New Lorentz-violating nonlocal field theory from string theory

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A four-dimensional field theory with a qualitatively new type of nonlocality is constructed from a setting where Kaluza-Klein particles probe toroidally compactified string theory with twisted boundary conditions. In this theory fundamental particles are not pointlike and occupy a volume proportional to their R-charge. The theory breaks Lorentz invariance but appears to preserve spatial rotations. At low energies, it is approximately $N = 4$ Super Yang-Mills theory, deformed by an operator of dimension seven. The dispersion relation of massless modes in vacuum is unchanged, but under certain conditions in this theory, particles can travel at superluminal velocities.

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I. INTRODUCTION

Violation of Lorentz invariance at high energies is an interesting theoretical possibility, and it is important to explore possible extensions of the standard model, and field theories in general, that incorporate it. Indeed, there exists a large body of work that covers various aspects of possible Lorentz-violating extensions of field theories (see [1–5] and references therein for a small sample of a vast literature). Many models of this kind start by adding to the Lagrangian a Lorentz-violating term that is an IR-irrelevant local operator (so that the low-energy behavior will be unaffected), and then there arises the question of whether a consistent UV-completion exists.

Consider, for example, adding a local Lorentz-violating term to $N = 4$ super Yang-Mills (SYM) theory. By themselves, terms of conformal dimension $\Delta > 4$ lead to a theory that is not UV-complete. Nonetheless, some examples of UV-complete Lorentz-violating deformations of SYM are known. One example is SYM on a space of noncommutative geometry [6]. There, at low energy the deformation operator is a 2-form of dimension $\Delta = 6$, which breaks the Lorentz group to $SO(2) \times SO(1, 1)$. Another example is dipole theory [7] where at low energy the deformation operator is a spacetime vector of dimension $\Delta = 5$, which breaks the Lorentz group to $SO(2, 1)$. In both examples, UV-completeness is maintained because, in addition to the leading deformation operator, the Lagrangian has an infinite number of nonrenormalizable local terms, which sum up to renormalizable *nonlocal* interactions. Both examples can be realized in string theory [8–10]. A general discussion of nonlocality and its relation to a consistent UV-completion of nonrenormalizable interactions appeared recently in [11].

For phenomenological applications, and also for theoretical exploration, it would be interesting to have new examples of Lorentz-violating theories that break $SO(3, 1)$ to $SO(3)$, thus preserving spatial rotations. In this paper I propose a string theoretic construction of such a nonlocal, Lorentz-violating field theory. The theory is a deformation of $N = 4$ SYM, and the deformation parameter has the dimensions of volume. This defines a new kind of nonlocality which is fundamentally different from the two examples mentioned above.

The construction, which involves brane probes in type-II string theory, is presented in Sec. II. In Sec. III Bogomol'nyi-Prasad-Sommerfield monopoles (BPS) bounds on energies of states with electric and magnetic fluxes are presented and interpreted. Section V concludes with a discussion of various novel effects in this theory, including superluminal velocities.

II. CONSTRUCTION

The formulation of the new nonlocal field theory is inspired by Douglas and Hull's construction of gauge theories on a noncommutative torus [8]. Douglas and Hull started with a compactification of type-IIA string theory on a small T^2 and considered n coincident D0-branes in the limit where the area of the T^2 approaches zero. This setting is T-dual to type-IIA on a large T^2 with n D2-branes, and can be described by a $U(n)$ Super Yang-Mills (SYM) theory at low energy. But if an NSNS 2-form flux B is turned on along the T^2, T-duality does not map a small T^2 to a large one. Rather, as Douglas and Hull argued, in an appropriate limit the D2-branes are described by a field theory with nonlocal interactions. It is a deformation of SYM theory that can be formally interpreted as a field theory on a torus whose coordinates are noncommutative.

Let us now turn to our construction. Start with type-IIA string theory on T^3, and let the compactification radii be R'_1, R'_2, R'_3. Denote the type-IIA string scale by $M'_{\rm st}$, and the type-IIA string coupling-constant by $g'_{\rm st}$. Now add a Kaluza-Klein particle with n units of momentum in the 1st direction, and take the limit

$$R'_1 \to 0, \qquad M'_{\rm st} R'_1 \to 0, \qquad M'^3_{\rm st} R'_1 \to \infty, \tag{1}$$

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$$g'_{\rm st} \to \text{finite}, \qquad M'_{\rm st} R'_k \to \text{finite}, \qquad (k = 2, 3). \tag{2}$$

An appropriate U-duality transformation transforms this setting to a configuration of n uncompactified D3-branes. Specifically, T-duality in the 1st direction, followed by S-duality, followed by T-duality in the 2nd and 3rd directions converts the Kaluza-Klein particle to n D3-branes compactified on T^3 with compactification radii

$$R_1 = \frac{1}{M'^2_{\rm st} R'_1}, \qquad R_k = \frac{g'_{\rm st}}{M'^3_{\rm st} R'_k R'_1}, \qquad (k = 2, 3) \tag{3}$$

and type-IIB string scale and coupling-constant

$$M_{\rm st} = M'_{\rm st}\left(\frac{M'_{\rm st} R'_1}{g'_{\rm st}}\right)^{1/2}, \qquad g_{\rm st} = \frac{1}{M'^2_{\rm st} R'_3 R'_2}. \tag{4}$$

Note that the original type-IIA coupling-constant is given, in terms of the type-IIB parameters $g_{\rm st}$, R_1, R_2, R_3, by

$$g'_{\rm st} = \left(\frac{R_2 R_3}{R_1^2 g_{\rm st}}\right)^{1/2}.$$

In the limit (1) and (2), the type-IIB T^3 becomes large,

$$M_{\rm st} R_1 = \frac{1}{(g'_{\rm st} M'_{\rm st} R'_1)^{1/2}} \to \infty, \tag{5}$$

$$M_{\rm st} R_k = \frac{1}{M'_{\rm st} R'_k}\left(\frac{g'_{\rm st}}{M'_{\rm st} R'_1}\right)^{1/2} \to \infty \qquad (k = 2, 3), \tag{6}$$

while the type-IIB string coupling-constant and the shape of the type-IIB T^3 stay fixed,

$$g_{\rm st} \to \text{finite}, \qquad \frac{R_k}{R_1} = \frac{g'_{\rm st}}{M'_{\rm st} R'_k} \to \text{finite}, \qquad (k = 2, 3).$$

Thus, in the limit (1) and (2), the Kaluza-Klein particle is described at low-energy by $U(n)$ $\mathcal{N} = 4$ SYM with finite coupling-constant $g^2_{\rm ym} = 2\pi g_{\rm st}$, compactified on a large T^3 of radii R_1, R_2, R_3.

Similarly to Douglas and Hull's B-field flux [8], we now add an obstruction that will prevent U-duality from producing a large T^3 on the type-IIB side. Unlike Douglas and Hull's construction, however, our obstruction will not be a flux but a *geometrical twist*. A geometrical twist in the 1st direction is defined as follows. Start with flat space $\mathbb{R}^{9,1}$, and let $t, x_1, \ldots, x_9$ be Minkowski coordinates so that the metric is

$$ds^2 = -dt^2 + \sum_{i=1}^{9} dx_i^2.$$

Now pick a constant matrix $\zeta' \in so(6)$ and make the global identification

$$x_1 \sim x_1 + 2\pi R'_1, \qquad x_{a+3} \sim \sum_{b=1}^{6} [\exp(2\pi\zeta')]^b{}_a x_{b+3} \qquad (a = 1, \ldots, 6). \tag{7}$$

In other words, we mod out $\mathbb{R}^{9,1}$ by a discrete group generated by a simultaneous translation in the 1st direction and rotation in directions 4,…,9. This group has no fixed points, and can easily be extended to act on the spin-structure of $\mathbb{R}^{9,1}$, by taking $\exp(2\pi\zeta')$ in a spinor representation of $so(6)$. (Such spaces have had many theoretical applications in string theory, a sample of which is listed in [12–22] and references therein.)

Next, we compactify the 2nd and 3rd directions on circles of radii R'_2, R'_3 with the usual identifications $x_k \sim x_k + 2\pi R'_k$ $(k = 2, 3)$ and use the resulting space as a type-IIA background.

We continue as in the beginning of this section; we probe the background with a Kaluza-Klein particle with n units of momentum in the 1st direction, and we take the limit (1) and (2), combined with

$$\zeta \equiv \frac{g'^2_{\rm st}}{M'^8_{\rm st} R'^3_1 R'_2 R'_3}\zeta' = \zeta' R_1 R_2 R_3 \to \text{finite}. \tag{8}$$

Here R_1, R_2, R_3 are still defined by (3), but they are no longer the geometrical compactification radii. The goal of this paper is to argue that in the limits (1), (2), and (8) the Kaluza-Klein particle is described at low-energy (below the string scale) by a nonlocal field theory that breaks Lorentz invariance but preserves rotational invariance. We will see that in the IR limit it can be approximated by $\mathcal{N} = 4$ SYM deformed by a dimension $\Delta = 7$ operator, the deformation parameter ζ having dimension (-3). For reasons to be explained in Sec. II B, I will refer to this conjectured field theory as *puffed field theory* (PFT). It will be useful to also consider the case where R_1, R_2, R_3 are large but finite: $R_1, R_2, R_3 \gg M^{-1}_{\rm st}$. I will refer to this theory as *PFT formulated on* T^3 and to R_1, R_2, R_3 as the formal compactification radii.

A. Supersymmetry

Before we proceed to explore the unique properties of PFT, we have to digress and discuss the conditions on ζ' that are required for PFT to be supersymmetric. Preserving some amount of supersymmetry is important, because non-perturbatively, the generic background (7) can be unstable. The stability of similar solutions has been analyzed in [13–16], and the generic background can decay either by a process of "bubble-of-nothing" nucleation or by a process reminiscent of Schwinger pair-production. However, these mechanisms do not destabilize the vacuum if supersymmetry is preserved.

It is not hard to see that the background defined by the identification (7) preserves 8 supersymmetry generators if $\zeta \in su(3) \subset so(6)$. We can choose a coordinate basis where ζ is of the form

$$\zeta = \begin{pmatrix} 0 & \beta_1 & 0 & 0 & 0 & 0 \\ -\beta_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 \\ 0 & 0 & -\beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta_3 \\ 0 & 0 & 0 & 0 & -\beta_3 & 0 \end{pmatrix} \in so(6). \tag{9}$$

Then, 8 linearly independent supersymmetry generators are preserved if $\beta_1 + \beta_2 + \beta_3 = 0$. If further $\beta_3 = 0$ then 16 supersymmetries are preserved. On the other hand, if for all combinations of $(\pm)$ signs $\beta_1 \pm \beta_2 \pm \beta_3 \neq 0$, then no supersymmetry is preserved. In what follows, unless stated differently, I will assume that $\beta_1 + \beta_2 + \beta_3 = 0$. The presence of n units of Kaluza-Klein momentum in the construction of PFT breaks additional supersymmetry, and thus PFT preserves 4 generators if $\beta_1 + \beta_2 + \beta_3 = 0$ and 8 if $\beta_1 = -\beta_2$ and $\beta_3 = 0$.

B. *R*-charge and nonlocality

What does PFT look like? I do not know the full Lagrangian description of PFT, but it is possible to make several observations without a full Lagrangian. In Sec. III exact results for some low-lying energy states are presented, and in Sec. IV a Lagrangian description up to order $O(\zeta)$ is discussed. These results suggest that PFT is a nonlocal theory with a unique structure of nonlocality. In a nutshell, it can be summarized as follows: *R-charge in PFT carries an intrinsic volume proportional to* ζ.

This means the following. In pure $N = 4$ SYM, the R-symmetry is $SU(4)$ and R-charge $\hat{J}$ is an element of the Lie algebra $su(4) \simeq so(6)$. The generic parameter ζ of PFT breaks $SU(4)$ down to its Cartan subalgebra $U(1)^3 \subset SU(4)$, because the Cartan subalgebra is the subgroup that commutes with a generic element $\zeta \in su(4)$. If ζ is such that $N = 2$ is preserved (see Sec. II A) then the R-symmetry is broken down to $U(1) \times U(2) \subset SU(4)$. To cast the "nutshell" statement above in a formula, we associate with R-charge $\hat{J}$, which is an element of the appropriate unbroken subalgebra of the Lie algebra $su(4)$, an intrinsic volume

$$\tilde{V} \equiv \tfrac{1}{2}(2\pi)^3 \operatorname{Tr}\{\hat{J}\zeta\}, \tag{10}$$

where both $\hat{J}$ and ζ are understood as elements of the Lie algebra $su(4) \simeq so(6)$, and the trace is taken in the representation **6**. The volume in (10) can be positive or negative, which corresponds to opposite orientations.

In pure $N = 4$ SYM, the six scalars are in the representation **6** of $su(4)$ and the fermions fit into the representations **4** and $\bar{\mathbf{4}}$. In PFT, if we take ζ of the form (9), then objects with the same R-charge as the components of the scalars acquire, according to (10), volume factors $\pm\beta_1$, $\pm\beta_2$, $\pm\beta_3$. Similarly, objects with the same R-charge as the components of the fermions acquire volume factors $(\pm\beta_1 \pm \beta_2 \pm \beta_3)/2$.

The heuristic picture advocated in (10) can be understood from the construction of Sec. II as follows. Consider first the geometric-twist background (7), and define the change of variables

$$\rho_a e^{i\theta_a} \equiv x_{2a+2} + i x_{2a+3}, \qquad a = 1, 2, 3.$$

An arbitrary scalar field in the geometry (7) can be expanded in a Fourier series as follows

$$\phi(t, x_1, \ldots, x_9) = \sum_{j_1, j_2, j_3} \sum_k \chi_{j_1 j_2 j_3 k}(t, x_2, x_3, \rho_1, \rho_2, \rho_3) \times e^{i\sum_{a=1}^3 j_a\theta_a} \exp\frac{i(k + \sum_{a=1}^3 \beta'_a j_a)x_1}{R'_1}, \tag{11}$$

where $\chi_{j_1 j_2 j_3 k}$ are arbitrary functions, and I have taken ζ' to be of the form

$$\zeta' = \begin{pmatrix} 0 & \beta'_1 & 0 & 0 & 0 & 0 \\ -\beta'_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta'_2 & 0 & 0 \\ 0 & 0 & -\beta'_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta'_3 \\ 0 & 0 & 0 & 0 & -\beta'_3 & 0 \end{pmatrix}, \tag{12}$$

which matches (8) and (9) if

$$\beta_a \equiv \frac{g'^2_{\rm st}}{M'^8_{\rm st} R'^3_1 R'_2 R'_3} \beta'_a, \qquad a = 1, 2, 3. \tag{13}$$

Equation (11) is the general expression that satisfies the periodic boundary conditions set in (7), and it can be interpreted as follows [22]. Let

$$P'_1 \equiv \frac{k + \sum_{a=1}^3 \beta_a j_a}{R'_1} \tag{14}$$

be the Kaluza-Klein momentum in the 1st direction, and let

$$\hat{J} = \begin{pmatrix} 0 & -j_1 & 0 & 0 & 0 & 0 \\ j_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -j_2 & 0 & 0 \\ 0 & 0 & j_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -j_3 \\ 0 & 0 & 0 & 0 & j_3 & 0 \end{pmatrix}, \tag{15}$$

be the angular momentum matrix. The unbroken rotation algebra is $so(2) \oplus so(2) \oplus so(2) \subset so(6)$, and I used the embedding in $so(6)$ to express $\hat{J}$ as a 6×6 matrix, which will make the notation more convenient. Equation (11) implies a linear relation between the fractional part of $P'_1 R'_1$ and the angular momentum $\hat{J}$,

$$P'_1 R'_1 = \tfrac{1}{2}\operatorname{Tr}\{\zeta'\hat{J}\} \qquad (\text{mod } \mathbb{Z}). \tag{16}$$

Now let us inspect (16) after the U-duality transformation (3) and (4) is performed, and after the limits (1), (2), and (8) are taken. $P'_1 R'_1$ becomes the *effective* number of D3-

branes $n_{\rm eff}$, and we learn from (16) that it is fractional, formally! The fractional part is given by

$$n_{\rm eff} = \tfrac{1}{2}{\rm Tr}\{\zeta'\hat{J}\} = \tfrac{1}{2}(R_1R_2R_3)^{-1}\,{\rm Tr}\{\zeta\hat{J}\} \qquad ({\rm mod}\ \mathbb{Z}), \tag{17}$$

where I have used (3), (4), and (8) to replace ζ' with ζ. The total volume that this effective fractional number of D3-branes occupies is

$$(2\pi)^3R_1R_2R_3n_{\rm eff} = \tfrac{1}{2}(2\pi)^3\,{\rm Tr}\{\zeta\hat{J}\}. \tag{18}$$

Thus, a state with R-charge $\hat{J}$ in PFT heuristically behaves as if it has an extra finite chunk of D3-brane of finite volume $4\pi^3\,{\rm Tr}\{\zeta\hat{J}\}$, as stated in (10). Of course, conventional type-IIB string theory does not have such an "open" D3-brane. We will, however, see below that thinking about PFT in this way is very convenient.

III. ELECTRIC AND MAGNETIC FLUXES

PFT depends on two parameters—the dimensionless coupling-constant $g_{\rm ym}$, and the dimension $\Delta = -3$ parameter ζ, which scales like volume. From here until almost the rest of this section, the discussion will be restricted to a value of ζ that preserves 8 supersymmetries (see Sec. II A). It is then possible to provide a BPS bound on the energy of a state in PFT (formulated on T^3) with given momentum, electric and magnetic flux, and R-charge. The R-charge is taken in the form of (15) with $j_1 = -j_2 \equiv j$ and $j_3 = 0$, and thus is specified by the single integer j.

The BPS bound can be easily derived from the central charge of the supersymmetry algebra in the flat supersymmetric background defined by the boundary conditions (7). Note that because of the presence of ζ' in (7), if we define the Kaluza-Klein charge to be an integer, the central charge will be augmented by a term proportional to R-charge, as in the numerator of the right-hand side of (14).

Before proceeding to the BPS formula, let me note that the BPS bound can also be derived by realizing the setting from the beginning of Sec. II as a decompactification limit of a certain configuration of charges in type-IIA string theory on T^6. Electric and magnetic flux can then be realized as fundamental string and D1-brane winding numbers. We also need to realize the R-symmetry charge j and the geometrical twist parameter β. A Kaluza-Klein monopole can do the job for us.

Take a Kaluza-Klein monopole with one unit of charge dual to Kaluza-Klein momentum in the 6th direction. For large R_6, in the absence of other excitations, it can be described by the Taub-NUT metric:

$$\begin{aligned} ds^2 &= -dt^2 + \sum_{i=1}^5 R_i^2dx_i^2 \\ &\quad + \left(1+\frac{R_6}{2r}\right)^{-1}R_6^2\left(dx_6 - \frac{1}{2}\sin\theta d\phi\right)^2 \\ &\quad + \left(1+\frac{R_6}{2r}\right)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \end{aligned} \tag{19}$$

This metric has an isometry corresponding to the Killing vector $\partial/\partial x_6$. The isometry has fixed points at $r = 0$ where it acts nontrivially as a rotation of the tangent space. By modifying the periodicity conditions on the coordinates to

$$x_6 \sim x_6 + 2\pi N_1 + 2\pi\beta' N_2, \qquad x_1 \sim x_1 + 2\pi N_2, \qquad (N_1, N_2 \in \mathbb{Z}), \tag{20}$$

where $\beta' \equiv \beta/(R_1R_2R_3)$, we can realize the geometry (7) near the origin $r = 0$ of (19), in the limit $R_6 \to \infty$. (A similar setting was also used in [23] to construct the dual of (p, q) 5-branes, and in [24] to solve the moduli space of certain gauge theories with twisted boundary conditions.) From the point of view of type-IIA string theory on T^6, all that (20) does is change the asymptotic metric on T^6 at infinity. Specifically, the T^2 in the 1st and 6th directions now has a complex structure given by $\tau = \beta' + i(R_1/R_6)$. We can then borrow BPS bounds [25] on the mass of a configuration of charges (a "black-hole") in toroidally compactified type-II string theory to construct BPS bounds on the energy of states in PFT. In particular, the Kaluza-Klein momentum in the 6th direction is $2j$.

I will now present the result, after the appropriate limits are taken. Let $k_1, k_2, k_3 \in \mathbb{Z}$ be the integer Kaluza-Klein charges (the quantized units of momenta), let $e_1, e_2, e_3 \in \mathbb{Z}$ be the number of units of electric flux, and let $m_1, m_2, m_3 \in \mathbb{Z}$ be the number of units of magnetic flux, in directions 1, 2, 3, respectively. Set

$$V \equiv R_1R_2R_3, \tag{21}$$

and let $\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{n}}_3$ be unit vectors in directions 1, 2, 3, respectively, and define the spatial momentum vector

$$\mathbf{P} \equiv \sum_{i=1}^3 \frac{k_i}{R_i}\hat{\mathbf{n}}_i, \tag{22}$$

and spatial electric and magnetic field vectors

$$\mathbf{E} \equiv \sum_{i=1}^3 \frac{e_iR_i}{2\pi V}\hat{\mathbf{n}}_i, \qquad \mathbf{B} \equiv \sum_{i=1}^3 \frac{m_iR_i}{2\pi V}\hat{\mathbf{n}}_i. \tag{23}$$

Then, the BPS bound on the energy turns out to be

$$\begin{aligned} E &= 2\frac{M_{\rm st}^4}{g_{\rm st}}j\beta + \frac{2\pi^2V^2}{|nV + 2j\beta|}\left(\frac{g_{\rm ym}^2}{2\pi}\mathbf{E}^2 + \frac{2\pi}{g_{\rm ym}^2}\mathbf{B}^2\right) \\ &\quad + \left|\mathbf{P} - \frac{4\pi^2V^2}{|nV + 2j\beta|}\mathbf{E}\times\mathbf{B}\right|. \end{aligned} \tag{24}$$

The first term on the right-hand side of (24) contains the string scale M_{st} and is dominant. This term is to be expected following the picture sketched at the beginning of Sec. II B: j units of R-charge carry an intrinsic volume of $2(2\pi)^3 j\beta$, which in turn accounts for extra energy. (Note that $(2\pi)^{-3}M_{st}^4/g_{st}$ is the tension of a D3-brane [26], and $2(2\pi)^3 j\beta$ is the effective extra volume.) This term can be eliminated by a redefinition of the Hamiltonian H of PFT:

$$H \to H - \frac{M_{st}^4}{2g_{st}} \mathrm{Tr}\{\zeta \hat{J}\}. \tag{25}$$

Since R-charge $\hat{J}$ is conserved, the extra term commutes with the Hamiltonian and therefore has no effect on the dynamics. The redefinition (25) is equivalent to a time dependent field redefinition, whereby each field is multiplied by a time dependent phase proportional to its R-charge.

The remaining terms in (24) reveal some of the peculiar features of PFT. Set $\tilde{E} \equiv E - 2(M_{st}^4/g_{st}) j\beta$, and let us assume that the BPS bound is attained for some BPS state. First note that if we set the electric and magnetic fluxes to zero in (24) we get $\tilde{E} = |\mathbf{P}|$, and so the dispersion relation of massless particles in vacuum is unchanged. Next, note that with the definition

$$\mathbf{E}_{\text{eff}} \equiv \left|1 + \frac{2j\beta}{nV}\right|^{-(1/2)} \mathbf{E}, \qquad \mathbf{B}_{\text{eff}} \equiv \left|1 + \frac{2j\beta}{nV}\right|^{-(1/2)} \mathbf{B}, \tag{26}$$

we can rewrite (24) as

$$\tilde{E} = \frac{2\pi^2}{n} V\left(\frac{g_{ym}^2}{2\pi}\mathbf{E}_{\text{eff}}^2 + \frac{2\pi}{g_{ym}^2}\mathbf{B}_{\text{eff}}^2\right) + \left|\mathbf{P} - \frac{4\pi^2}{n}\mathbf{E}_{\text{eff}} \times \mathbf{B}_{\text{eff}} V\right|. \tag{27}$$

This is the same expression as for undeformed $N = 4$ SYM, except with $\mathbf{E}$, $\mathbf{B}$ replaced by $\mathbf{E}_{\text{eff}}$, $\mathbf{B}_{\text{eff}}$. The first term in (27) is the energy stored in the electric and magnetic fluxes, and the second term is the energy associated with particles that carry momentum, in excess of the momentum stored in the electric and magnetic fields. The novelty in PFT is that the quantization condition on the effective electric and magnetic fluxes, as given by the combination of (23) and (26), depends on the total R-charge of the system, which is obviously a nonlocal effect.

The singularity in (24) when $nV + 2j\beta = 0$ requires some discussion. In the following, I will relax the supersymmetry restriction on ζ and allow β_1, β_2, β_3 to be generic. Let $\mathcal{H}(n, \hat{J}; \zeta, V)$ be the sector of the Hilbert space of $U(n)$ PFT with R-charge specified by $\hat{J}$, as in (15), and compactified on a T^3 of volume $(2\pi)^3 V$. According to (10), this sector can be thought of as having an effective net D3-brane volume of $(2\pi)^3 \times (nV + \frac{1}{2}\mathrm{Tr}\{\zeta\hat{J}\})$. This volume can be either bigger or smaller than the original sum of the volumes of all the D3-branes, $(2\pi)^3 nV$. Let (j_1', j_2', j_3') be a set of integers, and collect them into an $so(6)$ matrix $\hat{J}'$ as in (15). Then, the above consideration suggests that we should have an equivalence of Hilbert spaces:

$$\mathcal{H}(n, \hat{J}; \zeta + \hat{J}'V, V) \simeq \mathcal{H}(n + \tfrac{1}{2}\mathrm{Tr}\{\hat{J}'\hat{J}\}, \hat{J}; \zeta, V). \tag{28}$$

(Note that $\frac{1}{2}\mathrm{Tr}\{\hat{J}'\hat{J}\}$ is an integer.) As presented, the construction in Sec. II only depends on the fractional part of $\beta_1, \ldots, \beta_3$. But it is actually discontinuous in these parameters, because as we increase, say, β_1 continuously from the value of 0 to 1 we end up generating j_1 additional Kaluza-Klein particles, by a mechanism similar to *spectral-flow*. We can modify the construction of Sec. II and place $(n + \sum_{a=1}^3 j_a[\beta_a])$ Kaluza-Klein particles instead of n (where $[x]$ denotes the largest integer not exceeding x). Then, (28) holds.

Now, let us return to the supersymmetric case. If $nV + 2j\beta = 0$, the effective number of D3-branes is zero. The Hilbert space should then be trivial, and there are no states with nonzero electric or magnetic flux. If $nV + 2j\beta < 0$, the effective number of D3-branes is negative, and we should interpret it as $|nV + 2j\beta|$ anti D3-branes. (It may, in fact, also make sense to define the sector as empty if $nV + 2j\beta < 0$).

Equation (28) suggests that even the $n = 0$ PFT is meaningful, as long as $V < \infty$ and we restrict to sectors with R-charge that satisfies $\mathrm{Tr}\{\zeta\hat{J}\} > 0$. In the limit $V \to \infty$, we see from (24) that the energy levels with finite electric or magnetic flux have energies that scale with the volume like $V^{2/3}$ for $n = 0$ (compared to $V^{-1/3}$ for $n > 0$). Thus, in the limit $V \to \infty$ the $n = 0$ theory does not accommodate electric or magnetic flux. (It might, in fact, become empty altogether in that limit.)

IV. ADDITIONAL PROPERTIES OF PFT

This section is devoted to a few additional observations and conjectures regarding the properties of PFT. PFT is a deformation of $N = 4$ SYM, and the deformation parameter is ζ. By construction, this parameter is in the adjoint representation **15** of the R-symmetry group $SU(4)$, and it has dimension $\Delta = -3$.

The transformation of ζ under the $SO(3, 1)$ Lorentz group is less clear. The construction in Sec. II singles out both the time direction and the 1st direction. However, both the heuristic picture of Section II B as well as the BPS formula (24) suggest that ζ is the 123 component of a 3-form. If this conjecture is true, PFT preserves the $SO(3)$ symmetry of spatial rotations, and ζ transforms under $SO(3, 1)$ as the time component of a 4-vector. Another argument for $SO(3)$ symmetry is that the U-duality transformation that was used in the construction of PFT in Sec. II, after (1) and (2), can be applied in the presence of the Kaluza-Klein monopole (19). After the duality we then get n D3-branes at the origin of the Taub-NUT space, and the parameter ζ becomes a nonzero asymptotic value for a component of the Ramond-Ramond 4-form potential at infinity with indices 1236 [referring to Eq. (19)]. But in the presence of such a nonzero boundary condition, the topology of the Taub-NUT metric implies a nonzero Ramond-Ramond 5-form field- strength at the origin. In this way we transform the PFT setting into n D3-branes sitting at a place (the origin of the Taub-NUT space) with strong Ramond-Ramond 5-form field-strength that has a component with three indices parallel to the D3-branes (directions 123) and two indices perpendicular to the branes (see also [27] for related constructions). Additional arguments in favor of the $SO(3)$ symmetry of PFT will be presented in [28], where the supergravity dual is constructed using techniques similar to [10,22,29–31].

Although PFT is generally a nonlocal theory, order by order in ζ it has to be describable by a local Lagrangian. This would be a low-energy expansion. In particular, to first order in ζ the correction to the $N = 4$ SYM Lagrangian density has to be of the form $\mathrm{Tr}\{\zeta\mathcal{O}\}$, where $\mathcal{O}$ is a local operator of dimension $\Delta = 7$, and in the adjoint representation of $SU(4)$. Furthermore, if ζ transforms as the time component of a 4-vector, $\mathcal{O}$ must also be the time component of a 4-vector. In addition, if ζ is taken to preserve $N = 2$ supersymmetry (see Sec. II A) then, up to total derivatives, $\mathrm{Tr}\{\zeta\mathcal{O}\}$ must commute with the unbroken supersymmetry generators. These arguments suggest that $\mathcal{O}$ is in a protected supermultiplet. In fact, the list of local operators in short supersymmetry multiplets of $N = 4$ SYM [32] contains a unique natural candidate for $\mathcal{O}$. It is a descendant of a chiral primary operator of dimension $\Delta = 4$, and is obtained by acting on the chiral primary with supersymmetry generators 6 times. The explicit expression is rather long, and will be presented elsewhere [28].

Now set $n = 1$ in (24) and expand to first order in ζ to obtain

$$\tilde{E} = |\mathbf{P}'| + 2\pi^2 V\left(\frac{g_{\mathrm{ym}}^2}{2\pi}\mathbf{E}^2 + \frac{2\pi}{g_{\mathrm{ym}}^2}\mathbf{B}^2\right) + \mathrm{Tr}(\zeta J_\mu)T^{0\mu} + O(\zeta^2), \tag{29}$$

where

$$\mathbf{P}' \equiv \mathbf{P} - 4\pi^2 V(\mathbf{E}\times\mathbf{B}),$$

is the excess momentum in addition to the electromagnetic field, $T^{\mu\nu}$ is the energy-momentum tensor of the electromagnetic field, and the R-symmetry 4-current J^μ is defined to have components

$$J^\mu = \left(\frac{1}{V}\hat{J}, \frac{\mathbf{P}'}{|\mathbf{P}'|V}\hat{J}\right). \tag{30}$$

Therefore, in this case the correction $\mathcal{O}$ reduces to $T^{0\mu}J_\mu$. (In general $\mathcal{O}$ has more terms which do not contribute to the BPS state in this discussion.)

S-duality of $N = 4$ SYM is also preserved by PFT. It is obvious from (4) that the duality $g_{\mathrm{st}} \to 1/g_{\mathrm{st}}$ follows from T-duality in directions 2, 3 in the type-IIA setting, and this duality is not affected by the parameter ζ. A θ-angle can also be turned on by adding an NSNS 2-form flux in directions 2, 3.

Additional properties of PFT including a proposal for the supergravity dual of the theory and an investigation of the UV properties of the theory will be reported elsewhere [28].

V. DISCUSSION

On large scales, FRW cosmology breaks Lorentz invariance down to rotational invariance, and it is natural to wonder whether this Lorentz violation has a counterpart in high energy phenomena. If such a violation exists at an energy scale Λ, then it is quite reasonable to expect Λ to be of the order of the (3 + 1D) Planck scale M_{p}, in which case the effects involve quantum gravity. It is also possible, however, that $\Lambda \ll M_{\mathrm{p}}$. For example, Λ could be around the grand unified theory scale. It is then possible that an approximate description around that scale involves a Lorentz-violating quantum field theory that preserves spatial rotations.

The *puffed field theory* described in this paper is conjectured to be a UV-complete field theory which breaks Lorentz invariance but preserves spatial rotations. The Lorentz violation is parameterized by $\zeta \sim \Lambda^3$. Although it is not a phenomenologically realistic model because of, among other things, the high amount of supersymmetry, it is quite possible that more realistic models with like features can be similarly constructed.

At low energy, the theory contains a Lorentz-violating term that has a rather universal structure: it is proportional to a contraction of the energy-momentum tensor and the R-current, $T^{0\mu}J_\mu$. Such a term can have two interesting effects. First, suppose we have a soliton of characteristic size r and mass M. Inside the soliton $T^{00} \sim M/r^3$, and the term $\zeta T^{00}J_0$ translates into an effective potential of the order of $V_0 = \pm M/\Lambda^3 r^3$ for a particle of R-symmetry charge of the order of ± 1. If M is big enough, the potential might have a bound state. In a nonrelativistic order-of-magnitude analysis, the condition for a bound state is $1 \lesssim m|V_0|r^2 \sim mM/\Lambda^3 r$, where m is the mass of the R-charged particle. In this model, ignoring other interactions, there will be a bound state for particles with positive R-charge and mass $m \gtrsim \Lambda^3 r/M$.

Second, in a medium with nonzero R-charge density $\chi \equiv \langle J_0\rangle$, the term $T^{0\mu}J_\mu$ is dominated by χT^{00}, and this modifies the dispersion relation of massless particles so that the speed of light in such a material becomes approximately $1 + (\zeta\chi/2)$. Thus, superluminal velocities can be achieved. (This is also the case in noncommutative geometry, as was nicely demonstrated in [33,34]). However, it has to be mentioned that the interaction of PFT with gravity is not straightforward, and this issue has to be addressed before extensions to phenomenology can be discussed in a meaningful way. See [35] for some of the issues that can arise when a theory with a spontaneously broken Lorentz invariance is coupled to gravity.

The construction presented in this letter is reminiscent of the construction of dipole field theories in [10,17,22]. The difference is that in order to construct a dipole theory we need to probe the background (7) with a D0-brane, and take an appropriate limit of R_1, R_2, $R_3 \rightarrow 0$, while in the case of PFT we are probing the background with a Kaluza-Klein particle. (See [19,20] for a sample of literature discussing other configurations of brane-probes in Melvin universes.)

The dipole-theory has a *linear* nonlocality—R-symmetry charged objects expand to segments of length proportional to their R-charge. A similar nonlocality structure exists in field theories on noncommutative spaces. There, objects expand to segments in direction transverse to their momentum [36]. The nonlocality of PFT, on the other hand, might be described as *hyperplanar*—objects acquire a volume proportional to their R-charge and expand in a spacelike hyperplane. A theory with a linear nonlocality can often be constructed by defining a noncommutative product for fields. However, it is harder to construct a theory where the nonlocal objects are higher dimensional. Examples of this kind include open-membrane (OM) theory, which is a deformation of the six-dimensional (2, 0) theory, and the Open D-brane theories which are deformations of little string theories [37]. In these theories there are formally open membrane or open D-brane excitations on M5-branes or NS5-branes, respectively. Another example is "disc-pole-theory," which is also a deformation of the (2, 0) theory [10]. All these theories are deformations of already mysterious higher dimensional theories. PFT can formally be classified as an open D3-brane theory, but it is special in that the open brane is of the same type (D3) as the underlying branes. PFT might be easier to study, however, because it is a deformation of a $3 + 1$D Yang-Mills theory. Generalizations of PFT to other types of branes, such as M5-branes and M2-branes can be constructed along similar lines.

I will end this paper by mentioning a few recently discovered new research directions that might be of relevance.

First, as mentioned in Sec. IV and will be further explained in [28], PFT is formally related to the behavior of D3-branes in regions with strong RR 5-form flux. There might therefore be a connection between the Hamiltonian discovered in [38], which describes spherical D3-branes in pp-waves, and a sector of PFT (perhaps defined on S^3 rather than T^3).

Another recent development that is possibly related to PFT is an intriguing extension of classical geometry [39] that contains a nonassociative structure, and could potentially be parametrized by a 3-form. (I am grateful to Washington Taylor for suggesting this.) The definition of PFT requires a parameter ζ which can be thought of as a component of a spacetime vector, or a dual 3-form. Thinking about ζ as a 3-form is more natural, since it measures spatial volume. In a possibly related development, the effective theories on D-brane probes of nongeometric fluxes that are U-dual to 3-form fluxes [40] were studied in [41], where it was suggested that they involve a nonassociative structure. Lastly, another modification of geometry that also involves a spatial 3-form and superluminal velocities was recently described in [42]. It remains to be explored whether or not PFT is related to any of the above mentioned extensions of classical geometry.

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