\Delta^+) + \theta(-\delta\mu - E_\Delta^+)}{E_\Delta^+}\bigg[\frac{(E_\Delta^+ - p_0)E_\Delta^+ - E^+E^- - |\Delta|^2}{(E_\Delta^+ - p_0)^2 - (E_\Delta^-)^2} + \frac{(E_\Delta^+ + p_0)E_\Delta^+ - E^+E^- - |\Delta|^2}{(E_\Delta^+ + p_0)^2 - (E_\Delta^-)^2}\bigg]\bigg\}\\
&+ 8\int\frac{d^3q}{(2\pi)^3}\frac{E}{p_0^2 - 4E^2}[\theta(E - \mu_{db}) + \theta(E - \mu_{ub}) - \theta(-\mu_{ub} - E) - \theta(-\mu_{db} - E)]. \qquad (41)\end{aligned}$$

The function (41) is an even one, i.e. $\bar{\Gamma}_{\pi^0\pi^0}(p_0) = \bar{\Gamma}_{\pi^0\pi^0}(-p_0)$. Hence, it effectively depends on the variable p_0^2. Numerical investigations, performed both at $H = 3G/4$ and $H = G$, show that for each fixed value of $\mu_B > \mu_B^c$ the 1PI Green function (41) has a single zero on the positive p_0^2 semiaxis, which is just the mass squared of the π^0 meson. Its mass in the 2SC phase of the dense and neutral matter is presented graphically in Figs. 5 and 6.

The situation with the σ-meson mass is much more involved. Indeed, as it follows from the previous section, in the model under consideration the σ meson is mixed with $\Delta_2(x)$, $\Delta_2^*(x)$ diquarks (see also [29–31]). So, to get the particle masses in this case, one should find the zeros of the 3×3-matrix determinant, whose matrix elements are nothing but the 1PI Green functions $\bar{\Gamma}_{XY}(p_0)$ (we use the rest frame in the momentum space representation), where $X(x), Y(x) = \sigma(x), \Delta_2(x), \Delta_2^*(x)$. In general, it is a rather hard task, which, however, can be significantly simplified due to some reasons. It turns out that 1PI Green functions of the form $\Gamma_{\sigma X}$ $[X(x) = \Delta_2(x), \Delta_2^*(x)]$ are proportional to the gap Δ (see Fig. 2) and constituent quark mass M (see Fig. 1) as well. Hence, in the normal color symmetric phase (where $\Delta = 0$) there is no mixing between the σ meson and diquarks $\Delta_2(x)$, $\Delta_2^*(x)$ at all. The similar is true for the 2SC phase, if the current quark mass m is zero (in this case, as was pointed out in Footnote 2, the parameter M is also equal to zero in the 2SC phase). In our consideration the parameter M is a rather small quantity in comparison to the gap Δ in the 2SC phase (see Figs. 1 and 2). So, to have a grasp of the order of magnitude of the σ-meson mass, we ignore, for simplicity, the $\sigma - \Delta_2$ mixing effect in this case, too. As a result, we see that for both $H = 3G/4$ and $H = G$ the σ-meson mass is defined by the 1PI Green function $\Gamma_{\sigma\sigma}(x-y)$ which is approximately the 1PI Green function $\Gamma_{\pi^0\pi^0}(x-y)$. Hence, for both above-mentioned values of the coupling constant H, the σ-meson mass is approximately equal to the π^0-meson mass in the 2SC neutral matter (see Figs. 5 and 6).

Now some comments concerning the behavior of the π-meson masses in the neutral dense matter are in order.

First of all, note that in the normal phase (where $\mu_B < \mu_B^c$) the μ_Q-term of the Lagrangian (3) is zero. As a result, the ground state of this phase is an $SU(2)_I$-invariant one. Because of this symmetry, in the normal phase all π mesons have a common mass that is approximately 140 MeV for all $\mu_B < \mu_B^c$, both for $H = 3G/4$ and $H = G$ (the σ-meson mass is approximately equal to 700 MeV in the normal phase). Above the critical point, i.e. at $\mu_B > \mu_B^c$, the isotopic $SU(2)_I$ symmetry of the system is broken due to the appearance in (3) of a nonzero μ_Q-term. So, in the 2SC phase of electrically neutral matter the meson masses are allowed to have different values. Just this conclusion was supported by our numerical investigations (see Figs. 5 and 6), where a rather strong splitting of π-meson masses is observed. In contrast, if the electric charge neutrality is not imposed on the NJL system, then all mesons have a common mass in the 2SC phase [30,31].

Finally, note that in [38] a rather strong splitting of π-meson masses was shown to exist in electrically neutral and noncolor superconducting dense quark matter both with and without pion condensation phenomenon.

V. DIQUARK MASSES

As in the previous section, we will ignore, for simplicity, the mixture between diquarks $\Delta_2(x)$, $\Delta_2^*(x)$ and σ mesons (this is justified by moderately small values of the parameter M in the 2SC phase). In this case, in order to obtain the masses of diquarks, we need to analyze the 1PI Green functions, generated by the effective action $\mathcal{S}^{(2)}_{\text{diquarks}}$ (25):

$$\Gamma_{XY}(x-y) = -\frac{\delta^2 \mathcal{S}^{(2)}_{\text{diquarks}}}{\delta Y(y)\delta X(x)}, \tag{42}$$

where $X(x), Y(x) = \Delta_A(x), \Delta^*_{A'}(x)$.

A. Diquark masses in the 2SC phase

Because of the structure of (25), the diquarks, as such, are not mixed to one another in the framework of our model. So it is reasonable to study step-by-step the diquark excitations of the 2SC neutral matter in the $\Delta_2(x)$, $\Delta_2^*(x)$, $\Delta_5(x)$, $\Delta_5^*(x)$, and finally in the $\Delta_7(x)$, $\Delta_7^*(x)$ sectors of the model.

The investigations of the 1PI Green functions (42) in the $\Delta_5(x)$ and $\Delta_7(x)$ sectors of the model supply us with four excitations of the 2SC phase ground state. All of them have the common mass $3|\mu_8|$ for both $H = 3G/4$ and $H = G$, when color and electric charge neutrality constraints are imposed [22,31]. Evidently, these excitations form two real (or one complex) doublets of the $SU(2)_c$ ground state symmetry group of the 2SC phase.

To study the masses of the $SU(2)_c$-singlet diquark excitations, we should consider the 1PI Green functions in the $\Delta_2(x)$, $\Delta_2^*(x)$ sector of the model. It can be shown in the usual way that in the 2SC phase these quantities take the following form (here, again, the rest frame, i.e. $\vec{p} = 0$, in the momentum space representation is used):

$$\begin{aligned}
\bar{\Gamma}_{\Delta_2\Delta_2}(p_0) &= \bar{\Gamma}_{\Delta_2^*\Delta_2^*}(p_0) = 4\Delta^2 I_0(p_0^2),\\
\bar{\Gamma}_{\Delta_2^*\Delta_2}(p_0) &= \bar{\Gamma}_{\Delta_2\Delta_2^*}(-p_0)\\
&= (4\Delta^2 - 2p_0^2)I_0(p_0^2) + 4p_0 I_1(p_0^2),
\end{aligned} \tag{43}$$

where

$$\begin{aligned}
I_0(p_0^2) &= \int \frac{d^3q}{(2\pi)^3}\frac{\theta(E_\Delta^+ - |\delta\mu|)}{E_\Delta^+[4(E_\Delta^+)^2 - p_0^2]} + \int \frac{d^3q}{(2\pi)^3}\frac{\theta(E_\Delta^- - |\delta\mu|)}{E_\Delta^-[4(E_\Delta^-)^2 - p_0^2]},\\
I_1(p_0^2) &= \int \frac{d^3q}{(2\pi)^3}\frac{\theta(E_\Delta^+ - |\delta\mu|)E^+}{E_\Delta^+[4(E_\Delta^+)^2 - p_0^2]} - \int \frac{d^3q}{(2\pi)^3}\frac{\theta(E_\Delta^- - |\delta\mu|)E^-}{E_\Delta^-[4(E_\Delta^-)^2 - p_0^2]}.
\end{aligned} \tag{44}$$

From the expressions (43) it is possible to compose the inverse propagator matrix $\mathcal{G}^{-1}(p_0)$ for the diquarks $\Delta_2(x)$, $\Delta_2^*(x)$ moving in neutral 2SC matter:

$$\mathcal{G}^{-1}(p_0) = -\begin{pmatrix} \bar{\Gamma}_{\Delta_2\Delta_2}(p_0), & \bar{\Gamma}_{\Delta_2\Delta_2^*}(p_0)\\ \bar{\Gamma}_{\Delta_2^*\Delta_2}(p_0), & \bar{\Gamma}_{\Delta_2^*\Delta_2^*}(p_0)\end{pmatrix}. \tag{45}$$

Then, the mass spectrum in the $\Delta_2(x)$, $\Delta_2^*(x)$ sector of the model is defined by the equation

$$\begin{aligned}
\det\mathcal{G}^{-1}(p_0) &\equiv 4p_0^2\{(p_0^2 - 4\Delta^2)I_0^2(p_0^2) - 4I_1^2(p_0^2)\}\\
&\equiv 4p_0^2 F(p_0^2) = 0.
\end{aligned} \tag{46}$$

In the p_0^2 plane this equation has an evident zero, corresponding to a Nambu-Goldstone boson (NG boson), $p_0^2 = 0$. [Since $F(0) \neq 0$, it does not have another zero of the form $p_0^2 = 0$.][4] Since in the 2SC phase the chemical potential μ_8 is nonzero, it is clear that in this phase the initial color $SU(2)_c \times U(1)_{\lambda_8}$ symmetry of the Lagrangian (3) is spontaneously broken down to the $SU(2)_c$ group. So, there is only one broken symmetry generator, corresponding to the above-mentioned NG-boson solution of Eq. (46). Hence, it is possible to assert that in the 2SC phase of color and electrically neutral matter, described by the NJL model (3), there appears a *normal number* of NG bosons, corresponding to a spontaneous breaking of the color symmetry. In contrast, if neutrality requirements are not

[4] It might seem that the appearance of the NG boson in the mass spectrum of the model is strongly connected with the disregarding of the mixing between σ mesons and $\Delta_2(x)$, $\Delta_2^*(x)$ diquarks. However, as it was shown in [30], this NG boson is a native property of the model, since it exists in the mass spectrum even in the case, when the mixing is taken into account.

imposed and only the baryon chemical potential is taken into account, then in the NJL model there is an *abnormal number* of NG bosons in the mass spectrum of the 2SC phase [29]. (Note that an abnormal number of NG bosons is not an unexpected phenomenon. It is inherent to a variety of quantum models with broken Lorentz symmetry, which is provided by chemical potentials [28,39].)

For the further investigation of Eq. (46) some additional information about the functions I_0, I_1 (44), and consequently about the function F in (46), is required. It turns out that these functions are analytical in the complex p_0^2 plane, except for the cut $\varkappa < p_0^2$ along the real axis (the first Riemann sheet). It is easy to verify that $\varkappa = 4\Delta^2$ in the case of the gapped 2SC phase, i.e. at $H = G$, where $\Delta > |\delta\mu|$. However, $\varkappa = 4|\delta\mu|^2 \equiv \mu_Q^2 > 4\Delta^2$ in the case of the gapless 2SC phase, i.e. at $H = 3G/4$, where $\Delta < |\delta\mu|$. The quantity $F(p_0^2)$ is a complex-valued function in the whole first Riemann sheet, except for points on the real axis which do not belong to a cut, where $F(p_0^2)$ is a real-valued function.

Notice that, in the case of the gapless 2SC phase, the cut of the p_0^2 plane originates to the right of the point $4\Delta^2$, whereas in the case of the gapped 2SC phase it just starts at the point $4\Delta^2$. This circumstance is of decisive importance for the appearance of the diquark mass difference in the two types of color superconductivity. Indeed, at $H = G$ we did not manage to find any solution of the equation $F(p_0^2) = 0$ in the first Riemann sheet of the variable p_0^2. [In particular, it is evident from (46) that in the real points such that $p_0^2 < 4\Delta^2$ the function $F(p_0^2)$ is an exactly negative quantity.] So, using the procedure presented in the appendix of our previous paper [30], we continue the function $F(p_0^2)$ to the second Riemann sheet, where it takes

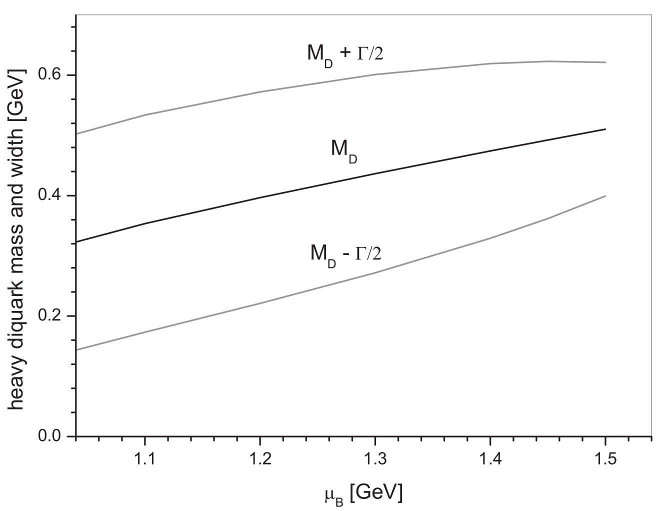

FIG. 7. The sketch of the $SU(2)_c$-singlet diquark resonance in the gapped ($H = G$) 2SC neutral matter ($\mu_B > \mu_B^c = 1.04$ GeV). The solid line is for its mass M_D vs μ_B, whereas the width of the strip between dashed lines $M_D \pm \Gamma/2$ is its width Γ vs μ_B.

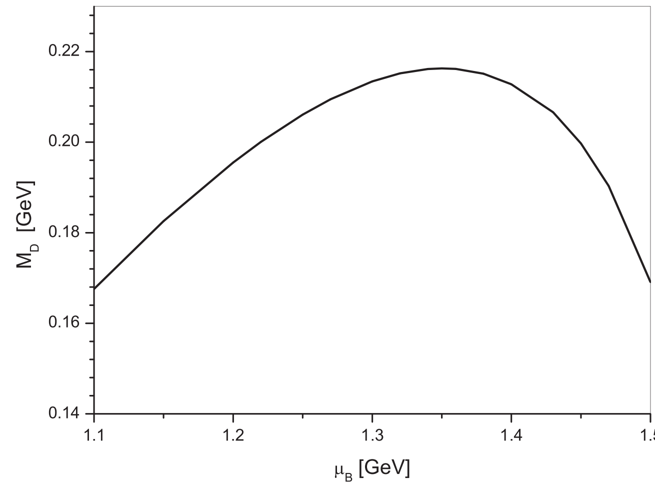

FIG. 8. The behavior of the stable $SU(2)_c$-singlet diquark mass M_D vs μ_B in the gapless ($H = 3G/4$) color superconducting neutral matter ($\mu_B > \mu_B^c = 1.08$ GeV).

a zero value in some complex point. This means that an $SU(2)_c$-singlet diquark resonance appears in the mass spectrum of the neutral gapped 2SC phase. Its mass M_D and width Γ are presented in Fig. 7 as functions of μ_B. For the gapless 2SC phase (at $H = 3G/4$) the situation is quite different. In this case the first Riemann sheet of the function $F(p_0^2)$ contains, in addition, the set of real points p_0^2 such that $4\Delta^2 \leq p_0^2 < \mu_Q^2$. Only among these points the zero M_D^2 of $F(p_0^2)$ is located. This means that the existence of an $SU(2)_c$-singlet stable diquark excitation with mass M_D such that $2\Delta \leq M_D < |\mu_Q|$ (see Fig. 8) is typical for the neutral gapless 2SC phase (at $H = 3G/4$), in contrast to the neutral gapped one (at $H = G$), where it is a resonance (see Fig. 7). In addition, it is easily concluded that for the same relation $H = 3G/4$ the $SU(2)_c$-singlet diquark is a heavy resonance in the 2SC (gapped) phase without a neutrality requirement [30,31], whereas it is a stable particle in the gapless neutral 2SC phase, if the neutrality constraints are fulfilled.[5]

Notice that in the present paper the dispersion relations of mesons and diquarks, i.e. the momentum dependence of their energies, are actually investigated for the particular value of the three-momentum, $\vec{p} = 0$. On the contrary, in

[5]Let us quote also another less rigorous, but more physical argument in favor of the stability of the $SU(2)_c$-singlet diquark excitation in the g2SC phase ($H = 3G/4$). Namely, the decay of the above-mentioned diquark excitation with mass M_D into a pair of u- and d-quark quasiparticles is forbidden due to the energy disbalance. Indeed, the total energy $E(p)$ of this quark pair in their center-of-mass system takes the form $E(p) = E_u(p) + E_d(p)$, where $p = |\vec{p}| \geq 0$ and $E_u(p) = E_\Delta^- + |\delta\mu|$, $E_d(p) = |E_\Delta^- - |\delta\mu||$. One can easily show that $E(p) \geq |\mu_Q|$. Since the mass M_D of the $SU(2)_c$-singlet diquark excitation in the g2SC phase is smaller than $|\mu_Q|$, we conclude that the decay of this excitation into a pair of u and d quasiparticles is forbidden.

the recent papers [24,25] the diquark dispersion relations were studied at $\vec{p} \neq 0$, and a conclusion about the Higgs instability of the NJL gapless 2SC phase was made. In this case the g2SC phase is unstable vs fluctuations of the Nambu-Goldstone diquark fields and, as a result, some of the diquarks acquire a nonphysical negative velocity squared [24]. (In addition, the g2SC phase of a gauged NJL model also suffers from a chromomagnetic instability [26].) In particular, it was shown that a sufficient condition for the Higgs instability in the gapped 2SC phase is the relation $\sqrt{2}|\delta\mu| > \Delta$. Since in the case $G = H$ this relation is not fulfilled, one may conclude that, in the gapped neutral 2SC phase considered above, the Higgs instability is absent.

Finally, it is necessary to note that, in the physical $SU(2)_c$-singlet diquark channel, we have found a stable massive mode in the case of the g2SC phase ($H = 3G/4$), whereas in [24] it was claimed that a gapless tachyon emerges in this channel. Actually, there is no contradiction between our result and the result of that paper. Indeed, in our case the diquark mass M_D obeys the relation $2\Delta < M_D = p_0$. In contrast, in [24] those solutions for the diquark dispersion relation that are constrained by $|p_0|, |\vec{p}| \ll \Delta$ were investigated. This means that in the gapless color superconductor there are two branches for the dispersion relation of the physical $SU(2)_c$-singlet diquark mode at small values of $|\vec{p}|$. The first is the massive heavy excitation with $2\Delta < p_0$ (see Fig. 8 of the present paper), and the other one corresponds to a gapless tachyon, located in a quite different kinematic region with $|p_0| \ll \Delta$. In addition, it was shown in [24] that the singularity $p_0/|\vec{p}|$ appears in the diquark two-point 1PI Green function in the gapless 2SC phase. In this case, owing to this singularity, it is quite possible to get, at $|\vec{p}| \to 0$, a result (gapless tachyon) that does not coincide with the diquark mass, directly calculated at $\vec{p} = 0$.

B. Diquarks in the normal phase ($\Delta = 0, \mu_8 = 0, \mu_Q = 0$)

At $\Delta = 0$ the three complex diquark fields $\Delta_A(x)$ ($A = 2, 5, 7$) are not mixed with other fields in the second order effective action (23) of the model. Since in addition the quantities μ_8, μ_Q are equal to zero in the normal phase, there is an $SU(3)_c$ symmetry of the ground state of this phase. So, in order to study the diquark masses at $\mu_B < \mu_B^c$, it is enough to consider, e.g., the Δ_2-diquark sector only. In this phase the inverse propagator matrix $\mathcal{G}^{-1}(p_0)$ for the diquarks $\Delta_2(x)$, $\Delta_2^*(x)$ looks, in contrast to the inverse propagator (45) of these fields in the 2SC phase, more simple [30,31] (again, we use the rest frame, $\vec{p} = 0$, in the momentum space representation):

$$\mathcal{G}^{-1}(p_0) = -\begin{pmatrix} 0, & \bar{\Gamma}_{\Delta_2\Delta_2^*}(p_0) \\ \bar{\Gamma}_{\Delta_2^*\Delta_2}(p_0), & 0 \end{pmatrix}, \qquad (47)$$

where $\bar{\Gamma}_{\Delta_2\Delta_2^*}(p_0) = \bar{\Gamma}_{\Delta_2^*\Delta_2}(-p_0)$,

$$\bar{\Gamma}_{\Delta_2^*\Delta_2}(p_0) = \frac{1}{4H} - 16\int\frac{d^3q}{(2\pi)^3}\frac{E}{4E^2 - (p_0 + 2\mu_B/3)^2} \equiv \frac{1}{4H} - \Phi(\epsilon), \qquad (48)$$

and $\epsilon = (p_0 + 2\mu_B/3)^2$. Since the determinant of the inverse propagator matrix (47) takes the form

$$\det\mathcal{G}^{-1}(p_0) \equiv \bar{\Gamma}_{\Delta_2^*\Delta_2}(p_0)\bar{\Gamma}_{\Delta_2\Delta_2^*}(p_0) = \bar{\Gamma}_{\Delta_2^*\Delta_2}(p_0)\bar{\Gamma}_{\Delta_2^*\Delta_2}(-p_0), \qquad (49)$$

it is clear that the mass spectrum is defined by the zeros of the 1PI Green function (48). Note that the function $\Phi(\epsilon)$ is analytical in the whole complex ϵ plane, except for the cut $4M^2 < \epsilon$ along the real axis. [In general, this function is defined on a complex Riemann surface which is to be described by several sheets. The integral representation for $\Phi(\epsilon)$, given in (48), defines its values on the first sheet only. To find a value of $\Phi(\epsilon)$ on the rest of the Riemann surface, a special procedure of analytical continuation is needed (see, e.g., [30]).] It turns out that, for the model parameter set (32) and a wide set of the coupling constant H values (see below), the equation

$$\Phi(\epsilon) = \frac{1}{4H} \qquad (50)$$

has, on the first Riemann sheet of the variable ϵ, a single root ϵ_0 on the real axis such that $0 < \epsilon_0 < 4M^2$. In this case the physical meaning of ϵ_0 is that $\epsilon_0 = (M_D^o)^2$, where M_D^o is the mass of a stable diquark at $\mu_B = 0$. Having a root ϵ_0, one can find two zeros of the 1PI Green function $\bar{\Gamma}_{\Delta_2^*\Delta_2}(p_0)$ as well as four zeros of the determinant (49). They provide us with the following mass squared in the Δ_2^*, Δ_2 sector of the model:

$$(M_\Delta)^2 = (M_D^o - 2\mu_B/3)^2, \quad (M_{\Delta^*})^2 = (M_D^o + 2\mu_B/3)^2. \qquad (51)$$

In particular, if $H = 3G/4$, then $M_D^o \approx 1.988M$, and if $H = G$, then $M_D^o \approx 1.746M$ [here M is the constituent quark mass, or gap, in the normal phase (see Fig. 1), i.e. $M \approx 0.350$ GeV]. We relate M_Δ in (51) to the mass of the diquark with the baryon number $B = 2/3$, and M_{Δ^*} to the mass of the antidiquark with $B = -2/3$. The difference between diquark and antidiquark masses in (51) is explained by the absence of a charge conjugation symmetry in the presence of a chemical potential μ_B.

Finally, due to the underlying color $SU(3)_c$ symmetry, the previous statement is valid also for Δ_5^*, Δ_5 and Δ_7^*, Δ_7. As a result, we have a color antitriplet of diquarks with the mass M_Δ (51) as well as a color triplet of antidiquarks with the mass M_{Δ^*} in the mass spectrum of the normal phase, i.e. at $\mu_B < \mu_B^c$.

It is clear from Eq. (50) that its own solution ϵ_0 lies inside the interval $0 < \epsilon_0 < 4M^2$ only if $H^* < H < H^{**}$, where H^* and H^{**} are defined by

$$H^* \equiv \frac{1}{4\Phi(4M^2)} = \frac{\pi^2}{4[\Lambda\sqrt{M^2+\Lambda^2} + M^2\ln((\Lambda+\sqrt{M^2+\Lambda^2})/M)]},$$
$$H^{**} \equiv \frac{1}{4\Phi(0)} = \frac{\pi^2}{4[\Lambda\sqrt{M^2+\Lambda^2} - M^2\ln((\Lambda+\sqrt{M^2+\Lambda^2})/M)]} = \frac{3MG}{2(M-m)} \qquad (52)$$

(here the values of Λ, m, and G are presented in (32), whereas $M \approx 0.350$ GeV is the dynamical quark mass in the normal phase). In this case, as was noted above, ϵ_0 defines the stable diquark mass M_D^o in the vacuum, i.e. at $\mu_B = 0$, through the relation $\epsilon_0 = (M_D^o)^2$. The values of M_D^o are depicted in Fig. 9 as a function of the parameter $\eta = H/G$. [Note that $\eta^* = H^*/G$, $\eta^{**} = H^{**}/G = 1.5M/(M-m)$]. For a rather weak interaction in the diquark channel ($H < H^*$ or $\eta < \eta^*$), ϵ_0 runs onto the second Riemann sheet, and unstable diquark modes (resonances) appear. Contrarily, a sufficiently strong interaction in the diquark channel ($H > H^{**}$ or $\eta > \eta^{**}$) pushes ϵ_0 towards the negative semiaxis, i.e. $(M_D^o)^2 < 0$ in this case. The latter indicates a tachyon singularity in the diquark propagator, evidencing that the $SU(3)_c$-color symmetric ground state is not stable. Indeed, at a very large H, as it has been shown in [35], the color symmetry is spontaneously broken even at a vanishing chemical potential. The fact that at $\eta \to \eta^{**}_-$ the diquark mass M_D^o tends to zero may be considered as a precursor of the spontaneous breaking of the $SU(3)_c$ symmetry, taking place at $H = H^{**}$.

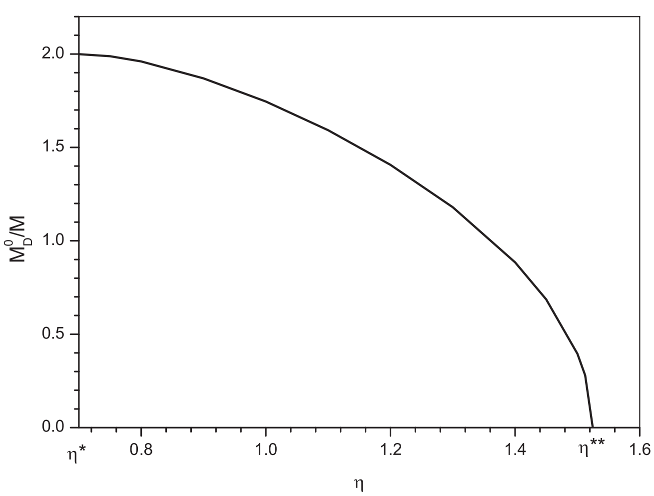

FIG. 9. The behavior of the stable diquark mass M_D^o vs $\eta \equiv H/G$ in the vacuum (at $\mu_B = 0$). Here $\eta^* = H^*/G \approx 0.698$, $\eta^{**} = H^{**}/G \approx 1.525$, $M \approx 0.350$ GeV. At $\eta < \eta^*$ there are no stable diquarks in the normal phase, and these particles are resonances. At $\eta > \eta^{**}$ an $SU(3)_c$ symmetric ground state of the normal phase (including the vacuum) is unstable in favor of color superconducting phases.

VI. SUMMARY AND DISCUSSION

The present paper is the last in the series of our papers [29–31], devoted to the investigation of the ground state bosonic excitations (mesons and diquarks) of color superconducting quark matter. The novel features of our present consideration, performed in the framework of the two-flavored NJL model, are the local electric charge neutrality constraint as well as the β equilibrium that is due to taking electrons into account. As a result, depending on the diquark channel coupling constant H, the gapless ($H = 3G/4$) or gapped ($H = G$) 2SC may exist in the system based on the Lagrangian (3). Since in (3) a new term with the electric charge chemical potential μ_Q appeared, we have an explicit breaking of the flavor SU(2) symmetry in both 2SC phases. So, in contrast to the non-neutral case [29–31], π mesons acquire a rather strong mass splitting in both gapless and gapped 2SC phases (see Figs. 5 and 6).

The diquark sector of the model consists of six modes. Since in the color and electrically neutral 2SC phase the chemical potential μ_8 is not equal to zero (see Fig. 3), the Lagrangian (3) is invariant under the color $SU(2)_c \times U(1)_{\lambda_8}$ group. It turns out that in the color superconducting phase this symmetry is spontaneously broken down to $SU(2)_c$, so, in accordance with general theorems, one of the diquark modes takes zero mass, i.e. the Nambu-Goldstone boson is an $SU(2)_c$ singlet. Besides, in this phase there are four very light diquark excitations that are composed into two real $SU(2)_c$ doublets with a common mass proportional to μ_8. In our opinion, when the NJL model is gauged, the above-mentioned diquark modes should be absent in the model. Instead, due to the Anderson-Higgs mechanism, five massive gluons will appear. So, we believe that five diquark excitations with zero or very small mass, observed in the 2SC phase of the nongauged model (3), are not physically interesting objects and might be ignored (see also the discussion in [32]).

The remaining one, an $SU(2)_c$-singlet diquark excitation of the 2SC ground state, has different properties under different external conditions. Indeed, if the electric charge neutrality requirement is not imposed, then, as it was shown in [30,31] at $H = 3G/4$, this diquark mode is a heavy resonance with mass approximately equal to 1100 MeV. However, if the neutrality and β-equilibrium conditions are imposed, then, on the one hand, at $H = 3G/4$ the gapless 2SC phase is realized, in which the above-mentioned diquark mode is already a stable particle, whose mass is evaluated around 200 MeV (see Fig. 8). Note that its mass is approximately equal to the value of $|\mu_Q|$ presented by Fig. 4. On the other hand, at $H = G$ we have the neutral gapped 2SC phase, in which the diquark is still a resonance but with a much smaller mass (see Fig. 7).

It was shown, e.g., in [40] that at sufficiently high baryon densities, comparable with densities inside neutron stars, the normal quark matter equation of state is significantly influenced by scalar mesons (σ mesons, etc.). Moreover, these mesons might change significantly the mass and radius of the neutron star as well as the role of hyperon degrees of freedom in dense matter. Since the masses of diquarks in the electrically neutral color superconducting quark matter are of the same order as those of mesons (compare Figs. 5–8), one might expect similar effects in the measurable parameters of neutron stars (e.g. their masses, radii, etc.), when diquarks will be taken into account properly in neutron star physics (see also [28]). So, the influence of rather light diquark excitations might, in principle, be checked by astrophysical observations.

In fact, in the present paper the dispersion relations for mesons and diquarks following from the NJL model (3) are investigated in the rest frame, i.e. at $|\vec{p}| = 0$. In the recent paper [24] (see also [25]) the diquark dispersion relations at small nonzero $|\vec{p}|$ were studied in the same model. It was found there that, in the gapless 2SC phase ($H = 3G/4$), Nambu-Goldstone diquark modes have a nonphysical negative velocity squared, i.e. the model is unstable against fluctuations of these fields. [In contrast, as it follows from our discussion at the end of Sec. VA, in the gapped 2SC phase ($H = G$) this kind of instability is absent.] Moreover, it was proved in [24] that at small $|\vec{p}| \neq 0$ in the $SU(2)_c$-singlet diquark channel of the g2SC phase there is a gapless tachyon with $|p_0| \ll \Delta$. Our result, i.e. the existence of a stable diquark excitation with mass $M_D \sim 200$ MeV (see Fig. 8) in the same channel, does not conflict with this, since $p_0 = M_D$ belongs to quite another kinematic region, where $p_0 = M_D > 2\Delta$. In spite of the fact that in our calculations $|\vec{p}| = 0$, we believe that at $|\vec{p}| \neq 0$ there is also a branch of the $SU(2)_c$-singlet diquark dispersion relation, which corresponds to a stable excitation of the g2SC phase with mass M_D (see Fig. 8).

Moreover, in the present paper the behavior of the diquark mass at vanishing μ_B as a function of the coupling constant H is also obtained (see Fig. 9).

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APPENDIX A: SOME FORMULAS

This appendix contains some useful formulas employed in the text.

(i) *Determinant*:

$$\det\begin{pmatrix} A, & B \\ C, & D \end{pmatrix} = \det[-CB + CAC^{-1}D] = \det[DA - DBD^{-1}C]. \quad (A1)$$

(ii) *Inverse matrix*:

$$\begin{pmatrix} A, & B \\ C, & D \end{pmatrix}^{-1} = \begin{pmatrix} C^{-1}DL, & -N \\ -L, & B^{-1}AN \end{pmatrix} = \begin{pmatrix} \bar{L}, & -A^{-1}B\bar{N} \\ -D^{-1}C\bar{L}, & \bar{N} \end{pmatrix}, \quad (A2)$$

where

$$L = [AC^{-1}D - B]^{-1}, \qquad N = [DB^{-1}A - C]^{-1},$$
$$\bar{L} = [A - BD^{-1}C]^{-1}, \qquad \bar{N} = [D - CA^{-1}B]^{-1}. \quad (A3)$$

(iii) *Variational derivatives*:
Let A, B be some operators in the coordinate space with matrix elements $A(x, y) \equiv A(x - y)$ and $B(x, y) \equiv B(x - y)$, respectively. Moreover, let $\sigma(x)$ and $\phi(x)$ be some fields. Then,

$$\mathrm{Tr}\{A\sigma B\phi\} \equiv \int dxdydzduA(x, z)\sigma(z)\delta(z - y) \times B(y, u)\phi(u)\delta(u - x) = \int dxdyA(x, y)\sigma(y)B(y, x)\phi(x). \quad (A4)$$

It follows from (A4) that

$$\frac{\delta^2 \mathrm{Tr}\{A\sigma B\phi\}}{\delta\sigma(y)\delta\phi(x)} = A(x, y)B(y, x) = A(x - y)B(y - x). \quad (A5)$$

APPENDIX B: QUARK PROPAGATOR IN THE NAMBU-GORKOV REPRESENTATION

In the Nambu-Gorkov representation, the inverse quark propagator matrix S_0^{-1} is defined in (19). Using the relation (A2) as well as the energy projection operator technique of [34], one can obtain the following expressions for the matrix elements $S_{ij}(x - y)$ of the quark propagator $S_0(z)$ (here $z = x - y$):

$$S_{11}(z) = A_{11}(z)\tau_+\tau_- + B_{11}(z)\tau_-\tau_+, \qquad S_{12}(z) = A_{12}(z)\tau_+ + B_{12}(z)\tau_-,$$
$$S_{22}(z) = A_{22}(z)\tau_+\tau_- + B_{22}(z)\tau_-\tau_+, \qquad S_{21}(z) = A_{21}(z)\tau_+ + B_{21}(z)\tau_-, \tag{B1}$$

where their explicit structure in the flavor space is presented with the help of $\tau_\pm \equiv (\tau_1 \pm \tau_2)/\sqrt{2}$ matrices (recall that τ_i are the Pauli matrices) and

$$A_{11}(z) = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} e^{-iqz}\Big\{\frac{(q_0+\delta\mu) - E^+}{(q_0+\delta\mu)^2 - (E_\Delta^+)^2}\gamma^0\bar{\Lambda}_+ + \frac{(q_0+\delta\mu)+E^-}{(q_0+\delta\mu)^2-(E_\Delta^-)^2}\gamma^0\bar{\Lambda}_-\Big\}P_{12}^{(c)} + \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^0\bar{\Lambda}_+}{q_0+E+\mu_{ub}} + \frac{\gamma^0\bar{\Lambda}_-}{q_0+\mu_{ub}-E}\Big\}P_3^{(c)} \equiv a_{11}^{(12)}(z)P_{12}^{(c)} + a_{11}^{(3)}(z)P_3^{(c)}, \tag{B2}$$

$$B_{11}(z) = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} e^{-iqz}\Big\{\frac{(q_0-\delta\mu) - E^+}{(q_0-\delta\mu)^2 - (E_\Delta^+)^2}\gamma^0\bar{\Lambda}_+ + \frac{(q_0-\delta\mu)+E^-}{(q_0-\delta\mu)^2-(E_\Delta^-)^2}\gamma^0\bar{\Lambda}_-\Big\}P_{12}^{(c)} + \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^0\bar{\Lambda}_+}{q_0+\mu_{db}+E} + \frac{\gamma^0\bar{\Lambda}_-}{q_0+\mu_{db}-E}\Big\}P_3^{(c)} \equiv b_{11}^{(12)}(z)P_{12}^{(c)} + b_{11}^{(3)}(z)P_3^{(c)}, \tag{B3}$$

$$A_{22}(z) = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} e^{-iqz}\Big\{\frac{(q_0-\delta\mu) - E^-}{(q_0-\delta\mu)^2 - (E_\Delta^-)^2}\gamma^0\bar{\Lambda}_+ + \frac{(q_0-\delta\mu)+E^+}{(q_0-\delta\mu)^2-(E_\Delta^+)^2}\gamma^0\bar{\Lambda}_-\Big\}P_{12}^{(c)} + \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^0\bar{\Lambda}_+}{q_0-\mu_{ub}+E} + \frac{\gamma^0\bar{\Lambda}_-}{q_0-\mu_{ub}-E}\Big\}P_3^{(c)} \equiv a_{22}^{(12)}(z)P_{12}^{(c)} + a_{22}^{(3)}(z)P_3^{(c)}, \tag{B4}$$

$$B_{22}(z) = \frac{1}{2}\int \frac{d^4q}{(2\pi)^4} e^{-iqz}\Big\{\frac{(q_0+\delta\mu) - E^-}{(q_0+\delta\mu)^2 - (E_\Delta^-)^2}\gamma^0\bar{\Lambda}_+ + \frac{(q_0+\delta\mu)+E^+}{(q_0+\delta\mu)^2-(E_\Delta^+)^2}\gamma^0\bar{\Lambda}_-\Big\}P_{12}^{(c)} + \frac{1}{2}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^0\bar{\Lambda}_+}{q_0-\mu_{db}+E} + \frac{\gamma^0\bar{\Lambda}_-}{q_0-\mu_{db}-E}\Big\}P_3^{(c)} \equiv b_{22}^{(12)}(z)P_{12}^{(c)} + b_{22}^{(3)}(z)P_3^{(c)}, \tag{B5}$$

$$A_{12}(z) = -\frac{\Delta\lambda_2}{\sqrt{2}}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^5\bar{\Lambda}_+}{(q_0+\delta\mu)^2-(E_\Delta^-)^2} + \frac{\gamma^5\bar{\Lambda}_-}{(q_0+\delta\mu)^2-(E_\Delta^+)^2}\Big\} \equiv a_{12}(z)\lambda_2, \tag{B6}$$

$$B_{12}(z) = \frac{\Delta\lambda_2}{\sqrt{2}}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^5\bar{\Lambda}_+}{(q_0-\delta\mu)^2-(E_\Delta^-)^2} + \frac{\gamma^5\bar{\Lambda}_-}{(q_0-\delta\mu)^2-(E_\Delta^+)^2}\Big\} \equiv b_{12}(z)\lambda_2, \tag{B7}$$

$$B_{21}(z) = \frac{\Delta^*\lambda_2}{\sqrt{2}}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^5\bar{\Lambda}_+}{(q_0+\delta\mu)^2-(E_\Delta^+)^2} + \frac{\gamma^5\bar{\Lambda}_-}{(q_0+\delta\mu)^2-(E_\Delta^-)^2}\Big\} \equiv b_{21}(z)\lambda_2, \tag{B8}$$

$$A_{21}(z) = -\frac{\Delta^*\lambda_2}{\sqrt{2}}\int\frac{d^4q}{(2\pi)^4}e^{-iqz}\Big\{\frac{\gamma^5\bar{\Lambda}_+}{(q_0-\delta\mu)^2-(E_\Delta^+)^2} + \frac{\gamma^5\bar{\Lambda}_-}{(q_0-\delta\mu)^2-(E_\Delta^-)^2}\Big\} \equiv a_{21}(z)\lambda_2. \tag{B9}$$

In the above formulas $P_{12}^{(c)} = \mathrm{diag}(1,1,0)$, $P_3^{(c)} = \mathrm{diag}(0,0,1)$ are the projectors on the red-green and blue subspaces of the color space, correspondingly; λ_2 is the Gell-Mann matrix; $\bar{\Lambda}_\pm = \frac{1}{2}(1 \pm \frac{\gamma^0(\vec{\gamma}\vec{q}-M)}{E})$ are projectors on the solutions of the Dirac equation with positive/negative energy. The other notations appearing in (B2)–(B9) are identical to those of (27). Note that, in (B2)–(B9) and similar integrals containing an integration over the energy variable, the symbol q_0 is a shorthand notation for $q_0 + i\varepsilon\cdot\mathrm{sign}(q_0)$, where $\varepsilon \to 0_+$. This prescription correctly implements the roles of μ_B, μ_8, and μ_Q as chemical potentials and preserves the causality of the theory (see, e.g., [37]). Introducing the new projectors $\Lambda_\pm = \frac{1}{2}(1 \pm \frac{\gamma^0(\vec{\gamma}\vec{q}+M)}{E})$, it is very convenient to use in trace calculations the following relations:

$$\gamma^5\bar{\Lambda}_\pm\gamma^5 = \Lambda_\pm, \qquad \gamma^0\bar{\Lambda}_\pm\gamma^0 = \Lambda_\mp, \qquad \Lambda_\pm^2 = \Lambda_\pm, \qquad \Lambda_\pm\Lambda_\mp = 0,$$
$$\mathrm{Tr}\Lambda_\pm = 2, \qquad \mathrm{Tr}(\Lambda_\pm\bar{\Lambda}_\pm) = \frac{2\vec{q}^2}{E^2}, \qquad \mathrm{Tr}(\Lambda_\pm\bar{\Lambda}_\mp) = \frac{2M^2}{E^2}. \tag{B10}$$

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