

Crossing $w = -1$ by a single scalar on a Dvali-Gabadadze-Porrati brane

Hongsheng Zhang and Zong-Hong Zhu*

Department of Astronomy, Beijing Normal University, Beijing 100875, China

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Recent type Ia supernovae data seem to favor a dark energy model whose equation of state $w(z)$ crosses -1 , which is a much more amazing problem than the acceleration of the universe. Either the case that $w(z)$ evolves from above -1 to below -1 or the case that $w(z)$ runs from below -1 to above -1 , is consistent with present data. In this paper we show that it is possible to realize the crossing behaviors of both of the two cases by only a single scalar field in frame of Dvali-Gabadadze-Porrati braneworld. At the same time we prove that there does not exist scaling solution in a universe with dust.

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I. INTRODUCTION

The existence of dark energy is one of the most significant cosmological discoveries over the last decades [1]. However, the nature of this dark energy remains a mystery. Various models of dark energy have been proposed, such as a small positive cosmological constant, quintessence, k -essence, phantom, holographic dark energy, etc., see [2] for recent reviews with fairly complete list of references of different dark energy models. A cosmological constant is a simple candidate for dark energy. However, following the more accurate data a more dramatic result appears: the recent analysis of the type Ia supernovas data indicates that the time varying dark energy gives a better fit than a cosmological constant, and, in particular, the equation of state (EOS) parameter w (defined as the ratio of pressure to energy density) may cross -1 [3]. The dark energy with $w < -1$ is called phantom dark energy [4], for which all energy conditions are violated. Here, it should be noted that the possibility that the dark energy behaves as phantom today is yet a matter under debate: the observational data, mainly those coming from the type Ia supernovae of high redshift and cosmic microwave background, may lead to different conclusions depending on what samples are selected, and what statistical analysis is applied [5]. By contrast, other researches imply that all classes of dark energy models are comfortably allowed by those very observations [6]. Presently all observations seem to not rule out the possibility of the existence of matter with $w < -1$. Even in a model in which the Newton constant is evolving with respect redshift z , the best fit $w(z)$ crosses the phantom divide $w = -1$ [7]. Hence the phenomenological model for phantom dark energy should be considered seriously. To obtain $w < -1$, scalar field with a negative kinetic term, may be a simplest realization. The model with phantom matter has been investigated extensively [8], and a test of such matters in solar system [9]. However, the EOS of phantom field is always less than -1 and can not cross -1 . It is easy to understand that if we put 2 scalar fields into the model, one

is an ordinary scalar and the other is a phantom: they dominate the universe by turns, under this situation the effective EOS can cross -1 [10]. As we have mentioned that either the case that $w(z)$ evolves from above -1 to below -1 (Case I) or the case that $w(z)$ runs from below -1 to above -1 (Case II) is consistent with present data. The observation data mildly favor dark energy of Case I, but also leave enough space for a dark energy of Case II [11]. It is worthy to point out that there exists some interacting models, in which the effective EOS of dark energy crosses -1 [12]. More recently it has been found that crossing -1 within one scalar field model is possible, the cost is the action contains higher derivative terms [13] (see also [14]). Also it is found that such a crossing can be realized without introducing ordinary scalar or phantom component in a Gauss-Bonnet brane world with induced gravity, where a four-dimensional curvature scalar on the brane and a five dimensional Gauss-Bonnet term in the bulk are present [15]. Other two related works on phantom divide crossing can be found in [16].

In this paper we suggest a possibility with effective EOS crossing -1 in brane world scenario with only a single scalar field. The brane world scenario is now one of the most important ideas in high energy physics and cosmology. In this scenario, the standard model particles are confined to the 3-brane, while the gravitation can propagate in the whole spacetime. As for cosmology in the brane world scenario, many works have been done over the last several years; for a review, see [17] and references therein. Brane world models admit a much wider range of possibilities for dark energy [18]. In an interesting braneworld model proposed by Dvali, Gabadadze and Porrati (DGP) [19] a late-time self-acceleration solution [20] appears naturally. In the DGP model, the bulk is a flat Minkowski spacetime, but a reduced gravity term appears on the brane, which is tensionless. In this model, gravity appears 4-dimensional at short distances but is altered at distance larger than some freely adjustable crossover scale r_c through the slow evaporation of the graviton off our 4-dimensional brane world universe into an unseen, yet large, fifth dimension. The late-time acceleration is driven by the manifestation of the excruciatingly slow leakage of gravity

*Electronic address: zhuzh@bnu.edu.cn

off our four-dimensional world into an extra dimension. In this scenario the universe eventually evolves into a de Sitter phase, and the effective EOS of dark energy never comes across -1 . The behavior of crossing $w = -1$ of the effective dark energy on a DGP brane with a cosmological constant and dust has been studied in [21].

In the present paper we study some interesting properties of the scalar, including ordinary scalar and phantom, as dark energy, in the late-time universe. We find effective EOS of the dark energy can cross -1 only by a single scalar: For ordinary scalar, the EOS runs from above -1 to below -1 , evolving as dark energy of Case I in the negative branch, and for phantom, the EOS runs from below -1 to above -1 , evolving as dark energy of Case II in the positive branch. As a byproduct, we prove that both for ordinary scalar and phantom, there does not exist scaling solution in a universe with dust in DGP brane world scenario.

In the next section we shall investigate our model in detail. And in Sec. III, we present the main conclusions and some discussions.

II. THE MODEL

Let us start from the action of the DGP model

$$S = S_{\text{bulk}} + S_{\text{brane}}, \quad (1)$$

where

$$S_{\text{bulk}} = \int_{\mathcal{M}} d^5 X \sqrt{-^{(5)}g} \frac{1}{2\kappa_5^2} {}^{(5)}R, \quad (2)$$

and

$$S_{\text{brane}} = \int_M d^4 x \sqrt{-g} \left[\frac{1}{\kappa_5^2} K^\pm + L_{\text{brane}}(g_{\alpha\beta}, \psi) \right]. \quad (3)$$

Here κ_5^2 is the 5-dimensional gravitational constant, ${}^{(5)}R$ is the 5-dimensional curvature scalar and the matter Lagrangian in the bulk. x^μ ($\mu = 0, 1, 2, 3$) are the induced 4-dimensional coordinates on the brane, K^\pm is the trace of extrinsic curvature on either side of the brane and $L_{\text{brane}}(g_{\alpha\beta}, \psi)$ is the effective 4-dimensional Lagrangian, which is given by a generic functional of the brane metric $g_{\alpha\beta}$ and matter fields ψ on the brane.

Consider the brane Lagrangian consisting of the following terms

$$L_{\text{brane}} = \frac{\mu^2}{2} R + L_m + L_\phi, \quad (4)$$

where μ is 4-dimensional reduced Planck mass, R denotes the curvature scalar on the brane, L_m stands for the Lagrangian of other matters on the brane, and L_ϕ represents the lagrangian of a scalar confined to the brane. Then assuming a mirror symmetry in the bulk, we have the Friedmann equation on the brane [20], see also [15],

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} \left[\rho + \rho_0 + \theta\rho_0 \left(1 + \frac{2\rho}{\rho_0} \right)^{1/2} \right] \quad (5)$$

where $H = \dot{a}/a$ is the Hubble parameter, a is the scale factor, k is the spatial curvature of the three dimensional maximally symmetric space in the FRW metric on the brane, and $\theta = \pm 1$ denotes the two branches of DGP model, ρ denotes the total energy density, including dust matter and scalar, on the brane,

$$\rho = \rho_\phi + \rho_{\text{dm}}, \quad (6)$$

and the term ρ_0 relates the strength of the 5-dimensional gravity with respect to 4-dimensional gravity,

$$\rho_0 = \frac{6\mu^2}{r_c^2}, \quad (7)$$

where the cross radius is defined as $r_c \triangleq \kappa_5^2 \mu^2$.

In quintom model [10], two scalar fields must be introduced for the crossing -1 of effective EOS of dark energy. By contrast, we shall show that in our model only one field is enough for this crossing behavior by aiding of the 5-dimensional gravity. Therefore, the accelerated expansion of the universe is due to the combined effect of the scalar and the competition between 4-dimensional gravity and the 5-dimensional gravity.

To explain the observed evolving of EOS of effective dark energy, we calculate the equation of state w of the effective ‘‘dark energy’’ caused by the scalar field and term representing brane world effect by comparing the modified Friedmann equation in the brane world scenario and the standard Friedmann equation in general relativity, because all observed features of dark energy are ‘‘derived’’ in general relativity. Note that the Friedmann equation in the four-dimensional general relativity can be written as

$$H^2 + \frac{k}{a^2} = \frac{1}{3\mu^2} (\rho_{\text{dm}} + \rho_{\text{de}}), \quad (8)$$

where the first term of RHS of the above equation represents the dust matter and the second term stands for the effective dark energy. Compare (8) with (5), one obtains the density of effective dark energy,

$$\rho_{\text{de}} = \rho_\phi + \rho_0 + \theta\rho_0 \left[\rho + \rho_0 + \theta\rho_0 \left(1 + \frac{2\rho}{\rho_0} \right)^{1/2} \right]. \quad (9)$$

Since the dust matter obeys the continuity equation and the Bianchi identity keeps valid, dark energy itself satisfies the continuity equation

$$\frac{d\rho_{\text{de}}}{dt} + 3H(\rho_{\text{de}} + p_{\text{eff}}) = 0, \quad (10)$$

where p_{eff} denotes the effective pressure of the dark energy. And then we can express the equation of state for the dark energy as

$$w_{\text{de}} = \frac{p_{\text{eff}}}{\rho_{\text{de}}} = -1 + \frac{1}{3} \frac{d \ln \rho_{\text{de}}}{d \ln(1+z)}, \quad (11)$$

where, from (10),

$$\frac{d \ln \rho_{\text{de}}}{d \ln(1+z)} = \frac{3}{\rho_{\text{de}}} \left[\rho_{\text{de}} + p_{\text{de}} + \theta \left(1 + 2 \frac{\rho_{\text{de}} + \rho_{\text{dm}}}{\rho_0} \right)^{-1/2} \times (\rho_{\text{de}} + \rho_{\text{dm}} + p_{\text{de}}) \right], \quad (12)$$

where p_{de} represents the pressure of the dark energy. Here we stress that it is different from p_{eff} . Clearly, if $\frac{d \ln \rho_{\text{de}}}{d \ln(1+z)}$ is greater than 0, dark energy evolves as phantom; if $\frac{d \ln \rho_{\text{de}}}{d \ln(1+z)}$ is less than 0, it evolves as quintessence; if $\frac{d \ln \rho_{\text{de}}}{d \ln(1+z)}$ equals 0, it is just cosmological constant. In a more intuitionistic way, if ρ_{de} decreases and then increases with respect to redshift (or time), or increases and then decreases, which implies that EOS of dark energy crosses phantom divide. It is known that the equations of sort (5) and (10) belong to so-called inhomogeneous EOS of FRW universe [22] or FRW universe with general EOS [23], which can cross the phantom divide.

In the DGP brane world model, the induced gravity correction arises because the localized massive scalars fields on the brane, which couple to bulk gravitons, can generate via quantum loops a localized four-dimensional world-volume kinetic term for gravitons [24]. Therefore, to investigate the behavior of classical level of such a field is not only interesting for cosmology, but also important for analysing the DGP model itself. The behavior of the scalar in the early universe, i.e., the inflation driven by a scalar field confined to a DGP brane, has been investigated in [25,26]. A proposal that the same phantom scalar plays the role of early time (phantom) inflaton and late-time dark energy is presented in [27]. In view of these progresses, it is interesting to study the behavior of a scalar field on a DGP brane at late-time universe.

In the following two subsections, we shall analyze the dynamics of an ordinary scalar and a phantom in the late-time universe on a DGP brane, respectively. We show both of them can cross the phantom divide: in the negative branch for an ordinary scalar; in the positive branch for a phantom.

A. Ordinary scalar field

For an ordinary scalar, the action in L_ϕ of (4) reads,

$$L_\phi = -\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi). \quad (13)$$

Varying the action with respect to the metric tensor in an FRW universe we reach to

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad (14)$$

$$p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad (15)$$

where a dot denotes derivative with respect to time. The exponential potential is an important example which can be solved exactly in the standard model. Also it has been shown that the inflation driven by a scalar with exponential potential can exit naturally in the warped DGP model [15]. In addition, we know that such exponential potentials of scalar fields occur naturally in some fundamental theories such as string/M theories. It is therefore quite interesting to investigate a scalar with such a potential in late-time universe on a DGP brane. Here we set

$$V = V_0 e^{-\lambda(\phi/\mu)}. \quad (16)$$

Here λ is a constant and V_0 denotes the initial value of the potential.

Then the derivation of effective density of dark energy with respect to $\ln(1+z)$ reads,

$$\frac{d \rho_{\text{de}}}{d \ln(1+z)} = 3 \left[\dot{\phi}^2 + \theta \left(1 + \frac{\dot{\phi}^2 + 2V + 2\rho_{\text{dm}}}{\rho_0} \right)^{-1/2} \times (\dot{\phi}^2 + \rho_{\text{dm}}) \right]. \quad (17)$$

If $\theta = 1$, both terms of RHS are positive, hence it never goes to zero at finite time. But if $\theta = -1$, the two terms of RHS carry opposite sign, therefore it is possible that the EOS of dark energy crosses phantom divide. In the following of this subsection we only consider the case of $\theta = -1$. For a more detailed research of the evolution of the variables in this model we write them in a dynamical system, which can be derived from the Friedmann Eq. (5) and continuity Eq. (10). We first define some new dimensionless variables,

$$x \triangleq \frac{\dot{\phi}}{\sqrt{6}\mu H}, \quad (18)$$

$$y \triangleq \frac{\sqrt{V}}{\sqrt{3}\mu H}, \quad (19)$$

$$l \triangleq \frac{\sqrt{\rho_m}}{\sqrt{3}\mu H}, \quad (20)$$

$$b \triangleq \frac{\sqrt{\rho_0}}{\sqrt{3}\mu H}. \quad (21)$$

The dynamics of the universe can be described by the following dynamical system with these new dimensionless variables,

$$x' = -\frac{3}{2} \alpha x(2x^2 + l^2) + 3x - \frac{\sqrt{6}}{2} \lambda y^2, \quad (22)$$

$$y' = -\frac{3}{2}\alpha y(2x^2 + l^2) + \frac{\sqrt{6}}{2}\lambda xy, \quad (23)$$

$$l' = -\frac{3}{2}\alpha l(2x^2 + l^2) + \frac{3}{2}l, \quad (24)$$

$$b' = -\frac{3}{2}\alpha b(2x^2 + l^2), \quad (25)$$

where

$$\alpha \triangleq 1 - \left(1 + 2\frac{x^2 + y^2 + l^2}{b^2}\right)^{-1/2}, \quad (26)$$

a prime stands for derivation with respect to $s \triangleq -\ln(1+z)$, and we have set $k=0$, which is implied either by theoretical side (inflation in the early universe), or observation side (CMB fluctuations [28]). One can check this system degenerates to a quintessence with dust matter in standard general relativity. Note that the 4 Eqs. (22)–(25), of this system are not independent. By using the Friedmann constraint, which can be derived from the Friedmann equation,

$$x^2 + y^2 + l^2 + b^2 - b^2\left(1 + 2\frac{x^2 + y^2 + l^2}{b^2}\right)^{1/2} = 1, \quad (27)$$

the number of the independent equations can be reduced to 3. There are two critical points of this system satisfying $x' = y' = l' = b' = 0$ appearing at

$$x = y = l = 0, \quad b = \text{constant}; \quad (28)$$

$$x = y = l = b = 0. \quad (29)$$

However, neither of them satisfies the Friedmann constraint (27). Hence we prove that there is no kinetic energy-potential energy scaling solution or kinetic

energy-potential energy -dust matter scaling solution on a DGP brane with quintessence and dust. Since the equation set (22)–(25) can not be solved analytically, we present some numerical results about the dark energy density. And one will see that in reasonable regions of parameters, the EOS of dark energy crosses -1 .

The stagnation point of ρ_{de} dwells at

$$\frac{b}{\sqrt{2} + b}\left(2 + \frac{l^2}{x^2}\right) = 2, \quad (30)$$

which can be derived from (17) and (27). One concludes from the above equation that a smaller r_c , a smaller Ω_m (which is defined as the present value of the energy density of dust matter over the critical density), or a larger Ω_{ki} (which is defined as the present value of the kinetic energy density of the scalar over the critical density) is helpful to shift the stagnation point to lower redshift region. We show a concrete numerical example of this crossing behaviors in Fig. 1–3. For convenience we introduce the dimensionless density and rate of change with respect to redshift of dark energy as below.

$$\begin{aligned} \beta &= \frac{\rho_{\text{de}}}{\rho_c} \\ &= \frac{\Omega_{r_c}}{b^2} \left[x^2 + y^2 + b^2 - b^2 \left(1 + 2\frac{x^2 + y^2 + l^2}{b^2}\right)^{1/2} \right], \end{aligned} \quad (31)$$

where ρ_c denotes the present critical density of the universe, and

$$\begin{aligned} \gamma &= \frac{1}{\rho_c} \frac{b^2}{\Omega_{r_c}} \frac{d\rho_{\text{de}}}{ds} \\ &= 3 \left[\left(1 + 2\frac{x^2 + y^2 + l^2}{b^2}\right)^{-1/2} (2x^2 + l^2) - 2x^2 \right]. \end{aligned} \quad (32)$$

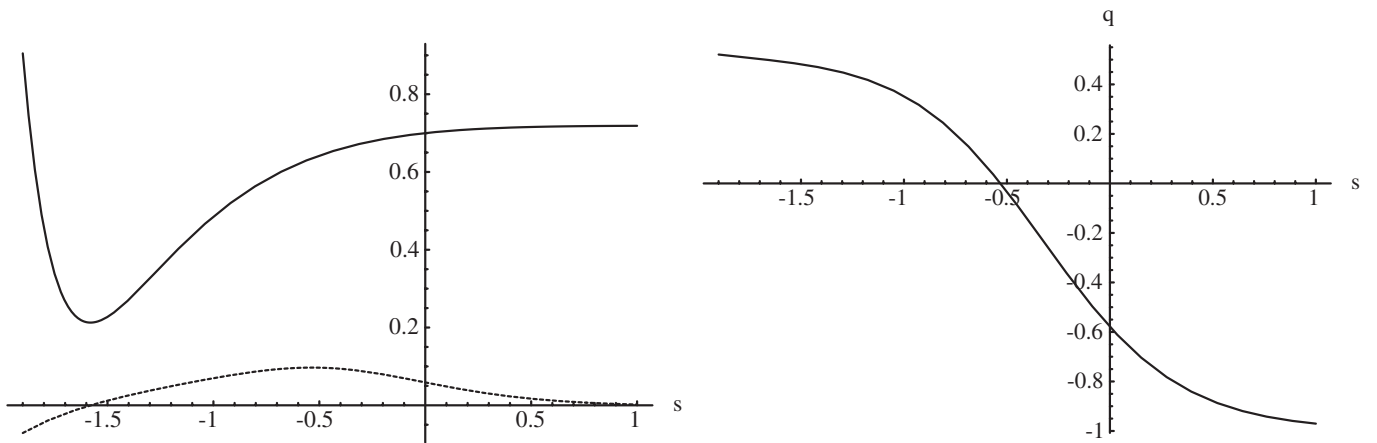


FIG. 1. For this figure, $\Omega_{\text{ki}} = 0.01$, $\Omega_{r_c} = 0.01$, $\lambda = 0.05$. (a) β and γ as functions of s , in which β resides on the solid line, while γ dwells at the dotted line. The EOS of dark energy crosses -1 at about $s = -1.6$, or $z = 3.9$. (b) The corresponding deceleration parameter, which crosses 0 at about $s = -0.52$, or $z = 0.68$.

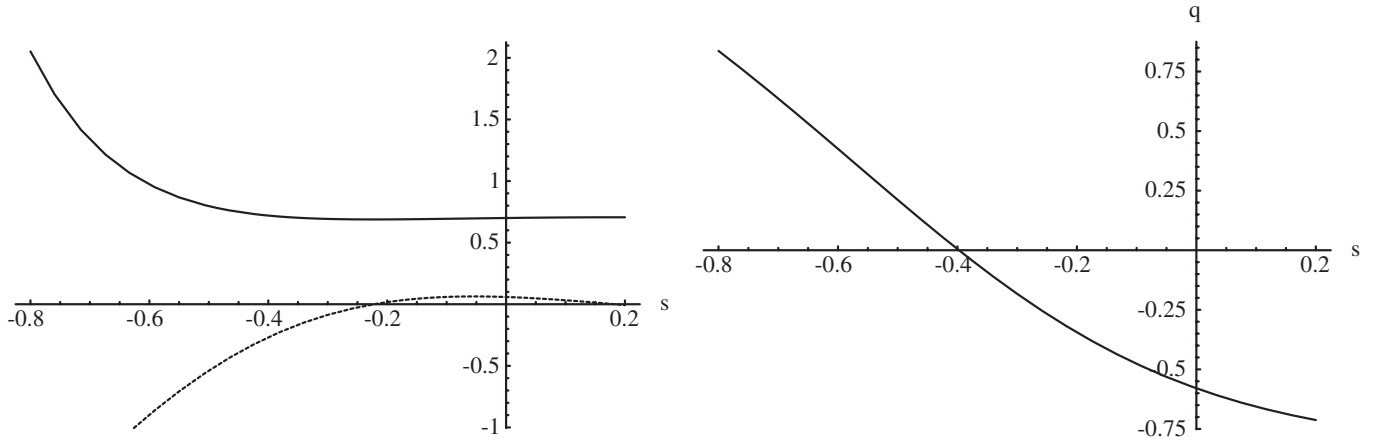


FIG. 2. For this figure, $\Omega_{ki} = 0.01$, $\Omega_{rc} = 0.01$, $\lambda = 0.5$. (a) β and γ as functions of s , in which β resides on the solid line, while γ dwells at the dotted line. The EOS of dark energy crosses -1 at about $s = -0.22$, or $z = 0.25$. (b) The corresponding deceleration parameter, which crosses 0 at about $s = -0.40$, or $z = 0.49$.

The most significant parameters from the viewpoint of observations is the deceleration parameter q , which carries the total effects of cosmic fluids. q is defined as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{3}{2}\alpha(2x^2 + l^2), \quad (33)$$

which is also plotted in these figures for corresponding density curve of dark energy. In all the figures we set $\Omega_m = 0.3$ but with different Ω_{ki} , λ , Ω_{rc} , in which Ω_{rc} is defined as the present value of the energy density of ρ_0 over the critical density $\Omega_{rc} = \rho_0/\rho_c$.

From Fig. 1–3, clearly, the EOS of effective dark energy crosses -1 as expected. At the same time the deceleration parameter is consistent with observations. As is well known, the EOS of a single scalar in standard general relativity never crosses the phantom divide, the induced term, through the “energy density” of r_c , ρ_0 , plays a critical in this crossing. We see that a small component

of ρ_0 , i.e. $\Omega_{rc} = 0.01$, is successfully to make the EOS of dark energy cross -1 . The other note is that here the EOS runs from above -1 to below -1 , which describes the dark energy of Case I.

B. Phantom field

In this subsection we investigate a phantom field in DGP model. Most of the discussions are parallel to the last subsection. For a phantom field, the action in L_ϕ of (4) reads,

$$L_\phi = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi). \quad (34)$$

A variation of the action with respect to the metric tensor in an FRW universe yields,

$$\rho_\phi = -\frac{1}{2} \dot{\phi}^2 + V(\phi), \quad (35)$$

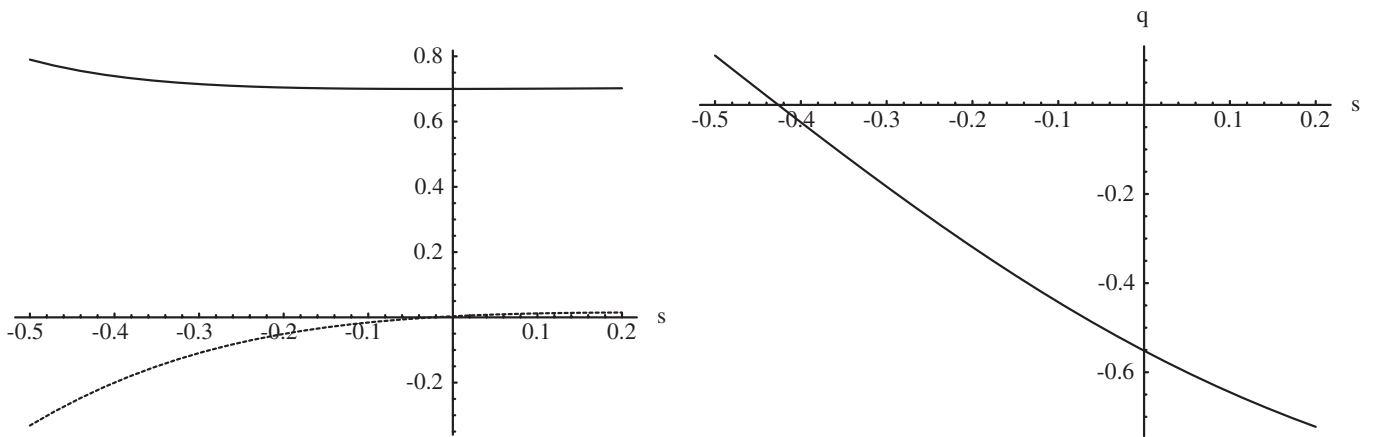


FIG. 3. For this figure, $\Omega_{ki} = 0.1$, $\Omega_{rc} = 0.01$, $\lambda = 0.05$. (a) β and γ as functions of s , in which β resides on the solid line, while γ dwells at the dotted line. The EOS of dark energy crosses -1 at about $s = -0.02$, or $z = 0.02$. (b) The corresponding deceleration parameter, which crosses 0 at about $s = -0.41$, or $z = 0.51$.

$$p_\phi = -\frac{1}{2}\dot{\phi}^2 - V(\phi). \quad (36)$$

To compare with the results of the ordinary scalar, here we set a same potential as before,

$$V = V_0 e^{-\lambda(\phi/\mu)}. \quad (37)$$

The ratio of change of effective density of dark energy with respect to $\ln(1+z)$ becomes,

$$\frac{d\rho_{de}}{d\ln(1+z)} = 3 \left[-\dot{\phi}^2 + \theta \left(1 + \frac{-\dot{\phi}^2 + 2V + 2\rho_{dm}}{\rho_0} \right)^{-1/2} \times (-\dot{\phi}^2 + \rho_{dm}) \right]. \quad (38)$$

To study the behavior of the EOS of dark energy, we first take a look at the signs of the terms of RHS of the above equation. $(-\dot{\phi}^2 + \rho_{dm})$ represents the total energy density of the cosmic fluids, which should be positive. The term $(1 + \frac{-\dot{\phi}^2 + 2V + 2\rho_{dm}}{\rho_0})^{-1/2}$ should also be positive. Hence if $\theta = -1$, both terms of RHS are negative: It never goes to zero at finite time. Contrarily, if $\theta = 1$, the two terms of RHS carry opposite sign: The EOS of dark energy is able to cross phantom divide. In the following of the present subsection we consider the branch of $\theta = 1$.

Similar to the case of an ordinary scalar, the dynamics of the universe can be described by the following dynamical system,

$$x' = -\frac{3}{2}\alpha'x(-2x^2 + l^2) + 3x - \frac{\sqrt{6}}{2}\lambda y^2, \quad (39)$$

$$y' = -\frac{3}{2}\alpha'y(-2x^2 + l^2) + \frac{\sqrt{6}}{2}\lambda xy, \quad (40)$$

$$l' = -\frac{3}{2}\alpha'l(-2x^2 + l^2) + \frac{3}{2}l, \quad (41)$$

$$b' = -\frac{3}{2}\alpha'b(-2x^2 + l^2), \quad (42)$$

where

$$\alpha' \triangleq 1 + \left(1 + 2\frac{-x^2 + y^2 + l^2}{b^2} \right)^{-1/2}, \quad (43)$$

and we have also adopted the spatial flatness condition. The definitions of x, y, l, b are the same as the last subsection. One can check this system degenerates to a phantom with dust matter in standard general relativity. Also the 4 Eqs. (39)–(42) of this system are not independent. Now the Friedmann constraint becomes

$$-x^2 + y^2 + l^2 + b^2 + b^2 \left(1 + 2\frac{-x^2 + y^2 + l^2}{b^2} \right)^{1/2} = 1, \quad (44)$$

with which there are 3 independent equations left in this system. Through a similar analysis as the case of an ordinary scalar, we can prove that there is no kinetic energy-potential energy scaling solution or kinetic energy-potential energy-dust matter scaling solution on a DGP brane with phantom and dust. The equation set (39)–(42) can not be solved analytically, therefore here we give some numerical results. Again, one will see that in reasonable regions of parameters, the EOS of dark energy crosses -1 , but from below -1 to above -1 .

The stagnation point of ρ_{de} inhabits at

$$\frac{b}{\sqrt{2}-b} \left(-2 + \frac{l^2}{x^2} \right) = 2, \quad (45)$$

which can be derived from (38) and (44). One concludes from the above equation that a smaller r_c , a smaller Ω_m , or a larger Ω_{ki} is helpful to shift the stagnation point to lower redshift region, which is the same as the case of an ordinary scalar. Then we show a concrete numerical example of the crossing behavior of this case in Fig. 4. The dimensionless density and rate of change with respect to redshift of dark

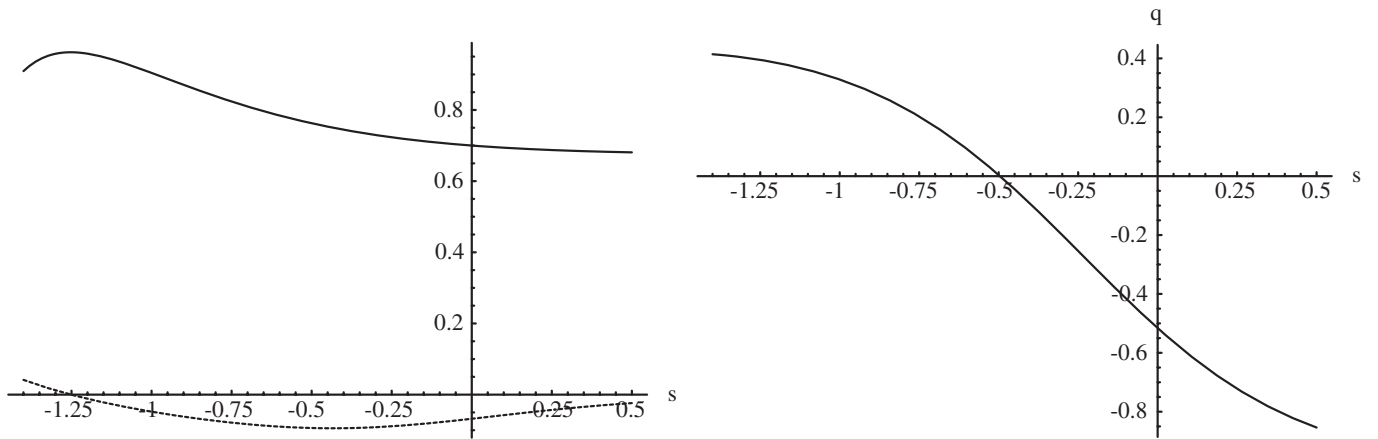


FIG. 4. For this figure, $\Omega_{ki} = 0.01$, $\Omega_{r_c} = 0.01$, $\lambda = 0.01$. (a) β and γ as functions of s , in which β resides on the solid line, while γ dwells at the dotted line. The EOS of dark energy crosses -1 at about $s = -1.25$, or $z = 1.49$. (b) The corresponding deceleration parameter, which crosses 0 at about $s = -0.50$, or $z = 0.65$.

energy become,

$$\beta = \frac{\Omega_{r_c}}{b^2} \left[-x^2 + y^2 + b^2 + b^2 \left(1 + 2 \frac{-x^2 + y^2 + l^2}{b^2} \right)^{1/2} \right], \quad (46)$$

and

$$\gamma = 3 \left[- \left(1 + 2 \frac{-x^2 + y^2 + l^2}{b^2} \right)^{-1/2} (-2x^2 + l^2) + 2x^2 \right]. \quad (47)$$

The deceleration parameter q becomes,

$$q = -1 + \frac{3}{2} \alpha' (-2x^2 + l^2), \quad (48)$$

which is plotted in the figure for corresponding density curve of dark energy. In this figures we also set $\Omega_m = 0.3$.

Figure 4 explicitly illuminates that the EOS of effective dark energy crosses -1 , as expected. At the same time the deceleration parameter is consistent with observations. The EOS of a single phantom field in standard general relativity always less than -1 , therefore, the 5 dimensional gravity plays a critical role in this crossing, though we see that the “geometric energy density” only takes a small component of the total density, i.e. $\Omega_{r_c} = 0.01$. Finally, the most important difference of the dynamics between an ordinary scalar and the phantom of the crossing behavior lies on the crossing manner: Here the EOS runs from below -1 to above -1 , which can describe the dark energy of Case II, while the effective EOS of an ordinary scalar evolves from above -1 to below -1 , as shown in Subsec. II A.

III. CONCLUSIONS AND DISCUSSIONS

To summarize, this paper displays that in frame of DGP brane world it is possible to realize crossing $w = -1$ for

the EOS of the effective dark energy of a single scalar. For an ordinary scalar in the negative branch, the EOS transits from $w > -1$ to $w < -1$, which can describe the dark energy of Case I, while for a phantom in the positive branch, the EOS transits from $w < -1$ to $w > -1$, which describes dark energy of Case II. The deceleration parameter can be consistent with observations. In standard general relativity, the EOS of a single scalar never crosses -1 , therefore, the five dimensional gravity plays an important role in this transition, although the competent of this “geometric energy density” ρ_0 over the critical energy density is very small. The other conclusion is that there does not exist scaling solution in a universe with dust on a DGP brane, neither in the positive branch nor in the negative branch.

Figure 2 shows that the deceleration parameter can exceed 0.5 in some high redshift region, which implies that the scalar may enter a kinetic energy dominated phase. And hence the energy density of scalar will exceed the density of dust in such region, which can spoil the successful predictions of structure formation theory. Therefore, we should treat the exponential potential as an approximation of a more appropriate potential, such as the tracker potential [29], in low redshift region. The present model may fail at very high redshift region (such as $z = 1000$). How to construct a potential which can describe both the early universe and the late universe is our further work.

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