Hawking radiation from general Kerr-(anti)de Sitter black holes

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We calculate the total flux of Hawking radiation from Kerr-(anti)de Sitter black holes by using gravitational anomaly method developed in [S. P. Robinson and F. Wilczek, Phys. Rev. Lett. 95, 011303 (2005)]. We consider the general Kerr-(anti)de Sitter black holes in arbitrary D dimensions with the maximal number $[D/2]$ of independent rotating parameters. We find that the physics near the horizon can be described by an infinite collection of $(1 + 1)$-dimensional quantum fields coupled to a set of gauge fields with charges proportional to the azimuthal angular momentums m_i. With the requirement of anomaly cancellation and regularity at the horizon, the Hawking radiation is determined.

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I. INTRODUCTION

Hawking radiation is one of the most important and intriguing effects in black hole physics. It shows that black hole is not really black, it radiates thermally like black body. Precisely speaking, Hawking radiation is the quantum effect of field in a background space-time with a future event horizon. It has a feature that the radiation is determined universally by the horizon properties. It has several derivations. The original one discovered by Hawking [1,2] is by directly calculating the Bogoliubov coefficients between in and out states of fields in a black hole background. This approach relies on the fact that in a curved background the choice of vacuum for incoming and outgoing particle is not unique. Later on a derivation based on the path-integral quantum gravity was given in [3]. A few years ago, Parikh and Wilczek [4] proposed a tunneling picture in which particle pair production happens near the horizon and Hawking radiation could be obtained by calculating WKB amplitudes for classically forbidden paths.

Very Recently, Robinson and Wilczek [5] have given a new derivation of Hawking radiation in the Schwarzschild black hole background through gravitational anomaly. This work is to some extent inspired by the work of Christensen and Fulling [6], in which the radiation created in the $(1 + 1)$-dimensional Schwarzschild black hole background was determined by the trace anomaly and the energy-momentum conservation law. In this approach, boundary conditions at the horizon and the infinity are required to specify the Unruh [7] vacuum. Moreover the method in [6] could not be applied to the cases in more than $(2 + 1)$ dimensions. Robinson and Wilczek found that by dimension reduction, the physics near the horizon can be described by an infinite collections of free $(1 + 1)$-dimensional fields because the mass and interaction terms of quantum fields in the background are suppressed. If one only consider the effective field theory outside the horizon, the theory become chiral since classically all ingoing modes can not affect physics outside the horizon. Quantum mechanically, the effective theory is anomalous with respect to gauge or general coordinate symmetries. The anomaly should be cancelled by the quantum effects of the classically irrelevant incoming modes. The condition for chiral and gravitational anomaly cancellation and regularity requirement at the horizon, combining with the energy-momentum conservation law, determines Hawking fluxes of the charge and energy-momentum. Robinson and Wilczek's treatment once again shows that Hawking radiation (if we neglect the back-reaction on the background) is universal, it only depends on the property of the event horizon.

In the further development, Iso *et al.* [8,9] investigated the charged and rotating black hole. By using a dimensional reduction technique, they found each partial wave of quantum fields in $d = 4$ rotating black hole background can be interpreted as a $(1 + 1)$-dimensional charged field with a charge proportional to the azimuthal angular momentum m. The total flux of Hawking radiation can be determined by demanding gauge invariance and general coordinate covariance at the quantum level. And the boundary conditions are clarified. The results are consistent with the effective action approach. Murata *et al.* [10] extended the method to Myers-Perry black holes [11] with only one rotating axis and also clarified the boundary condition. The Hawking radiation from general spherically symmetric black holes [12] and BTZ black holes [13] have also been investigated.

In this paper, we further extend Robinson and Wilczek's derivation of Hawking radiation to general Kerr-(anti)de Sitter(K(A)dS) black holes [14] in D dimensions. For a general K(A)dS metric, there are at most $N = [\frac{D-1}{2}]$ Killing symmetries corresponding to the rotational symmetries in N orthogonal spatial 2-planes. For a quantum field in such backgrounds, the physics near the future event horizon could still be effectively described by an infinite collection of $(1 + 1)$-dimensional fields coupled to N $U(1)$ gauge fields. We discuss such dimensional reduction in detail in Sec. II. In Sec. III, we obtain the Hawking fluxes

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by requiring anomaly cancellation and regularity condition. The final section is devoted to the conclusion.

II. QUANTUM FIELDS IN GENERAL KERR-(ANTI)DE SITTER BLACK HOLES

In this section, we will discuss the quantum fields in general Kerr-(anti)de Sitter blacks holes and its effective dimensional reduction near the horizon. The Kerr-(anti)de Sitter metric in D-dimension has been studied carefully in [14]. Here we just give a brief review of its basic property. The metric takes the form in an Boyer-Lindquist coordinates

$$\begin{aligned} ds^2 = & -W(1-\lambda r^2)dt^2 + \frac{2M}{VF}\left(Wdt - \sum_{i=1}^{N}\frac{a_i\mu_i^2 d\varphi_i}{1+\lambda a_i^2}\right)^2 \\ & + \sum_{i=1}^{N}\frac{r^2+a_i^2}{1+\lambda a_i^2}\mu_i^2 d\varphi_i^2 + \frac{VFdr^2}{V-2M} \\ & + \sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{1+\lambda a_i^2}d\mu_i^2 + \frac{\lambda}{W(1-\lambda r^2)} \\ & \times\left(\sum_{i=1}^{N+\epsilon}\frac{r^2+a_i^2}{1+\lambda a_i^2}\mu_i d\mu_i\right)^2 \end{aligned} \tag{2.1}$$

where

$$\epsilon = \begin{cases} 1, & D \text{ even} \\ 0, & D \text{ odd} \end{cases} \qquad W \equiv \sum_{i=1}^{N+\epsilon}\frac{\mu_i^2}{1+\lambda a_i^2}$$

$$V \equiv r^{\epsilon-2}(1-\lambda r^2)\prod_{i=1}^{N}(r^2+a_i^2),$$

$$F \equiv \frac{1}{1-\lambda r^2}\sum_{i=1}^{N+\epsilon}\frac{r^2\mu_i^2}{r^2+a_i^2}.$$

Here N is the integral part of $(D-1)/2$. There are N independent rotation parameters a_i in N orthogonal spatial 2-planes. The φ_i's are azimuthal angular coordinates. And the μ_i's are the latitudinal coordinates obeying a constraint $\sum_{i=1}^{N+\epsilon}\mu_i^2 = 1$, so only $N+\varepsilon-1$ latitudinal coordinates μ_i are independent. The λ is the cosmological constant. Up to the sign of λ, the above metric describes different Kerr black holes in D dimensions:

$$\begin{cases} \lambda > 0, & \text{Kerr-de Sitter metric} \\ \lambda = 0, & \text{Myers-Perry metric[14]} \\ \lambda < 0, & \text{Kerr-Anti-de Sitter metric} \end{cases}$$

Hawking fluxes of Myers-Perry black holes with only one azimuthal angular momentum has been discussed in [10]. In this paper, we will discuss Hawking radiation of the other two cases with any permissible angular momentums. Certainly our discussion apply to the Myers-Perry black holes with more than one angular momentums.

It is remarkable that for the black holes in de Sitter space-time, there exist a cosmological event horizon. However, our motivation is to study the Hawking radiation of the black hole so we focus on the physics near the black hole event horizon.

The metric (2.1) could be cast into a generalized Boyer-Lindquist form,

$$\begin{aligned} ds^2 = & Xdt^2 + 2Y_i dt d\varphi^i + Z_{ij}d\varphi^i d\varphi^j + g_{ab}dx^a dx^b \\ & + \frac{1}{B}dr^2 \end{aligned} \tag{2.2}$$

where φ^i, $i = 1, \cdots N$ are periodic with period 2π and x^a, $a = 1, \cdots n$ with $n = N + \epsilon - 1$ are independent latitudinal coordinates. All the metric components only depend on $x^a(\mu_i)$ and the radial coordinate r. The Z_{ij} and g_{ab} are positive definite and their corresponding inverses are Z^{ij} and g^{ab} with $Z^{ij}Z_{jk} = \delta^i_k$, $g^{ab}g_{bc} = \delta^a_c$.

From the discussion in [14], the angular velocities of the horizon are given by $\Omega^i = \Omega^i_H$

$$\Omega^i_H = -Y^i|_{r=r_H} = \frac{a_i(1-\lambda r_H^2)}{r_H^2+a_i^2} \tag{2.3}$$

where $Y^i = Z^{ij}Y_j$ and r_H is the radius of the horizon. Using (2.1), r_H is just the largest positive root of equation $V - 2M = 0$ or in the metric (2.2) a largest positive root of equation $B = 0$. The angular velocities, relative to a non-rotating frame at infinity, is a little different from (4.4) in [14], which is defined relative to a rotating frame at infinity. The null generator l of the horizon is a linear combination of the Killing vector fields

$$l = \frac{\partial}{\partial t} + \Omega^i_H\frac{\partial}{\partial\varphi^i} \tag{2.4}$$

The surface gravity on the horizon is

$$\kappa^2 = (\nabla^\mu L)(\nabla_\mu L)|_{r=r_H} \tag{2.5}$$

where

$$-L^2 = l^\mu l_\mu = X + 2Y_i\Omega^i_H + Z_{ij}\Omega^i_H\Omega^j_H \tag{2.6}$$

Note that L and B vanish on the horizon but $\partial_r L$ and $\partial_r B$ are nonzero. So near the horizon, we have

$$L^2 \approx (\partial_r L^2)|_{r=r_H}(r-r_H), \qquad B \approx (\partial_r B)|_{r=r_H}(r-r_H) \tag{2.7}$$

Thus the surface gravity is:

$$\kappa = \frac{1}{2}\sqrt{(\partial_r L^2)(\partial_r B)|_{r=r_H}} = \frac{1}{2}(1-\lambda r_H^2)\frac{V'(r_H)}{V(r_H)} \tag{2.8}$$

The property that κ is a constant and $Y^iY_i - X = 0$ on the horizon are very important to the following discussions.

In order to do dimensional reduction, we need some other properties of the metric near the horizon. Define $A = Y^iY_i - X$ then

$$L^2 - A = -Z_{ij}(\Omega_H^i + Y^i)(\Omega_H^j + Y^j) \tag{2.9}$$

From the definition (2.3) $\Omega_H^i + Y^i|_{r=r_H} = 0$, so near the horizon $\Omega_H^i + Y^i \approx C^i(r - r_H)$, then we have

$$L^2 - A \approx -Z_{ij}C^iC^j(r - r_H)^2, \qquad \partial_r L^2 = \partial_r A|_{r=r_H} \tag{2.10}$$

When $A \neq 0$, the inverse of the metric (2.2) can be written as

$$\begin{aligned} g^{tt} &= -\frac{1}{A}, \qquad g^{ij} = -\frac{1}{A}Y^iY^j + Z^{ij}, \\ g^{ti} &= g^{it} = \frac{1}{A}Y^i, \qquad g^{rr} = B, \end{aligned} \tag{2.11}$$

Note that near the horizon $A \to 0$, Y^i, Z^{ij} and g^{ab} are finite. This property is essential to the dimensional reduction. The metric (2.2) can be written in another form

$$ds^2 = -Adt^2 + Z_{ij}(d\varphi^i + Y^i dt)(d\varphi^j + Y^j dt) + \frac{1}{B}dr^2 + g_{ab}dx^a dx^b \tag{2.12}$$

with

$$\sqrt{-g} = \sqrt{Ag_1g_2\frac{1}{B}} \tag{2.13}$$

where $g = \det(g_{\mu\nu})$, $g_1 = \det(Z_{ij})$, $g_2 = \det(g_{ab})$

Now let us consider a free complex scalar field for simplicity in the general Kerr-(anti)de Sitter black hole background. Using the inverse of metric (2.11), the free part of the action is

$$\begin{aligned} S_{\rm free} &= -\int d^Dx\sqrt{-g}g^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi \\ &= -\int dtdrd^N\varphi^i d^nx^a\sqrt{-g}\bigg(-\frac{1}{A}\partial_t\phi^*\partial_t\phi \\ &\quad + \frac{1}{A}Y^i\partial_t\phi\partial_i\phi^* + \frac{1}{A}Y^i\partial_i\phi\partial_t\phi^* - \frac{1}{A}Y^iY^j\partial_i\phi^*\partial_j\phi \\ &\quad + Z^{ij}\partial_i\phi^*\partial_j\phi + B\partial_r\phi^*\partial_r\phi + g^{ab}\partial_a\phi^*\partial_b\phi\bigg) \end{aligned} \tag{2.14}$$

where ∂_i denotes $\frac{\partial}{\partial\varphi^i}$ and ∂_a denotes $\frac{\partial}{\partial x^a}$.

In order to consider the physics near the horizon, we make a coordinate transformation $\frac{dr_*}{dr} = f(r)^{-1}$, where $f(r) \equiv \sqrt{A'B'}|_{r=r_H}(r - r_H) = 2\kappa(r - r_H)$. In this frame, considering the region near the outer horizon r_H, the finite terms $Z^{ij}\partial_i\phi^*\partial_j\phi$ and $g^{ab}\partial_a\phi^*\partial_b\phi$ are suppressed by the factor $f(r(r_*))$, vanishing exponentially fast near the horizon. We can also substitute $\sqrt{AB}$, g_1g_2 by $f(r)$, $g_1g_2|_{r=r_H}$ because the omitting terms are suppressed for the same reason. Similarly, one can redefine Y^i by $-\frac{a_i(1-\lambda r^2)}{r^2+a_i^2}$. So the action with dominant terms is

$$\begin{aligned} S = -\int dtdrd^N\varphi^i d^nx^a\sqrt{g_1(r_H)g_2(r_H)}[-f(r)^{-1} \\ \times(\partial_t - Y^i\partial_i)\phi^*(\partial_t - Y^i\partial_i)\phi + f(r)\partial_r\phi^*\partial_r\phi] \end{aligned} \tag{2.15}$$

We can expand ϕ by a complete set of orthogonal functions of (φ^i, x^a) with the measure $\sqrt{g_1(r_H)g_2(r_H)}$. As we know, the angles φ^i are periodic with period 2π and coordinates x^a come from μ_i which obey a constraint $\sum_i\mu_i^2 = 1$. So (φ^i, x^a) describe a compact manifold with a metric

$$ds^2 = Z_{ij}(r_H)d\varphi^i d\varphi^j + g_{ab}(r_H)dx^a dx^b \tag{2.16}$$

whose measure is $\sqrt{g_1(r_H)g_2(r_H)}$. Then the eigenfunctions of the operator ∇^2 of the compact manifold with the metric (2.16) comprise a complete orthogonal functions. Note that there are N killing vectors $\frac{\partial}{\partial\varphi^i}$ which generate isometry. The eigenfunctions can be given by

$$Y_{m_1\cdots m_N\alpha} = \prod_{j=1}^N \exp^{im_j\varphi^j}f_\alpha(x^a) \tag{2.17}$$

satisfying

$$\begin{aligned} \int dx^a d\varphi^i\sqrt{g_1(r_H)g_2(r_H)}Y^*_{m_1\cdots m_N\alpha}Y_{n_1\cdots n_N\beta} \\ = \delta_{m_1n_1}\cdots\delta_{m_Nn_N}\delta_{\alpha\beta} \end{aligned} \tag{2.18}$$

Performing the partial wave decomposition of ϕ in terms of these functions,

$$\phi = \sum_{m_1,\cdots,m_N,\alpha}\phi_{m_1\cdots m_N\alpha}Y_{m_1\cdots m_N\alpha}, \tag{2.19}$$

the theory is reduced to a two-dimensional effective theory with an infinite collection of fields with quantum numbers $(m_1, \cdots, m_N, \alpha)$, simply denoted as ϕ_n. It is straightforward to show that the physics near the outer horizon can be effectively described by an infinite collection of massless $(1 + 1)$-dimensional fields with the following action

$$\begin{aligned} S = -\int dtdr[-f(r)^{-1}(\partial_t - im_jY^j)^*\phi_n^*(\partial_t - im_jY^j)\phi_n \\ + f(r)\partial_r\phi_n^*\partial_r\phi_n] \end{aligned} \tag{2.20}$$

III. ANOMALIES AND HAWKING FLUXES

In this section, we will try to obtain the Hawking fluxes. We will follow the approach firstly proposed in [8,9]. The basic point is that the Hawking fluxes can be determined by the anomaly cancellation of the effective chiral theory.

From the effective action (2.20), near the horizon, each partial wave mode of the scalar field ϕ_n can be considered as $(1 + 1)$-dimensional complex scalar field in the backgrounds of the metric $g_{\mu\nu}$ and N gauge potentials A_μ^i

$$g_{tt} = -f(r), \qquad g_{rr} = f(r)^{-1}, \qquad g_{rt} = 0 \\ A_t^i = Y^i, \qquad A_r^i = 0 \tag{3.1}$$

In this case, there are N $U(1)$ gauge symmetries and N gauge currents which actually relate to angular momentum currents. Each gauge symmetry originates from the axial isometry along φ^i direction. With respect to gauge fields A_μ^i, the field ϕ_n has charges m_i, which is the azimuthal quantum number rotating along φ^i direction. The corresponding $U(1)$ currents J_i^r can be defined from the D-dimensional energy-momentum tensor.

$$J_i^r = -\int d^n x^a d^N \varphi^i \sqrt{-g} T^r_{\varphi^i}. \tag{3.2}$$

In effect, performing a partial wave decomposition and an integral, we find the result of right side of the above equation is just the current obtained from the two-dimensional effective action. Similarly the energy-momentum tensor in two-dimensional effective action is the reduction of the one in D-dimension

$$T^r_{t(2)} = \int d^n x^a d^N \varphi^i \sqrt{-g} T^r_t. \tag{3.3}$$

Without bringing any confusion, from now on we denote $T^r_{t(2)}$ as T^r_t for simplicity.

As shown in [8], we can divide the region $r \in [r_H, \infty]$ into two regions. One is $r \in [r_H + \varepsilon, \infty]$ which is apart from the horizon and the other is $r \in [r_H, r_H + \varepsilon]$ which is near the horizon. In the region $r \in [r_H + \varepsilon, \infty]$, each current is conserved. So we have

$$\partial_r J^r_{i(o)} = 0 \tag{3.4}$$

On the other hand, in the near horizon region, the effective two-dimensional theory become chiral since classically the ingoing modes are irrelevant and there are only outgoing modes. In this effective chiral theory, the gauge symmetries and general coordinate transformation symmetries become anomalous quantum mechanically. The anomaly equation for each $U(1)$ current near the horizon is [15,16]

$$\partial_r J^r_{i(H)} = \frac{m_i}{4\pi} \partial_r \mathcal{A}_t \tag{3.5}$$

where $\mathcal{A}_t = m_i A_t^i$ is the sum of N $U(1)$'s.

Actually one can take $\mathcal{A}(r)$ more seriously. From effective action, we can take the point of view that the scalar field coupled to a single gauge potential $\mathcal{A}(r)$. There exist a $U(1)$ gauge symmetry associated with gauge potential $\mathcal{A}(r)$. The corresponding current denoted as $\mathcal{J}(r)$ can be constructed from the original N $U(1)$ gauge symmetries. Note that each J_i^r is not independent for a fixed azimuthal angular momentum m_i. Their expectation values are related as $\frac{1}{m_i} J_i^r = \frac{1}{m_j} J_j^r = \mathcal{J}^r$. The anomaly equation for $\mathcal{J}(r)$ is

$$\partial_r \mathcal{J}^r = \frac{1}{4\pi} \partial_r \mathcal{A}_t, \tag{3.6}$$

so we have anomaly Eq. (3.5). Solving the above equations in each region, we have,

$$J^r_{i(o)} = C_{i(o)} \qquad J^r_{i(H)} = C_{i(H)} + \frac{m_i}{4\pi}(\mathcal{A}_t(r) - \mathcal{A}_t(r_H)) \tag{3.7}$$

where $C_{i(o)}$ and $C_{i(H)}$ are two integration constants. $C_{i(H)}$ is the value of the consistent current of the outgoing modes at the horizon and $C_{i(o)}$ is the value of the angular momentum flux at infinity. It is our goal to determine $C_{i(o)}$, which encodes the information of Hawking radiation.

Under gauge transformations, the variation of the effective action is given by

$$-\delta W = \int \sqrt{-g_{(2)}} \lambda \nabla_\mu J_i^\mu \tag{3.8}$$

where λ is a gauge parameter, and

$$J_i^\mu = J^\mu_{i(o)} \theta_+(r) + J^\mu_{i(H)} H(r). \tag{3.9}$$

Here $\theta_+(r) = \theta(r - r_H - \varepsilon)$ and $H(r) = 1 - \theta_+(r)$, where $\theta(x)$ is a step function. Note that we have not take the contribution of the ingoing modes into account. Using Eqs. (3.4) and (3.5) we have

$$-\delta W = \int d^2 x \lambda \bigg[\delta(r - r_H - \epsilon)\Big(J^\mu_{i(o)} - J^\mu_{i(H)} + \frac{m_i}{4\pi}\mathcal{A}_t\Big) + \partial_r\Big(\frac{m_i}{4\pi}\mathcal{A}_t H(r)\Big)\bigg] \tag{3.10}$$

Since the underlying theory must be gauge invariant, so $\delta W = 0$. Actually the last term is cancelled by quantum effects of the classically irrelevant ingoing modes [5]. Then the coefficient of the delta-function should vanish. With the results (3.7), we can obtain a relation between the two constants

$$C_{i(o)} = C_{i(H)} - \frac{m_i}{4\pi}\mathcal{A}_t(r_H) \tag{3.11}$$

In order to determine the value of $C_{i(o)}$, one need to impose the regularity condition. As discussed in [8,9], the regularity requires that the covariant current is zero on the horizon,

$$\tilde{J}^r_i = J_i^r + \frac{m_i}{4\pi}\mathcal{A}_t H(r), \qquad \tilde{J}_i^r(r_H) = 0 \tag{3.12}$$

Then the flux of the angular momentum is obtained as

$$C_{i(o)} = -\frac{m_i}{2\pi}\mathcal{A}_t(r_H) = \frac{m_i}{2\pi}\sum_{j=1}^N m_j \frac{a_j(1 - \lambda r_H^2)}{r_H^2 + a_j^2} \tag{3.13}$$

Similarly we can determine the flux of the energy-momentum tensor radiated from general Kerr-(anti)de Sitter black holes. In the presence of the effective gauge potentials $A_t^i(r)$, the conservation equation outside the horizon is modified to be

$$\partial_r T^r_{t(o)} = \mathcal{F}_{rt} \mathcal{J}^r_{(o)} \tag{3.14}$$

where $\mathcal{F}_{rt} = \partial_r \mathcal{A}_t$. Note that the right hand side of the above relation depends simply on $\mathcal{A}_t$. With the definition of $\mathcal{J}^r$, we have $\mathcal{J}^r_{(o)} = -\frac{1}{2\pi}\mathcal{A}_t(r_H) \equiv C_o$, where

$$C_o = \sum_{j=1}^{N} m_j \frac{a_j(1-\lambda r_H^2)}{r_H^2 + a_j^2}. \tag{3.15}$$

The solution of the above equation gives the value of the energy flux at spatial infinity

$$T^r_{t(o)} = a_o + C_o \mathcal{A}_t(r) \tag{3.16}$$

where a_o is an integration constant. Physically, it could be taken as the value of the total energy flow of radiation measured at spatial infinity.

On the other hand, there are gauge and gravitational anomalies near the horizon and the anomaly equation is now as

$$\partial_r T^r_{t(H)} = \mathcal{F}_{rt}\mathcal{J}^r_{(H)} + \mathcal{A}_t \nabla_\mu \mathcal{J}^\mu_{(H)} + \partial_r N^r_t \tag{3.17}$$

where $N^r_t = (f'^2 + ff'')/192\pi$ [8]. The second term indicates gauge anomaly while the third term is gravitational anomaly [17] for the consistent energy-momentum tensor. From the definition of $\mathcal{J}^r$ and Eqs. (3.7) and (3.11) we have $\mathcal{J}^r_{(H)} = C_o + \frac{1}{4\pi}\mathcal{A}_t(r)$. $T^r_{t(H)}$ can be solved as

$$T^r_{t(H)} = a_H + \int_{r_H}^{r} dr \partial_r \left(C_o \mathcal{A}_t + \frac{1}{2\pi}\mathcal{A}_t^2 + N^r_t \right) \tag{3.18}$$

where a_H is an integration constant.

Under the general coordinate transformation, the variation of the effective action is

$$\begin{aligned} -\delta W &= \int d^2x \sqrt{-g_{(2)}} \xi^t \nabla_\mu T^\mu_t \\ &= \int d^2x \xi^t \Big[C_0 \partial_r \mathcal{A}_t(r) + \partial_r \Big(\frac{1}{4\pi}\mathcal{A}_t^2 H(r) \\ &\quad + N^r_t H(r) \Big) + \Big(T^r_{t(o)} - T^r_{t(H)} + \frac{1}{4\pi}\mathcal{A}_t^2 + N^r_t \Big) \\ &\quad \times \delta(r - r_H - \epsilon) \Big] \end{aligned} \tag{3.19}$$

where ξ^t is the transformation parameter and $T^\mu_\nu = T^\mu_{\nu(o)}\theta_+(r) + T^\mu_{\nu(H)}H(r)$. The first term is generated by classical current. The second term should be cancelled by the quantum effect of the ingoing modes. As we discussed before, the last term should vanish because the underlying theory is general coordinate transformation covariant. So we have:

$$a_o = a_H + \frac{1}{4\pi}\mathcal{A}_t^2(r_H) - N^r_t(r_H) \tag{3.20}$$

Similarly we need to impose the regularity condition, which requires that the covariant energy-momentum flux vanish on the future horizon. The covariant energy-momentum is defined by [18,19]

$$\tilde{T}^{\,r}_t = T^r_t + \frac{1}{192\pi}(ff'' - 2f'^2) \tag{3.21}$$

Combining with (3.18), the condition reads

$$a_H = \kappa^2/24\pi = 2N^r_t(r_H), \tag{3.22}$$

where $\kappa = 2\pi/\beta$ is the surface gravity of the black hole. Therefore the total flux of the energy-momentum tensor is given by

$$a_o = \frac{\mathcal{A}_t^2(r_H)}{4\pi} + N^r_t(r_H) = \frac{1}{4\pi}\left(\sum_{i=1}^{N} m_i \Omega_H^i\right)^2 + \frac{\pi}{12\beta^2} \tag{3.23}$$

IV. CONCLUSION

In this paper we studied the Hawking radiation of general Kerr-(anti)de Sitter black holes. We considered a complex scalar field in a general Kerr-(anti)de Sitter black hole background. Near the horizon, the field can be described by an infinite collection of $(1+1)$-dimensional fields. A formal derivation of the dimensional reduction based on the properties of the horizon is given and each term in the effective action has obvious physical meaning. Although there are no gauge fields in the original D dimensional theory, the metric has N isometries, which induce N $U(1)$ gauge symmetries in the effective two-dimensional theory. Each partial wave mode is charged under the gauge symmetries with charge m_i's, where the m_i's are angular quantum numbers. We have shown that Hawking radiation from general Kerr-de Sitter space can be determined by the cancellation condition of the gravitational anomaly and gauge anomaly, combining with the boundary condition required by regularity at the horizon. We obtained the Hawking flux of each angular momentum $C_{i(o)}$ and energy-momentum tensor a_o for each partial wave mode:

$$C_{i(o)} = \frac{m_i}{2\pi}\sum_{j=1}^{N} m_j \frac{a_j(1-\lambda r_H^2)}{r_H^2 + a_j^2} \tag{4.1}$$

$$a_o = \frac{1}{4\pi}\left(\sum_{i=1}^{N} m_i \Omega_H^i\right)^2 + \frac{\pi}{12\beta^2}. \tag{4.2}$$

Our work shows that the proposal in [5,8,9] can be applied to more general blackhole backgrounds in higher dimensions which have more than one angular momentum. It would be interesting to apply the gravitational anomaly method to study other problems in blackhole physics.

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