PHYSICAL REVIEW D **75,** 021302(R) (2007)

How well can (renormalized) perturbation theory predict dark matter clustering properties?

Niayesh Afshordi*

Institute for Theory and Computation, Harvard-Smithsonian Center for Astrophysics, MS-51, 60 Garden Street, Cambridge, Massachusetts 02138, USA

(Received 2 November 2006; published 12 January 2007)

There has been some recent activity in trying to understand the dark matter clustering properties in the quasilinear regime, through resummation of perturbative terms, otherwise known as the renormalized perturbation theory [M. Crocce and R. Scoccimarro, Phys. Rev. D **73**, 063519 (2006).], or the renormalization group method [P. McDonald, astro-ph/0606028.]. While it is not always clear why such methods should work so well, there is no reason for them to capture nonperturbative events such as shell-crossing. In order to estimate the magnitude of nonperturbative effects, we introduce a (hypothetical) model of *sticky dark matter*, which only differs from collisionless dark matter in the shell-crossing regime. This enables us to show that the level of nonperturbative effects in the dark matter power spectrum at $k \sim 0.1\ \mathrm{Mpc}^{-1}$, which is relevant for baryonic acoustic oscillations, is about a percent, but rises to order unity at $k \sim 1\ \mathrm{Mpc}^{-1}$.

DOI: 10.1103/PhysRevD.75.021302 PACS numbers: 95.35.+d, 98.80.−k

In the era of precision cosmology, an accurate understanding of dark matter clustering properties is an essential ingredient of relating observations of clustering in galaxy or weak lensing maps to the fundamental properties of our Universe, such as its geometry or linear growth history. This need is underlined by the recent discovery of baryonic acoustic oscillations in the power spectrum of galaxies [1,2], and the prospects of its application as a cosmic standard ruler (e.g. [3]). While semianalytic methods, most famously within the context of the halo model [4–6], have been successfully used to model both galaxy and dark matter clustering properties, they suffer from their phenomenological nature, which inevitably leads to a plethora of parameters that need to be calibrated using numerical simulations or observations. Moreover, the number of these parameters is expected to increase through more in-depth studies, which will certainly lead to the surfacing of more subtle effects (e.g. [7,8]). Thus, the ultimate dilemma in the era of precision cosmology will become if we are probing the fundamental properties of our universe, or rather further constraining the intricate phenomenology of nonlinear gravitational gastrophysics.

Numerical simulations go a long way in elucidating the complicated process of gravitational collapse of dark matter and baryonic gas. However, they still suffer from our poor understanding of star and galaxy formation, and its feedback on the surrounding intergalactic medium. Moreover, they may also suffer from finite resolution and box size effects, as well as potential ill-understood numerical artifacts that can limit the accuracy of numerical studies.

The latter has motivated analytic studies, which are most systematically done in the context of perturbation theory (see [9] for a review). However, consecutive terms in a perturbative series become comparable in the quasilinear regime, when overdensities become of order unity, signaling the breakdown of the perturbative expansion. Inspired by the use of renormalization methods in quantum field theory, some studies [10,11] have introduced renormalization or resummation technics that, in principle, capture the dominant terms in the perturbative series, and can yield an accurate result, even when the perturbation theory breaks down.

While the theoretical validity of the technics advocated in these studies is not transparent to this author, it is possible to put a limit on the degree of accuracy that any method, based on perturbation theory, can achieve. This limit comes from the nonperturbative effects involved in the gravitational collapse process, and is the subject of this note.

The cosmological perturbation theory for cold dark matter [9] is based on the pressureless Euler equation, where the fluid starts from a nearly homogenous expanding initial conditions, and then evolves under its own gravity. The equations break down at shell crossing, which is when perturbations grow to the limit that multiple streams cross each other, and thus the assumption of zero pressure fails.

As long as different streams of dark matter do not cross each other, the cross-section for their collision does not enter the equations. Therefore the perturbative framework, which is the basis for the recently suggested renormalization methods, is insensitive to dark matter collisional properties. Even though in the minimal dark matter model, the particles have a negligible self-interaction cross-section, one may also envisage an opposite regime, where the collisions have a huge cross-section, and are maximally inelastic. We dub this particular model for dark matter as *sticky dark matter*.

Of course, sticky dark matter is a terrible candidate for the cosmological dark matter. This is because, as a result of frequent and inelastic collisions, all the dark matter particles would sink into the center of a halo, probably forming

*Electronic address: nafshordi@cfa.harvard.edu

RAPID COMMUNICATIONS

a black hole, or a small compact disk (if prevented by the initial angular momentum of the halo). However, it provides an interesting theoretical test study to examine the nonperturbative effect of shell-crossing, as sticky and collisionless dark matter obey the exact same equations within the (single-stream) perturbation theory.

In order to make this comparison, we resort to the simplest version of the halo model [5], where dark matter haloes cluster according to the linear matter power spectrum with a constant bias, which only depends on their mass. The structure within a halo of collisionless dark matter is well approximated by an NFW form [12]:

$$\rho(r) = \frac{\rho_s}{(r/r_s)(1 + r/r_s)^2}, \tag{1}$$

which extends out to $r_{\rm vir} = c r_s$, where the concentration parameter, c, as a function of the total enclosed mass is also constrained from simulations.

It is known that a combination of halo-halo correlations, plus single halo autocorrelation terms gives a reasonable approximation to the dark matter nonlinear power spectrum in numerical simulations (e.g. [13]). With this qualitative picture at hand, we can thus examine the case of sticky dark matter, by collapsing each NFW halo into a point.

Figure 1 compares the dimensionless power spectra, $\Delta^2(k) = \frac{k^3 P(k)}{2\pi^2}$, for collisionless and sticky dark matter models, in the context of the halo model. Here, we assume the WMAP 1st year best fit concordance cosmological parameters [14]. We see that the difference between the two models, which characterizes the level of nonperturbative effects in the dark matter power spectrum is $\sim 1\%$ at $k \sim 0.1\ \mathrm{Mpc}^{-1}$, which is the region relevant for probing baryonic acoustic oscillations. This level rises to order unity at $k \sim 1\ \mathrm{Mpc}^{-1}$, indicating the complete breakdown of (renormalized) perturbation theory, or any single-stream renormalization group method.

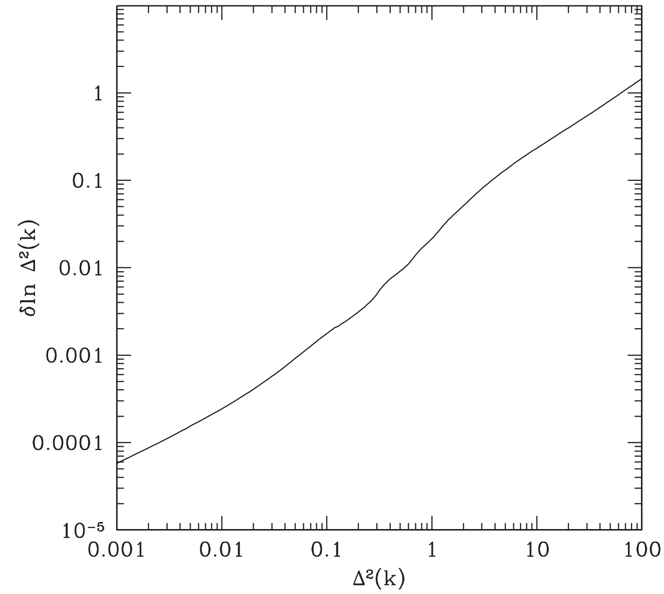

FIG. 2. The level of nonperturbative effects in the nonlinear power spectrum (see Fig. 1), as a function of nonlinear power.

Figure 2 shows the same difference between the sticky and collisionless power spectra, as a function of the non-

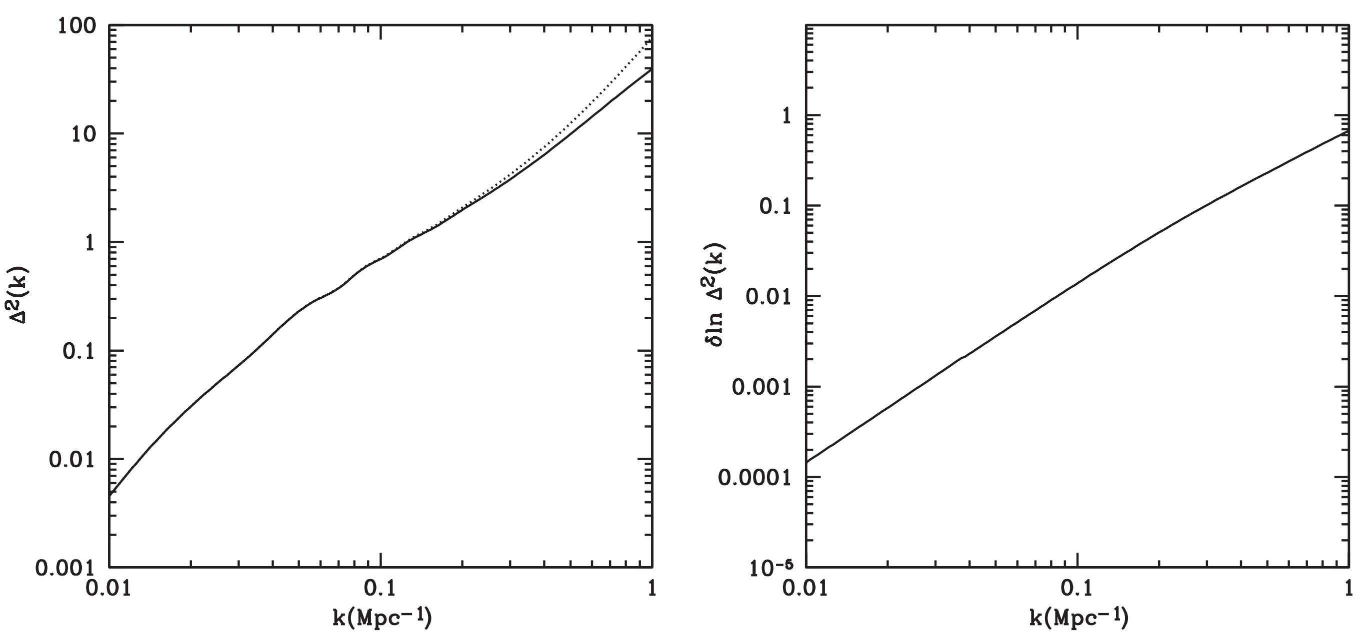

FIG. 1. *Left*: Halo model dark matter power spectrum for collisionless (solid) and sticky (dotted) dark matter. *Right*: Relative difference between power spectra of collisionless and sticky dark matter models. This shows the level of nonperturbative effects in the dark matter power spectrum.

linear power. It is interesting to notice that nonperturbative effects do not dominate the power until $\Delta^2(k) \sim 100$, which is far into the nonlinear regime. This may explain why renormalization methods based on the perturbation theory, such as [10,11], can do so well in the quasilinear regime.

A question that may arise is if we are stretching the halo model beyond its level of applicability. It is true that the halo model has had significant phenomenological success in fitting simulations and observations (e.g. [13,15]). However, it suffers from unphysical features, such as spurious power on very large scales, which is caused by the presence of the 1-halo term [5,16]. However, the difference shown in Figs. 1 and 2 is dominated by the 2-halo term for $k \lesssim 0.3\ \mathrm{Mpc}^{-1}$ (at least down to near-horizon scales). Even without any reference to the halo model, it is clear that the large scale correlation function is, to some extent, affected by the change in the profile of the virialized regions, say from an NFW (for collisionless dark matter) to a pointlike profile (for sticky dark matter). The difference in the 2-halo term, in the context of the halo model, gives an approximation for the magnitude of this effect.

As vorticity cannot be locally generated in the single stream regime of dark matter collapse, a different test for the breakdown of the single-stream perturbation theory is when the vorticity and the divergence of the velocity field become comparable [17]. This happens at $k \sim 2\ \mathrm{Mpc}^{-1}$, which is consistent with our results. However, in contrast to our approach, it is not clear how the vorticity to divergence ratio is related to the level of nonperturbative effects in the power spectrum within the quasilinear regime.

What if a single-stream perturbative method agrees with numerical N-body simulations at a better level than predicted by our simple exercise? Then I would argue that this level of accuracy is not warranted for any single-stream calculation, and thus any such agreement is *accidental*, unless it is justified based on a nonperturbative method.

Finally, we should point out that sticky dark matter is very similar to the so-called adhesion model [18,19], that has been introduced in the context of Zel'dovich approximation [20]. Adhesion model, which is realized as the zero/small viscosity limit of a (single stream) fluid, has been introduced to enable the use of analytic Zel'dovich approximation beyond shell crossing (or pancake formation). However, direct comparison of the adhesion model to the sticky dark matter might be misleading, as it mixes the perturbative terms, missing in the Zel'dovich approximation, with the nonperturbative adhesive feature.

In conclusion, in this note, we have pointed out that any renormalized perturbation theory method is ultimately limited to the context of single stream fluid equations, and thus does not capture the nonperturbative shell-crossing events. To estimate the level of nonperturbative effects, we introduced a toy model of sticky dark matter, which only differs from collisionless dark matter in the shell-crossing regime. This provides us with a framework to estimate the level of accuracy expected from renormalized perturbation theory, showing that nonperturbative effects could be $\sim 1\%$ at scale of baryonic acoustic oscillations, but do not dominate until deep in the nonlinear regime.

I would like to thank Simon DeDeo, Roya Mohayaee, Roman Scoccimarro, Anze Solsar, and especially Patrick McDonald for helpful comments and stimulating discussions. I also thank the organizers of the Benasque Cosmology Workshop, for providing the atmosphere that inspired this work.

[1] D. J. Eisenstein *et al.*, Astrophys. J. **633**, 560 (2005).
[2] S. Cole *et al.*, Mon. Not. R. Astron. Soc. **362**, 505 (2005).
[3] H.-J. Seo and D. J. Eisenstein, Astrophys. J. **598**, 720 (2003).
[4] U. Seljak, Mon. Not. R. Astron. Soc. **318**, 203 (2000).
[5] A. Cooray and R. K. Sheth, Phys. Rep. **372**, 1 (2002).
[6] A. A. Berlind *et al.*, Astrophys. J. **593**, 1 (2003).
[7] L. Gao, V. Springel, and S. D. M. White, Mon. Not. R. Astron. Soc. **363**, L66 (2005).
[8] R. H. Wechsler, A. R. Zentner, J. S. Bullock, and A. V. Kravtsov, Astrophys. J. **652**, 71 (2006).
[9] F. Bernardeau, S. Colombi, E. Gaztanaga, and R. Scoccimarro, Phys. Rep. **367**, 1 (2002).
[10] M. Crocce and R. Scoccimarro, Phys. Rev. D **73**, 063519 (2006).
[11] P. McDonald, astro-ph/0606028.
[12] J. F. Navarro, C. S. Frenk, and S. D. M. White, Astrophys. J. **490**, 493 (1997).
[13] R. E. Smith *et al.*, Mon. Not. R. Astron. Soc. **341**, 1311 (2003).
[14] D. N. Spergel *et al.*, Astrophys. J. Suppl. Ser. **148**, 175 (2003).
[15] I. Zehavi *et al.*, Astrophys. J. **608**, 16 (2004).
[16] R. Scoccimarro, R. K. Sheth, L. Hui, and B. Jain, Astrophys. J. **546**, 20 (2001).
[17] R. Scoccimarro, *The Onset of Nonlinearity in Cosmology, 2001,* edited by J. N. Fry, J. R. Buchler, and H. Kandrup (New York Academy of Sciences, New York, 2001) p. 13.
[18] A. L. Melott, S. F. Shandarin, and D. H. Weinberg, Astrophys. J. **428**, 28 (1994).
[19] L. Kofman, D. Pogosyan, S. F. Shandarin, and A. L. Melott, Astrophys. J. **393**, 437 (1992).
[20] I thank Roya Mohayaee for pointing this out.