Charged shells in Lovelock gravity: Hamiltonian treatment and physical implications

Gonçalo A. S. Dias*

Centro Multidisciplinar de Astrofísica—CENTRA Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

Sijie Gao†

Department of Physics, Beijing Normal University, Beijing 100875, China

José P. S. Lemos‡

Centro Multidisciplinar de Astrofísica—CENTRA Departamento de Física, Instituto Superior Técnico, Universidade Técnica de Lisboa, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal

(Received 14 November 2006; published 23 January 2007)

Using a Hamiltonian treatment, charged thin shells, static and dynamic, in spherically symmetric spacetimes, containing black holes or other specific types of solutions, in d dimensional Lovelock-Maxwell theory are studied. The free coefficients that appear in the Lovelock theory are chosen to obtain a sensible theory, with a negative cosmological constant appearing naturally. Using an Arnowitt-Deser-Misner (ADM) description, one then finds the Hamiltonian for the charged shell system. Variation of the Hamiltonian with respect to the canonical coordinates and conjugate momenta, and the relevant Lagrange multipliers, yields the dynamic and constraint equations. The vacuum solutions of these equations yield a division of the theory into two branches, namely $d - 2k - 1 > 0$ (which includes general relativity, Born-Infeld type theories, and other generic gravities) and $d - 2k - 1 = 0$ (which includes Chern-Simons type theories), where k is the parameter giving the highest power of the curvature in the Lagrangian. There appears an additional parameter $\chi = (-1)^{k+1}$, which gives the character of the vacuum solutions. For $\chi = 1$ the solutions, being of the type found in general relativity, have a black hole character. For $\chi = -1$ the solutions, being of a new type not found in general relativity, have a totally naked singularity character. Since there is a negative cosmological constant, the spacetimes are asymptotically anti-de Sitter (AdS), and AdS when empty (for zero cosmological constant the spacetimes are asymptotically flat). The integration from the interior to the exterior vacuum regions through the thin shell takes care of a smooth junction, showing the power of the method. The subsequent analysis is divided into two cases: static charged thin shell configurations, and gravitationally collapsing charged dust shells (expanding shells are the time reversal of the collapsing shells). In the collapsing case, into an initially nonsingular spacetime with generic character or an empty interior, it is proved that the cosmic censorship is definitely upheld. Physical implications of the dynamics of such shells in a large extra dimension world scenario are also drawn. One concludes that, if such a large extra dimension scenario is correct, one can extract enough information from the outcome of those collisions as to know, not only the actual dimension of spacetime, but also which particular Lovelock gravity, general relativity or any other, is the correct one at these scales, in brief, to know d and k.

DOI: 10.1103/PhysRevD.75.024030 PACS numbers: 04.50.+h, 04.70.−s

I. INTRODUCTION

In a world with extra large space dimensions, of the order of some microns or less, as postulated in [1], the gravitational field sees all the dimensions, whereas the standard model fields are confined to the usual world, or brane, of three space dimensions. Thus, the four-dimensional spacetime of the brane can be seen as embedded in the d-dimensional spacetime of the whole world. In this setup several different problems are tackled. For instance, one solves, in a way, the hierarchy problem, since both the Planck scale and the electroweak scale can be made of the same order, and so quantum gravity, being an electroweak scale phenomenon, can be experimentally tested. Indeed, by smashing particles against each other in experiments with the new generation accelerators or in eventual cosmic ray collisions, one can produce black holes, or perhaps other spacetimes with different causal structure, with tiny masses and radii. As this is a gravitational phenomenon, and the gravitational field spreads into all dimensions, the newly created spacetimes probe all the spacetime dimensions, and in addition render visible some quantum effects (see, e.g., [2–5]).

Now, since in this setting one might expose quantum gravity and large extra dimension phenomena, in order to study the higher dimensional black holes, or other spacetimes with different causal structure, formed in the collision of particles, one should consider studying these objects in possible natural extensions to the theory of general relativity, conceivably appropriate to a further

*Electronic address: gadias@fisica.ist.utl.pt
†Electronic address: sijie@bnu.edu.cn
‡Electronic address: lemos@fisica.ist.utl.pt

quantum framework development. The action of general relativity in d spacetime dimensions is proportional to the integral of the cosmological constant plus the Ricci scalar, and possible extensions should include powers of the Riemann tensor, such as the Kretschmann scalar, powers of the Ricci tensor, and powers of the Ricci scalar [6]. In principle one has to choose a criterion to pick up the right combination of curvature scalars. For instance, adopting the criterion of keeping the same degrees of freedom, one may wonder what is the most natural generalization of general relativity for other dimensions. Such a generalization is given by the Lovelock action [7], where essentially one keeps the field equations second order. In more detail, the Lovelock theory looks for an action which when properly varied yields symmetric tensors that are functions of the metric and its first and second spacetime derivatives, and divergence free. In four dimensions the corresponding action is the Einstein-Hilbert action, proportional to the cosmological constant and the Ricci scalar. When varied, the Einstein-Hilbert action yields the Einstein tensor and the metric tensor, the only tensors that display the above properties and contribute to the equations of motion. In dimensions higher than four Lovelock found that only certain precise combinations of higher powers of the curvature scalars could enter in the action [7]. The interpretation in physical, geometric, and topological terms of these precise combinations was put forward by Teitelboim and Zanelli [8] (see also [9,10]). In this context, one can first argue that in zero spacetime dimensions, a spacetime point, there is a zero-dimensional topological invariant which is a numerical constant, called the cosmological constant. When this term is integrated in one or in two dimensions to form an action, i.e., when the term is dimensionally continued to the next odd or even dimension, it contributes to the equations of motion, which in this case are somewhat trivial, giving either a zero metric or a zero cosmological constant with an indeterminate metric. Now, in two dimensions, there is also the corresponding topological invariant. This invariant, the Euler characteristic, is obtained by integrating the so-called Gauss-Bonnet term. In two dimensions the Gauss-Bonnet term is the two-dimensional Ricci scalar, which can thus be consistently considered the corresponding Euler density. This term, being topological, does not contribute to the equations of motion in two dimensions. However, when both these terms, the cosmological constant (the topological invariant in zero dimensions) and the Ricci scalar (the topological invariant in two dimensions) are integrated in three or in four dimensions, i.e., when they are dimensionally continued to the next odd or even dimensions, they contribute nontrivially to the equations of motion, and indeed in three and four dimensions, yield the standard general relativity. Now, given general relativity in four dimensions, one can add a generalized Gauss-Bonnet (i.e., a generalized Euler density) term, in four dimensions, first discovered by Lanczos [11], which when integrated in the four-dimensional action gives essentially the corresponding Euler characteristic, also a topological invariant not contributing to the classical field equations. In turn, in five and six dimensions, not only an action with the cosmological constant and the Ricci scalar, mentioned above, contribute to the equations of motion, but also one can dimensionally continue the previous generalized Gauss-Bonnet term in order to have a meaningful extended action, which gives the desired equations of motion of second order. Of course, in six dimensions there is a new generalized Gauss-Bonnet term, which when integrated gives the corresponding Euler characteristic, not contributing to the equations of motion. Then, repeating this process of adding a generalized Gauss-Bonnet term of the previous even dimension to the next two dimensions, odd and then even, one gets the Lovelock gravity of that specific dimension. Thus, the Lovelock theory can also be considered as a dimensional continuation of the Euler characteristics of lower dimensions [8–10]. The theory has, in addition to Newton's constant and the cosmological constant, new arbitrary dimensionful parameters, which should be chosen carefully. We have given a classical motivation for including certain special new terms in the gravitational action in d dimensions, but there are also quantum motivations to study the dimensional continuation of the Euler characteristics. Indeed, in a string theory context, in order to prevent the existence of ghosts in a low energy limit of the theory, one has to add to the d-dimensional general relativity action all the previous lower dimensional Euler characteristics [12,13].

Suppose now the world has indeed extra dimensions, and that the whole bulk spacetime obeys Lovelock gravity rather than Einstein gravity. Thus, in this case, it is important to study generic properties of black holes in the Lovelock theory. Solutions of spherical black holes in d spacetime dimensions for Einstein-Hilbert action plus a generalized Gauss-Bonnet term, i.e., a specially truncated Lovelock theory, were found in [14–18]. Thermodynamics were often studied in these works. In [19], where black hole solutions were also found, no relevant Lovelock term was put to zero, rather a special choice was made for all the Lovelock coefficients, which depend only on the cosmological constant and the dimension of the spacetime. All nontopological terms are included in the action, and the theory can thus be called dimensionally continued gravity. This choice is one among infinite, but it leads to a natural outcome, where the action in even dimensions has a Born-Infeld form, i.e., it may be regarded as the gravitational analogue of the Born-Infeld electrodynamics, and in odd dimensions has a Chern-Simons form. Also in [19], Lovelock theory was coupled to Maxwell electromagnetism, and the corresponding black hole solutions with electric charge and cosmological constant were found. Now, a natural extension of this prescription was advanced in [20], where it was allowed that the topological term plus some nontopological terms, starting from the top, could be put to zero. For instance, in $d = 10$ dimensions in this setting one may work just with the general relativity term and the following generalized Gauss-Bonnet term, by putting to zero the other higher dimensional Euler densities. For the nonzero terms, the choice of the coefficients of the theory is the same as in [19]. This extension has as a special case the dimensionally continued gravity of [19]. By coupling this extension to Maxwell electromagnetism, a general class of charged vacuum spacetimes with a negative cosmological constant, including of course the black holes found in [19], were exhibited [20]. These vacuum solutions yield a natural division of the Lovelock theory into two branches, namely $d - 2k - 1 > 0$ (which includes general relativity, Born-Infeld type theories, and other generic gravities) and $d - 2k - 1 = 0$ (which includes Chern-Simons type theories), where k is the parameter that gives the highest power of the curvature in the Lagrangian. In the solutions there appears an additional parameter χ, with $\chi = (-1)^{k+1}$. This implies that, in addition, the solutions can be subdivided into two families of different characters, one family $\chi = 1$, comprising solutions of the type found in general relativity, has a black hole character (meaning that for a correct choice of the parameters, such as mass and charge, the solution is a black hole solution, although, of course, for other choices of parameters it can be an extremal black hole or a naked singularity), and the other family $\chi = -1$, being of a new type not found in general relativity, has a naked singularity character (meaning there is no possible choice of the parameters that gives a black hole solution, the full vacuum solution is always singular without horizons). Of course the solutions also include empty spacetimes. Since in general there is a negative cosmological constant the spacetimes are asymptotically anti-de Sitter (AdS), or when empty they are AdS itself or some form of it (for zero cosmological constant the spacetimes are asymptotically flat).

Given that one has a family of spacetimes with either black hole or naked singularity character, it is now important to study matter effects, either static or dynamic (collapsing or expanding), on these solutions. Such a phenomenon could be interesting in the aftermath of a collision between charged particles where the debris formed in the collision can be accreted or excreted in the newly formed charged spacetime. Accretion and excretion processes are usually technically elaborate, so to turn the analysis simpler one can study the case of a thin shell in the background spacetime, be it a black hole or otherwise. Static and dynamic solutions on such backgrounds are then of interest. However, in Lovelock theories, even a thin shell and its dynamics can bring complications. Indeed, in general relativity for instance, to study a thin shell in a given background one has to make a smooth junction from the interior to the exterior spacetime, as was done in [21]. Now, the junction conditions, in general, depend on the theory one is studying, and for theories with higher powers on the Riemann and other tensors this can be nontrivial [22]. An elegant and useful approach, that bypasses in a way the junction conditions for a spacetime with a thin shell, uses a Hamiltonian formalism for the theory under study. This approach was developed by Hájíček and Kijowski [23] for the theory of general relativity with matter in four dimensions, and was subsequently explored by Crisóstomo and Olea in d-dimensional general relativity coupled to Maxwell theory and matter with applications mainly in three spacetime dimensions [24,25]. The method requires that one puts the theory in a Hamiltonian form and then one directly integrates the canonical constraints, producing the shell dynamics for the background one wants to study. The payoff is that, not only one avoids to have to develop covariant junction conditions for each particular theory one is studying, but moreover the full spacetime, comprised of interior, shell, and exterior components, is treated as whole. This method is particularly well suited for symmetric configurations. The great power of the formalism was brought up in [26] where it was applied in pure Lovelock gravity to thin shell collapse in the background of the uncharged spacetimes found in [20]. Now, since the newly formed spacetime, black hole or otherwise, generated from a collision between charged particles, is most probably charged, and for the same reason the collapsing or expanding debris are also charged, it is of interest to study, using the Hamiltonian formalism advanced in [26], the gravitational dynamics of a charged thin shell in the charged sector of the spacetime solutions, black hole or otherwise, found in [20], of Lovelock gravity coupled to Maxwell electromagnetism. In the usual extra dimension scenarios the electromagnetic and matter fields, being confined to the brane, do not probe the extra dimensions, so that an axisymmetric shell, static or collapsing, around a spherically symmetric black hole is an important configuration to study. However, in order to further simplify the analysis we study instead, as a zero order approximation, a spherically symmetric shell around a spherically symmetric black hole (such a configuration would be quite realistic for charged fields and particles that can probe the extra dimensions). Of course, within general relativity, a very special case of Lovelock gravity, such an analysis should recover the earlier results, obtained through a junction condition formalism [21] of charged shell dynamics in a charged black hole spacetime [27–29]. All the works just mentioned are classical, and an extension of full Lovelock gravity into the quantum domain, or even within a semiclassical approximation, although important, seems beyond reach.

In the present article we extend the treatment of the classical dynamics of an uncharged thin shell in an uncharged spacetime, black hole or otherwise, with a negative cosmological constant background in Lovelock-Maxwell theory, given in [26], to the classical dynamics of a charged thin shell in a charged spacetime, black hole or otherwise, with a negative cosmological constant background in Lovelock-Maxwell theory, and among other things, show the power of the Hamiltonian method in unifying in a single unit the three sectors of the problem, namely, the gravitational, the electrodynamic, and the matter sectors, as well as taking into account automatically the smooth junction of the several regions of spacetime in question, the interior, the exterior, and the thin shell in between. In Sec. II the system under study, namely, charged vacuum plus charged thin shell, is initially put in an action and Lagrangian framework, in tensor language. The field content of the action is composed of three sectors, the gravitational, the electrodynamic, and the matter term. In the gravitational action we establish the coupling constants and make the choice of Lovelock coefficients, cutting off the Lovelock polynomial at the highest possible power of the curvature for which the theory is sensible. In the electrodynamic action we write the Maxwell term and add a current term, which describes the current of charged matter in the shell, and precede each term with its coupling constant. In the matter action we define the energy-momentum tensor of a perfect fluid matter source. We then write the same action in the Hamiltonian form in tensorial language. For each of the above mentioned sectors of the action we define the canonical coordinates and its conjugate momenta, up to surface terms. As the system under study is a system with constraints, we write the constraints and their respective Lagrange multipliers for each of the sectors of the action. Next, we obtain the Hamiltonian field equations from the Hamiltonian action by using spherically symmetry in the action and varying it with respect to the canonical coordinates and momenta, for the evolution equations, and the Lagrange multipliers, for the constraint equations. In Sec. III we analyze the vacuum solutions of the derived Hamiltonian equations and describe their properties. Afterwards we establish the geometrical framework of the thin shell and its Hamiltonian description. We divide spacetime into three parts, interior spacetime, thin shell, and exterior spacetime, and describe their geometric setup. Then we specify the matter properties of the thin shell, which we define as being that of a perfect fluid. We write its energy-momentum tensor and its relevant projections. We then solve the complete Hamiltonian equations around the thin shell. First, we note again that the vacuum solutions yield a natural division of the Lovelock theory into two branches, namely $d - 2k - 1 > 0$ and $d - 2k - 1 = 0$, where k is the parameter that gives the highest power of the curvature in the Lagrangian. There appears an additional parameter χ, with $\chi = (-1)^{k+1}$, which gives the character of the solutions, namely, the vacuum solutions may have a black hole character ($\chi = 1$) or a naked singularity character ($\chi = -1$). Since there is a negative cosmological constant, the spacetimes are asymptotically AdS, and AdS when empty, or some form of it (for zero cosmological constant the spacetimes are asymptotically flat). Second, the whole integration of the equations from the interior to the exterior vacuum regions through the thin shell is performed, where the smooth junction of the several regions of spacetime in question is automatically taken into account by the integration, showing definitely the efficacy of the method. The shell's dynamics in the vacuum spacetimes is then derived and the electrodynamic constraint equation recovers the electric charge conservation. An analysis of the equations of the thin shell is performed by studying two interesting cases, namely, a static thin shell in equilibrium and the gravitational collapse of a thin shell (the study of gravitational expansion is simply the time reversal of gravitational collapse, and so it is not necessary to analyze it in any detail). For the static shell we determine the radii at which the shell is in equilibrium and the pressure necessary to maintain the shell at this radius. For the gravitational collapse of a thin shell we start studying a simple example of dust matter in an empty interior and then prove cosmic censorship in a general case. More specifically, following the natural division of the Lovelock theory into the two branches mentioned above, we study the collapse of a thin shell onto an empty interior without cosmological constant, and give as examples collapse in four and ten dimensions in general relativity, collapse in ten dimensions in Born-Infeld type theories, and collapse in ten dimensions in general relativity with a single generalized Gauss-Bonnet term. We also study gravitational collapse in five dimensions onto an empty interior with a cosmological constant as an example of Chern-Simons type theories. Plots are drawn for these cases just mentioned, which illustrate the dynamics of the collapse and exhibit the cosmic censorship at work. This example of collapse of a shell into an empty interior, in a number of theories specified by d and k, shows that electric charge provides a mechanism for cosmic censorship for all relevant cases, including those which, in the uncharged case, did not conform to it. Then, more generally, for gravitational collapse onto black hole interiors, it is proven that the equations respect the cosmic censorship hypothesis. In Sec. IV we conclude and draw some physical implications in connection with the extra large dimensions scenario. We put $c = 1$.

II. ACTION, LAGRANGIAN, HAMILTONIAN, AND EQUATIONS OF MOTION IN LOVELOCK GRAVITY COUPLED TO MAXWELL ELECTRODYNAMICS AND A CHARGED THIN SHELL IN A SPHERICALLY SYMMETRIC BACKGROUND

A. Action, Lagrangian, and Hamiltonian in a general form

1. Action and Lagrangian

For our purposes of studying charged matter in a d-dimensional charged background spacetime, the field content is divided into three sectors, the gravitational, the electrodynamic, and the matter sectors. Then, the action I can be written as the sum of a gravitational action $I^{(g)}$, an electrodynamic action $I^{(e)}$, and a generic matter field action $I^{(m)}$, which will be specialized later on to be a charged thin shell. Thus,

$$I = I^{(g)} + I^{(e)} + I^{(m)}. \tag{1}$$

The gravitational action and Lagrangian:

The gravitational sector of the action, $I^{(g)}$, depends on which theory one wants to adopt. General relativity, first formulated in four spacetime dimensions, can be trivially extended to higher d spacetime dimensions by changing the action from a four-dimensional integral of the cosmological constant term and the Ricci scalar, to a d-dimensional integral of both terms. However, this extension is no longer unique. Another natural extension is given by the Lovelock gravity [7], in which its action results from demanding that the Euler-Lagrange equations derived from the corresponding Lagrangian yield all tensors $A^{\mu\nu}$ symmetric in $\mu\nu$, occurring concurrently with the metric tensor $g_{\mu\nu}$ and its first two derivatives, i.e., $A^{\mu\nu} = A^{\mu\nu}(g_{\rho\sigma}; g_{\rho\sigma;\gamma}; g_{\rho\sigma;\gamma\delta})$, and divergence free. In four dimensions only the cosmological constant and the Ricci scalar yield an action with these properties, which is precisely the Einstein-Hilbert action. As one goes to higher dimensions new generalized Gauss-Bonnet terms, topological in nature in the previous dimension, make their appearance. As mentioned in the introduction, the Lovelock theory can then be considered as a dimensional continuation of the topological Euler characteristics of lower dimensions [8–10]. The theory has, in addition to Newton's constant and the cosmological constant, new arbitrary dimensionful parameters, which should be chosen carefully. Because of its interest and generality, we work here with Lovelock gravity. The gravitational part $I^{(g)}$ in (1) is then the Lovelock action, which turns out to be a polynomial in the curvature tensor, of degree $[d/2]$, where the brackets $[d/2]$ represent the integer part of $d/2$. This action, and the corresponding Lagrangian $\mathcal{L}$ defined through $I = \int d^d x \mathcal{L}$, can be written as

$$I^{(g)} = \kappa \int_{\mathcal{M}} d^d x \sum_{p=0}^{[d/2]} \alpha_p \sqrt{-\mathfrak{g}}\, 2^{-p} \delta^{\mu_1 \cdots \mu_{2p}}_{\nu_1 \cdots \nu_{2p}} R^{\nu_1 \nu_2}_{\mu_1 \mu_2} \cdots R^{\nu_{2p-1} \nu_{2p}}_{\mu_{2p-1} \mu_{2p}}, \tag{2}$$

where κ is inversely proportional to the Newton's constant (which will be appropriately chosen below), $\mathcal{M}$ stands for the spacetime manifold, $\mathfrak{g}$ is the determinant of the spacetime metric, and $\mu_1, \mu_2, \ldots$ are spacetime indices. The generalized δ function is antisymmetric in all of the upper indices and all of the lower indices, and $R^{\nu_1 \nu_2}_{\mu_1 \mu_2}$ is the Riemann tensor. The coefficients α_p are arbitrary in general, apart from the first two α_0 and α_1. Indeed, in the gravitational action (2), the first term of the integrand is the cosmological constant Λ and the second term of the integrand is the Ricci scalar R, i.e., the terms of the Einstein-Hilbert action. This shows that general relativity is contained in the Lovelock theory as a particular case, namely, by putting in the action all the $\alpha_p = 0$ for $p \geq 2$. For even dimensions, the term $p = d/2$ in the action (2) is the Euler characteristic of that d-dimensional manifold and does not contribute to the field equations. However, although not relevant for the purposes of this paper, the presence of the Euler term guarantees the existence of a well-defined variational principle for asymptotically locally AdS spacetimes (see [20]). Lovelock gravity, from the way it is constructed, has the same essence as general relativity. However, the theory is more complicated, it sets extra problems not present in general relativity and yields new interesting features. For instance, for $d > 4$ the fields may evolve in a nonunique manner, such that, given initial values for the fields at $t = t_0$, at $t > t_0$ the equations of motion do not determine those fields completely. This is due to the presence in the Lagrangian of high powers in the first derivative of the metric tensor with respect to time [8–10]. An interesting aspect of Lovelock gravity is that, although its linearized approximation is classically equal to the corresponding linearized approximation in general relativity [13], in the full strong gravity regime of the theory, the higher powers of curvature in the Lagrangian yield solutions that are different, and such type of solutions in Lovelock theory cannot be reached through a solution in general relativity [14–26].

Another feature is the fact that, in addition to the constant κ (inversely proportional to an appropriate generalization of Newton's constant) and the cosmological constant Λ, there are the $[(d+1)/2]$ arbitrary dimensionful parameters. Now, for a given dimension and an arbitrary choice of the coefficients α_p, the dynamical evolution can become unpredictable [8–10], thus it is advantageous to restrict these coefficients in order to be able to construct meaningful black hole and other solutions. An important step in this direction was taken first in [19] and generalized in [20], where the coefficients α_p were chosen so that unique sensible solutions, with well-defined perturbations could be found. One way to meet these requirements is to demand, in a consistent way, that the theory has a unique cosmological constant Λ as shown in [19,20], which leads to the choice

$$\alpha_p := c_p^k = \begin{cases} \frac{l^{2(p-k)}}{(d-2p)} \binom{k}{p}, & p \leq k \\ 0, & p > k \end{cases}, \tag{3}$$

where l is a length scale of the theory given in terms of the cosmological constant Λ by

$$\Lambda = -\frac{(d-1)(d-2)}{2l^2}. \tag{4}$$

As seen in the last expression, in this setting the cosmological constant is negative throughout, and so the solutions will be asymptotically AdS solutions. We are then left with a set of theories labeled by an integer k, with $1 \leq k \leq [(d-1)/2]$. One should explain better the dependence of the theories on k. In [19] k was put equal to its maximum value always, $k = [(d-1)/2]$, i.e., no pertinent Lovelock term is put to zero, and one has the corresponding special choice made in Eq. (3). In this case, all nontopological terms, plus the topological nondynamical term in the even dimensional case, are included in the action, and the theory can thus be called dimensionally continued gravity, since as one goes one dimension up, what was a topological term in the previous dimension, is now a term that contributes dynamically when one continues the dimension. In even dimensions, $d \geq 4$, this choice leads to a Born-Infeld gravitational action, the gravitational analogue of the Born-Infeld electrodynamics, and in odd dimensions, $d \geq 5$, it leads to a Chern-Simons gravitational action [19]. This dimensionally continued theory [19] can be naturally extended by canceling higher order terms, from a given integer k upwards. This was done in [20]. For instance one can set up a theory in which only the terms $p = 0$ and $p = 1$ appear, thus $k = 1$. This theory is general relativity. If one puts $k = 2$ one gets general relativity plus the corresponding generalized Gauss-Bonnet term, and so on, up to the dimensionally continued gravity where $k = [(d-1)/2]$. In brief, for a given dimension d, depending on the choice of the integer k, one generates different theories, with k giving the highest power of the curvature in the Lagrangian. Now, the other fundamental constant κ can be related to a generalized Newton's constant G_k, labeled by k, with units $[G_k] = (\text{length})^{d-2k}$, through

$$\kappa = \frac{1}{2(d-2)!\Omega_{(d-2)}G_k}, \tag{5}$$

where Ω_{d-2} is the area of the unit sphere in $d-2$ dimensions. These theories have well-defined black hole configurations.

The electrodynamic action and Lagrangian:

The electrodynamic sector of the action (1), the electrodynamic action $I^{(e)}$, can be chosen to be the electromagnetic Maxwell term plus a matter electric current, and can be written as

$$I^{(e)} = -\frac{1}{4\epsilon\Omega_{d-2}}\int_{\mathcal{M}} d^dx\sqrt{-\mathfrak{g}}F_{\mu\nu}F^{\mu\nu} - \frac{1}{\epsilon}\int_{\mathcal{M}} d^dx\sqrt{-\mathfrak{g}}J^\mu A_\mu, \tag{6}$$

where ϵ is related to the vacuum permitivity ϵ_0 through the expression $\epsilon = \frac{\epsilon_0}{\Omega_{d-2}}$, $\mathfrak{g}$ is the determinant of the spacetime metric $\mathfrak{g}_{\mu\nu}$, $F^2 = F_{\mu\nu}F^{\mu\nu}$ is the square of $F_{\mu\nu} = \nabla_\mu A_\nu - \nabla_\nu A_\mu$, which is the Maxwell tensor, with A_μ being the electromagnetic potential, and J^μ is the electric current. For vacuum solutions J^μ is zero, and one has pure Maxwell electromagnetism.

The matter action and Lagrangian:

The last term in the action (1), the matter action $I^{(m)}$ is defined as

$$I^{(m)} = \int d^dx\sqrt{-\mathfrak{g}}\mathcal{L}^m, \tag{7}$$

where the matter Lagrangian $\mathcal{L}_m$ is defined through the energy-momentum tensor of the matter $T_{\mu\nu}$ by

$$T_{\mu\nu} = -\frac{(d-2)!\Omega_{(d-2)}}{4\pi\sqrt{-\mathfrak{g}}}\frac{\delta\mathcal{L}^{(m)}}{\delta g^{\mu\nu}}, \tag{8}$$

where δ here means variation. In vacuum $I^{(m)} = 0$. Later on we will specify the matter as being made of a charged thin shell.

2. Hamiltonian

In order to apply the Hamiltonian framework, we use the Arnowitt-Deser-Misner (ADM) formulation [30] where there is a foliation of spacetime into $t =$ constant hypersurfaces, denoted by Σ_t. In this foliation of the spacetime the metric, both inside and outside, is written generically as

$$ds^2 = -(N^\perp)^2dt^2 + g_{ij}(N^idt + dx^i)(N^jdt + dx^j), \tag{9}$$

where $N^\perp$ and N^i are the lapse and shift functions of the foliation, respectively. The g_{ij} are the metric coefficients of the intrinsic geometry of the hypersurfaces Σ_t, where i, j run only on the spatial components. It is known that the action $I = \int d^dx\mathcal{L}$, with $\mathcal{L}$ the Lagrangian, can be written as

$$I = \int dt \int d^{d-1}x(\pi^{ij}\dot{g}_{ij} - \mathcal{H}), \tag{10}$$

where $\pi^{ij} = \delta\mathcal{L}/\delta\dot{g}_{ij}$ are the momentum components conjugate to the metric components g_{ij}, $\dot{g}_{ij}$ are the respective coordinate time derivatives, and $\mathcal{H}$ is the Hamiltonian of the system. It is then useful to write the Hamiltonian in the form

$$\mathcal{H} = N^\perp\mathcal{H}_\perp + N^i\mathcal{H}_i, \tag{11}$$

where $\mathcal{H}_\perp$ is the normal Hamiltonian constraint that generates the time translations normal to the hypersurface Σ_t, and $\mathcal{H}_i$ are the tangential constraints that generate the translations in each of the hypersurfaces Σ_t, which is the same as saying that $\mathcal{H}_i$ is the generator of hypersurface diffeomorphisms, or that it generates coordinate transformations in Σ_t. In addition, one can write

$$\begin{aligned} \mathcal{H}_\perp &= \mathcal{H}_\perp^{(g)} + \mathcal{H}_\perp^{(e)} + \mathcal{H}_\perp^{(m)}, \\ \mathcal{H}_i &= \mathcal{H}_i^{(g)} + \mathcal{H}_i^{(e)} + \mathcal{H}_i^{(m)}, \end{aligned} \tag{12}$$

where the superscript (g) refers to the gravitational sector, the superscript (e) refers to the electrodynamic sector, and the superscript (m) refers to the matter sector of the Hamiltonian. The lapse function $N^{\perp}$ and the shift functions N^i of the ADM metric act here as Lagrange multipliers.

The gravitational Hamiltonian:

For the Lovelock theory, the Hamiltonian components of the gravitational field can be taken from the action (2), the momenta conjugate to the corresponding Lagrangian $\pi^{ij} = \delta\mathcal{L}/\delta\dot{g}_{ij}$, and $\dot{g}_{ij}$, with the result [8–10]

$$\mathcal{H}_{\perp}^{(g)} = -\kappa\sqrt{g}\sum_{p}\frac{(d-2p)!\alpha_p}{2p}\delta^{i_1\cdots i_{2p}}_{j_1\cdots j_{2p}}R^{j_1j_2}_{i_1i_2}\cdots R^{j_{2p-1}j_{2p}}_{i_{2p-1}i_{2p}}, \tag{13}$$

$$\mathcal{H}_{i}^{(g)} = -2\pi^{j}_{i|j}. \tag{14}$$

The R_{ijkl} in (13) are the spatial components of the curvature tensor in the d-dimensional spacetime. The Gauss-Codazzi equations give us the relation between this tensor and $\hat{R}_{ijkl}$, the Riemann tensor intrinsic to the surface Σ_t,

$$R_{ijkl} = \hat{R}_{ijkl} + K_{ik}K_{jl} - K_{il}K_{jk}, \tag{15}$$

where K_{ij} is the extrinsic curvature tensor of the surface, given by the expression

$$K_{ij} = \left(\frac{1}{2N^{\perp}}\right)(N_{i;j} + N_{j;i} - \dot{g}_{ij}). \tag{16}$$

The semicolon in the indices denotes the intrinsic covariant derivative in the hypersurface Σ_t, and the dot is the derivative of the spatial-spatial components of the metric with respect to the time coordinate. The conjugate momentum to g_{ij} can be found through $\delta\mathcal{L}/\delta\dot{g}_{ij}$, which in this case gives [8–10]

$$\pi^{i}_{j} = -\kappa\sqrt{g}\sum_{p}\frac{p!(d-2p)!\alpha_p}{2^{p+1}}\sum_{s=0}^{p-1}D_{s(p)}\delta^{ii_1\cdots i_{2s}\cdots i_{2p-1}}_{jj_1\cdots j_{2s}\cdots j_{2p-1}} \times R^{j_1j_2}_{i_1i_2}\cdots R^{j_{2s-1}j_{2s}}_{i_{2s-1}i_{2s}}K^{j_{2s+1}}_{i_{2s+1}}\cdots K^{j_{2p-1}}_{i_{2p-1}}, \tag{17}$$

where

$$D_{s(p)} = \frac{(-4)^{p-2}}{s![s(p-s)-1]!!}. \tag{18}$$

Here the double factorial is defined by:

$$s!! \equiv \begin{cases} s\cdot(s-2)\ldots 5\cdot 3\cdot 1 & s>0 \text{ odd} \\ s\cdot(s-2)\ldots 6\cdot 4\cdot 2 & s>0 \text{ even}. \\ 1 & s=-1,0 \end{cases} \tag{19}$$

The electrodynamic Hamiltonian:

The action in a Hamiltonian form of the electrodynamic field in a curved background is given by $I_e = \int dt\int d^{D-1}x[p^i\dot{A}_i - \frac{1}{2}N^{\perp}(\Omega_{d-2}\frac{1}{\sqrt{g}}p^ip_i + \frac{\sqrt{g}}{2\Omega_{d-2}}F^{ij}F_{ij}) + \varphi p^i_{,i} - \varphi J^0]$, where p^i is the momentum conjugate to the spatial components of the gauge field A_i, $\varphi \equiv A_0$, F_{ij} are the spatial components of the Maxwell tensor, J^0 is the time component of the electric current, and Ω_{d-2} is the area of the $(d-2)$-dimensional unit sphere. We are ignoring surface terms since they will be automatically taken into account in this setting. From the electrodynamic action, and using the definition given in Eqs. (11) and (12), we have

$$\mathcal{H}_{\perp}^{(e)} = \frac{1}{2}\Omega_{d-2}\frac{1}{\sqrt{g}}p^ip_i + \frac{\sqrt{g}}{2\Omega_{d-2}}F^{ij}F_{ij}, \tag{20}$$

$$\mathcal{H}_{i}^{(e)} = 0. \tag{21}$$

Besides the usual constraints, there is also a new constraint E_φ, associated with the Lagrangian multiplier φ

$$E_\varphi \equiv p^i_{,i} - J^0. \tag{22}$$

The matter Hamiltonian:

The matter sector of the Hamiltonian constraint is written as

$$\mathcal{H}_{\perp}^{(m)} = \sqrt{g}T_{\perp\perp}, \tag{23}$$

$$\mathcal{H}_{i}^{(m)} = 2\sqrt{g}T_{\perp i}, \tag{24}$$

where $T_{\mu\nu}$ is the matter energy-momentum tensor, and where the index $\perp$ means that the tensor has been contracted with the hypersurface normal $n_\mu = (-N^{\perp}, 0, 0, 0)$. In the following we will specify the matter as being made of a charged thin shell.

B. Hamiltonian and field equations for thin shells with interior and exterior static vacua in spherically symmetric spacetimes

1. Gravitational, electrodynamic, and thin shell Hamiltonians in spherically symmetric spacetimes

Using the Hamiltonian formalism, we now want to set up the field equations appropriate for charged thin shells in static spherically symmetric charged Lovelock backgrounds. The formalism advanced here can be used for both static and dynamic thin shells. In the next section we will make full use of the formalism when we apply it first to find the pressure necessary to maintain a charged thin shell in static equilibrium in a vacuum background interior, black hole or otherwise, and second to the study of the collapse of a charged thin shell into a vacuum background interior, black hole or otherwise, in Lovelock gravity coupled to Maxwell electromagnetism. Since the three sectors that enter the problem are the gravitational background, the electrodynamic interaction, and the matter that constitutes the shell, we have to set up the metrics for the interior and exterior to the shell, we have to give the form of vector potential field, and we have also to give the energy-momentum tensor.

The gravitational Hamiltonian in spherically symmetric spacetimes:

We assume now the shell traces its spacetime trajectory in a static spherically symmetric background. Thus the generic ansatz both for the exterior and interior to the shell can be written as

$$ds^2 = -N^2(r)f^2(r)dt^2 + \frac{dr^2}{f^2(r)} + r^2 d\Omega_{d-2}^2, \qquad (25)$$

where Schwarzschild type coordinates $\{t, r, \theta^1, \theta^2, \cdots, \theta^{(d-2)}\}$ have been chosen, $N(r)$ is the lapse function exclusively dependent on the radial coordinate, $f(r)$ is the metric function, dependent on r only, and $d\Omega_{d-2}^2$ is the line element of the $d-2$-dimensional unitary sphere, written explicitly as $d\Omega_{d-2}^2 = (d\theta^1)^2 + \sin(\theta^1)^2(d\theta^2)^2 + \sin(\theta^1)^2\sin(\theta^2)^2(d\theta^3)^2 + \cdots + \prod_{i=1}^{d-3}\sin(\theta^i)^2(d\theta^{(d-2)})^2$. With the ansatz (25), the first gravitational constraint (13) is written as

$$\mathcal{H}_\perp^{(g)} = -\kappa\frac{(d-2)!}{r^{d-2}}\sqrt{g}\frac{d}{dr} \times \left\{r^{d-1}\sum_p (d-2p)\alpha_p\left(\frac{1-f^2}{r^2}\right)^p\right\}, \qquad (26)$$

and the other gravitational constraints (14) yield the equation

$$\mathcal{H}_i^{(g)} = \kappa\sqrt{\Omega}(d-2)!\sum_{p=0}^{k}\frac{p!(d-2p)!\alpha_p}{2^{p+1}(d-2p-1)!} \times \sum_{s=0}^{p-1}2^s D_{s(p)} f^{2(s-p)}(1-f^2)^s \times \frac{d}{dr}(r^{d-2p-1}(\alpha_p \dot{r})^{2p-2s-1}), \qquad (27)$$

where Ω is the angular part of the determinant g of the intrinsic metric g_{ij} of the hypersurface Σ_t, with $g = g_{rr}r^{d-2}\Omega$, and we use the fact that the metric is diagonal. The function $D_{s(p)}$ is defined in Eq. (18).

The electrodynamic Hamiltonian in spherically symmetric spacetimes:

For a static system with spherical symmetry the electromagnetic field is purely electric and radial for an outside observer at rest. In this case, the vector potential has only one nonzero component, which depends exclusively on the radial coordinate

$$A_t = A(r). \qquad (28)$$

Given the symmetries and the fact that we are working on a static background, the totally spatial components of the Maxwell tensor are null, i.e., $F_{ij} = 0, i, j = r, \theta^1, \cdots, \theta^{(d-2)}$. The only nonvanishing component of the Maxwell tensor is $F_{tr} = -\partial_r A_t(r)$, with also $F_{rt} = \partial_r A_t(r)$. Thus, electrically charged, static, spherically symmetric vacuum solutions imply

$$F_{ij} = 0, \qquad (29)$$

$$p^i = (0, p^r, 0, \ldots, 0), \qquad (30)$$

$$\dot{A}_i = 0 \quad \text{and} \quad \dot{p}^i = 0, \qquad (31)$$

where Eq. (29) means there is no magnetic field, Eq. (30) means the electric field is spherically symmetric, and Eq. (31) means the field is static. From Eqs. (20)–(22), the electrodynamic constraints, associated with the Lagrange multipliers $N^\perp$ and φ, are, respectively,

$$\mathcal{H}_\perp^{(e)} = \frac{1}{2\sqrt{g}}\Omega_{d-2}p^r p_r = \frac{1}{2\sqrt{g}}\Omega_{d-2}(p^r)^2 f^{-2}(r), \qquad (32)$$

$$\mathcal{H}_i^{(e)} = 0, \qquad (33)$$

$$E_\varphi = p^r_{,r} - J^0, \qquad (34)$$

where $f(r)$ comes from (25).

The thin shell Hamiltonian in spherically symmetric spacetimes:

We want to spell out completely the matter constraints, namely, $\mathcal{H}_\perp^{(m)} = \sqrt{g}T_{\perp\perp}$ and $\mathcal{H}_i^{(m)} = 2\sqrt{g}T_{\perp i}^{(m)}$, for a charged thin shell in a charged vacuum background geometry. More precisely we want to develop a Hamiltonian framework for this situation and set up a natural junction which arises in this formalism. For this to be made we have first to describe the geometrical setup in order to be able to write the projected components of the energy-momentum tensor, namely, $T_{\perp\perp}$ and $T_{\perp i}$.

First we state the nomenclature for the thin shell and for the interior and exterior spacetimes. The thin shell is a $d-2$-dimensional spacelike surface, which evolves in time, see Fig. 1. This time evolution of the thin shell can be represented by a timelike hypersurface ∂V_ξ, a boundary surface, which divides spacetime into two regions, the interior, denoted by $V^{(-)}$, and the exterior, denoted by $V^{(+)}$. At each point on the boundary surface there exists a unit spacelike vector ξ, with components ξ^μ, normal to ∂V_ξ, and pointing from the interior $V^{(-)}$ to the exterior $V^{(+)}$. In ∂V_ξ there is a set of intrinsic spacetime coordinates ρ^a, where a runs from 0 to $d-2$. In the regions $V^{(-)}$ and $V^{(+)}$ independent coordinates are introduced, x_-^μ and x_+^μ, respectively (where μ runs from 0 to $d-1$), and so the parametric equations for ∂V_ξ in these coordinates are $x_-^\mu(\rho^a)$ and $x_+^\mu(\rho^a)$. Thus, ∂V_ξ has tangential vectors e_a with components given by $e_a^\mu = \frac{\partial x^\mu}{\partial \rho^a}$. Since ∂V_ξ represents the spacetime evolution of the $d-2$-dimensional thin shell, the d-velocity u, with components u^μ, of the matter of the shell is tangential to ∂V_ξ, with u being orthogonal to ξ, and vanishing outside the shell. One can also consider an intrinsic velocity vector $\bar{u}$, with components $\bar{u}^a$, related to the velocity u through the relation $u^\mu = e_a^\mu \bar{u}^a$, where $\bar{u}$ is built by considering the evolution of matter in the shell using coordinates $(\rho^1, \cdots, \rho^{d-2})$ intrinsic to the shell, and the intrinsic time ρ^0.

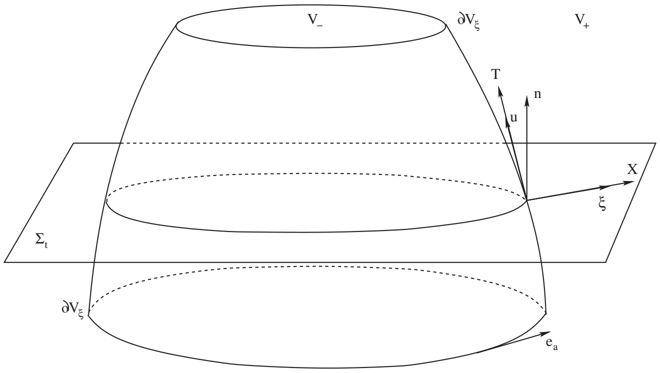

FIG. 1. The shell trajectory manifold ∂V_ξ in spacetime is depicted. V_- and V_+ are the spacetime regions interior and exterior to ∂V_ξ, respectively, and Σ_t is the hypersurface intersecting ∂V_ξ. Also shown are the normal vector n (normal to Σ_t), the velocity vector u (tangent to ∂V_ξ), the vector ξ (the spacelike normal to ∂V_ξ), and the vector e_a (the generic tangent vector to ∂V_ξ). The vectors indicating the origin and direction of the adapted coordinate system $\{T, X\}$ are depicted together with the u and ξ vectors.

Working with the spherically symmetric metric (25), we choose the intrinsic coordinates of the thin shell surface ∂V_ξ as the proper time τ, and the spherical angles of the Schwarzschild-like coordinates, so that $\rho^a = (\tau, \theta^1, \ldots, \theta^{d-2})$. The motion of the shell is governed by the radial function of the proper time $r = R(\tau)$. The derivative with respect to τ will henceforth be denoted by a dot, e. g. $\frac{d}{d\tau}R(\tau) \equiv \dot{R}(\tau)$. The line element for ∂V_ξ in these coordinates is

$$ds^2_\Sigma = -d\tau^2 + R^2(\tau)d\Omega^2_{d-2}. \tag{35}$$

We want to study the spacetime evolution of a thin shell in a vacuum background with metrics of the type given in (25). Thus, the interior $V^{(-)}$ and exterior $V^{(+)}$ spacetimes have their general line element given by

$$ds^2_- = -f^2_-(r)dt^2_- + f^{-2}_-(r)dr^2 + r^2 d\Omega^2_{d-2}, \tag{36}$$

$$ds^2_+ = -f^2_+(r)dt^2_+ + f^{-2}_+(r)dr^2 + r^2 d\Omega^2_{d-2}. \tag{37}$$

The radial and angular coordinates match continuously at the shell, but the time coordinates do not in general. The radial and angular coordinates match on account of the fact that the surface area of the shell is independent of the coordinates used to determine it, and it should be the same on both sides, as we are working in the limit of infinitesimal thickness, the thin shell limit. As the area of the shell is a function of the radial position of the shell $R(\tau)$, there should not be a different radius for the shell inside and outside, as we would be attributing two different areas to the shell. Also, spherical symmetry allows us to induce coordinate charts on the manifold ∂V_ξ merely by restricting the charts of $V^{(-)}$ and $V^{(+)}$ to their boundary on ∂V_ξ. As, due to symmetry, the radial and angular coordinates on both sides of the shell are the same, their restriction to the shell should be the same, and hence match perfectly. However, the restriction of the inside line element (36) to the surface of the shell must match the restriction of the outside line element (37), all having the general form (35) on the shell. In these coordinates the vectors u and ξ defined above (see also Fig. 1) have the following components,

$$u^\mu = \left(\frac{\gamma}{f^2}, \dot{R}, 0, 0\right), \tag{38}$$

$$\xi^\mu = \left(\frac{\dot{R}}{f^2}, \gamma, 0, 0\right), \tag{39}$$

where

$$\gamma = \sqrt{f^2 + \dot{R}^2}, \tag{40}$$

where $\gamma_\pm$ is a generalized Lorentz factor. These vectors were obtained through the matching of (36) and (37) to the surface line element (35), using the relation $u^\mu = \frac{dx^\mu}{d\tau}$, where the $t_\pm$ coordinates and the radial coordinate are treated as functions of the proper time τ. We have also used $u^\mu u_\mu = -1$, $\xi^\mu u_\mu = 0$, and $\xi_\mu \xi^\mu = 1$ (as ξ is the spacelike normal to ∂V_ξ).

The shell matter properties are described by the surface energy-momentum tensor T_{ab}, which is orthogonal to ξ and vanishes outside the hypersurface ∂V_ξ. For an observer in the shell, T_{ab} is written in intrinsic coordinates, which for a perfect fluid is given by

$$T_{ab} = \sigma \bar{u}_a \bar{u}_b + P(h_{ab} + \bar{u}_a \bar{u}_b), \tag{41}$$

where σ is the rest mass surface density, P is the surface pressure (or, when negative, the surface tension), and $h_{ab} \equiv g_{\mu\nu} e^\mu_a e^\nu_b$. T_{ab} is confined to the hypersurface ∂V_ξ and it satisfies the conservation equation, $T^b_{a|b} = 0$, where a $|$ denotes covariant differentiation on the hypersurface, which implies the explicit equation

$$(\sigma \bar{u}^a)_{|a} + P\bar{u}^a_{|a} = 0, \tag{42}$$

this last being obtained by multiplying the tensor T_{ab} by $\bar{u}^a$.

To apply the Hamiltonian formalism we should pass to a spacetime characterization of the energy-momentum tensor. In order to do so we define the spacetime energy-momentum tensor as

$$T^{\mu\nu} = T^{ab} e^\mu_a e^\nu_b. \tag{43}$$

Then, in order to write $T_{\perp\perp}$ and $T_{\perp i}$ we must first write the energy-momentum tensor in an adapted coordinate system $\{T, X, \theta^1, \cdots, \theta^{d-2}\}$ (see Fig. 1), where T and X are the time and radial adapted coordinates, and $\theta^1, \cdots, \theta^{d-2}$ are the usual angular coordinates. In more detail, the origin of the new adapted coordinate X is at the shell, and the direction of any vector in X alone is the same as that of the outward spacetime normal to the surface, ξ, and the direction of any vector in T alone is the same as that of the tangent vector u. Then,

$$T^{\mu\nu} = \{\sigma u^\mu u^\nu + P(h^{\mu\nu} + u^\mu u^\nu)\}\delta(X), \tag{44}$$

where $h^{\mu\nu}$ is the intrinsic metric of the shell written as a bulk d-dimensional spacetime tensor, $h^{\mu\nu} = g^{\mu\nu} - \xi^\mu \xi^\nu$, and u^μ is the velocity of the particles of the shell written as a bulk d-dimensional vector. When written as a bulk tensor, the energy-momentum tensor of the thin shell is proportional to a delta function. Now, $T_{\perp\perp}$ is given by $T^{\mu\nu}n_\mu n_\nu$, which means that $T^{\mu\nu}n_\mu n_\nu = [\sigma u^\mu u^\nu n_\mu n_\nu + P(h^{\mu\nu}n_\mu n_\nu + u^\mu u^\nu n_\mu n_\nu)]\delta(X)$. Knowing that $n^\mu n_\mu = -1$, we have that $(N^\perp)^2 = f^2$. So $u_\perp u_\perp = \gamma^2/f^2$. In the same way, we obtain $h^{\mu\nu}n_\mu n_\nu = -1 - (\dot{R}^2)/(f^2)$. We arrive thus at $T_{\perp\perp} = \sigma(\gamma^2/f^2)\delta(X)$. It is, however, necessary to write the delta function as a function of the radial coordinate r. This means we have to return to the original Schwarzschild type coordinates of the metrics (36) and (37). For that, we write $dt = u^t dT + \xi^t dX$, $dr = u^r dT + \xi^r dX$. As we are interested in the energy-momentum tensor on the spacelike hypersurface Σ_t, we make $dt = 0$ and obtain the differential relation $dr/dX = \xi^r - (u^r/u^t)\xi^t = f^2/\gamma$. Taking into consideration the relation $\delta(f(x)) = \Sigma_i \delta(x - x_i)/|f'(x_i)|$, where x_i are the zeros of a general function $f(x)$, and $'$ denotes differentiation with respect to the argument of the function, we have $\delta(X) = (f^2/\gamma)\delta[r - R(\tau)]$. Plugging one equation into the other we get

$$T_{\perp\perp} = \sigma\gamma\delta[r - R(\tau)], \tag{45}$$

where σ is the energy surface density on the shell, γ is given in (40), and $R(\tau)$ is the shell radial function.

For the other components of the matter tensor, $T_{\perp i}$, we have first to define $z_i^\mu = \frac{\partial x^\mu}{\partial y^i}$ as the projection vectors onto the hypersurface Σ_t, where y^i are the intrinsic coordinates of Σ_t. Then, writing the intrinsic metric of ∂V_ξ as $h^{\mu\nu} = g^{\mu\nu} - \xi^\mu \xi^\nu$, and knowing that $u^\mu \xi_\mu = 0$, we arrive at

$$T_{\perp i} = \frac{f^2}{\gamma}[(\sigma + P)(-N^\perp f^{-2}\gamma)u_i + Pu_i]\delta[r - R(\tau)], \tag{46}$$

where u_i are the components of the velocity vector projected onto Σ_t. It is then possible to write completely the matter constraints as

$$\mathcal{H}_\perp^{(m)} = \sqrt{g}\sigma\gamma\delta[r - R(\tau)], \tag{47}$$

and

$$\mathcal{H}_i^{(m)} = 2\sqrt{g}\frac{f^2}{\gamma}[(\sigma + P)(-N^\perp f^{-2}\gamma)u_i + Pu_i] \times \delta[r - R(\tau)]. \tag{48}$$

One remark is due, regarding the foliation of spacetime. From the way we have sliced the spacetime, by looking at the ADM metric (9) and the fact that we are working with spherical symmetry and static background (cf. (25)), we see that the shift Lagrange multipliers N^i are equal to zero. Therefore, every constraint with components in the Σ_t hypersurface $\mathcal{H}_i$ will not be relevant to our discussion. In the following we will need neither $T_{\perp i}$ nor $\mathcal{H}_i^{(m)}$.

2. The equations of motion

By varying the Hamiltonian action, written with the constraints spelt out previously, with respect to g, $N^\perp$, φ, and p^r the field equations will be, respectively,

$$\frac{dN}{dr} = 0, \tag{49}$$

$$-\kappa\frac{(d-2)!}{r^{d-2}}\frac{d}{dr}\left(r^{d-1}\left[F(r) + \frac{1}{l^2}\right]^k\right) = \frac{p^2}{2\Omega_{d-2}} + T_{\perp\perp}, \tag{50}$$

$$\frac{d}{dr}(r^{d-2}p) = r^{d-2}j^0, \tag{51}$$

$$\frac{d\varphi}{dr} + Np = 0, \tag{52}$$

where $F(r) = (1 - f^2(r))/r^2$, $N = N^\perp(g_{rr})^{1/2}$, and $p(r)$ is a redefinition of the canonical momentum radial component p^r through

$$p^r = r^{d-2}\frac{\Omega^{1/2}}{\Omega_{d-2}}p, \tag{53}$$

where Ω is the angular part of the determinant g of the intrinsic metric g_{ij} of the hypersurface Σ_t, with $g = g_{rr}r^{d-2}\Omega$, and we have used the fact that the metric is diagonal. In the same way we have redefined the current's time component

$$J^0 = r^{d-2}\frac{\Omega^{1/2}}{\Omega_{d-2}}j^0. \tag{54}$$

We also have used (3) to write

$$\sum_p (d - 2p)\alpha_p\left(\frac{1 - f^2}{r^2}\right)^p = \left(F + \frac{1}{l^2}\right)^k. \tag{55}$$

One should note that varying with respect to $N^\perp$ and φ is requiring that the constraints be made equal to zero, because both are Lagrange multipliers. That is, in the case of $N^\perp$, the variation implies $\mathcal{H}_\perp = 0$, and varying with respect to φ implies $E_\varphi = 0$.

Equations (49)–(52) are valid for any thin spherically symmetric charged shell in Lovelock gravity coupled to Maxwell electromagnetism. One has only to give the thin shell delta-function energy-momentum tensor and integrate the equations to find the solutions. Note that Eq. (50) is valid only for a delta-function energy-momentum tensor. Indeed this equation when integrated gives the equation of motion of the shell itself, but if one has a continuous distribution of matter, instead of a thin shell, then within the matter there exists time dependence in the metric function, which should be taken into account. In the case of the thin shell we were able to ignore this time dependence, since inside and outside the shell it is possible to set up static backgrounds.

III. THIN SHELL SOLUTIONS IN A VACUUM BACKGROUND WITH SPHERICAL SYMMETRY

A. Spherically symmetric vacuum solutions: solutions with black hole character, with naked singularity character, and empty solutions

The vacuum solutions are obtained when we consider the matter action equal to zero, $T_{\perp\perp} = 0$, and when there is no vector current, $J^\mu = 0$. So, after integrating the field equations above, one obtains the vacuum solutions [20]

$$N = N_\infty = 1, \tag{56}$$

$$f^2(r) = 1 + \frac{r^2}{l^2} - \chi g_k(r), \tag{57}$$

$$p(r) = \epsilon \frac{Q}{r^{(d-2)}}, \tag{58}$$

$$\varphi(r) = \varphi_\infty + \frac{\epsilon}{(d-3)} \frac{Q}{r^{(d-3)}}. \tag{59}$$

The function $g_k(r)$ is defined by

$$g_k(r) = \left(\frac{2G_k M + \delta_{d-2k,1}}{r^{d-2k-1}} - \frac{\epsilon G_k}{(d-3)} \frac{Q^2}{r^{2(d-k-2)}}\right)^{1/k}, \tag{60}$$

where the integration constants M and Q are the mass and electric charge of the solutions, and where we have used Eq. (5) for the definition of κ in terms of G_k. From Eqs. (55)–(60), in particular, from Eqs. (58)–(60), one notes that the vacuum solutions yield a natural division of the Lovelock theory into two branches, namely $d - 2k - 1 > 0$ and $d - 2k - 1 = 0$, with d being the dimension of the spacetime and k the parameter that gives the highest power of the curvature in the Lagrangian. The branch $d - 2k - 1 > 0$ embodies general relativity when $k = 1$ (any d), Born-Infeld when $k = [\frac{d-1}{2}]$, and other generic cases. The branch $d - 2k - 1 = 0$ embodies Chern-Simons type theories alone. Worth mentioning now is the fact that in the $d - 2k - 1 > 0$ branch the empty vacuum solutions have $M = 0$ and $Q = 0$, whereas in the $d - 2k - 1 = 0$ branch the empty vacuum solutions have $M = -(2G_k)^{-1}$ and $Q = 0$. In the solutions, there appears an additional unusual parameter χ. The parameter χ is the spacetime character, given by

$$\chi = (\pm 1)^{k+1}. \tag{61}$$

This means that the vacuum solutions may have a black hole character or a naked singularity character. For $\chi = 1$ the solutions, being of the type found in general relativity, have a black hole character, since for a correct choice of the parameters, such as mass and charge, the solution is a black hole solution, although, of course, for other choices of parameters it can be an extremal black hole or a naked singularity. For $\chi = -1$ the solutions, being of a new type not found in general relativity, have a naked singularity character, as there is no possible choice of the parameters that gives a black hole solution, the full vacuum solution is always singular without horizons. Note also that Eq. (61) implies that if k is odd (such as in general relativity, where $k = 1$) the character $\chi = 1$ only, whereas if k is even (such as in general relativity with a Gauss-Bonnet term, where $k = 2$) the character χ can have both values ± 1. Note also that in the $d - 2k - 1 = 0$ Chern-Simons theory, the black hole vacuum spacetime, i.e., the vacuum of the $\chi = 1$ character black hole solution, given by $M = 0$ and $Q = 0$, is different from the usual AdS spacetime, and there is a mass gap between these spacetimes, with the latter being obtained for $M = -(2G_k)^{-1}$, as mentioned above. Equations (56)–(61) provide the electrically charged solutions of the Lovelock theory defined by the parameters d and k coupled to Maxwell electromagnetism, with $d \geq 4$. The dimension $d = 3$ yields singular solutions in the charged sector, and so we do not consider this dimension in this work, although in the uncharged sector it yields perfectly sensible solutions [19].

Now, given a spacetime solution one should search for horizons and singularities. For $\chi = 1$ the solution has a black hole character and can have horizons and singularities, i.e., the solutions represent black holes when the parameters are appropriately chosen. For $\chi = -1$ the solution has a naked singularity character, and should have only singularities with no horizons, independently of the choice of the other parameters. First we locate the horizons for the $\chi = 1$ solutions, and then we locate the singularities for both $\chi = \pm 1$. For $\chi = 1$, the zeros of $f(r)$, in the coordinates used, give the horizons. Upon close scrutiny, the properties of these solutions have many similarities with the d-dimensional Reissner-Nördstrom-AdS black holes in pure general relativity. Moreover, this set of black hole solutions reduces to the d-dimensional Reissner-Nördstrom-AdS black holes for $k = 1$, and the charged Born-Infeld and charged Chern-Simons black hole solutions are recovered for $d = 2k + 2$ and $d = 2k + 1$, respectively [19]. For generic values of d and k, in analogy with the Reissner-Nordström geometry, the black hole solutions possess, in general, two horizons located at the roots of $f^2(r)$, one is the event horizon r_+, and the other the Cauchy horizon r_-, with $r_- < r_+$. When the two horizons merge the black hole is extremal as usual. If the solution is overcharged one has a naked singularity for some value of Q in terms of M [20]. Now one finds the location of the singularities, which can be treated together for both characters $\chi = \pm 1$. The scalar curvature is

$$R = \frac{1}{r^{d-2}} \frac{d^2}{dr^2} \left[r^{d-2} \left(g_k(r) - \frac{r^2}{l^2} \right) \right]. \tag{62}$$

For $k = 1$, general relativity with its Reissner-Nördstrom-AdS black holes, one finds that $R = 0$ as it should, thus in this case the singularities are located when the Kretschmann scalar, $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}$, blows up, which is at $r = 0$. For $k > 1$ the Ricci scalar is no more zero, $R \neq 0$. In this case, Eq. (62) has a singularity at the zero of the function $g_k(r)$, and is due to the existence of an electric field. This is a real timelike singularity located at r_e

$$r_e = \left[\frac{\epsilon}{2(d-3)} \frac{Q^2}{(M + (2G_k)^{-1}\delta_{d-2k,1})} \right]^{1/(d-3)}, \qquad k > 1. \tag{63}$$

This singularity can be reached in a finite proper time interval. However, an external observer is protected from it because both horizons cover it, $r_e < r_- < r_+$. Moreover, for even k, regions where $r < r_e$ have a metric with complex functions, so there is no solution in this region. For odd k, the metric can be defined in the region $0 < r < r_e$, where $r = 0$ is also a spacetime timelike singularity. This solution has no horizons, and is defined between two timelike singularities and so is of no interest to us. For all purposes the solutions of interest, for $k > 1$, are defined in the interval $r_e < r < \infty$, and for $k = 1$ (general relativity) in the interval $0 < r < \infty$. The corresponding Carter-Penrose diagrams can be easily sketched.

B. Spherically symmetric thin shell solutions through the Hamiltonian formalism

1. Shell dynamics

a. The two master equations—We now want to find the equations governing the motion of a thin shell in a vacuum background of a given Lovelock theory specified by the dimension d of the spacetime and the parameter k, which gives the highest power in the curvature terms in the Lagrangian of the theory. In order to obtain shell solutions in a vacuum background, as opposed to pure vacuum solutions, we study the complete field equations, Eqs. (49)–(52), and take into consideration the matter term $\mathcal{H}^{(m)}$ and the electric current J^μ in those equations. We know that inside the shell the spacetime solution, with mass M_- say, is obtained by integrating the constraint $\mathcal{H}_\perp = 0$, i.e., the Eq. (50) with $T_{\perp\perp} = 0$, with the vacuum solution given in (57). In the same way, outside the shell the spacetime vacuum solution, with mass M_+ say, is obtained by integrating the constraint $\mathcal{H}_\perp = 0$, i.e., the Eq. (50) again with $T_{\perp\perp} = 0$, with the vacuum solution given in (57). It remains now to integrate the constraint $\mathcal{H}_\perp = 0$ in the neighborhood of the shell, from $R - \epsilon$ to $R + \epsilon$, where ϵ is the infinitesimal thickness of the shell, with $\epsilon \to 0$. That is, we have to impose $\int_{R-\epsilon}^{R+\epsilon} dr \mathcal{H}_\perp = 0$. This integration should be performed in the asymptotic region, outside of the black hole, since there the normal to the $t =$ constant hypersurfaces is timelike, and thus the Hamiltonian formalism is directly applicable. In addition, one has to fix from the start the same value of the character χ on both sides of the shell (one has to choose either $\chi = 1$ or $\chi = -1$), i.e., one has to choose which of the two types of spacetime one is working with. Using then Eq. (50) we obtain

$$\int_{R-\epsilon}^{R+\epsilon} dr \left(-\kappa \frac{(d-2)!}{r^{d-2}} \frac{d}{dr} \left\{ r^{d-1} \left[F + \frac{1}{l^2} \right]^k \right\} - \frac{p^2}{2\Omega_{d-2}} - T_{\perp\perp} \right) = 0, \tag{64}$$

where $T_{\perp\perp}$ is given in (45). Knowing (3) and (5), using (55), and defining the energy-density content of the shell $m \equiv \sigma\Omega_{d-2}R^{d-2}$, the integration yields

$$\frac{1}{2} m(\gamma_+ + \gamma_-) = (M_+ - M_-) - \frac{\epsilon(Q_+^2 - Q_-^2)}{2(d-3)} \frac{1}{R^{d-3}}, \tag{65}$$

where following Eq. (40)

$$\gamma_+ \equiv \sqrt{f_+^2 + \dot{R}^2} \quad \text{and} \quad \gamma_- \equiv \sqrt{f_-^2 + \dot{R}^2}, \tag{66}$$

$\gamma_\pm$ being the generalized Lorentz factor, and where the different indices $+$ and $-$ stem from the fact that the integration is made both in the $V^{(-)}$ (from $R - \epsilon$ to R) and $V^{(+)}$ (from R to $R + \epsilon$) spacetimes. Note that when integrating Eq. (64), we have used the fact that the radial electric field is zero inside a uniformly charged sphere, due to Gauss' theorem, and that outside it is $(Q_+ - Q_-)/(r^2 + \epsilon^2)$, where ϵ denotes the distance from the surface on the outside to the point of measurement, r denotes the radius of the sphere, and $f^{-2}(r)$ is continuous in the domain of integration. When there are black holes this domain should be exterior to the horizon of the black hole solution, where the solutions are static. Then, the integration in r of the $\mathcal{H}_\perp^{(e)}$ between the limits $R - \epsilon$ and $R + \epsilon$ is, in the limit of $\epsilon \to 0$, equal to zero. Thus, the electric field does not contribute to the shell equation through the radial integration of the Hamiltonian constraint. It is sometimes useful to write Eq. (65) in another way, namely, to multiply both sides by $\gamma_- - \gamma_+$. This yields the following equivalent equation

$$m\chi(g_{k+}(R) - g_{k-}(R)) = [g_{k_+}(R)^k - g_{k-}(R)^k] \times R^{d-2k-1}(\gamma_- - \gamma_+). \quad (67)$$

Equation (65), and its equivalent (67), were deduced for the asymptotic region, where the vacuum spacetime is static, but in the case a black hole is present or being formed, with some care, those equations can be extended, by continuity, to the interior black hole region.

We proceed our study by analyzing now the electrodynamic constraint (51). Inside the shell the electric field solution, Q_-/r^{d-2}, is obtained by integrating the constraint $E_\varphi = 0$, i.e., Eq. (51) with $j^0 = 0$, yielding the vacuum solution (58). In the same way, outside the shell the electric field solution, Q_+/r^{d-2}, is obtained by integrating the constraint $E_\varphi = 0$, i.e., Eq. (51) again with $j^0 = 0$, yielding the vacuum solution (58). It remains now to integrate the constraint E_φ in the neighborhood of the shell, from $R - \epsilon$ to $R + \epsilon$, where ϵ is the infinitesimal thickness of the shell, with $\epsilon \to 0$. That is, we have to impose $\int_{R-\epsilon}^{R+\epsilon} drE_\varphi = 0$. Using then Eq. (51), we obtain

$$\int_{R-\epsilon}^{R+\epsilon} dr \frac{d}{dr}(r^{d-2}p) = \int_{R-\epsilon}^{R+\epsilon} dr\, r^{d-2} j^0, \quad (68)$$

where j^0 is the time component of the vector current, and $j^0 = \sigma_e \delta(r - R)u^0$, where σ_e is the surface charge density. Defining the charge of the shell as $q \equiv \sigma_e \Omega_{d-2} R^{d-2}$ yields

$$Q_+ - Q_- = q. \quad (69)$$

Relation (69) is Gauss' law, and is a trivial consequence of the conservation of charge.

The two master equations are then Eq. (65) (or its equivalent (67)) and Eq. (69). The former can be seen as a dynamic equation for $\dot{R}^2$, the latter is a static equation that simply expresses the conservation of charge. Thus in the rest of our study we concentrate on Eq. (65) (or equivalently on Eq. (67)).

b. Rewriting the equations to simplify the analysis—Equation (65) (or Eq. (67)) was derived for the asymptotic flat region, thus if there is a black hole present it was derived for the region exterior to the event horizon. We should now transform it in order to have it written in a more workable manner. For that we have to square Eq. (65) appropriately. To simplify the notation let us define the right-hand side of Eq. (65) as E, i.e.,

$$E \equiv (M_+ - M_-) - \frac{\epsilon(Q_+^2 - Q_-^2)}{2(d-3)} \frac{1}{R^{d-3}}. \quad (70)$$

Note that from (65) one sees that this quantity E is positive for positive shell mass m (i.e., positive energy density $\sigma > 0$), a condition we always assume throughout. Now we obtain from Eq. (65) a set of two equations which give the dynamics of the shell in detail. First, squaring Eq. (65) and using the definition (70), we get for $\dot{R}^2$ the following expression

$$\dot{R}^2 = \left[\frac{E}{m} - \frac{m}{4E}(f_+^2 - f_-^2)\right]^2 - f_-^2, \quad (71)$$

or, alternatively

$$\dot{R}^2 = \left[\frac{E}{m} + \frac{m}{4E}(f_+^2 - f_-^2)\right]^2 - f_+^2. \quad (72)$$

Second, there is one last step needed for the entire set of relevant equations, in the asymptotic region, to be written down. This has to do with the fact that squaring Eq. (65) yields Eq. (71) (or, alternatively, (72)) making the solutions of (65) also solutions of (71) (or, alternatively, (72)) but not all solutions of the (71) (or, alternatively, (72)) are solutions of (65). So, a sufficient condition is found by putting Eqs. (71) and (72) appropriately back into Eq. (65). One arrives then at the following conditions

$$f_+^2 - f_-^2 \geq -\frac{4}{m^2}E^2, \quad (73)$$

$$f_+^2 - f_-^2 \leq \frac{4}{m^2}E^2. \quad (74)$$

Equations (73) and (74) define implicitly a constraint radius r_c, meaning that, in the asymptotic region, a solution for $R(\tau)$ is a solution of the equation of motion only if it obeys (71) (or, alternatively, (72)) and $R(\tau) > r_c$ for all R. We need now to choose which of the two relations, Eqs. (73) and (74), hold for the system in question. In order to do so, we have to know what signal the difference $f_+^2 - f_-^2$ carries. From (57) one sees that the most general expression is $f_+^2 - f_-^2 = \chi(g_{k-}(r) - g_{k+}(r))$, where $g_k(r)$ comes from (60). The signal on the right-hand side of this equation depends on the difference $\chi(g_{k-}(r) - g_{k+}(r))$ being positive or negative, and this determines which relation in (73) and (74) is the relevant one to use. For example, in the case where the interior is flat, $g_{k-}(r) = 0$, the right-hand side is $-\chi g_{k+}(r)$. In this case when $-\chi g_{k+}(r) < 0$ the relation in (73) is the relevant one, when $-\chi g_{k+}(r) > 0$ the relation in (74) is the appropriate one. (Note that for pure vacuum, no shell, $-\chi g_{k+}(r) > 0$, represents a solution with naked singularity character, while $-\chi g_{k+}(r) < 0$ represents a solution with black hole character). In brief, Eq. (73) defines a constraint radius r_c which is of use in the asymptotically exterior region. So, in the asymptotically exterior region, Eqs. (71) (or (72)) and (73) and (74) recover the same solutions as those of the shell equation (65), with (73) and (74) being a constraint equation. With this set of equations, containing no square roots, one can analyze the spacetime evolution of the shell, in the asymptotic region, outside the black hole.

It is also useful to derive the acceleration of the shell radius. Differentiating equations (71) and (72) with respect to the proper time of the shell τ, we get

$$m\ddot{R} = \frac{(Q_+^2 - Q_-^2)\gamma_+\gamma_- m}{2ER^{d-2}} + (d-2)R^{d-3}\Omega_{d-2}P\gamma_+\gamma_- - \frac{m^2}{4E}\times\left(\gamma_-\frac{df_+^2}{dR} + \gamma_+\frac{df_-^2}{dR}\right), \quad (75)$$

where we have used the conservation equation (42) to arrive at the result $\frac{dm}{d\tau} = -(d-2)R^{d-3}P\dot{R}$, which was used in (75).

2. Analysis of two cases: matter in equilibrium and gravitational collapse

We can now apply the above equations of motion to a variety of physical situations. We choose two interesting cases, namely, static matter in equilibrium and gravitational collapse of matter.

a. Matter in equilibrium—To find a solution for a static shell we have to determine the radii where both the velocity of the shell and its acceleration are null, i.e., $\dot{R} = 0$ and $\ddot{R} = 0$, in the Eqs. (71) (or, alternatively, (72)) and (75), respectively. Forcing $\dot{R} = 0$ in (71) (or, alternatively, (72)) yields

$$R_0^{d-3}\left[\frac{1}{2}m(\sqrt{f_+^2} + \sqrt{f_-^2}) - (M_+ - M_-)\right] = \frac{\epsilon(Q_+^2 - Q_-^2)}{2(d-3)}, \quad (76)$$

where R_0 is the radius at which the shell is static, and f_+^2 and f_-^2 are evaluated at R_0. To be in equilibrium we have to impose, in addition, $\ddot{R} = 0$, which when combined simultaneously with (76) gives the pressure necessary to hold the thin shell in equilibrium. This pressure is then given by

$$P = -\frac{1}{(d-2)R_0^{d-3}\Omega_{d-2}\gamma_+\gamma_-}\left[\frac{(Q_+^2 - Q_-^2)\gamma_+\gamma_- m}{2ER_0^{d-2}} - \frac{m^2}{4E}\left(\gamma_-\frac{df_+^2}{dR}\bigg|_{R_0} + \gamma_+\frac{df_-^2}{dR}\bigg|_{R_0}\right)\right], \quad (77)$$

where the functions are evaluated at R_0, given by (76). Thus, we obtain the pressure in terms of the parameters of the problem, such as d, k, m, M_+, M_-, Q_+, Q_-, and the radius of the static shell R_0, for a static configuration. As a particular simple case we may apply the above expressions to find the pressure necessary to hold the shell in static equilibrium in general relativity ($k = 1$, $\chi = 1$), without charge, and for zero cosmological constant and flat interior. This gives

$$P = \frac{(d-3)}{2(d-2)}\frac{Gm^2}{\Omega_{d-2}R_0^{d-2}(R_0^{d-3} - m)}, \quad (78)$$

where R_0 is the radius of the static shell, given by $R_0 = (Gm^2/[2(m-M)])^{1/(d-3)}$, and G is the Newton's constant for $k = 1$, i.e., we have done $G_{k=1} \equiv G$, whatever the dimension d. In four dimensions, $d = 4$, Eq. (78) reduces to $P = (Gm^2)/(16\pi R_0^2(R_0 - m))$, with $R_0 = Gm^2/[2(m-M)]$, confirming the result given in [27].

b. Gravitational collapse—In this analysis of gravitational collapse in Lovelock gravity coupled to Maxwell electromagnetism, we first study a simple example of charged dust matter collapsing into an empty interior and then prove cosmic censorship in the generic case of shell collapse into an interior free of naked singularities. An analysis of gravitational expansion, not worked out here, follows straightforwardly by performing a time reversal operation on the collapsing solutions.

i. Gravitational collapse of dust matter in an empty interior:—In order to have an idea of the main features of gravitational collapse in Lovelock gravity coupled to Maxwell electromagnetism, rather than analyzing the full problem, which can be daunting since there are a great number of parameters to play with, we investigate the simplest problem, namely, of gravitational collapse of charged dust matter into an empty interior. Even then, in this case the task of studying it in full detail is enormous, since one has first to set the dimension of the spacetime, the dimension of the k parameter, and then all the other parameters. So we resort to study it somewhat generically, and then give some particular examples in the diverse theories that we chose to study. The aim is to show that gravitational collapse is possible in this subset of theories of Lovelock gravity, reinforcing some similarities which this has with general relativity, as well as showing some new features. So, in the following we analyze the case where the matter is composed of dust particles with $P = 0$, in which case m is a constant and can be identified with the shell's rest mass, and where the interior is empty, in which case the function $f_-(R)$ is given through $f_-^2(R) = R^2/l^2 + 1$.

Before proceeding, it is necessary to emphasize that the Lovelock theory we are analyzing is separable into two branches: (i) the $d - 2k - 1 > 0$, which can have even and odd dimensions, and includes general relativity when $k = 1$ (any d), includes the Born-Infeld case of the dimensionally continued theory when $k = [\frac{d-1}{2}]$, and includes other generic cases, and (ii) the $d - 2k - 1 = 0$, which can only have odd dimensions and is precisely the Chern-Simons case of the dimensionally continued theory. We study each branch in turn.

The branch $d - 2k - 1 > 0$ (general relativity when $k = 1$ (any d), Born-Infeld when $k = [\frac{d-1}{2}]$, and other generic cases):

The $d - 2k - 1 > 0$ branch allows theories in even and odd dimensions. For instance in $d = 6$ one can have $k = 1$ (general relativity) and $k = 2$ (general relativity with a generalized Gauss-Bonnet term) theories, which in the case $k = 2$ gives a Born-Infeld theory. On the other hand, in $d = 7$ one can also have theories with $k = 1$ (general relativity) and $k = 2$ (general relativity with a generalized Gauss-Bonnet term), but none of these is a Born-Infeld theory. Indeed, as already alluded to in Sec. II, the Born-Infeld case is the realization of this Lovelock theory in even dimensions, for which $k = [\frac{d-1}{2}]$.

Considering thus an empty interior, with $M_- = 0$ and $Q_- = 0$, one has $f_-^2(R) = R^2/l^2 + 1$. Putting then $M_+ = M$, $Q_+ = Q$, and $\chi = (\pm 1)^{k+1}$ (see (61)) one finds from Eqs. (71) and (72) that the shell equation for $d - 2k - 1 > 0$ is,

$$\dot{R}^2 = \Bigg(\frac{M - \frac{\epsilon Q^2}{2(d-3)R^{d-3}}}{m} + \frac{m}{4(M - \frac{\epsilon Q^2}{2(d-3)R^{d-3}})} \times \chi\bigg(\frac{2G_k}{R^{d-2k-1}}\bigg)^{1/k}\bigg(M - \frac{\epsilon}{2(d-3)}\frac{Q^2}{R^{d-3}}\bigg)^{1/k}\Bigg)^2 - \bigg(1 + \frac{R^2}{l^2}\bigg). \tag{79}$$

In order to have physical solutions, we have to demand $\dot{R}^2 > 0$.

The equation of the acceleration of the thin shell (75), allows us to understand the forces acting on the shell in collapse. Expanding the $\gamma_\pm$ defined in (66), with $\dot{R}^2$ replaced by the right-hand side of (71) or (72), appropriately chosen, we have

$$\gamma_\pm = \Bigg|\frac{M - \frac{\epsilon Q^2}{2(d-3)R^{d-3}}}{m} \pm \frac{m}{4(M - \frac{\epsilon Q^2}{2(d-3)R^{d-3}})} \times \chi\bigg(\frac{2G_k}{R^{d-2k-1}}\bigg)^{1/k}\bigg(M - \frac{\epsilon}{2(d-3)}\frac{Q^2}{R^{d-3}}\bigg)^{1/k}\Bigg|. \tag{80}$$

Then Eq. (75) for an empty interior turns into

$$\ddot{R} = -\frac{2R}{l^2} + \frac{Q^2\gamma_+\gamma_-}{2ER^{d-2}} - \chi\frac{m}{2E} \times \bigg(\gamma_- \frac{1}{k}\bigg(\frac{2G_kM}{R^{d-2k-1}} - \frac{\epsilon G_k}{d-3}\frac{Q^2}{R^{2(d-k-2)}}\bigg)^{(1-k)/k} \times \bigg(\frac{2(d-2k-1)G_kM}{R^{d-2k}} - \frac{2(d-k-2)\epsilon G_k}{d-3} \times \frac{Q^2}{R^{2(d-k)-3}}\bigg)\bigg), \tag{81}$$

where the functions $\gamma_\pm$ are given in (80), and we have used Eq. (65). The first term on the right-hand side of Eq. (81) is proportional to the radius of the thin shell, meaning that this term dominates the acceleration for large values of R, it tends to $-\infty$ as $R \to \infty$ dominating all the others, and since the term is negative it points towards the center. The term proportional to the square of the electric charge Q^2 includes a $\dot{R}^2$ term and a $|\dot{R}|$ term (due to the product $\gamma_-\gamma_+$). This means that the charge term is positive for a thin shell, works to push the shell to higher radii, and it tends to zero as $R \to \infty$. The third term has several terms included, like gravitational and viscosity forces per unit mass terms. A simple example where the terms in Eq. (81) are drastically reduced is when we put $d = 4$, $k = 1$, $l^2 \to \infty$, $Q = 0$, i.e., the case of an uncharged shell collapse in usual general relativity in four dimensions. Then we get for the acceleration of the shell, $\ddot{R} = -M/R^2 + m^2/(4R^3)$. The term $m^2/(4R^3)$ is a correction to the Newtonian dynamics, representing a self-gravitational potential energy.

Since, in general, the character χ has two values, $\chi = \pm 1$, for the branch we are analyzing, we study separately both cases. Note first that for $\chi = 1$ one has from (57) that the shell spacetime solution has a black hole character (i.e., for certain choices of the parameters one has a black hole solution), whereas for $\chi = -1$ one has from (57) that the shell spacetime solution has a naked singularity character (i.e., there are no possible choices of parameters that give a black hole). So, since when k is odd (such as in general relativity where $k = 1$) the character $\chi = 1$ for sure, odd k allows black hole character solutions only, and since when k is even (such as in general relativity with a Gauss-Bonnet term where $k = 2$) the character χ can have both values ± 1, even k can have both black hole and naked singularity character solutions.

For $\chi = 1$ the spacetime has a black hole character. When the shell collapses it will pass through its horizon radius, defined by the equation $f_+^2(r_h) = 0$. So for $\chi = 1$ shell collapse implies the formation of a black hole. To see this more clearly, note that from Eq. (57) the relation $f_+^2(r_h) = 0$ defining the horizon turns into $(1 + r_h^2/l^2) = (2G_k/r_h^{d-2k-1})^{1/k}(M - (\epsilon Q^2)/(2(d-3)r_h^{d-3}))^{1/k}$. One can show by inspection that the horizon radius is always larger than the turning radius r_t, $r_h > r_t$. So in this type of Lovelock gravity the bounce is always inside the event horizon, which implies the shell expands into another universe, a result already obtained in pure general relativity [29].

For $\chi = -1$ the spacetime has a naked singularity character. We can now give an argument which shows that cosmic censorship holds in the case of $d - 2k - 1 > 0$, with an empty interior. The spacetime would be singular if the shell would hit r_e, given in (63). Through the shell equation, Eq. (65), we see that $M - (\epsilon Q^2)/(2(d-3)R^{d-3}) > 0$. However, the singularity at r_e makes the right-hand side of the shell equation (65) zero. This shows that the radius of this singularity is out of the region of validity of the shell equation, preventing the collapse of the charged shell to form a naked singularity, and validating here the cosmic censorship hypothesis. Note that this is only applicable to k even.

In Fig. 2 the branch $d - 2k - 1 > 0$ (general relativity when $k = 1$, Born-Infeld when $k = [\frac{d-1}{2}]$, and other generic cases) is studied in some instances. In each plot the effective potential $\dot{R}^2$ as a function of R (which gives the turning points and the allowed region for the shell motion),

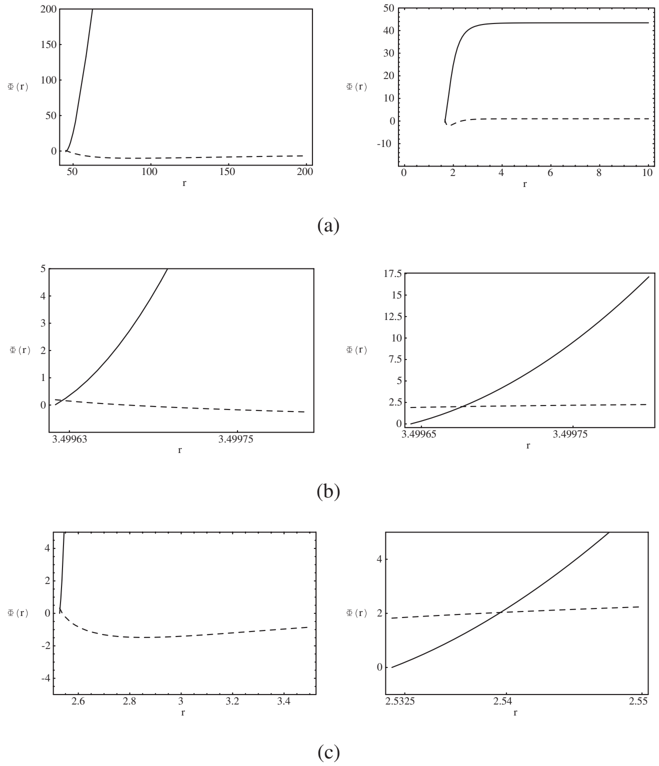

FIG. 2. In this figure the branch $d - 2k - 1 > 0$ (general relativity when $k = 1$, Born-Infeld when $k = [\frac{d-1}{2}]$, and other generic cases) is studied in some particular instances. In each plot the effective potential $\dot{R}^2$ as a function of r (which gives the turning points and the allowed regions for the shell), and the metric function $f^2(r)$ as a function of r (which gives the horizon formation, for several different values of the parameters) are displayed. The vertical axis $\Phi(r)$ represents both $\dot{R}^2(r)$ (full line) and $f_+^2(r)$ (dashed line). Note that the thin shell trajectory is allowed only in the region where $\dot{R}^2$ is positive. In all the plots the respective Newton's constant is put equal to the unity, $G_k = 1$, so that radii and energies are measured in the respective Planck units ($\hbar = 1$, $c = 1$). (a) General relativity, $k = 1$, left plot is for $d = 4$ and right plot for $d = 10$, with $\chi = 1$ in both plots ($\chi = -1$ is not possible for k odd), (b) Born-Infeld, $k = [\frac{d-1}{2}]$, here $k = 4$ and $d = 10$, left plot is for $\chi = 1$ and right plot for $\chi = -1$, (c) General relativity with a generalized Gauss-Bonnet term, $k = 2$, a particular case of $1 < k < [\frac{d-1}{2}]$, here $d = 10$, left plot is for $\chi = 1$ and right plot for $\chi = -1$. See text for details.

and the metric function $f^2(R)$ as a function of R (which gives the horizon formation), for several different values of the parameters, are shown. The vertical axis $\Phi(r)$ in the figure represents both $\dot{R}^2(r)$ (full line) and $f_+^2(r)$ (dashed line). The thin shell trajectory is allowed only in the region where $\dot{R}^2$ is positive. In all the plots we have made the respective Newton's constant equal to the unity, $G_k = 1$, so that radii and energies are measured in the respective Planck units ($\hbar = 1$, $c = 1$). We have not plotted $\ddot{R}$ since it yields a totally different scale in the vertical axis, rendering the other two functions, $\dot{R}^2$ and $f^2(R)$, almost invisible. We now look at each choice of plot in turn. (a) General relativity, $k = 1$. Left plot: $d = 4$, $l = \infty$, $M = 1000$, $Q = 300$, $m = 20$. Right plot: $d = 10$, $l = \infty$, $M = 200$, $Q = 300$, $m = 30$ (since k is odd, one has $\chi = 1$ for both plots). More specifically, the left plot is the usual general relativity with $d = 4$ (which incidentally is also a Born-Infeld type theory), and the right plot is general relativity with $d = 10$. There is always collapse to a black hole, a result confirmed by both plots. Indeed, since $\dot{R}^2$ is nonzero at the horizon, where $f_+^2(r) = 0$, the shell passes smoothly through the horizon itself. So there is no bounce outside the horizon, the bounce being inside the horizon, and into another universe. For example, in $d = 4$ the value of the radius of the bounce, or turning point, inside the horizon is given by $r_t = (m^2 - Q^2)/(2(m - M)) = 45.7$ in the units defined above. This agrees with the expressions given in [29], where using Israel's formalism [21], as opposed to the Hamiltonian methods used here, a full study of the equation of motion and of the turning points in d-dimensional general relativity was performed. (b) Born-Infeld, $k = [\frac{d-1}{2}]$. Left plot: $d = 10$, $k = 4$, $\chi = 1$, $l = \infty$, $M = 10\,000$, $Q = 30\,000$, $m = 1$. Right plot: $d = 10$, $k = 4$, $\chi = -1$, $l = \infty$, $M = 10\,000$, $Q = 30\,000$, $m = 1$. More specifically, the left plot is for $\chi = 1$, and we find there is always collapse to a black hole, since $\dot{R}^2$ is nonzero at the horizon, where $f_+^2(r) = 0$, and the shell passes smoothly through the horizon itself. There is a bounce inside the horizon into another universe. The right plot is for $\chi = -1$, with the same choices of the other parameters as for $\chi = 1$. In this case the external spacetime has a naked singularity character. Here there is no horizon because the metric function $f_+^2(r)$ has no zero. There is a zero of $\dot{R}^2$, which limits the region $\dot{R}^2 > 0$, the region where the shell has its trajectory. The zero is larger than the radius at which there is a singularity. Thus the collapse never forms a naked singularity. (c) General relativity with a generalized Gauss-Bonnet term, $k = 2$, a particular case of $1 < k < [\frac{d-1}{2}]$ (here the intermediate cases, ($1 < k < [\frac{d-1}{2}]$), where k is neither minimum (general relativity), nor maximum (Born-Infeld), are studied). Left plot: $d = 10$, $k = 2$, $\chi = 1$, $l = \infty$, $M = 1000$, $Q = 300$, $m = 20$. Right plot: $d = 10$, $k = 2$, $\chi = -1$, $l = \infty$, $M = 1000$, $Q = 300$, $m = 20$. The choice of the coefficients α_p are given in Eq. (3). More specifically, the left plot is for $\chi = 1$, and we see there is always collapse to a black hole, since $\dot{R}^2$ is nonzero at the horizon, where $f_+^2(r) = 0$, and the shell crosses the horizon smoothly. There is also a bounce inside the horizon. The right plot is for $\chi = -1$, where the external spacetime has a naked singularity character. There is no horizon, because there are no zeros of $f_+^2(r)$. The region where $\dot{R}^2 > 0$, where the shell has its trajectory, is limited on the left by the zero of $\dot{R}^2$. The zero of $\dot{R}^2$ is larger than the radius at which there is a singularity. There is thus no collapse to form a naked singularity. In all the figures 2(a)–2(c) we have shown bouncing solutions only, although one could easily find parameters for which the collapsing shell suffers no bounce and goes all the way down to the singularity. Note that a general setup for the study of the turning points and the shell's trajectory in a Carter-Penrose diagram for the causal structure (with a corresponding careful analysis of the normal to the shell along the trajectory), as was done analytically for d-dimensional general relativity in [28,29], could also be performed here case to case, i.e., giving d and k. However, this is beyond the scope of this work.

The branch $d-2k-1=0$ (Chern-Simons):

The $d-2k-1=0$ branch implies d is odd always. Moreover the theory is of Chern-Simons type. The $d-2k-1>0$ branch and the Chern-Simons $d-2k-1=0$ branch, are quite distinct. First, in this latter branch, in this charged setting, there is no general relativity ($k=1$), because the theory is defined only for $d>3$, and thus $k\geq 2$ (however in the uncharged case the theory is well-defined for $d=3$, yielding three dimensional general relativity, see [19]). Second, for the $d-2k-1>0$ branch one has that the vacuum mass is $M_-=0$, whereas for the $d-2k-1=0$ Chern-Simons branch the vacuum mass is $M_-=-(2G_k)^{-1}$. From (57) and (60) one finds that the interior for the Chern-Simons theory with $M_-=-(2G_k)^{-1}$ is characterized by $f_-^2(R)=R^2/l^2+1$ also.

From Eq. (65) the shell equation for $d-2k-1=0$, $M_-=-(2G_k)^{-1}$, $M_+=M$, $Q_-=0$, $Q_+=Q$, and $\chi=(\pm 1)^{k+1}$ is

$$\dot{R}^2=\Bigg(\frac{M+(2G_k)^{-1}-\frac{\epsilon Q^2}{2(d-3)R^{d-3}}}{m}+\frac{m}{4(M+(2G_k)^{-1}-\frac{\epsilon Q^2}{2(d-3)R^{d-3}})}\chi(2G_k)^{1/k}\times\left(M+(2G_k)^{-1}-\frac{\epsilon}{2(d-3)}\frac{Q^2}{R^{d-3}}\right)^{1/k}\Bigg)^2-\left(1+\frac{R^2}{l^2}\right). \tag{82}$$

In order to have physical solutions, we have to demand $\dot{R}^2>0$.

Since, in general, $\chi=\pm 1$, we study separately both cases. Again note first that for $\chi=1$ one has from (57) that the shell spacetime solution has a black hole character, whereas for $\chi=-1$ one has from (57) that the shell spacetime solution has a naked singularity character. So, since when k is even (such as in general relativity with a generalized Gauss-Bonnet term, where $k=2$ and $d=5$) χ can have both values ± 1, even k can have both black hole and naked singularity character solutions, and since when k is odd (such as $k=3$, and $d=7$) $\chi=1$ for sure, odd k allows solutions of black hole character only.

For $\chi=1$ (so k can be odd or even) the horizon is located at the radius r_h given implicitly by the equation (see (57)) $(1+r_h^2/l^2)=(2G_k)^{1/k}(M+(2G_k)^{-1}-(\epsilon Q^2)/(2(d-3)r_h^{d-3}))^{1/k}$. When the parameters m, M, Q, l yield a solution for the collapsing shell then the shell will pass through its own event horizon at r_h and form a black hole.

For $\chi=-1$ (so k can be even only), the shell solution has a naked singularity character. We can again give an argument to show that cosmic censorship in the particular case of Chern-Simons ($d-2k-1=0$) with an empty interior holds. Indeed, from Eq. (65) one finds that the following relation holds $M+(2G_k)^{-1}-(\epsilon Q^2)/(2(d-3)R^{d-3})>0$. Noting that r_e, given in (63), makes the right-hand side zero, this means that this singularity is unattainable through gravitational collapse. Thus cosmic censorship holds in charged collapse in a Chern-Simons type theory. This is in contrast to uncharged collapse in the same theory where naked singularities can form [26].

Finally we write the equation of the acceleration of the thin shell, which allows us to understand the forces acting on the collapsing shell. Expanding the $\gamma_\pm$ defined in (66), with $\dot{R}^2$ replaced by the right-hand side of (71) or (72), appropriately chosen, we have

$$\gamma_\pm=\left|\frac{M+(2G_k)^{-1}-\frac{\epsilon Q^2}{2(d-3)R^{d-3}}}{m}\pm\frac{m}{4(M+(2G_k)^{-1}-\frac{\epsilon Q^2}{2(d-3)R^{d-3}})}\times\chi(2G_k)^{1/k}\left(M+(2G_k)^{-1}-\frac{\epsilon}{2(d-3)}\frac{Q^2}{R^{d-3}}\right)^{1/k}\right|. \tag{83}$$

As before, the constraint radius marks the lower bound of validity of the shell's equations. Then the Eq. (75) for an empty interior turns into

$$\begin{aligned}\ddot{R}&=\frac{Q^2\gamma_+\gamma_-}{2ER^{d-2}}-\frac{m}{2E}\times\left(\gamma_-\left[\frac{2R}{l^2}-\chi\frac{2}{d-1}\left(2G_kM+1-\frac{\epsilon G_k}{d-3}\frac{Q^2}{R^{d-3}}\right)^{-(d-3)/(d-1)}\left(\epsilon G_k\frac{Q^2}{R^{d-2}}\right)\right]+\gamma_+\frac{2R}{l^2}\right)\\&=-\frac{2R}{l^2}+\frac{Q^2\gamma_+\gamma_-}{2ER^{d-2}}+\chi\frac{m}{2E}\times\left(\gamma_-\frac{2}{d-1}\left(2G_kM+1-\frac{\epsilon G_k}{d-3}\frac{Q^2}{R^{d-3}}\right)^{-(d-3)/(d-1)}\left(\epsilon G_k\frac{Q^2}{R^{d-2}}\right)\right),\end{aligned} \tag{84}$$

where the functions $\gamma_\pm$ are given in (83), and we used Eq. (65) in the second equality. For uncharged collapse $Q=0$ the acceleration is always negative, as can be easily checked from Eq. (84). In the charged case $Q\neq 0$, there is no definite sign on the right-hand side of Eq. (84), it depends on the radius at which the shell is located. Thus, reexpansion, after a collapsing phase, is allowed.

In Fig. 3 we show the effective potential $\dot{R}^2$ as a function of R (which gives the turning points and the allowed regions for the shell), and the metric function $f^2(R)$ as a function of R (which gives the horizon formation), for several different values of the parameters. The vertical axis $\Phi(r)$ in the figure represents both $\dot{R}^2(r)$ (full line) and $f_+^2(r)$ (dashed line). The thin shell trajectory is allowed only in the region where $\dot{R}^2$ is positive. In all the plots we have made the respective Newton's constant equal to the unity, $G_k = 1$, so that radii, energies, and forces are measured in the respective Planck units ($\hbar = 1$, $c = 1$). Left plot: $d = 5$, $k = 2$, $\chi = 1$, $l = 50$, $M = 20$, $Q = 610$, $m = 5$. Right plot: $d = 5$, $k = 2$, $\chi = -1$, $l = 50$, $M = 20$, $Q = 300$, $m = 5$. More specifically, the left plot shows a region limited by two zeros of $\dot{R}^2$, the region where a trajectory of a thin shell is possible. The horizon, given by the larger zero of $f^2(R)$, is inside the region where $\dot{R}^2 > 0$, which means that the shell passes smoothly through the horizon. The shell then suffers a bounce inside the horizon into another universe. The plot on the right shows a naked singularity character exterior spacetime, which means that there is no zero of $f^2(R)$, hence no horizon. There is, however, no collapse to a naked singularity, because the region where $\dot{R}^2 > 0$ is limited by two zeros, the smaller being larger than the constraint radius. Thus the shell bounces between two extreme values of R, and so does not collapse at all. In the figure we have shown bouncing solutions only, although as before one could produce totally collapsing solutions. As in the $d - 2k - 1 > 0$ branch, a general setup for the study of the turning points and the shell's trajectory in a Carter-Penrose diagram for the causal structure could be performed here, but this will not be done.

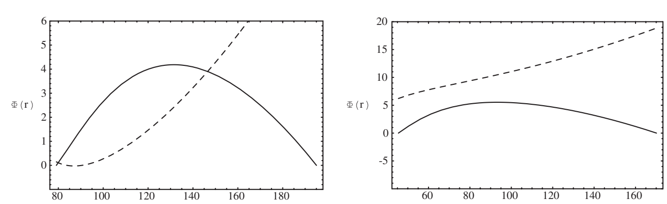

FIG. 3. In this figure the branch $d - 2k - 1 = 0$ (Chern-Simons, d always odd) is studied in some particular instances. In each plot the effective potential $\dot{R}^2$ as a function of r (which gives the turning points and the allowed regions for the shell), and the metric function $f^2(r)$ as a function of r (which gives the horizon formation, for several different values of the parameters) is displayed. The vertical axis $\Phi(r)$ represents both $\dot{R}^2(r)$ (full line) and $f_+^2(r)$ (dashed line). Note that the thin shell trajectory is allowed only in the region where $\dot{R}^2$ is positive. In the plots the respective Newton's constant is put equal to the unity, $G_k = 1$, so that radii and energies are measured in the respective Planck units ($\hbar = 1$, $c = 1$). Left plot is for $k = 2$, $d = 5$, and $\chi = 1$, and right plot is for $k = 2$, $d = 5$, and $\chi = -1$. See text for details.

ii. Cosmic censorship:—Given the experience we have acquired with the above examples, one can now study cosmic censorship directly from Eq. (65) or (67). Cosmic censorship holds if no naked singularity forms from gravitational collapse in an initially nonsingular spacetime or if no naked singularity forms from gravitational collapse in a spacetime initially containing a black hole. We assume $Q_+ \neq 0$ and $Q_- \neq 0$. There are three such cases in our study: first, both spacetime regions on each side of the collapsing shell have character $\chi = 1$, and the interior solution is a black hole spacetime; second, both spacetime regions on each side of the collapsing shell have character $\chi = 1$, and the interior solution is an empty vacuum spacetime; and third, both spacetime regions on each side of the collapsing shell have character $\chi = -1$, and the interior solution is an empty vacuum spacetime. On all these cases cosmic censorship holds. For the first case, that both spacetime regions on each side of the collapsing shell have character $\chi = 1$, and the interior solution is a black hole spacetime, given the above dynamic equation one can directly prove that the collapse of a charged shell in such a background never yields a naked singularity spacetime. To start with we have a shell collapsing into an interior black hole spacetime. This means that the χ term on the left of (67) is positive (otherwise the interior would not be that of a black hole spacetime). From Eq. (65) it is known that $g_{k+}(R)^k - g_{k-}(R)^k > 0$, because this expression is the right-hand side of (65). If then the shell is collapsing onto an existing black hole, and it is such that the result would be a naked singularity, i.e., the collapsing shell is sufficiently overcharged, then $f_-^2(r_h) = 0$ for a certain r_h, and $f_+^2(r)$ would always be larger than zero (because in the exterior spacetime there would be no horizon). It is then clear that the term $(\gamma_- - \gamma_+)$ is negative if the shell reaches r_h. However, as we are working in the asymptotically outside region, then $g_k(r) > 0$. This means that the signal of $g_k(r)$ is the same as that of $g_k(r)^k$, and so the signal of $g_{k+}(R)^k - g_{k-}(R)^k$ is the same as that of $g_{k+}(R) - g_{k-}(R)$. This results in the fact that the left-hand side of (67) is positive at the horizon of the inner black hole, defined by $f_-^2(r_h) = 0$, and that the right-hand side of the same Eq. (67) is negative, which implies a contradiction. For the second case, that both spacetime regions on each side of the collapsing shell have character $\chi = 1$, and the interior solution is an empty vacuum spacetime, one can treat it as a limiting case of the first case, as the mass of the interior black hole goes to zero, or directly using similar arguments as above, with the result that a collapsing shell, if undercharged forms a black hole, if overcharged has a bounce back from where it came. For the third case, that both spacetime regions on each side of the collapsing shell have character $\chi = -1$, and the interior solution is an empty vacuum spacetime, one can write

Eq. (67) as $m = -g_{k+}(R)^{k-1}R^{d-2k-1}(\gamma_- - \gamma_+)$. One sees that when the shell approaches r_e, its right-hand side approaches zero, whereas the left-hand side approaches a finite value. So there is a bounce and the shell never collapses to a singularity. These three cases prove the result, and yield cosmic censorship in charged backgrounds in the Lovelock theory we are studying coupled to Maxwell electromagnetism. We now briefly comment on the uncharged case $Q_+ = Q_- = 0$. One can prove that in this particular case, in the $d - 2k - 1 > 0$ branch one has no formation of naked singularities, whereas in the $d - 2k - 1 = 0$ uncharged branch there is formation of naked singularities, so that one can say that electrical charge acts really as a cosmic censor.

Thus, a charged shell never develops a naked singularity. This is rather like having a shell with some angular momentum, a situation which is much harder to study directly in theories with $d > 3$. Nonetheless, the special case of an uncharged collapsing shell in $d = 3$ with angular momentum was studied in [24,25], with the result that indeed the angular momentum also prevents the formation of a naked singularity.

IV. CONCLUSIONS AND PHYSICAL IMPLICATIONS

There are two main conclusions from this work. One conclusion is that the Hamiltonian formalism is a powerful method to treat in a unified way spacetimes composed of several pieces, such as several vacua and thin shells, in theories much more complicated than general relativity such as the subset of theories derived from Lovelock gravity coupled to Maxwell electromagnetism we have studied. The other conclusion is that, when the spacetimes in question have the same character of those spacetimes provided by general relativity ($\chi = 1$), the collapse of the thin shells in the backgrounds, black hole or otherwise, of each different type of Lovelock theory is in many ways similar to the collapse in general relativity itself, and when the spacetimes in question have the opposite character ($\chi = -1$), some other new features appear. This in turn has the following physical implications: if indeed there are extra dimensions with a relative large size, as exposed in the introduction, then the new and the old features, when confronted with experimental data, can provide the signature to the uncovering not only of the actual spacetime dimension d, but also of the value of the parameter k, i.e., of which particular Lovelock gravity nature picks up at the appropriate scales, whether be it general relativity ($k = 1$, any d), Born-Infeld ($k =$ maximum, even d), Chern-Simons ($k =$ maximum, odd d), or other generic gravity (other k, any d). Of course, a full quantum treatment, or even a semiclassical approximation, would be much more appropriate for this kind of questions, but for Lovelock type gravities the technical difficulties are exponentiated easily, and thus it is advisable to start up with a classical analysis, as we did here.

ACKNOWLEDGMENTS

This work was partially funded by Fundação para a Ciência e a Tecnologia (FCT) of the Ministry of Science, Portugal, through Project No. POCTI/FIS/57552/2004. G. A. S. D. is supported by Grant No. SFRH/BD/2003 from FCT. S. G. was supported by NSFC Grants No. 10605006 and No. 10373003 and the Scientific Research Foundation for the Returned Overseas Chinese Scholars, State Education Ministry. J. P. S. L. thanks Observatório Nacional do Rio de Janeiro for hospitality.

[1] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998).
[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Rev. D **59**, 086004 (1999).
[3] M. Cavaglia, Int. J. Mod. Phys. A **18**, 1843 (2003).
[4] P. Kanti, Int. J. Mod. Phys. A **19**, 4899 (2004).
[5] V. Cardoso, M. Cavaglia, and L. Gualtieri, Phys. Rev. Lett. **96**, 071301 (2006).
[6] C. P. Burgess, Living Rev. Relativity **7**, 5 (2004).
[7] D. Lovelock, J. Math. Phys. (N.Y.) **12**, 498 (1971).
[8] C. Teitelboim and J. Zanelli, *Constraint Theory and Relativistic Dynamics*, edited by G. Longhi and L. Lusanna (World Scientific, Singapore, 1987).
[9] C. Teitelboim and J. Zanelli, Classical Quantum Gravity **4**, L125 (1987).
[10] M. Henneaux, C. Teitelboim, and J. Zanelli, Phys. Rev. A **36**, 4417 (1987).
[11] C. Lanczos, Ann. Math. **39**, 842 (1938).
[12] B. Zwiebach, Phys. Lett. B **156**, 315 (1985).
[13] B. Zumino, Phys. Rep. **137**, 109 (1986).
[14] J. T. Wheeler, Nucl. Phys. **B273**, 732 (1986).
[15] D. G. Boulware and S. Deser, Phys. Rev. Lett. **55**, 2656 (1985).
[16] B. Whitt, Phys. Rev. D **38**, 3000 (1988).
[17] R. C. Myers and J. Simon, Phys. Rev. D **38**, 2434 (1988).
[18] D. L. Wiltshire, Phys. Rev. D **38**, 2445 (1988).
[19] M. Bañados, C. Teitelboim, and J. Zanelli, Phys. Rev. D **49**, 975 (1994).
[20] J. Crisóstomo, R. Troncoso, and J. Zanelli, Phys. Rev. D **62**, 084013 (2000).
[21] W. Israel, Nuovo Cimento B **44**, 1 (1966).
[22] S. C. Davis, Phys. Rev. D **67**, 024030 (2003).
[23] P. Hájíček and J. Kijowski, Phys. Rev. D **57**, 914 (1998).

[24] J. Crisóstomo and R. Olea, Phys. Rev. D **69**, 104023 (2004).
[25] R. Olea, Mod. Phys. Lett. A **20**, 2649 (2005).
[26] J. Crisóstomo, S. del Campo, and J. Saavedra, Phys. Rev. D **70**, 064034 (2004).
[27] K. Kuchař, Czechoslovak Journal of Physics, Section B **18**, 435 (1968).
[28] D. G. Boulware, Phys. Rev. D **8**, 2363 (1973).
[29] S. Gao and J. P. S. Lemos (unpublished).
[30] R. Arnowitt, S. Deser, and C. W. Misner, in *Gravitation: An Introduction to Current Research*, edited by L. Witten (Wiley, New York, 1962); gr-qc/0405109.