Supersymmetry without the desert

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Naturalness of electroweak symmetry breaking in weak scale supersymmetric theories may suggest the absence of the conventional supersymmetric desert. We present a simple, realistic framework for supersymmetry in which (most of) the virtues of the supersymmetric desert are naturally reproduced without having a large energy interval above the weak scale. The successful supersymmetric prediction for the low-energy gauge couplings is reproduced due to a gauged R symmetry present in the effective theory at the weak scale. The observable sector superpotential naturally takes the form of the next-to-minimal supersymmetric standard model, but without being subject to the Landau pole constraints up to the conventional unification scale. Supersymmetry breaking masses are generated by the F-term and D-term vacuum expectation values of singlet and $U(1)_R$ gauge fields, as well as by anomaly mediation, at a scale not far above the weak scale. We study the resulting pattern of supersymmetry breaking masses in detail and find that it can be quite distinct. We construct classes of explicit models within this framework, based on higher-dimensional unified theories with TeV-sized extra dimensions. A similar model based on a non-R symmetry is also presented. These models have a rich phenomenology at the TeV scale, and allow for detailed analyses of, e.g., electroweak symmetry breaking.

I. INTRODUCTION

Weak scale supersymmetry provides an elegant framework for explaining the origin of electroweak symmetry breaking. In its simplest realization, one assumes that the fundamental scale of nature is extremely large, of order the Planck scale $M_{\rm Pl}$, and that supersymmetry is (dynamically) broken at a hierarchically small scale [1]. This picture of a supersymmetric desert, in fact, seems to be supported by the apparent unification of the three gauge couplings at a scale of $M_U \approx 10^{16}$ GeV. Suppressions of various higher-dimension operators, such as the ones leading to proton decay, are also naturally explained in this picture.

On the other hand, the fact that no definite sign of supersymmetry has been seen so far has put models based on the picture described above in a somewhat unpleasant situation. Given the current experimental constraints, parameters of these models must typically be fine-tuned to reproduce the correct scale for electroweak symmetry breaking. Looking at this more carefully, the problem typically arises for (one of) the following reasons:

(i) The Higgs mass squared parameter receives radiative corrections that are proportional to the top squark squared masses. These corrections arise from the entire energy interval between the weak scale and the scale where the supersymmetry breaking masses are generated and so are enhanced by the logarithm of the ratio of these two scales. For example, in the case where supersymmetry breaking is mediated by gravitationally suppressed interactions [2], the logarithm is inevitably large, giving a large negative contribution to the Higgs mass squared parameter. This leads to fine-tuning, since the large negative contribution must be canceled to a high

degree by some positive contribution, such as the one coming from the supersymmetric mass term.

(ii) The amount of cancellation required to reproduce the correct scale for electroweak symmetry breaking becomes smaller if the mass of the physical Higgs boson, $M_{\rm Higgs}$, becomes larger. In the minimal supersymmetric standard model (MSSM), the value of $M_{\rm Higgs}$ is bounded from above by the Z boson mass, m_Z, at tree level, so that we need an extra source of $M_{\rm Higgs}$ to make the Higgs boson significantly heavier than m_Z. This is, however, not so easy to achieve, because the extra coupling (the Higgs quartic coupling) needed to make $M_{\rm Higgs}$ large is subject to the Landau pole constraint, and thus often not large enough to push up $M_{\rm Higgs}$ to the level enough to eliminate fine-tuning.

It is, of course, possible to evade these difficulties and eliminate fine-tuning within the supersymmetric desert framework. For example, the large logarithm between the weak and Planck scales may be avoided due to a special renormalization group property of moduli and anomaly mediated supersymmetry breaking, leading to a natural model of supersymmetry [3,4]. Alternatively, a large Higgs quartic coupling needed to obtain large $M_{\rm Higgs}$ may arise as a result of strong gauge dynamics, giving some of the low-energy states as composite particles [5–8]. Nevertheless, it is still true that none of these models are particularly simple. The physics of electroweak symmetry breaking would be much simpler if all of the relevant physics occurred at energies not far from the weak scale.

In this paper, we thus take the viewpoint that the difficulties described above are suggestive hints for the absence of the supersymmetric desert. Note that these difficulties

are associated with the existence of the supersymmetric desert, and not that of weak scale supersymmetry itself. Without the large desert, weak scale supersymmetry indeed allows for the possibility of constructing a fully natural model of electroweak symmetry breaking, adopting, for example, the scheme discussed recently in Ref. [9]. A natural question then is to what extent the successes of the conventional supersymmetric desert picture are preserved in such a scenario. These include, in particular, a simple understanding of the weakness of gravity, the successful unification prediction for the low-energy gauge couplings, with the apparent unification scale close to the gravitational scale, and natural suppressions of certain higher-dimension operators such as the ones leading to rapid proton decay and large neutrino masses.

In this paper we present a simple, realistic framework for supersymmetry in which (most of) the virtues of the supersymmetric desert are naturally reproduced without a large energy interval above the weak scale. We show that the features usually attributed to the successes of the supersymmetric desert can in fact be preserved in a relatively simple setup with a low fundamental scale of $O(10 \sim 100 \text{ TeV})$. Lowering the fundamental scale to the TeV region was first proposed in the context of solving the gauge hierarchy problem without supersymmetry [10]. (For earlier work on lowering the fundamental scale, see [11].) The possibility of obtaining a prediction for the low-energy gauge couplings with a lowered fundamental scale was discussed in various (unification) scenarios [12–14], and supersymmetry breaking with a TeV-sized extra dimension was studied in [15–17]. Mechanisms of suppressing unwanted operators were also considered, for example, in [18]. We present in this paper a complete, effective field theory framework in which all of these issues are simultaneously addressed in a consistent manner. The framework has several general implications, for example, on the form of the observable sector superpotential and the pattern of supersymmetry breaking masses, leading to various interesting phenomenological consequences. Classes of explicit models can be constructed within this framework. We find it significant that fully realistic theories with a low fundamental scale are obtained, which allow for detailed phenomenological analyses, including electroweak symmetry breaking.

We construct our framework in two steps. We first identify the structure of the theory at the weak scale. This theory is supposed to be an effective field theory describing physics below the cutoff scale M_c. The value of M_c can be restricted by various phenomenological requirements; in particular, lower bounds of order a few TeV are obtained from the precision electroweak data in the context of particular models. In this paper we mainly take M_c to be between of order a few and a hundred TeV, to make the resulting theory the most natural in terms of electroweak symmetry breaking. Higher values of M_c,

however, are also possible. We find that an $U(1)_R$ symmetry that assigns the same charge for all the matter and Higgs supermultiplets plays an important role in this framework. In particular, we consider the gauging of this symmetry by canceling its mixed anomalies with the standard model gauge group by a nontrivial shift of a singlet (moduli) field. We then find that the successful supersymmetric prediction for the low-energy gauge couplings is automatically reproduced, if the singlet field has a certain vacuum expectation value (VEV). Generating the required VEV within the regime of effective field theory is nontrivial, but we demonstrate that it is possible. An important consequence of this setup is that the superpotential for the Higgs fields takes the form of the next-to-minimal supersymmetric standard model: $W = \lambda S H_u H_d + (\kappa/3) S^3$, and this is true even though the $U(1)_R$ symmetry is spontaneously broken at the cutoff scale. (It is possible to consider a MSSM-type superpotential if there is additional dynamics generating the supersymmetric mass for the Higgs doublets.) Since the fundamental scale of our theory is now of $O(10 \sim 100 \text{ TeV})$, however, the couplings λ and κ are not subject to the strong Landau pole constraint up to the conventional unification scale of $\approx 10^{16}$ GeV. This is an interesting result. Since the scale where the superparticle masses are generated is also very low (of order M_c), and so there is no large logarithmic running, the present framework provides a perfect platform for realizing the λSUSY models of Ref. [9], discussed recently from the viewpoint of eliminating fine-tuning in electroweak symmetry breaking. We discuss possible sources of supersymmetry breaking in our framework and study their implications on the pattern of supersymmetry breaking masses.

We next seek possible theories above the scale M_c, which reduce to the effective theory described above below the energy scale of M_c. We construct classes of realistic models based on higher-dimensional unified field theories, in which the standard model gauge fields propagate in an extra dimension(s) of size $\approx M_c^{-1}$. These models provide an understanding of the universal contribution to the tree-level standard model gauge kinetic terms, which cannot be fully understood in the effective theory below M_c. They also provide an explicit framework which allows us to relate supersymmetry breaking masses to the fundamental parameters of the theories. We discuss the consistency of these models as effective field theories, including suppressions of proton decay and the absence of unphysical modes, and study the resulting pattern of supersymmetry breaking masses as well as the masses of the lightest Kaluza-Klein (KK) excitations.

The paper is organized as follows. In the next section we describe the structure of the theory below M_c. We show that the successful prediction for the gauge couplings is reproduced in this theory and that a consistent vacuum can be obtained. In Sec. III we discuss phenomenological implications of the theory, especially those on supersym-

metry breaking. In Sec. IV we present classes of higher-dimensional models describing physics above M_c up to the fundamental scale M_*. We study their phenomenological implications, including the pattern of supersymmetry breaking masses and the spectrum of the lightest KK states. Discussion and conclusions are given in Sec. V. Appendix A gives the expressions for soft supersymmetry breaking masses in the minimal higher-dimensional model of Sec. IV, and Appendix B presents an alternative class of theories in which a non-R $U(1)$ gauge symmetry is used instead of the gauged $U(1)_R$ symmetry.

II. FRAMEWORK

In this section we present our framework. We first present a basic physical picture and then discuss some details about the viability of the framework.

A. Basic picture

We consider that physics above a few hundred GeV is described by a four-dimensional (4D) supersymmetric standard model. The quadratic divergence for the Higgs mass squared parameter is then cut off, as usual, by loops of superparticles. We assume that this theory is an effective field theory valid only below the cutoff scale $M_c = O(10 \sim 100 \text{ TeV})$, which is close (or equal) to the fundamental scale of nature, M_*.[1] Our first question then is, what is the basic structure of this effective field theory? In particular, we ask if there is a simple way of reproducing the successful supersymmetric prediction for the low-energy gauge couplings, and if so, what are the generic implications of it.

We consider, as usual, that the standard model quarks, leptons, and Higgs boson are embedded into chiral superfields: Q_i, U_i, D_i, L_i, E_i, H_u, and H_d, where $i = 1, 2, 3$ is the generation index [19]. The Yukawa couplings are then given by the superpotential:

$$W_{\text{Yukawa}} = (y_u)_{ij}Q_iU_jH_u + (y_d)_{ij}Q_iD_jH_d$$
$$+ (y_e)_{ij}L_iE_jH_d. \tag{1}$$

Note that these are the only superpotential terms whose existence is (almost certainly) required from the low-energy data. The "fundamental" mass term for the Higgsinos, $W = \mu H_uH_d$, may or may not exist. For example, if there is a singlet field S, the effective Higgsino mass term can arise from the superpotential term $W = \lambda SH_uH_d$ through the VEV of S, so that we do not need the term $W = \mu H_uH_d$.

The superpotential of Eq. (1) possesses an $U(1)_R$ symmetry under which all the chiral superfields have a charge of $+2/3$, in the normalization that the superpotential carries a charge of $+2$. This is, obviously, the unique $U(1)_R$ symmetry if we require that all the chiral superfields carry the same charge, which may have some motivation in the ultraviolet theory. Now, suppose that we consider gauging this $U(1)_R$ symmetry (or its discrete subgroup $Z_{N,R}$ with sufficiently large N,[2] e.g. $N = 5$). We then find that $U(1)_R$ (or $Z_{N,R}$) has the following mixed anomalies with respect to the standard model gauge group, $SU(3)_C \times SU(2)_L \times U(1)_Y$:

$$U(1)_R\text{-}U(1)_Y^2\text{:}\ A_1 \equiv 3(-\tfrac{1}{3})(\tfrac{3}{5})(\tfrac{1}{6} + \tfrac{4}{3} + \tfrac{1}{3} + \tfrac{1}{2} + 1)$$
$$+ (-\tfrac{1}{3})(\tfrac{3}{5})(\tfrac{1}{2} + \tfrac{1}{2})$$
$$= -\tfrac{11}{5},$$
$$U(1)_R\text{-}SU(2)_L^2\text{:}\ A_2 \equiv 3(-\tfrac{1}{3})(\tfrac{1}{2} \times 4) + (-\tfrac{1}{3})(\tfrac{1}{2} \times 2) + 2$$
$$= -\tfrac{1}{3},$$
$$U(1)_R\text{-}SU(3)_C^2\text{:}\ A_3 \equiv 3(-\tfrac{1}{3})(\tfrac{1}{2} \times 4) + 3$$
$$= 1, \tag{2}$$

which must somehow be canceled. (Note that the gauginos have a charge of $+1$ under $U(1)_R$, while the quarks, leptons, and Higgsinos have a charge of $-1/3$.) Here, we have taken the "$SU(5)$-normalization" for $U(1)_Y$, and have assumed that the MSSM quark, lepton, and Higgs superfields are the only states charged under the standard model gauge group. The $U(1)_Y$-$U(1)_R^2$ anomaly is automatically vanishing, and we do not consider the $U(1)_R^3$ or $U(1)_R$-(gravity)^2 anomalies because they depend on unknown fields that are singlet under the standard model gauge group.

The anomalies of Eq. (2) can be canceled by the (generalized) Green-Schwarz mechanism [20]. Assuming that a single moduli field M is responsible for the cancellation, the interactions between M and the standard model gauge fields are given by

$$\mathcal{L} = -\sum_{I=1,2,3} \frac{A_I}{c} \int d^2\theta M \mathcal{W}_I^{a\alpha} \mathcal{W}_{I\alpha}^a + \text{H.c.}, \tag{3}$$

where $I = 1, 2, 3$ represents $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively, $\mathcal{W}_I^{a\alpha}$ the field strength superfields with a representing the adjoint indices, and c a real constant.[3] The moduli field M is normalized such that the coefficient of the $M^\dagger M$ term in the Kähler potential is of order the fundamental scale M_*^2, and transforms as $M \rightarrow M + i\alpha c/16\pi^2$ under $U(1)_R$, where α represents the $U(1)_R$ transformation parameter in the normalization that

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a chiral superfield with a charge q transforms as $\Phi(x, \theta) \rightarrow e^{iq\alpha}\Phi(x, \theta e^{-i\alpha})$. The coefficients in Eq. (3) are then determined such that the anomalies of Eq. (2) are canceled by the classical transformation of Eq. (3). (In the case of a discrete $Z_{N,R}$ symmetry, $\alpha = 2\pi/N$.) Here we choose the constant c to be of $O(1)$. This corresponds to giving a small "charge" of $O(1/16\pi^2)$ to M and is necessary for the theory to stay within the regime of effective field theory (see the next subsection). With the terms in Eq. (3), we can gauge the R symmetry, under which all the MSSM chiral superfields have a charge of $+2/3$.

We now consider the gauge couplings in this theory. Since the coefficients of the terms in Eq. (3) have indefinite signs, we need other positive gauge kinetic terms for (at least some of) the standard model gauge fields. We assume that these terms are universal for the standard model gauge group [in the $SU(5)$ normalization], and that the gauge kinetic functions for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$ are given by the sum of the universal contribution and the ones from Eq. (3). Assuming that the universal piece arises from the VEV of a chiral superfield, which we denote by T, we obtain

$$\mathcal{L} = \sum_{I=1,2,3} \int d^2\theta \left(\frac{1}{4}T - \frac{A_I}{c}M\right) \mathcal{W}_I^{a\alpha}\mathcal{W}_{I\alpha}^a + \text{H.c.} \quad (4)$$

This form requires some justification from the theory above the cutoff scale M_c; in particular, the normalization (coupling to T) of $U(1)_Y$ should be explained. We will give examples of such theories in Sec. IV. Here we simply note that the form of Eq. (4) is technically natural. As long as the VEV of T is of order unity or larger (which is the case we are interested; see below), corrections to the gauge kinetic functions that do not respect the form of Eq. (4) are of order $1/8\pi^2$ or smaller, and thus are negligible for our purpose.

We assume that the VEVs of T and M are stabilized (dynamically) with $\langle T \rangle, \langle M \rangle = O(1)$. (Note that the T and M fields are dimensionless in our convention.) The standard model gauge couplings, g_I, at the scale $M_c = O(10 \sim 100 \text{ TeV})$ are then given by

$$\frac{1}{g_I^2}(M_c) = \langle T \rangle - \frac{4A_I}{c}\langle M \rangle. \quad (5)$$

An important point here is that the anomaly coefficients A_I are exactly proportional to the corresponding MSSM beta-function coefficients b_I:

$$\begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} 33/5 \\ 1 \\ -3 \end{pmatrix} = -3 \begin{pmatrix} -11/5 \\ -1/3 \\ 1 \end{pmatrix} = -3 \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix}. \quad (6)$$

We then find that the gauge couplings of Eq. (5) satisfy exactly the same relation as that arising in the conventional supersymmetric desert picture:

$$\frac{1}{g_I^2}(M_c) = \frac{1}{g_U^2} + \frac{b_I}{8\pi^2} \ln\frac{M_U}{M_c}, \quad (7)$$

where $g_U \simeq 0.7$ is the unified gauge coupling at the unification scale $M_U \simeq 2 \times 10^{16}$ GeV. (The gauge couplings in both Eqs. (5) and (7) are the holomorphic gauge couplings.) The explicit correspondence between the two theories is given by

$$\langle T \rangle \leftrightarrow \frac{1}{g_U^2}, \quad (8)$$

$$\langle M \rangle \leftrightarrow \frac{3c}{32\pi^2} \ln\frac{M_U}{M_c}, \quad (9)$$

and the relation among the low-energy gauge couplings by

$$\frac{1}{g_3^2} = \frac{12}{7}\frac{1}{g_2^2} - \frac{5}{7}\frac{1}{g_1^2}. \quad (10)$$

The relation of Eq. (10) is, in fact, renormalization group invariant and well reproduces the observed QCD coupling, g_3, in terms of the electroweak gauge couplings, g_1 and g_2, at m_Z. The correspondence of Eq. (9) implies that M should be stabilized with $\langle M \rangle$ taking a positive value of $O(1)$. This is not entirely trivial to achieve and will be discussed in the next subsection.

In general, if we assign the R charge of $+2/3$ for all the (charged) chiral superfields in the theory, we always obtain the relation $A_I = -b_I/3$ for any gauge group I present in the theory. Then, assuming that the mixed anomalies for the R symmetry are canceled by the (generalized) Green-Schwarz mechanism with a single modulus M, as in Eq. (4), we always obtain the correspondence between the threshold effects from $\langle M \rangle$ and the running effect, given by Eq. (9). This originates from the relation between R and dilatation symmetries in supersymmetric theories, although the R symmetry considered here is not the exact supersymmetric partner of the (broken) dilatation symmetry.[4] It is fortunate that the Yukawa couplings are dimen-

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[4]Here we consider an R symmetry that is not the exact supersymmetric partner of the (broken) dilatation symmetry but is a (unbroken) linear combination of it with some other $U(1)$ symmetry. The dependence of the superspace density, or the Kähler potential, on the M field is then "arbitrary" in the effective field theory; in particular, the $U(1)_R$ gauge field V_R and the combination $-(8\pi^2/c)(M + M^\dagger)$ can be used interchangeably as far as $U(1)_R$ gauge invariance is concerned. Here we consider the case that the M field appears simply in the gauge kinetic functions to cancel the mixed anomalies, so that its VEV leads to the large threshold effects without (much) affecting the superspace density. In particular, we assume that $U(1)_R$ gauge invariance of the (observable sector) superspace density is recovered by the appropriate appearance of the $U(1)_R$ gauge field V_R, including the "anomalous" pieces. (Note that the cutoff scale is "charged" under a part of the supersymmetric $U(1)_R$ gauge symmetry.) This assumption should ultimately be understood in the ultraviolet theory above M_* (or it perhaps suggests a certain structure for the ultraviolet theory), but it is a stable assumption in the framework of effective field theory.

sionless and thus allow this particular R charge assignment. (Other possible charge assignments, preserving the unification prediction, will be discussed briefly in Sec. V.)

The prediction of Eq. (10) receives corrections at higher orders if we take the g_I's to be the canonically defined gauge couplings. In particular, the prediction in our framework differs from that in the desert picture at higher orders, since some of the two-loop running effects between M_U and M_c are absent in our case. (Part of the effects are retained through rescaling anomalies associated with the gauge multiplets [21].) The difference, however, is small, of $O(1/8\pi^2)$, and is the same size as the effect arising from incalculable, nonuniversal corrections to Eq. (4).

Similar dynamics relating the low-energy gauge couplings to chiral anomalies were considered earlier in the context of string theory. A pseudoanomalous $U(1)$ gauge symmetry was considered in Ref. [22] to obtain the weak mixing angle without grand unification, through the universal nature of the mixed gauge anomalies in weakly coupled heterotic string theory. A pseudoanomalous gauge symmetry with nonuniversal mixed anomalies was considered in Ref. [14] in the context of (more general) string theory, in an attempt to lower the string scale (mainly) to an intermediate scale, although a proper implementation of the dynamics was not successfully realized. Here we present a viable and realistic effective field theory framework, in which the fundamental scale can be lowered to the $(10 \sim 100)$ TeV region, preserving automatically the successful supersymmetric prediction for gauge coupling unification. As we have seen and will see in more detail in the next subsection, this provides nontrivial constraints on physics associated with M, e.g. the transformation property and the stabilization dynamics.

An important consequence of the present way of obtaining the prediction for the low-energy gauge couplings is that we cannot write a direct mass term for the Higgs doublets, $W = \mu H_u H_d$, since it is forbidden by the (gauged) R symmetry. Here we have assumed that the M field does not appear in the tree-level superpotential, which may be justified in a theory above M_c. The simplest way to generate the Higgsino mass term, then, is to introduce a singlet field S which has a charge of $+2/3$ under the R symmetry. The most general superpotential is then given by

$$W = \lambda S H_u H_d + \frac{\kappa}{3} S^3 + W_{\text{Yukawa}}, \qquad (11)$$

where W_{Yukawa} is given by Eq. (1), and we have imposed the standard matter parity, or R parity, under which Q_i, U_i, D_i, L_i, and E_i are odd while H_u, H_d, and S are even. It is interesting that we are naturally led to the form of the superpotential of the next-to-minimal supersymmetric standard model. The Higgsino mass then arises from the VEV of S, which should be generated through supersymmetry breaking. It is also interesting that higher-dimension operators in the superpotential, such as the ones leading to rapid proton decay and large Majorana neutrino masses,

are suppressed by the R symmetry.[5] (The possibility of generating mass terms, e.g. $W = \mu H_u H_d$, without using a singlet VEV will be discussed in the next subsection.)

Since the fundamental scale in our framework, M_*, is of order $10 \sim 100$ TeV, the couplings λ and κ appearing in Eq. (11) are not subject to the Landau pole constraint up to the unification scale M_U. This allows us to have large couplings, e.g. $\lambda \lesssim 2$ and $\kappa \lesssim 1$, at the weak scale, which in turn allows us to have a large mass for the lightest Higgs boson, reducing fine-tuning. In fact, it has recently been shown that a large value of λ can eliminate fine-tuning in electroweak symmetry breaking while naturally preserving consistency with the precision electroweak data, because of extra contributions to the electroweak T parameter coming from the Higgs boson and Higgsino states [9]. For fine-tuning to be really eliminated, however, it is also necessary that there is no large logarithm between the weak scale and the scale where the superparticle masses are generated. Our framework also addresses this issue. Since supersymmetry will be broken and mediated at the scale $\approx M_c$ (or M_*), there is no large logarithm between the mediation scale and the weak scale. The explicit pattern of supersymmetry breaking masses, and thus the form of the Higgs potential, depends on how supersymmetry is broken. In fact, there are many possible ways to incorporate supersymmetry breaking in our framework, and some of them will be discussed in the next section. An explicit analysis of electroweak symmetry breaking in some of these supersymmetry breaking scenarios will be given in a separate publication [23].

We finally discuss physics associated with the R symmetry and the moduli fields T and M. If the gauged R symmetry is a continuous $U(1)_R$ symmetry, a nonvanishing and positive Fayet-Iliopoulos term of $O(M_{\text{Pl}}^2)$ will be generated for $U(1)_R$ [24]. Here, M_{Pl} appears because the observed gravitational scale is large, $M_{\text{Pl}} \simeq 10^{18}$ GeV, which may arise from the fact that gravity propagates in (large) spatial dimensions in which the MSSM states do not propagate [10]. We assume that this term is canceled by the VEV of some field ϕ that has a negative charge under $U(1)_R$.[6] The kinetic term for this field must be enhanced by

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[5]It will be necessary to have a constant term in the superpotential to cancel the cosmological constant after supersymmetry breaking, which can be regarded as a soft breaking term of the R symmetry (arising dynamically). This term can affect the form of the Kähler potential but not that of the superpotential, because of the supersymmetric nonrenormalization theorem, and our discussions are not affected by its existence. The term, however, may affect the mass of the light pseudo-Goldstone boson state, which could arise from spontaneous R symmetry breaking occurring associated with supersymmetry breaking. The constant term in the superpotential will be discussed further in Sec. III.

[6]Neither the MSSM scalars nor the S scalar will obtain (disastrously large) VEVs from the $U(1)_R$ D-term, since they all have an $U(1)_R$ charge of $+2/3$.

$M_{\rm Pl}^2/M_*^2$ so that the ϕ VEV does not far exceed the fundamental scale. Such an enhancement occurs if ϕ propagates in the same spacetime dimensions as the gravitational multiplet. The $U(1)_R$ gauge supermultiplet then becomes massive, absorbing the ϕ supermultiplet. The generated mass is only of order M_*, because the $U(1)_R$ gauge coupling must also receive a volume suppression of order $M_*/M_{\rm Pl}$ and the $U(1)_R$ gauge boson mass is given by the product of the $U(1)_R$ gauge coupling and the canonically normalized ϕ VEV. (These issues will be elaborated on further in the next subsection.) Depending on the R charge of ϕ, an unbroken discrete R symmetry may remain at low energies.[7]

Supersymmetry breaking will also contribute to $U(1)_R$ breaking, because the gaugino masses violate $U(1)_R$, although by a much smaller amount than that from the ϕ (or M) VEV. A potential R axion from supersymmetry breaking will obtain a mass from operators involving the ϕ VEV, or from a (effective) constant term in the superpotential that should arise as a soft symmetry breaking term of $U(1)_R$ (see footnote 5). Masses for the T and M fields can be generated, i.e. their VEVs can be stabilized, through couplings of these fields to a gauge group(s) other than that in the standard model. This issue will be studied further in the next subsection.

The story will be similar in the case that the gauged R symmetry is a discrete $Z_{N,R}$ symmetry, except for the issues related to the $U(1)_R$ gauge multiplet and the D-term potential. The $Z_{N,R}$ symmetry will be spontaneously broken through supersymmetry breaking (to the $Z_{2,R}$ subgroup), and the moduli fields T and M can still be stabilized by some gauge dynamics.

We note that all the VEVs and masses appearing in the analysis above can stay within the regime in which the effective field theoretic description is applicable (see the next subsection for more details). Despite the apparent appearance of the scale $M_{\rm Pl}$ ($\gg M_*$), no knowledge about physics above M_* is required to describe the phenomena discussed above.

B. Stabilizing moduli: producing the effective desert

As we have seen in the previous subsection, it is crucial that the M field can be stabilized with $\langle M \rangle$ taking a positive value $\langle M \rangle/c \simeq 0.25$ [see Eq. (9)]. The T field should also be stabilized with $\langle T \rangle \simeq 2$ [see Eq. (8)]. In this subsection

we discuss the issue of stabilization of these fields, concentrating on the case that the gauged R symmetry is a continuous $U(1)_R$ symmetry.

For definiteness, we consider the case that the M field does not propagate in large gravitational extra dimensions. (The case in which it does can be treated similarly.) In our convention, the M field is dimensionless. The Kähler potential for this field is then given by

$$ K = M_*^2 \mathcal{F}\Big(M + M^\dagger + \frac{c}{8\pi^2} V_R\Big), \qquad (12) $$

where V_R is the $U(1)_R$ gauge supermultiplet and $\mathcal{F}(x)$ is an arbitrary polynomial in x with the coefficients of $O(1)$ up to symmetry factors. The origin of M, $M = 0$, is chosen such that the standard model gauge kinetic functions take the form of Eq. (4). The form of Eq. (12) immediately tells us that c cannot be much larger than of order unity, since then the required value of $\langle M \rangle \simeq 0.25c$ would exceed $\simeq 1$, going outside the regime of effective field theory.

Let us first address the issue of the stabilization of T. In general, the stabilization of T is related dynamically to that of M, leading to a complicated potential minimization problem. It is, however, possible that the two dynamics are practically decoupled. The most straightforward way to realize that is to consider a gauge group(s) G which does not have a mixed anomaly with $U(1)_R$. This can be easily arranged by assigning appropriate $U(1)_R$ charges for the fields that transform nontrivially under G. We can then use conventional mechanisms for dilaton stabilization to stabilize T. For example, we can adopt one of the models discussed in Ref. [25], which do not violate $U(1)_R$ invariance. Alternatively, the stabilization of M can be much stronger than that of T, in which case the stabilization of T can be analyzed independently from that of M, after M is fixed. We thus focus only on the stabilization of M below, assuming that T is independently stabilized.[8]

We consider the possibility that the M field is stabilized strongly without using supersymmetry breaking effects. This requires that the superpotential contains the effect of $U(1)_R$ breaking, since otherwise the M dependence of the

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[7]An interesting alternative would be to cancel the Fayet-Iliopoulos term by the VEV of M without introducing the ϕ field, which is *a priori* possible if the kinetic term of M is enhanced by the factor of $M_{\rm Pl}^2/M_*^2$. This is because the kinetic term of M takes the form $\propto \int d^4\theta (M + M^\dagger + (c/8\pi^2)V_R)^2 + \cdots$, where V_R is the $U(1)_R$ gauge supermultiplet and thus gives a term linear in M in the auxiliary component of V_R. This, however, fixes the VEV of M such that $c\langle {\rm Re} M \rangle < 0$, leading to a wrong prediction of $\ln(M_U/M_c) < 0$ in the correspondence of Eq. (9).

[8]An interesting, alternative possibility is to introduce extra vectorlike matter states that are neutral under $U(1)_R$ and charged under the standard model gauge group. These states then give extra contributions to the anomaly coefficients A_I in Eq. (2). Assuming that they have the quantum numbers of 2 pairs of **5** + **5*** of $SU(5) \supset SU(3)_C \times SU(2)_L \times U(1)_Y$, the extra contributions are $\delta A_I = -2$, leading to $(A_1, A_2, A_3) = (-21/5, -7/3, -1)$. This can give the observed values of the standard model gauge couplings for $\langle M \rangle/c \simeq 0.25$, without ever introducing the $T \mathcal{W}_I^{a\alpha} \mathcal{W}_{I\alpha}^a$ term. Masses for the extra states of order the weak scale or somewhat larger can be generated through Kähler potential terms (see discussions in Sec. IV). While this possibility is not realized in the explicit models of Sec. IV, where T always appears as a field parameterizing the size of an extra dimension(s), it may be a viable option if the theory just below M_* is 4D (other than the gravitational dimensions).

superpotential is completely fixed by $U(1)_R$, which does not allow the strong stabilization of M. How does the effect of $U(1)_R$ breaking appear in the superpotential? It can appear through the VEV of the field ϕ that absorbs the large Fayet-Iliopoulos term of $U(1)_R$: $\xi \simeq 2M_{\rm Pl}^2$. For the ϕ field to be able to absorb ξ, ϕ must propagate in large gravitational extra dimensions. This is because if ϕ does not propagate in these dimensions, the ϕ VEV is simply bounded as $\langle\phi\rangle \lesssim M_*$ in the effective field theory, so that it cannot absorb $\xi \gg M_*^2$. On the other hand, if ϕ propagates in the large gravitational dimensions, its kinetic term is enhanced by the volume factor $M_{\rm Pl}^2/M_*^2$, so that the canonically normalized 4D (zero-mode) field $\hat{\phi} \equiv (M_{\rm Pl}/M_*)\phi$ can take a VEV as large as $M_{\rm Pl}$ and thus can absorb the large Fayet-Iliopoulos term of $O(M_{\rm Pl}^2)$. Note that the $U(1)_R$ D-term potential is then given by

$$V_D = \frac{g_R^2}{2}\frac{1}{(1-|\hat{\phi}|^2/3M_{\rm Pl}^2)^2}\Big\{\xi + \Big(r_\phi - \frac{2}{3}\Big)|\hat{\phi}|^2\Big\}^2, \quad (13)$$

where r_ϕ (< 0) is the $U(1)_R$ charge of the ϕ field [26].[9] Here, we have assumed the minimal form of the superspace density for ϕ, and g_R is the 4D $U(1)_R$ gauge coupling, which receives a volume suppression of order $M_*/M_{\rm Pl}$. The generated $U(1)_R$ gauge boson mass is of order $g_R\langle\hat{\phi}\rangle \lesssim M_*$, which is compatible with the effective field theory treatment of the dynamics.

We now present an explicit example of a model stabilizing the M field with the correct value of $\langle M\rangle$. We consider a supersymmetric $SU(2)$ gauge theory with four "quark" chiral superfields Q_i ($i = 1, \cdots, 4$). We assume that the $U(1)_R$ charge of Q_i's is universal, which we denote by r. The mixed $U(1)_R$-$SU(2)^2$ gauge anomaly coefficient is then given by $A = 4(1/2)(r-1) + 2 = 2r$, and the gauge kinetic function for $SU(2)$ is given by

$$\mathcal{L} = \int d^2\theta\Big(\frac{1}{4g_0^2} - \frac{A}{c}M\Big)\mathcal{W}^{a\alpha}\mathcal{W}_\alpha^a + \text{H.c.}, \quad (14)$$

where $\mathcal{W}^{a\alpha}$ is the field strength superfield for $SU(2)$ with a representing the adjoint index. The tree-level term $1/g_0^2$ may come from the VEV of some moduli field, which may or may not be T, and which we assume to be stabilized independently with M. The values of g_0 can be naturally of $O(1)$, as for the standard model gauge group.

There are six gauge-invariant meson operators constructed out of Q_i, which can be decomposed into a 5-plet $(QQ)_m$ ($m = 1, \cdots, 5$) and a singlet (QQ) under the $SP(4)$ subgroup of the flavor $SU(4)$ symmetry. Nonperturbative $SU(2)$ dynamics induce VEVs for these operators $(QQ)_m^2 + (QQ)^2 = \Lambda^4$, where Λ is the dynamical scale of $SU(2)$ [27]. We now introduce the superpotential term $W = kZ_m(QQ)_m$, where Z_m is a

$SU(2)$-singlet chiral superfield and k a coupling constant. This leads to $\langle(QQ)_m\rangle = 0$ and $\langle(QQ)\rangle = \Lambda^2$, which can be used as a general scale generation mechanism through the (QQ) operator [28]. For a sufficiently large value of k, this does not disturb possible other dynamics associated with (QQ).

We now use the dynamics described above to stabilize M. We assume that the ϕ field, which absorbs the large Fayet-Iliopoulos term, has an $U(1)_R$ charge of $-2r$. We then introduce the superpotential

$$W = \eta X\Big(\frac{\hat{\phi}}{M_{\rm Pl}}(QQ) - M_{\tilde{X}}^2\Big), \quad (15)$$

where X is a chiral superfield with an $U(1)_R$ charge of $+2$, $\hat{\phi}$ is the (canonically normalized) 4D mode of ϕ, and η and $M_{\tilde{X}}^2$ are coefficients of $O(1)$ and $O(M_*^2)$, respectively. Here, the X field, as well as the $SU(2)$ sector, are supposed not to propagate in the gravitational dimensions, and the $M_{\rm Pl}$ suppression in the first term in the bracket arises from the large volume factor associated with the ϕ field. As discussed above, the $SU(2)$ dynamics effectively replaces (QQ) with the square of the dynamical scale Λ, which in turn is given by

$$\Lambda = M_*e^{8\pi^2/bg^2} = M_*e^{-2\pi^2((1/g_0^2)-8r(M/c))}, \quad (16)$$

where $b = -4$ is the beta-function coefficient for $SU(2)$, and $1/g^2 = 1/g_0^2 - 4AM/c$ is the inverse-squared $SU(2)$ gauge coupling at the scale M_*. We then obtain the supersymmetric minimum from the vanishing of the D- and F-term potential, given by Eqs. (13) and (15). In particular, the vanishing of $F_X = -(\partial W/\partial X)^*$ leads to $\exp[-4\pi^2(1/g_0^2 - 8rM/c)] = (M_{\tilde{X}}^2/M_*^2)(M_{\rm Pl}/\langle\hat{\phi}\rangle) = O(1)$, giving

$$\frac{\langle M\rangle}{c} = \frac{1}{8rg_0^2} + O\Big(\frac{1}{32\pi^2r}\Big), \quad (17)$$

which stabilizes M. The VEVs of the other fields are given by $\langle X\rangle = 0$ and $\langle\hat{\phi}\rangle = O(M_{\rm Pl})$, and the masses of the excitations around the minimum are all of order M_*, implying that the stabilization can be very strong. The result obtained here, including the value of $\langle M\rangle$ given in Eq. (17), is not affected if we replace the first term in the bracket of Eq. (15) by an arbitrary function of the $U(1)_R$-invariant combination $(\hat{\phi}/M_{\rm Pl})(QQ)$.

We find from Eq. (17) that the phenomenologically required value of $\langle M\rangle/c \simeq 0.25$ can be obtained with a natural choice of parameters, $rg_0^2 \simeq 0.5$. In particular, having $\langle M\rangle/c = O(1)$ is quite natural in the present stabilization mechanism. This implies that the apparent closeness of the unification scale M_U and the gravitational scale $M_{\rm Pl}$ can be naturally explained in the present context. For example, if $g_0 \simeq g_U$ [e.g. due to certain unification of $SU(2)$ with the standard model gauge group at or above M_*], then a natural choice of $r \simeq 1$ leads to the correct value for the apparent unification scale, $M_U \simeq 2 \times 10^{16}\,\text{GeV}$, in

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[9]The $U(1)_R$ charge of the ϕ field, r_ϕ, should be negative in order for the graviton kinetic term to have the correct sign at the vacuum.

Eq. (9). In fact, the origin of this desired property is very simple and general. Let us imagine that the gauge coupling of the stabilizing gauge group G, as well as those of the standard model gauge group, are given by the sum of $O(1)$ contributions and the contributions from $\langle M \rangle$. Then, if there is a superpotential interaction relating the dynamical scale of G with some scale of order M_* [as in Eq. (15)], the value of $\langle M \rangle/c$ is fixed to be of order unity, which in turn implies that the apparent unification scale is hierarchically larger (or smaller) than the weak scale. The gauge coupling of the stabilizing group G is generically large at M_*, but can still stay within the field theory regime, for example, by taking the relevant mass parameter in the superpotential [M_X in Eq. (15)] somewhat smaller than M_*.

We now discuss the robustness of our Higgs sector superpotential in Eq. (11), and more generally the implication of $U(1)_R$ on the form of the observable sector superpotential. Since $U(1)_R$ is broken strongly by the VEV of ϕ, one might think that $U(1)_R$ invariance does not give any constraint on the form of the superpotential. However, since the superpotential is holomorphic in fields and the $U(1)_R$ charge of ϕ is negative, we find that no linear or quadratic term can appear in the observable sector superpotential through the VEV of ϕ. The form of the Higgs sector superpotential, Eq. (11), is thus robust at the renormalizable level. On the other hand, higher-dimension operators can in general be induced through the ϕ VEV. For example, if we choose $r = 1$ in the example of the M stabilization discussed above, the $U(1)_R$ charge of ϕ becomes -2, allowing e.g. the operator $W = k(LH_u)^2 H_u H_d/M_*^3$, which leads to Majorana masses for the observed neutrinos of $O(0.1 \text{ eV})$ for $k \sim 10^{-3}$ and $M_* \sim 100$ TeV. (Possible proton decay operators, e.g. $W \sim QQQLH_u H_d$, should somehow be forbidden.) The existence of higher-dimension operators, however, is model dependent. For example, if we choose the $U(1)_R$ charge of ϕ to be irrational, then no higher-dimension operators are induced in the observable sector superpotential through the ϕ VEV.

The argument given above does not entirely exclude the possibility of having linear and/or quadratic terms in the observable sector superpotential. For example, we can consider a supersymmetric $SU(2)$ gauge sector which has an identical structure to that used above in stabilizing M, but with the $U(1)_R$ charge of the quark fields $\mathcal{Q}_i$ fixed to be $+1/3$. This can lead to a mass term for the Higgs doublet $\mu \sim \Lambda'^2/M_*$ through the tree-level superpotential coupling $W \sim (\mathcal{Q}\,\mathcal{Q})H_u H_d/M_*$, where Λ' is the dynamical scale of $SU(2)$. By choosing Λ' and/or the coefficient of the superpotential operator appropriately, this allows us to reproduce the weak scale supersymmetric mass for the Higgs doublets without introducing a singlet field. The dynamics described here, in fact, can also be applied in a theory with a singlet field S. In this case, we obtain the Higgs sector superpotential $W_{\text{Higgs}} = \lambda S H_u H_d + \mu H_u H_d +$

$L_S^2 S + (M_S/2)S^2 + (\kappa/3)S^3$, where the second, third, and forth terms arise from couplings to a $SU(2)$ gauge-invariant operator $(\mathcal{Q}\,\mathcal{Q})$. An interesting property of this theory is that the mass parameters appearing in the superpotential are naturally of the same order, $\mu \sim L_S \sim M_S \sim \Lambda'^2/M_*$, which can be taken to be of order the weak scale by appropriately choosing the value of Λ'. This, therefore, can be used to realize a general λSUSY setup discussed in Ref. [9]. In the rest of the paper, however, we focus for simplicity on the case of Eq. (11), which does not require additional dynamics generating dimensionful parameters in the observable sector superpotential.

III. PHENOMENOLOGICAL IMPLICATIONS

In this section we discuss general phenomenological implications of the framework described in the previous section.

A. Supersymmetry breaking

There are in general many possible ways to incorporate supersymmetry breaking in our framework. Here we identify several sources of supersymmetry breaking, intrinsic to our setup. We study the resulting superparticle spectrum and its phenomenological implications.

In general, any field that is singlet under the standard model gauge group has the potential to provide supersymmetry breaking effects in the observable sector, through its auxiliary component VEV. In our context, natural candidates are given by the VEVs of the auxiliary components of the T and M supermultiplets, F_T and F_M. Nonvanishing values for F_T and F_M can be generated through the stabilization mechanisms of T and M. Here we study their phenomenological implications without specifying explicit dynamics generating these VEVs. For earlier related studies, see e.g. [29].

The couplings of the T and M superfields to the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge multiplets are given by Eq. (4). This gives a definite prediction for the gaugino masses. At the scale M_c, the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gaugino masses, M_I ($I = 1, 2, 3$), are given by

$$\frac{M_I}{g_I^2} = -\frac{1}{2}F_T - \frac{2b_I}{3c}F_M, \qquad (18)$$

where g_I are the standard model gauge couplings at M_c, and we have used Eq. (6). In fact, since M_I/g_I^2 are renormalization group invariants, the gaugino masses at an arbitrary scale μ_R are given by Eq. (18) with g_I interpreted as the standard model gauge couplings at the scale μ_R. In the limit $F_M \to 0$, these gaugino masses reproduce the ones arising from the standard "unified gaugino mass assumption": $M_I \propto g_I^2$. In the other extreme limit of $F_T \to 0$, the gaugino masses satisfy $M_I \propto b_I g_I^2$, the same relation as that in anomaly mediated supersymmetry breaking [30]. The effects of real anomaly mediation in the present context will be discussed later.

Since the three gaugino masses, M_I, are determined by two free parameters F_T and F_M/c, we have one relation among them:

$$\frac{M_3}{g_3^2} = \frac{12}{7}\frac{M_2}{g_2^2} - \frac{5}{7}\frac{M_1}{g_1^2}, \qquad (19)$$

regardless of the values of F_T and F_M/c, where we have used $(b_1, b_2, b_3) = (33/5, 1, -3)$. The ratios between two of the M_I's, e.g. M_2 and M_3, depend on the ratio between F_T and F_M/c. An interesting property for the gaugino masses in Eq. (18) [or Eq. (19)] which appears if F_T and F_M are real (more precisely, if the complex phases of F_T and F_M are the same) is that when these masses, as well as the gauge couplings, are extrapolated naïvely to high or low energies using the MSSM renormalization group equations, the three gaugino masses M_I (not M_I/g_I^2) meet at a point at some (fictitious) energy scale. The scale where they meet depends on the ratio between F_T and F_M/c, and is given by

$$M_U^\lambda = M_U \exp\left(-\frac{8\pi}{3\alpha_U}\frac{F_M}{cF_T}\right), \qquad (20)$$

where M_U is the conventional gauge coupling unification scale, $M_U \simeq 2 \times 10^{16}$ GeV, and $\alpha_U \equiv g_U^2/4\pi$ the unified gauge coupling strength, $\alpha_U \simeq 1/24$. From a purely low-energy point of view, this phenomenon is reminiscent of that in mixed moduli-anomaly mediated supersymmetry breaking [31], although the underlying physics picture is very different. For $F_T F_M > 0$ (<0), the *effective gaugino mass unification scale*, M_U^λ, is below (above) the effective gauge coupling unification scale M_U. In particular, for $F_T/(F_M/c) \gtrsim 6$, M_U^λ is between M_U and the weak scale ≈ 100 GeV. Note, however, that from a theoretical point of view there is no particular reason that M_U^λ must be in this region.

The couplings of the T and M superfields to the matter and Higgs superfields are not determined within the effective field theory below M_c. In Sec. IV we give models in which T is identified as the radion superfield associated with an extra dimension(s) in which the standard model gauge fields propagate. The couplings of T to the matter and Higgs fields then depend on the wave function profiles of these fields in the extra dimension(s), as well as the higher-dimensional spacetime curvature. The couplings of M can also contain arbitrary functions of $M + M^\dagger + (c/8\pi^2)V_R$ in the effective theory. The issue of supersymmetric flavor changing neutral currents should thus be addressed in a theory at or above M_c. The models in Sec. IV provide examples of such a framework. An alternative possibility is to consider some flavor symmetry, ensuring flavor universality for the squark and slepton masses.

Another interesting source of supersymmetry breaking in our framework comes from a possible nondecoupling $U(1)_R$ D term. (For earlier work on supersymmetry breaking in a theory with gauged $U(1)_R$, see e.g. [32].) Suppose,

for example, that the field canceling the $U(1)_R$ Fayet-Iliopoulos term, ϕ, has a supersymmetry breaking mass squared m^2 of order the weak scale or somewhat larger (in the basis where ϕ is canonically normalized). Such a mass can arise from the VEV of F_T (and/or F_M) if the T (and/or M) field propagates in the large gravitational dimensions, in which case the size of m can naturally be of the same order as other supersymmetry breaking masses arising from F_T (and/or F_M). In this case, the minimization of the potential leads to a nonvanishing D-term VEV for $U(1)_R$, $D_R = O(m^2)$, *regardless of the value of the $U(1)_R$ gauge coupling*. This gives a supersymmetry breaking squared mass of $(r_i - 2/3)(-D_R)$ to a scalar field that has an $U(1)_R$ charge of r_i through the $U(1)_R$ D-term potential ($-D_R$ is positive in our notation). Since all the quark, lepton, and Higgs superfields have an $U(1)_R$ charge of $+2/3$, however, this contribution is absent in our theory.[10] A nonvanishing contribution may arise if there are direct couplings of the form $\int d^4\theta(\phi^\dagger e^{2r_\phi V_R}\phi) \times (Q_i^\dagger e^{4V_R/3}Q_j)$ in the superspace density, where Q_i represents generic quark, lepton, and Higgs chiral superfields. Since these couplings are not flavor universal in general, we may need to impose a nontrivial flavor symmetry if they give nonnegligible contributions.

A nonvanishing $U(1)_R$ D-term VEV also gives a contribution of $(2/3)\gamma_i D_R$ to the scalar squared masses, since the cutoff is charged under a part of the supersymmetric $U(1)_R$ gauge symmetry [i.e. we must include terms involving V_R to cancel anomalous variations of the superspace density under $U(1)_R$ transformations]. Here, γ_i represents the anomalous dimension of Q_i, defined by $d\ln Z_i/d\ln\mu_R \equiv -2\gamma_i$ with Z_i the wave function renormalization for Q_i. This gives a positive and approximately flavor universal contribution to the first two generation squark and slepton squared masses (at the scale $\approx M_c$), which becomes important if the value of $\sqrt{|D_R|}$ is somewhat larger than the weak scale and if the direct couplings between ϕ and Q_i are suppressed in the superspace density, e.g. by locality in an extra dimension.[11] While this contribution leads to some amount of flavor violation, especially in the top squark sector, it is sufficiently small. Note that the contributions from a nonvanishing $U(1)_R$ D-term VEV discussed above preserve the gaugino mass prediction of Eq. (19).

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[10]The theory allows us to write a kinetic mixing term between the $U(1)_R$ and $U(1)_Y$ gauge fields at tree level. The supersymmetry breaking squared masses for the scalars can then obtain contributions proportional to their $U(1)_Y$ hypercharges. If the mixing term has an $O(1)$ coefficient in the basis where the gauge couplings appear in front of the kinetic terms, these contributions can be of order m^2. Note that a coefficient of $O(1)$ is phenomenologically harmless, since this term is suppressed by $g_R = O(M_*/M_{\rm Pl})$ in the basis where the gauge fields are canonically normalized.

[11]In fact, this contribution can naturally be the dominant one if we do not introduce ϕ from the beginning, since then $D_R = O(M_*^2)$. In that case, however, we must come up with an alternative model for the M stabilization.

The contribution from anomaly mediation [30] can also be sizable if the T (and/or M) field propagates in the large gravitational dimensions. Suppose that (one of) the dominant contribution to observable sector supersymmetry breaking comes from F_T, and that T propagates in all the gravitational dimensions. This is indeed the case if T parameterizes the size of an extra dimension(s), as in the models discussed in the next section. Let us now consider the 4D effective theory obtained after integrating out all the extra dimensions. In this theory, the positive contribution to the vacuum energy from supersymmetry breaking δV is of order $F_T^2 M_{\rm Pl}^2$, which is canceled by a (effective) constant term in the superpotential $\langle W \rangle = O(F_T M_{\rm Pl}^2)$, where F_T is of order the weak scale. (Both F_T and $\langle W \rangle$ should arise from dynamical breaking of the R symmetry.) Note that these values of δV and $\langle W \rangle$ ($\delta V \gg M_*^4$ and $\langle W \rangle \gg M_*^3$) are consistent with the effective field theory treatment of the dynamics, as the apparent large scales arise simply from the large volume factor associated with the large gravitational dimensions. This implies that the effective bulk cosmological constant is negative before supersymmetry breaking. The F-term VEV of a chiral compensator field F_C, which controls the size of anomaly mediation, depends on the mechanism of T stabilization, but it typically takes a value in the range $F_T \lesssim F_C \lesssim 8\pi^2 F_T$. (For analyses in the context of the conventional desert framework, see e.g. [33].) Since the anomaly mediated contribution to the observable sector superparticle masses, $m_{\rm AMSB}$, is of order $F_C/8\pi^2$, we obtain

$$\frac{F_T}{8\pi^2} \lesssim m_{\rm AMSB} \lesssim F_T. \qquad (21)$$

We thus find that the contribution from anomaly mediation can be comparable to that from F_T (and $\sqrt{|D_R|}$) in the present framework.[12] An interesting point is that since the anomaly mediated contribution satisfies $M_I \propto b_I g_I^2$, it does not destroy the gaugino mass prediction of Eq. (19). [F_M/c in Eq. (20) should be replaced by $F_M/c + 3F_C/32\pi^2$.] The gravitino mass is given by $m_{3/2} \simeq F_C$, which is typically in the range of $O(100\,{\rm GeV} \sim 10\,{\rm TeV})$. Note that if the dominant contribution to observable sector supersymmetry breaking comes from a field that does not propagate in the gravitational dimensions, the gravitino mass is very small $m_{3/2} = O(m_{\rm weak}^2/M_{\rm Pl})$, and the anomaly mediated contribution, $m_{\rm AMSB} \simeq m_{3/2}/8\pi^2$, is completely negligible.

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[12]It is possible that the dynamics stabilizing T is localized to a subspace in the direction of the gravitational dimensions. In this case the mass of the T field is of order $M_c^2/M_{\rm Pl} = O(0.01 \sim 10\,{\rm eV})$, because of the volume suppression factor associated with the gravitational dimensions. The wave function of this state can be (highly) nontrivial if the mass is close to the scale of the gravitational dimensions L^{-1}, which can occur if the number of these (flat) dimensions is two. On the other hand, the VEVs of T and F_T always tend to have constant profiles along these dimensions (in the flat space case), because of the associated kinetic energy. Therefore, the low-energy, or zero-energy, 4D consideration leading to Eq. (21) is still valid in this case.

We finally comment on other possibilities for supersymmetry breaking in our framework. We can consider that the dominant source of supersymmetry breaking comes from the auxiliary field VEV(s) of a chiral superfield(s) other than T and M. For example, we can consider a chiral superfield Z which couples to the standard model gauge fields as $\int d^2\theta Z \mathcal{W}_I^{a\alpha} \mathcal{W}_{I\alpha}^a$ with arbitrary coefficients for $SU(3)_C$, $SU(2)_L$, and $U(1)_Y$. (The lowest component VEV of Z should be small/vanishing in order not to contribute to the gauge couplings.) This can give arbitrary masses for the gauginos, which do not respect Eq. (19). In fact, this scenario can be naturally accommodated in a higher-dimensional scenario discussed in Sec. IV B. The couplings of Z to the matter and Higgs fields can be naturally suppressed, leading to the spectrum of gaugino mediation [34] but with a very low compactification scale $M_c = O(10 \sim 100\,{\rm TeV})$. The Higgs fields and (a part of) the third generation scalars may have different masses than the other scalars. An interesting property of this model is that the gauginos are significantly heavier than the scalar particles, typically by a factor of a few. We leave detailed studies of these and related possibilities for future work.

B. Gravity, proton decay, and neutrino masses

Since the fundamental scale of nature M_* is of order $M_c \approx (10 \sim 100)\,{\rm TeV}$ in our framework, suppressions of various operators and interactions must be explained without using energy scales larger than M_*. Here we list several possibilities for achieving this.

The weakness of gravity can be explained if there are large gravitational dimensions in which the MSSM or S states do not propagate [10]. Assuming that the sizes of these dimensions are (approximately) equal, we find $L^{-1} \approx (M_*^{2+n}/M_{\rm Pl}^2)^{1/n}$, giving $L^{-1} \approx 10^{-11} - 1\,{\rm GeV}$ ($L \approx 10^{-16} - 10^{-5}$ m) for $n = 2, \cdots, 6$ and $M_* = (10 \sim 100)\,{\rm TeV}$. Here, n is the number of extra gravitational dimensions, and we have assumed that the extra space is flat. (In the case that supersymmetry is broken by the auxiliary field VEV of a bulk supermultiplet, such as a radion, this implies that the bulk cosmological constant is negative in the limit of unbroken supersymmetry, which is canceled by a positive contribution from bulk supersymmetry breaking.) Dimensions of these sizes are not constrained by the existing submillimeter gravitational experiments, although there are astrophysical constraints for $n = 2$ [35].

Proton decay should be suppressed much more strongly than what is naïvely expected based on the scale M_*. In Sec. IV, we present models above M_c based on higher-dimensional unified field theories. In these models there exist KK states for the unified gauge fields and colored Higgs multiplets, and the requirement that proton decay should not be caused by the exchange of these states gives nontrivial constraints on the structure of the models. There are also possible tree-level proton decay operators. These

can be suppressed if the quark and lepton supermultiplets are localized at different positions in extra dimensions. The relevant dimension for the separation can be one of the large gravitational dimensions as originally considered in [18], but can also be a dimension of $O(M_c^{-1})$, orthogonal to the ones used in explaining the weakness of gravity. A model accommodating such a possibility will be presented in Sec. IV B. Alternative ways of suppressing these operators include imposing an appropriate (gauged) discrete symmetry, or a continuous gauged baryon and/or lepton number broken on a distant brane.

Small neutrino masses can be generated by introducing right-handed neutrino superfields N having an R charge of $+2/3$. Note that dangerous superpotential operators $W \sim (LH_u)^2$, giving too large Majorana neutrino masses, are forbidden by the R symmetry, and they are not regenerated unless there is a nonvanishing VEV carrying an R charge of $-2/3$ (assuming that the M field does not appear in the superpotential). If the N fields propagate in (a part of) the large gravitational dimensions, the 4D neutrino Yukawa couplings $W \sim LNH_u$ are suppressed by a factor of $(M_*/M_{\mathrm{Pl}})^{m/n}$, where m is the number of dimensions in which N propagates, giving naturally small Dirac neutrino masses [36]. For example, a neutrino mass of $O(0.01 \sim 0.1 \text{ eV})$ relevant for atmospheric neutrino oscillations is naturally obtained for $m = n$ and $M_* \simeq 100$ TeV. Alternatively, small Majorana neutrino masses may be generated from higher-dimension operators, such as $W \sim (LH_u)^2 H_u H_d$, through the ϕ VEV, as discussed in Sec. II B.

In general, large mass scales can be obtained effectively for any higher-dimension operators by making the relevant field(s) propagate in large extra dimensions. For example, the QCD axion can be obtained by coupling the axion superfield Φ to the $SU(3)_C$ gauge field as $\int d^2\theta \Phi \mathcal{W}_3^{a\alpha} \mathcal{W}_{3\alpha}^a$ and making Φ propagate in (a part of) the large gravitational bulk [37]. The (effective) axion decay constant is then given by $f_a \approx M_*(M_{\mathrm{Pl}}/M_*)^{m/n}$, where m is the number of dimensions in which Φ propagates.

With $M_* = O(10 \sim 100 \text{ TeV})$, most other higher-dimension operators are phenomenologically harmless. The operators leading to flavor changing neutral currents, however, also need some suppressions. For instance, the coefficients of the operators leading to the K^0-$\bar{K}^0$ mixing must be smaller than of order $10^{-2}(M_*/100 \text{ TeV})^2$ in units of M_*. [The coefficients must be even smaller by a factor of ≈ 100 if they have $O(1)$ phases.] The origin of these suppressions is presumably related to the physics giving the Yukawa couplings. For example, we can consider the situation in which the wave function renormalization factors for lighter generation quarks and leptons are enhanced compared to the heavier ones. After canonically normalizing the fields, this leads to realistic Yukawa couplings as well as suppressions of flavor changing higher-dimension

operators. In fact, this situation can easily be realized if lighter generation quarks and leptons propagate in extra dimensions somewhat larger than M_*^{-1}. An alternative possibility to suppress flavor changing neutral currents is to impose a flavor symmetry whose breaking resides only in the Yukawa couplings.

IV. EXPLICIT MODELS

In this section we present possible theories above M_c which reproduce the effective theory below M_c discussed in the previous sections. In particular, we present models in which the structure of Eq. (4), especially the universal coupling of the standard model gauge fields to T, is naturally reproduced. Below we provide models based on 5D spacetime, in which the standard model gauge fields propagate. The existence of additional (orthogonal) large gravitational dimensions, however, should be understood as discussed in Sec. III B. It is also straightforward to extend these models to higher dimensions, which will be discussed.

A. Minimal model

Let us consider a 5D supersymmetric gauge theory. We consider that the fifth dimension, y, is compactified on an S^1/Z_2 orbifold, $0 \leq y \leq \pi R$, and that the gauge group in the bulk is $SU(5)$. We assume that the compactification radius is stabilized with $R^{-1} = O(10 \sim 100 \text{ TeV})$, which we typically take to be a factor of a few smaller than the fundamental scale M_*.

The 5D gauge supermultiplet can be decomposed into a 4D $N = 1$ vector superfield $V(A_\mu, \lambda)$ and a 4D $N = 1$ chiral superfield $\Sigma(\sigma + iA_5, \lambda')$, where both V and Σ are in the adjoint representation of $SU(5)$. We assume that these fields obey the following boundary conditions:

$$V : \begin{pmatrix} (+,+) & (+,+) & (+,+) & (-,+) & (-,+) \\ (+,+) & (+,+) & (+,+) & (-,+) & (-,+) \\ (+,+) & (+,+) & (+,+) & (-,+) & (-,+) \\ \hline (-,+) & (-,+) & (-,+) & (+,+) & (+,+) \\ (-,+) & (-,+) & (-,+) & (+,+) & (+,+) \end{pmatrix}, \quad (22)$$

$$\Sigma : \begin{pmatrix} (-,-) & (-,-) & (-,-) & (+,-) & (+,-) \\ (-,-) & (-,-) & (-,-) & (+,-) & (+,-) \\ (-,-) & (-,-) & (-,-) & (+,-) & (+,-) \\ \hline (+,-) & (+,-) & (+,-) & (-,-) & (-,-) \\ (+,-) & (+,-) & (+,-) & (-,-) & (-,-) \end{pmatrix}, \quad (23)$$

where $+$ and $-$ represent Neumann and Dirichlet bound-

ary conditions, respectively, and the first and second signs in parentheses represent boundary conditions at $y = 0$ and $y = \pi R$, respectively. This reduces the gauge symmetry at low energies to $SU(3) \times SU(2) \times U(1)$, which we identify as the standard model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ (321). The active gauge group on the $y = 0$ brane is 321, while that in all other places in the extra dimension is $SU(5)$ (see e.g. [38]). The typical mass scale for the KK towers is $R^{-1} = O(10 \sim 100 \text{ TeV})$, which we identify as M_c in the previous sections.

The gauge couplings for the low-energy 321 gauge fields, g_I ($I = 1, 2, 3$), receive contributions both from the bulk and brane gauge couplings. Here we assume that the brane contributions are small, giving only $O(1/8\pi^2)$ corrections to the inverse square couplings $1/g_I^2$. This assumption is technically natural, and may be justified from physics above M_*. The 321 gauge couplings at the scale $M_c = 1/R$ are then given by the bulk contribution. An important point is that this contribution is $SU(5)$ symmetric: $1/g_I^2 = \pi R/g_*^2$, where g_* is the 5D $SU(5)$ gauge coupling. Denoting the radion chiral superfield associated with the fifth dimension as T and appropriately choosing the normalization for this field, we find that we exactly reproduce the first term of Eq. (4). In particular, this explains the particular normalization for the coupling of $U(1)_Y$ gauge field to T.

We now gauge an $U(1)_R$ symmetry in this theory. Since $U(1)_R$ does not commute with the 5D Lorentz symmetry [$U(1)_R$ contains the subgroup of $SU(2)_R$ in the 5D supersymmetry algebra that commutes with the 4D Lorentz symmetry], gauging it is associated with breaking of the 5D Lorentz symmetry to the 4D one. Specifically, it will lead to a (small) nontrivial warping along the fifth dimension. Here we assume that the resulting warping is small such that we can treat our 5D spacetime approximately flat, which can be the case depending on the explicit 5D setup. [For an analysis of gauged $U(1)_R$ symmetries in 5D spacetime, see e.g. [39].] The case with nontrivial warping will be discussed in Sec. IV C.

In general, an $U(1)_R$ symmetry gauged in our system could have anomalous matter content. The resulting anomalies depend on the $U(1)_R$ charge assignment for matter, i.e. on what linear combination of the geometric $U(1)_R$ [the subgroup of $SU(2)_R$] and other "flavor" $U(1)$'s we gauge as our $U(1)_R$ gauge symmetry. To reproduce the effective theory of Sec. II below M_c, we introduce three generations of quark and lepton supermultiplets, as well as a pair of Higgs doublet supermultiplets and a singlet supermultiplet, in the bulk. Each of these 5D supermultiplets (hypermultiplets) is decomposed into two 4D $N = 1$ chiral superfields $\Phi(\phi, \psi)$ and $\Phi^c(\phi^c, \psi^c)$ with opposite gauge transformation properties. The $U(1)_R$ charge assignment is then given by $\Phi(2/3)$ and $\Phi^c(4/3)$ for all matter and Higgs hypermultiplets (the 4D matter and Higgs superfields arise from Φ; see below), leading to mixed $U(1)_R$ anomalies

given in Eq. (2). The $U(1)_R$ charges of V and Σ are zero. The cancellation of these anomalies can then occur through the (generalized) Green-Schwarz mechanism with a modulus M, as discussed in Sec. II. Assuming that M is a brane-localized chiral superfield for simplicity, the M field as well as the couplings of M to the standard model gauge multiplets must be located on the $y = 0$ brane (the 321 brane), since they do not respect $SU(5)$.[13] The coefficients of these couplings are determined by the anomaly cancellation conditions. This essentially reproduces the second term of Eq. (4) at the scale M_c, although it may have a slight modification arising from the existence of extra moduli and/or matter fields. Such a modification may, in fact, be necessary to ensure the consistency of the effective field theory. A more detailed analysis of the anomaly cancellation in higher dimensions, as well as the consistency of the higher-dimensional theory, will be given later.

We now discuss matter and Higgs fields in more detail. We impose boundary conditions on these fields such that each $SU(3)_C \times SU(2)_L \times U(1)_Y$ multiplet arises as the zero mode of a single 5D hypermultiplet that transforms as a definite representation under $SU(5)$. Consider, for example, a hypermultiplet $\{\mathcal{D}, \mathcal{D}^c\}$ transforming as $\mathbf{5^*}$ of $SU(5)$. [In our notation, a conjugated field has the opposite transformation property from a nonconjugated field, and we specify the transformation property of a hypermultiplet by that of the nonconjugated chiral superfield; for instance, $\mathcal{D}$ and $\mathcal{D}^c$ transform as $\mathbf{5^*}$ and $\mathbf{5}$ under $SU(5)$, respectively.] We choose the boundary conditions for this hypermultiplet as follows:

$$\mathcal{D} = \mathcal{D}_D^{(+, +)}(\mathbf{3^*}, \mathbf{1})_{1/3} \oplus \mathcal{D}_L^{(-, +)}(\mathbf{1}, \mathbf{2})_{-1/2}, \qquad (24)$$

$$\mathcal{D}^c = \mathcal{D}_D^{c(-, -)}(\mathbf{3}, \mathbf{1})_{-1/3} \oplus \mathcal{D}_L^{c(+, -)}(\mathbf{1}, \mathbf{2})_{1/2}. \qquad (25)$$

The right-hand side of these equations shows the decomposition of $\mathcal{D}$ and $\mathcal{D}^c$ into representations of 321 (in an obvious notation), as well as the boundary conditions imposed on each component [in the same notation as that in Eqs. (22) and (23)]. With these boundary conditions, the only massless state arising from $\{\mathcal{D}, \mathcal{D}^c\}$ is the zero mode of $\mathcal{D}_D(\mathbf{3^*}, \mathbf{1})_{1/3}$, which we identify as the low-energy down-type quark superfield D. The other quark and lepton superfields are also obtained similarly. Specifically, we introduce three generations of hypermultiplets $\{\mathcal{Q}_i, \mathcal{Q}_i^c\}(\mathbf{10})$, $\{\mathcal{U}_i, \mathcal{U}_i^c\}(\mathbf{10})$, $\{\mathcal{D}_i, \mathcal{D}_i^c\}(\mathbf{5^*})$, $\{\mathcal{L}_i, \mathcal{L}_i^c\}(\mathbf{5^*})$, and $\{\mathcal{E}_i, \mathcal{E}_i^c\}(\mathbf{10})$ ($i = 1, 2, 3$) for the quarks and leptons, obeying the following boundary conditions:

$$\begin{aligned} \mathcal{Q} &= \mathcal{Q}_Q^{(+, +)}(\mathbf{3}, \mathbf{2})_{1/6} \oplus \mathcal{Q}_U^{(-, +)}(\mathbf{3^*}, \mathbf{1})_{-2/3} \\ &\oplus \mathcal{Q}_E^{(-, +)}(\mathbf{1}, \mathbf{1})_1, \end{aligned} \qquad (26)$$

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[13]The case with a bulk M field can also be considered with M completed into an appropriate 5D (vector or tensor) supermultiplet; see e.g. [40].

$$\mathcal{U} = \mathcal{U}_Q^{(-,+)}(\mathbf{3}, \mathbf{2})_{1/6} \oplus \mathcal{U}_U^{(+,+)}(\mathbf{3^*}, \mathbf{1})_{-2/3}$$
$$\oplus \, \mathcal{U}_E^{(-,+)}(\mathbf{1}, \mathbf{1})_1, \tag{27}$$

$$\mathcal{D} = \mathcal{D}_D^{(+,+)}(\mathbf{3^*}, \mathbf{1})_{1/3} \oplus \mathcal{D}_L^{(-,+)}(\mathbf{1}, \mathbf{2})_{-1/2}, \tag{28}$$

$$\mathcal{L} = \mathcal{L}_D^{(-,+)}(\mathbf{3^*}, \mathbf{1})_{1/3} \oplus \mathcal{L}_L^{(+,+)}(\mathbf{1}, \mathbf{2})_{-1/2}, \tag{29}$$

$$\mathcal{E} = \mathcal{E}_Q^{(-,+)}(\mathbf{3}, \mathbf{2})_{1/6} \oplus \mathcal{E}_U^{(-,+)}(\mathbf{3^*}, \mathbf{1})_{-2/3} \oplus \mathcal{E}_E^{(+,+)}(\mathbf{1}, \mathbf{1})_1, \tag{30}$$

where we have omitted the generation index i. [The boundary conditions for the conjugated fields are given by $+ \leftrightarrow -$, as in Eqs. (24) and (25).] The only massless states arising from these hypermultiplets are the zero modes of $\mathcal{Q}_Q(\mathbf{3}, \mathbf{2})_{1/6}$, $\mathcal{U}_U(\mathbf{3^*}, \mathbf{1})_{-2/3}$, $\mathcal{D}_D(\mathbf{3^*}, \mathbf{1})_{1/3}$, $\mathcal{L}_L(\mathbf{1}, \mathbf{2})_{-1/2}$, and $\mathcal{E}_E(\mathbf{1}, \mathbf{1})_1$, which we identify as Q, U, D, L, and E in Sec. II. For the Higgs fields, we introduce $\{\mathcal{H}, \mathcal{H}^c\}(\mathbf{5})$, $\{\bar{\mathcal{H}}, \bar{\mathcal{H}}^c\}(\mathbf{5^*})$, and $\{\mathcal{S}, \mathcal{S}^c\}(\mathbf{1})$, obeying the boundary conditions:

$$\mathcal{H} = \mathcal{H}_C^{(-,+)}(\mathbf{3}, \mathbf{1})_{-1/3} \oplus \mathcal{H}_F^{(+,+)}(\mathbf{1}, \mathbf{2})_{1/2}, \tag{31}$$

$$\bar{\mathcal{H}} = \bar{\mathcal{H}}_C^{(-,+)}(\mathbf{3^*}, \mathbf{1})_{1/3} \oplus \bar{\mathcal{H}}_F^{(+,+)}(\mathbf{1}, \mathbf{2})_{-1/2}, \tag{32}$$

$$\mathcal{S} = \mathcal{S}_S^{(+,+)}(\mathbf{1}, \mathbf{1})_0. \tag{33}$$

(Again the boundary conditions for the conjugated fields are given by $+ \leftrightarrow -$.) The massless states arise from the zero modes of $\mathcal{H}_F(\mathbf{1}, \mathbf{2})_{1/2}$, $\bar{\mathcal{H}}_F(\mathbf{1}, \mathbf{2})_{-1/2}$, and $\mathcal{S}_S(\mathbf{1}, \mathbf{1})_0$, which we identify as H_u, H_d, and S in Sec. II. Note that the boundary conditions of Eqs. (22), (23), and (26)–(33) can be imposed consistently with the interactions of the theory.[14]

A bulk hypermultiplet $\{\Phi, \Phi^c\}$ can generically have a mass term in the bulk, which is written as

$$S = \int d^4x \int_0^{\pi R} dy \int d^2\theta M_\Phi \Phi \Phi^c + \text{H.c.}, \tag{34}$$

in the basis where the kinetic term is given by $S_{\text{kin}} = \int d^4x \int dy [\int d^4\theta (\Phi^\dagger \Phi + \Phi^c \Phi^{c\dagger}) + \{\int d^2\theta \Phi^c \partial_y \Phi + \text{H.c.}\}]$. The parameter M_Φ controls the wave function profile of the zero mode. For $M_\Phi > 0$ (< 0) the wave function of a zero mode arising from Φ is localized to the $y = 0$ ($y = \pi R$) brane; for $M_\Phi = 0$ it is flat. [If a zero mode arises from Φ^c, its wave function is localized to the $y = \pi R$ ($y = 0$) brane for $M_\Phi > 0$ (< 0) and is flat for $M_\Phi = 0$.] Explicit constraints on M_Φ in our theory depend on the detailed setup, e.g., on the source of supersymmetry breaking. There is, however, one constraint that generically applies regardless of these details. Suppose, for example, that we want to localize the zero mode of $\mathcal{D}_D(\mathbf{3^*}, \mathbf{1})_{1/3}$ to the $y = \pi R$ brane by taking $M_\mathcal{D} \to -\infty$. In this case, however, we find that the lightest KK state from $\mathcal{D}_L(\mathbf{1}, \mathbf{2})_{-1/2}$ and $\mathcal{D}_L^c(\mathbf{1}, \mathbf{2})_{1/2}$ becomes exponentially light, with the former (latter) degrees of freedom localized to the $y = \pi R$ ($y = 0$) brane. This is, in fact, expected because the $\mathcal{D}_D$ state is localized to the $y = \pi R$ brane, where the active gauge group is $SU(5)$, so that it locally requires a $SU(5)$ partner, which is provided by the $\mathcal{D}_L$ state. Since the $\mathcal{D}_L$ state is massive in 4D (due to the boundary conditions), it must be in a vectorlike representation, hence the existence of $\mathcal{D}_L^c$ localized at $y = 0$. Since any extra vectorlike state is not observed in nature, this gives a constraint on $M_\mathcal{D}$ from below. Applying similar considerations also to the other multiplets, we find

$$M_\Phi R \gtrsim -(1 \sim 2), \tag{35}$$

for $\Phi = \mathcal{Q}_i, \mathcal{U}_i, \mathcal{D}_i, \mathcal{L}_i, \mathcal{E}_i, \mathcal{H}$, and $\bar{\mathcal{H}}$, implying that the wave functions of the low-energy states Q_i, U_i, D_i, L_i, E_i, H_u. and H_d should not be strongly localized to the $y = \pi R$ brane [the $SU(5)$ brane].[15]

With the structure for the matter and Higgs sectors described above, no rapid proton decay is induced by an exchange of the bulk $SU(5)$ gauge boson, whose mass is only of order $1/R \approx (10 \sim 100)$ TeV. This is because quarks and leptons that would be unified into a single $SU(5)$ representation in standard grand unified theories now arise from different $SU(5)$ multiplets in the bulk. [Note that this preserves a $SU(5)$ understanding of the quark and lepton quantum numbers, especially quantization of $U(1)_Y$ hypercharges.] The colored-Higgs KK states with masses of $O(10 \sim 100$ TeV) do not induce proton

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[14]Note that the signs $\pm$ for the boundary conditions in these equations represent the Neumann/Dirichlet boundary conditions in the interval y: $[0, \pi R]$. In the orbifold picture, the boundary conditions of e.g. Eq. (27) can be obtained effectively as follows. We prepare a hypermultiplet obeying the boundary conditions $\mathcal{U} = \mathcal{U}_Q^{(-,+)}(\mathbf{3}, \mathbf{2})_{1/6} \oplus \mathcal{U}_U^{(+,+)}(\mathbf{3^*}, \mathbf{1})_{-2/3} \oplus \mathcal{U}_E^{(+,+)}(\mathbf{1}, \mathbf{1})_1$, where the first and second signs in the parentheses represent transformation properties under the reflection $y \leftrightarrow -y$ and $(y - \pi R) \leftrightarrow -(y - \pi R)$, respectively. We then introduce a 321-brane-localized chiral superfield transforming as $(\mathbf{1}, \mathbf{1})_{-1}$ under 321, and couple it to the $\mathcal{U}_E^{(+,+)}(\mathbf{1}, \mathbf{1})_1$ state from $\mathcal{U}$. This reproduces the boundary conditions of Eq. (27) in the limit that this coupling (brane mass term) becomes large. A similar (or more straightforward) construction also applies to the other multiplets. The fact that the boundary conditions of Eqs. (22), (23), and (26)–(33) can be reproduced in the orbifold picture by taking a consistent limit guarantees their consistency.

[15]One may think that for a $\{\mathcal{D}, \mathcal{D}^c\}$ multiplet with $M_\mathcal{D} \to -\infty$, we can introduce a 321-brane-localized field $L'(\mathbf{1}, \mathbf{2})_{-1/2}$ and couple it to $\mathcal{D}_L^c$ on the 321 brane, leading to the low-energy states $\mathcal{D}_D$ and $\bar{\mathcal{D}}_L$ (with a slight mixture from L'), which may be identified as D and L. (The $\{\mathcal{L}, \mathcal{L}^c\}$ multiplet should then be eliminated.) In fact, this construction can work for a $\mathbf{5^*}$ ($\supset D + L$) state, although a similar construction for a $\mathbf{10}$ ($\supset Q + U + E$) state does not because it would lead to rapid proton decay caused by an exchange of the bulk $SU(5)$ gauge boson. This opens a possibility in which (some of) the $\mathbf{5^*}$ states are strongly (or exactly) localized to the $SU(5)$ brane.

decay either, because of the special form of the mass matrices for these states, dictated by higher-dimensional gauge invariance. (See Ref. [38] for details.) Possible tree-level proton decay operators may be forbidden by imposing an appropriate discrete (gauge) symmetry or if the quarks and leptons are separated in an extra dimension orthogonal to the dimension y (see Sec. IV B).

The superpotential interactions arise from the $y = 0$ (321) and/or $y = \pi R$ [$SU(5)$] brane(s). Because of the gauged $U(1)_R$ symmetry, these interactions must be cubic in fields. Locating them on the 321 brane for simplicity, we obtain

$$S = \int d^4x \int_0^{\pi R} dy \, \delta(y) \int d^2\theta (\mathcal{Q}\,\mathcal{U}\mathcal{H} + \mathcal{Q}\,\mathcal{D}\bar{\mathcal{H}}$$
$$+ \mathcal{L}\mathcal{E}\bar{\mathcal{H}} + \mathcal{S}\mathcal{H}\bar{\mathcal{H}} + \mathcal{S}^3) + \text{H.c.}, \quad (36)$$

where we have omitted generation indices as well as (dimensionful) coefficients, and we have assumed the standard matter parity (R parity). These interactions reproduce the interactions of Eq. (11) below M_c. (Small neutrino masses can be generated by introducing a brane or bulk right-handed neutrinos $\mathcal{N}$ together with a superpotential term of the form $\mathcal{L}\mathcal{N}\mathcal{H}$; see Sec. III B.) Note that there is no unwanted unified mass relation between the quarks and leptons, since different 321 multiplets come from different bulk multiplets.

We now discuss anomaly cancellation for the $U(1)_R$ symmetry in the present theory in more detail. In general, when one performs an $U(1)_R$ transformation, variations of the Lagrangian caused by the anomalous matter content are confined to the branes at $y = 0$ and $y = \pi R$ [41]. In our theory, the generated anomalies take the form

$$\begin{pmatrix} A_1^{(0)} \\ A_2^{(0)} \\ A_3^{(0)} \end{pmatrix} = \begin{pmatrix} -\frac{11}{5} - a \\ -\frac{1}{3} - a \\ 1 - a \end{pmatrix}, \qquad \begin{pmatrix} A_1^{(\pi)} \\ A_2^{(\pi)} \\ A_3^{(\pi)} \end{pmatrix} = \begin{pmatrix} a \\ a \\ a \end{pmatrix}, \quad (37)$$

where $A_I^{(0)}$ and $A_I^{(\pi)}$ ($I = 1, 2, 3$) represent the mixed $U(1)_R$-321 anomalies located at the $y = 0$ and $y = \pi R$ branes, respectively. The constant a is given by $a = -5/12$ in the model described above, although the value of a changes in general if we locate (some of) the quark, lepton, and Higgs multiplets strictly on the 321 brane. The mixed anomalies of Eq. (37) can be canceled by a combination of the Green-Schwarz mechanism and anomaly transfer in the bulk. By introducing a bulk Chern-Simons term with an appropriate coefficient, we can "transfer" the mixed anomalies from $y = \pi R$ to $y = 0$ by an amount of a. The rest of the anomalies can then be canceled by introducing the terms

$$S = \int d^4x \int_0^{\pi R} dy \, \delta(y) \left\{ -\sum_{I=1,2,3} \frac{\zeta_I}{c} \int d^2\theta M \, \mathcal{W}_I^{a\alpha} \mathcal{W}_{I\alpha}^a \right.$$
$$\left. + \text{H.c.} \right\}, \quad (38)$$

with the coefficients chosen to be $\zeta_I = A_I \equiv A_I^{(0)} + A_I^{(\pi)}$. Here, $\int_0^\epsilon \delta(y)dy \equiv 1$ for $\epsilon > 0$, and M transforms as $M \to M + i\alpha c/16\pi^2$ under $U(1)_R$. After integrating out the fifth dimension, the interactions of Eq. (38) lead exactly to the second term of Eq. (4) at the scale M_c. (The first term arises from the bulk gauge kinetic term.)

A (potential) problem with the setup just described is that the interactions of Eq. (38) give a negative brane-localized kinetic term for $SU(3)_C$ after the modulus M obtains the required VEV of Eq. (9). (Note that $\langle M \rangle/c > 0$ and $\zeta_3 = 1$.) While the zero mode of the $SU(3)_C$ gauge field has a positive gauge kinetic term, the negative brane kinetic term could cause problems in processes involving higher KK states. Suppose that the coefficients of the bulk and brane kinetic terms for a bulk gauge field are given by $1/g_5^2$ and $1/\tilde{g}^2$, respectively. [For the $SU(3)_C$ gauge field considered here, $1/g_5^2 = 1/g_*^2$ and $1/\tilde{g}^2 = -4\langle M \rangle/c$.] We then find that for $1/\tilde{g}^2 < 0$, the KK decomposition leads to a mode that has a negative kinetic term (ghost), whose "mass" μ_0 (> 0) is given by the solution to

$$\tanh(\pi R \mu_0) = -\frac{g_5^2 \mu_0}{\tilde{g}^2}. \quad (39)$$

In order for the 5D effective field theory to be consistent, this mass must be larger than the cutoff scale: $\mu_0 \gtrsim M_*$, leading to the condition $1/\tilde{g}^2 \gtrsim -1/g_5^2 M_*$. For the case of $SU(3)_C$, however, the values of $1/\tilde{g}^2$ and $1/g_5^2$ are determined by the phenomenological requirements of Eqs. (8) and (9) as $1/\tilde{g}^2 \simeq -1$ and $\pi R/g_5^2 \simeq 2$. This yields $M_* \lesssim 2/\pi R \approx 1/R$, implying that the cutoff scale of the 5D theory should be at or below the scale of the masses of the first KK excitations. This clearly casts doubt on the viability of the 5D theory as a theory describing physics "above M_c."

There are essentially two approaches we could take to deal with this issue. One is to consider that the size of the extra dimension, πR, is in fact not much larger than the cutoff length M_*^{-1}. In this case, the 5D theory described above may not be a fully viable effective field theory. However, we can still take the view that it suggests the basic structure, e.g. the gauge symmetry structure and matter content, of the fundamental theory at M_*, e.g. string theory. This is an interesting proposal for future string model-building. The other approach is to consider that the problem arose because of the particular (too minimal) structure of the model described above, and that we can (slightly) modify the theory so that it does not suffer from the problem. Below we take this latter approach and find ways to avoid the problem within effective field theory.

A simple way of avoiding the sizable negative gauge kinetic term for $SU(3)_C$ on the $y = 0$ brane is to introduce another modulus M', which is localized on the $y = \pi R$ brane and has the interaction

$$S = \int d^4x \int_0^{\pi R} dy \, \delta(y - \pi R)$$
$$\times \left\{ -\frac{\zeta'}{c'} \int d^2\theta M' \mathcal{W}^{a\alpha} \mathcal{W}^a_\alpha + \text{H.c.} \right\}, \quad (40)$$

where ζ' and c' are real constants, $\mathcal{W}^{a\alpha}$ is the field strength superfield for $SU(5)$, which contains 321 as a subgroup, and the field M' transforms as $M' \rightarrow M' + i\alpha c'/16\pi^2$ under $U(1)_R$. Note that the active gauge group on the $y = \pi R$ brane is $SU(5)$, so that the coefficient ζ' is universal for 321. In this case, the coefficient for the bulk Chern-Simons term should be chosen such that mixed anomalies of the amount $a - \zeta'$ are transferred from $y = \pi R$ to $y = 0$, and the coefficients ζ_I in Eq. (38) chosen as

$$\zeta_I = A_I - \zeta', \quad (41)$$

where $(A_1, A_2, A_3) = (-11/5, -1/3, 1)$. Now, let us consider, for example, that $\zeta' = 1$. In this case the coefficients of the 321-brane-localized interactions take the values $(\zeta_1, \zeta_2, \zeta_3) = (-16/5, -4/3, 0)$, so that the VEV of $\langle M \rangle / c > 0$ does not lead to a negative brane kinetic term for $SU(3)_C$, $SU(2)_L$, or $U(1)_Y$. In fact, assuming that $\langle M' \rangle / c' \lesssim 1/16\pi^2$ (not necessarily $|\langle M' \rangle / c'| \lesssim 1/16\pi^2$), we can make our 5D theory a viable effective field theory in a (moderately) large energy interval above $M_c \approx 1/R$. To reproduce the observed gauge couplings, we must have

$$\frac{\langle M \rangle}{c} \simeq \frac{3}{32\pi^2} \ln \frac{M_U}{M_*} \simeq 0.25, \quad (42)$$

$$\frac{\pi R}{g_*^2} - \frac{4\zeta'}{c'} \langle M' \rangle = \langle T \rangle - 4\frac{\langle M' \rangle}{c'} \simeq 1. \quad (43)$$

By choosing the 5D $SU(5)$ gauge coupling to be strong at the scale M_*, i.e. $1/g_*^2 \approx CM_*/16\pi^3$ with $C \simeq 5$ a group theoretical factor, we find that M_*R can be as large as ≈ 30 for $|\langle M' \rangle / c'| \ll 1$. These choices of parameters do not disturb any of the arguments before, e.g. technical naturalness for the smallness of the tree-level brane gauge kinetic operators. The fields M and M' can be stabilized easily with the desired values of $\langle M \rangle / c$ and $\langle M' \rangle / c'$ along the lines of Sec. II B. For instance, we can consider two supersymmetric $SU(2)$ gauge sectors each localized on the $y = 0$ and $y = \pi R$ branes, which are responsible for the stabilizations of M and M', respectively.

An alternative way of avoiding the problem is to introduce extra matter fields that are vectorlike under 321 and obtain masses through $U(1)_R$ breaking. Let us, for example, introduce chiral superfields $\Phi(\mathbf{5}) + \bar{\Phi}(\mathbf{5}^*)$ on the $y = \pi R$ brane which have a vanishing $U(1)_R$ charge. Here, the numbers in parentheses represent the transformation properties under $SU(5)$. In this case, these fields produce the mixed anomalies of -1 localized on the $y = \pi R$ brane, so that $A_I^{(\pi)}$ in Eq. (37) are replaced as $A_I^{(\pi)} = a \rightarrow a - 1$. The anomaly transfer by a Chern-Simons term should then be $a - 1$, and the coefficients ζ_I in Eq. (38) become $\zeta_I =$

$A_I - 1 \leq 0$, avoiding the problem. The required values of $\langle T \rangle$ and $\langle M \rangle$ are given by Eqs. (42) and (43) (with $\langle M' \rangle$ set to zero), and we find, following the argument below Eqs. (42) and (43), that there can be a (moderately) large energy interval up to a factor of $M_*R \approx 30$ in which the effective 5D field theory is applicable. A mass for the Φ and $\bar{\Phi}$ states of order the weak scale or somewhat larger can be generated through the Kähler potential term $\int d^4\theta C^\dagger C \Phi \bar{\Phi}$ on the $y = \pi R$ brane, where C represents the chiral compensator field. (This requires that supersymmetry is broken in the bulk of the gravitational dimensions, in which case F_C is of order the weak scale or somewhat larger; see Sec. III A.) To preserve the successful prediction for the gauge couplings, the absence of similar vectorlike states on the 321 brane which do not fill a complete $SU(5)$ multiplet must be assumed. A similar comment also applies to states that obtain masses through the VEV of ϕ, the field absorbing the large Fayet-Iliopoulos term of $U(1)_R$.

We emphasize that the mechanisms presented above for avoiding a sizable negative brane kinetic term for $SU(3)_C$ are actually simple—much simpler than how they might naïvely look. We simply assume that the mixed $U(1)_R$ anomalies are canceled by a combination of the M field and the M' field (or extra vectorlike states). The interactions (quantum numbers) of M' (vectorlike states) are universal for 321 because of the location of the field(s), so that the successful supersymmetric prediction for the low-energy gauge couplings is preserved. Note that this requires some reinterpretations of the formulas given in the previous sections, for example F_T in Eqs. (18) and (20) should be replaced by $F_T + (4\zeta'/c)F_M$, but the essential physics is unchanged. The size of the 5D energy interval M_*R takes a value between a factor of a few and ≈ 30. We thus arrive at the picture given in Fig. 1. This completes our discussion on the basic construction of the model reproducing the effective theory of Sec. II below M_c.

Let us now discuss supersymmetry breaking and its implications in this theory. As discussed in Sec. III A, one of the natural possibilities is that the T field, the radion supermultiplet associated with the dimension y, obtains a nonvanishing VEV in the auxiliary component, $F_T = O(100 \text{ GeV})$. This induces supersymmetry breaking masses for the gauginos as well as the bulk scalar fields. (For flat spacetime, this is equivalent to the Scherk-Schwarz mechanism [42]. For earlier work on Scherk-Schwarz supersymmetry breaking, see e.g. [15–17].) Since the generated scalar masses and scalar trilinear interactions depend on bulk mass parameters M_Φ, this generically introduces the supersymmetric flavor problem. One way to avoid this problem is to assume that the bulk masses are flavor universal, which may be the result of some flavor symmetry. This can lead to interesting phenomenology, with a variety of spectra for the squarks and sleptons depending on arbitrary bulk mass parameters.
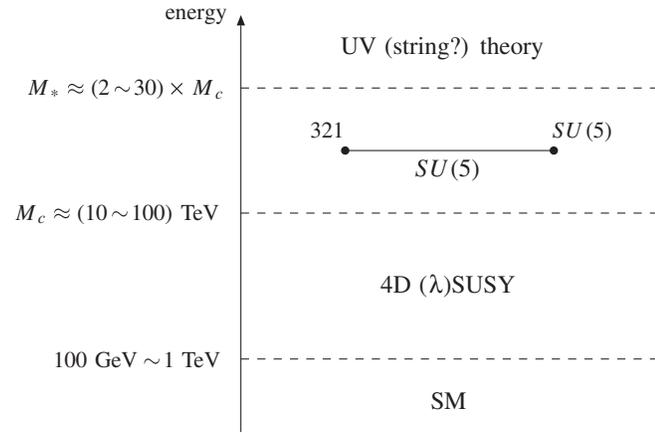
FIG. 1. The schematic picture of our minimal theory. The standard model (SM) is the effective theory up to a scale of 100 GeV $\sim$ 1 TeV, where it is replaced by a 4D (λ)SUSY model: a 4D $N = 1$ supersymmetric standard model with the superpotential given by Eq. (11) (in the minimal case). This model is further replaced at $M_c \approx (10 \sim 100)$ TeV by the minimal 5D $SU(5)$ theory described in the text, which is the effective theory for the next factor of $(2 \sim 30)$. Finally, at the scale $M_* \approx (2 \sim 30) \times M_c$, the theory is embedded into a fundamental ultraviolet theory, which may be string theory.

Another possible way, which we focus on below, is to strongly localize the quark and lepton multiplets to a brane, since then the generated tree-level supersymmetry breaking masses for the squarks and sleptons are exponentially suppressed. Flavor universal squark and slepton masses can be generated by gauge loops through the gaugino masses (approximately flavor universal contributions can also come from the $U(1)_R$ D-term VEV), and scalar trilinear interactions proportional to the Yukawa matrices are also generated by gauge loops as well as by the tree-level contribution through the Higgs fields. Since we have a constraint on M_Φ in Eq. (35), we should then take

$$M_{\mathcal{Q}_i}, M_{\mathcal{U}_i}, M_{\mathcal{D}_i}, M_{\mathcal{L}_i}, M_{\mathcal{E}_i} \gg \frac{1}{\pi R}, \quad (44)$$

implying that the low-energy Q_i, U_i, D_i, L_i, and E_i states are localized to the 321 brane.[16] The required amount of localization, however, is not very strong; $MR \gtrsim 2$ is enough for the first two generations, and the degree of localization can be even milder for the third generation. For the Higgs and S fields, there are no strong constraints on their bulk masses from flavor violating processes. The constraints, however, may arise from electroweak symmetry breaking, depending on details of the setup, for ex-

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[16]One of the main reasons we did not localize these states strictly on the 321 brane from the beginning is that we would then lose the $SU(5)$ understanding of the matter quantum numbers in the 5D effective theory. The correct quantum numbers, however, can arise naturally if the fundamental theory is higher dimensional and has a larger gauge group, as in Ref. [43].

ample, to avoid too large volume suppressions for the low-energy λ and κ couplings in Eq. (11) and/or to have a sufficiently large scalar trilinear coupling between the S and Higgs fields.

In Fig. 2 we present a schematic picture of the model described here. The wave function profiles for the S and Higgs fields are depicted arbitrarily. The structure of this theory is somewhat similar to that in Ref. [44], although we now have a low compactification scale of $1/R \approx (10 \sim 100)$ TeV and the gauged R symmetry. These two ingredients provide extra constraints on the location of the matter and Higgs fields, as well as on the form of the superpotential, given in Eqs. (35) and (36). Supersymmetry breaking masses also show a characteristic pattern. Motivated by suppressions of flavor changing neutral currents, let us consider the situation in which the source of supersymmetry breaking resides in nonvanishing F_T, D_R, and anomaly mediation (see Sec. III A). In this case, soft supersymmetry breaking masses at the scale $M_c = 1/R = O(10 \sim 100)$ TeV are given as follows. For the gaugino masses M_I ($I = 1, 2, 3$), we have

$$M_I = \frac{g_I^2}{g_U'^2} \hat{m}_T + \frac{b_I g_I^2}{16\pi^2} \hat{m}_C, \quad (45)$$

where g_I are the 321 gauge couplings at M_c, $g_U'^2 \equiv g_*^2/\pi R$ with g_* the 5D $SU(5)$ gauge coupling, and b_I are the beta-function coefficients defined by $d(1/g_I^2)/d\ln\mu_R = -b_I/8\pi^2$. (The value of g_U' would agree with the unified gauge coupling in the conventional supersymmetric desert, $g_U \simeq 0.7$, for $\zeta' = 0$.) The mass parameters $\hat{m}_T$ and $\hat{m}_C$ are given by

$$\hat{m}_T \equiv -\frac{g_U'^2}{2} F_T, \qquad \hat{m}_C \equiv -F_C. \quad (46)$$

The two terms in Eq. (45) give comparable contributions for $\hat{m}_T \approx \hat{m}_C/8\pi^2$ [$\approx O(100$ GeV)]; otherwise, one
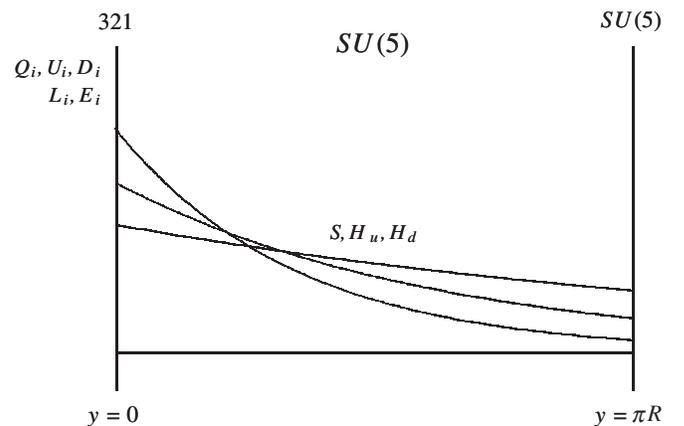

FIG. 2. The schematic picture of the minimal model described in the text. The quark and lepton supermultiplets are localized (strongly) to the $y = 0$ brane. The wave function profiles for the S and Higgs fields are depicted arbitrarily.

dominates the other. The scalar trilinear interactions, defined generally by $\mathcal{L}_{\text{soft}} = -\sum_{A,B,C}(a_{ABC}/6)\phi_A\phi_B\phi_C + \text{H.c.}$, are given by

$$a_{ABC} = -y_{ABC}\{(a_{\Phi_A} + a_{\Phi_B} + a_{\Phi_C})\hat{m}_T \\ + (\gamma_{\Phi_A} + \gamma_{\Phi_B} + \gamma_{\Phi_C})\hat{m}_C\}, \qquad (47)$$

where y_{ABC} are the Yukawa couplings $W = \sum_{A,B,C}(y_{ABC}/6)\Phi_A\Phi_B\Phi_C$, with Φ_A and ϕ_A representing a generic 4D chiral superfield and its scalar component, respectively.[17] The coefficient a_Φ is given in flat space by

$$a_\Phi = \frac{2\pi R M_\varphi}{e^{2\pi R M_\varphi} - 1}, \qquad (48)$$

where M_φ is the bulk mass of the hypermultiplet $\{\varphi, \varphi^c\}$ giving Φ as the zero mode of φ, and γ_Φ is the anomalous dimension of Φ, defined by $d\ln Z_\Phi/d\ln\mu_R = -2\gamma_\Phi$ with Z_Φ the wave function renormalization for Φ.[18] The soft supersymmetry breaking scalar mass squared m_ϕ^2 for a 4D chiral superfield Φ is given by

$$m_\phi^2 = -\gamma_\Phi\hat{m}_D^2 + c_\Phi|\hat{m}_T|^2 + \frac{1}{2}\frac{d\gamma_\Phi}{d\ln\mu_R}|\hat{m}_C|^2 \\ + \left\{\left(\sum_{y_{ABC}}|y_{ABC}|^2(a_{\Phi_A} + a_{\Phi_B} + a_{\Phi_C})\frac{\partial\gamma_\Phi}{\partial|y_{ABC}|^2}\right.\right. \\ \left.\left. + \sum_{g_I}\frac{g_I^4}{g_U'^2}\frac{\partial\gamma_\Phi}{\partial g_I^2}\right)\hat{m}_T\hat{m}_C^* + \text{H.c.}\right\}, \qquad (49)$$

where the mass parameter $\hat{m}_D$ is given by

$$\hat{m}_D^2 = -\frac{2}{3}D_R, \qquad (50)$$

which is positive and can take a value of order the weak scale, or somewhat larger,[19] and the coefficient c_Φ is given (in the flat space limit) by

$$c_\Phi = \left(\frac{\pi R M_\varphi}{\sinh(\pi R M_\varphi)}\right)^2. \qquad (51)$$

The last term in the right-hand side of Eq. (49) is the interference term between F_T and F_C [31], and we have assumed the absence of direct couplings between the observable sector fields and ϕ, the field absorbing the Fayet-Iliopoulos term of $U(1)_R$. More explicit expressions for the scalar trilinear interactions, a_{ABC}, and scalar squared masses, m_ϕ^2, in the present setup are given in Appendix A.

The soft supersymmetry breaking masses derived above, Eqs. (45), (47), and (49), do not lead to the supersymmetric flavor problem, as long as the conditions of Eq. (44) are satisfied (at least for the first two generations). They do not lead to the supersymmetric CP problem either, if the complex phases of $\hat{m}_T$ and $\hat{m}_C$ (F_T and F_C) are aligned, which is indeed the case if T is stabilized through a single gaugino condensation [31,46], or if one of $\hat{m}_T$ and $\hat{m}_C$ dominates the other. (All the soft supersymmetry breaking masses, as well as the couplings λ and κ, can be made real by choosing the appropriate phase convention for S and H_uH_d and by using the appropriate R-rotation.) These soft supersymmetry breaking masses show a variety of interesting patterns. For example, if the contribution from anomaly mediation is small, $\hat{m}_C/8\pi^2 \ll \hat{m}_T$, $\hat{m}_D/4\pi$, we find that the gaugino masses satisfy the standard "unified mass relation" $M_I \propto g_I^2$ while the squark and slepton masses take the form $m_{\tilde{F}}^2 \simeq -\gamma_F\hat{m}_D^2$, where $F = Q$, U, D, L, E (at least for the first two generations).[20] The Higgs squared masses are arbitrary, and there are scalar trilinear terms (A terms) proportional to the Yukawa matrices. Note that these masses are given at the scale $M_c = O(10 \sim 100 \text{ TeV})$, only a few orders of magnitude above the weak scale. In the case that F_T is suppressed, on the other hand, we find that the spectrum is given by the sum of the anomaly mediated contributions and the scalar masses from the $U(1)_R$ D-term VEV, again at the scale $M_c = O(10 \sim 100 \text{ TeV})$. This can happen if there is a supersymmetry breaking field Z in the gravitational bulk, which does not directly couple to the MSSM states. In general, the superparticle spectra derived in Eqs. (45), (47), and (49) have a very rich structure, including the possibility of nonuniversality in the third generation sfermion masses caused by nontrivial profiles of the wave functions of these states in the extra dimension. In addition, the gaugino masses may also have a contribution from F_M, which affects the scale of effective gaugino mass unification. We can even consider some interesting variations of the model. For example, we can change the boundary conditions of $\mathcal{S}$ (and/or $\mathcal{H}_F$, $\bar{\mathcal{H}}_F$) to $(+, -)$ or $(-, +)$ [only $(+, -)$ is available for $\mathcal{H}_F$, $\bar{\mathcal{H}}_F$]. In this case, the tree-level contribution to the supersymmetry breaking mass

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[17]Our sign convention for the soft supersymmetry breaking parameters agrees with that of SUSY Les Houches Accord [45].

[18]Note that the 4D Yukawa couplings y_{ABC} also receive suppressions with factors $z_{\Phi_A}z_{\Phi_B}z_{\Phi_C}$, where $z_\Phi = (2M_\varphi/(1 - e^{-2\pi R M_\varphi})M_*)^{1/2}$. The suppression factors, however, could differ if these couplings receive nonnegligible contributions from the brane couplings located at $y = \pi R$. The expressions for the scalar trilinear couplings a_{ABC} also change in this case from that in Eq. (47). This can happen, for example, for the couplings λ and κ, practically making the corresponding scalar trilinear interactions, a_λ and a_κ, independent free parameters.

[19]In the presence of an $U(1)_R$-$U(1)_Y$ kinetic mixing term on the 321 brane, the scalar squared masses receive contributions proportional to their $U(1)_Y$ hypercharges. Writing the gauge kinetic terms as $\int d^2\theta\{(1/4g_R^2)\mathcal{W}_R^\alpha\mathcal{W}_{R\alpha} + (1/4g_1^2)\mathcal{W}_1^\alpha\mathcal{W}_{1\alpha} + (\epsilon/2)\mathcal{W}_R^\alpha\mathcal{W}_{1\alpha}\} + \text{H.c.}$ in 4D, the first term of Eq. (49) is then modified to $(-\gamma_\Phi - 3\epsilon g_1^2 Y_\Phi/2)\hat{m}_D^2$, where Y_Φ is the $U(1)_Y$ hypercharge of the chiral superfield Φ in the $SU(5)$ normalization. Although it is (technically) natural to have a small value for ϵ, it can also be of $O(1)$ without contradicting phenomenological constraints.

[20]This is the case if the Fayet-Iliopoulos term for $U(1)_R$ is not absorbed.

squared m_S^2 (and/or $m_{\mathcal{H}}^2$, $m_{\bar{\mathcal{H}}}^2$) can be negative in a certain parameter region. A detailed study of electroweak symmetry breaking for some of these spectra will be given in Ref. [23].

We finally discuss physics associated with the KK states, which have masses of order $M_c = 1/R = O(10 \sim 100 \text{ TeV})$. These states form multiplets of 4D $N = 2$ supersymmetry, with small mass splittings inside each supermultiplet due to F_T. Interesting quantities among others are masses of the lightest KK excitations for the gauge fields. These are determined independently of bulk mass parameters, and thus provide relatively model-independent predictions. We find that the masses of the KK gauge states associated with $SU(3)_C$, $SU(2)_L$, $U(1)_Y$, and $SU(5)/321$ are given by $M_3' = (g_3^2/g_U'^2)R^{-1}$, $M_2' = (g_2^2/g_U'^2)R^{-1}$, $M_1' = (g_1^2/g_U'^2)R^{-1}$, and $M_X' = (1/2)R^{-1}$, giving

$$M_3':M_2':M_1':M_X' = g_3^2:g_2^2:g_1^2:\frac{g_U'^2}{2}, \qquad (52)$$

where g_I are the standard model gauge couplings at M_c, and $g_U'^2 = g_*^2/\pi R$ with g_* the 5D gauge coupling. The masses of the KK states for the matter and Higgs fields are highly model dependent, since they depend on bulk masses for these supermultiplets.

B. Models in higher dimensions

In the previous subsection we have presented a model based on $SU(5)$ in 5D. There are a variety of ways to extend this to higher dimensions and/or a larger gauge group, as was the case in higher-dimensional grand unified theories at a high scale of order M_U [47,48]. There are, however, new constraints in our framework. First, proton decay caused by the exchange of higher-dimensional unified gauge fields, as well as colored-Higgs multiplets, must be suppressed. This can be achieved, for example, by extracting different standard model multiplets from different bulk multiplets, as in the minimal model in the previous subsection. Another constraint comes from the $[U(1)]$ R symmetry which must exist to reproduce the successful supersymmetric prediction for the low-energy gauge couplings. This restricts the form of possible superpotential terms, giving potential constraints on the Higgs sector, including the sector breaking the unified symmetry (if any), as well as on ways of obtaining realistic fermion masses (although terms violating the R symmetry could be generated through spontaneous breaking of R).

To illustrate an example of new possibilities that open up by going to higher dimensions, here we consider a $SU(5)$ unified theory in 6D. We consider that the theory possesses $N = 2$ supersymmetry in 6D, which corresponds to $N = 4$ supersymmetry in 4D, and that the extra two dimensions, x^5 and x^6, are compactified on a $T^2/(Z_2 \times Z_2')$ orbifold: $0 \leq x^5 \leq 2\pi R_5$ and $0 \leq x^6 \leq 2\pi R_6$. The 6D $N = 2$ supersymmetry guarantees that the gauge anomalies in the 6D bulk automatically cancel. It also requires that the only

bulk field is the 6D $SU(5)$ gauge supermultiplet, which can be decomposed into a 4D $N = 1$ vector superfield V and three 4D $N = 1$ chiral superfields Σ_5, Σ_6 and Φ, where Σ_5 and Σ_6 contain the fifth- and sixth-dimensional components of the gauge field, A_5 and A_6 [49]. We now impose the following boundary conditions on these fields. Along the sixth direction, x^6, we impose

$$V(+, +), \qquad \Sigma_5(+, +), \qquad \Sigma_6(-, -), \qquad \Phi(-, -),$$
$$(53)$$

where $+$ and $-$ represent Neumann and Dirichlet boundary conditions, respectively, and the first and second signs in parentheses represent boundary conditions at $x^6 = 0$ and $x^6 = \pi R_6$. Along the fifth direction, x^5, we impose the ones in Eq. (22) for V and Σ_6 and the ones in Eq. (23) for Σ_5 and Φ, with the first and second signs in parentheses representing boundary conditions at $x^5 = 0$ and $x^5 = \pi R_5$, respectively. These boundary conditions reduce the low-energy theory to be the 4D $N = 1$ supersymmetric $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge theory. The only massless state arising from the 6D gauge multiplet is the 321 component of V.

The resulting supersymmetry and gauge symmetry structures in the extra two dimensions are quite rich. There are four 5D fixed lines $x^6 = 0$, $x^6 = \pi R_6$, $x^5 = 0$, and $x^5 = \pi R_5$, each having $SU(5)$, $SU(5)$, 321, and $SU(5)$ gauge symmetries with 5D $N = 1$ (4D $N = 2$) supersymmetry, and there are four 4D fixed points $(x^5, x^6) = (0, 0)$, $(0, \pi R_6)$, $(\pi R_5, 0)$, and $(\pi R_5, \pi R_6)$, each having 321, 321, $SU(5)$, and $SU(5)$ gauge symmetries with 4D $N = 1$ supersymmetry. The theory possesses an $U(1)_R$ symmetry analogous to the one in the previous sections, which is a linear combination of an $U(1)$ subgroup of $SU(4)_R$ in the 6D supersymmetry algebra and certain flavor $U(1)$'s. The $U(1)_R$ charge assignment for the gauge multiplet is given by $V(0)$, $\Sigma_5(0)$, $\Sigma_6(0)$, and $\Phi(2)$, and that for the matter and Higgs fields, which are introduced on 5D or 4D subspaces, is essentially identical to the one in the minimal model of Sec. IVA. Upon gauging this symmetry, we find the mixed $U(1)_R$ anomalies given by Eq. (2). These are canceled (essentially) by a shift of a single modulus M through the generalized Green-Schwarz mechanism, with the couplings of M to the 321 gauge fields located at the $(x^5, x^6) = (0, 0)$ or $(0, \pi R_6)$ fixed point (for brane-localized M). Together with an extra moduli field M' or vectorlike states located on a $SU(5)$-preserving brane, we can have a consistent 6D effective field theory describing physics above the compactification scale M_c, as discussed in the previous subsection.

This setup can be used for various purposes. Let us assume, for simplicity, that the shape modulus, R_5/R_6, is fixed (strongly) such that R_5 is (somewhat) larger than R_6. In this case, the low-energy theory below R_6^{-1} is essentially the 5D $SU(5)$ theory described in Sec. IVA. However, there are now a variety of possibilities for where to locate

the fields. For example, we can introduce the quark hypermultiplets $\{\mathcal{Q}_i, \mathcal{Q}_i^c\}(\mathbf{10})$, $\{\mathcal{U}_i, \mathcal{U}_i^c\}(\mathbf{10})$, and $\{\mathcal{D}_i, \mathcal{D}_i^c\}(\mathbf{5}^*)$ on the $x^6 = \pi R_6$ fixed line and the lepton hypermultiplets $\{\mathcal{L}_i, \mathcal{L}_i^c\}(\mathbf{5}^*)$ and $\{\mathcal{E}_i, \mathcal{E}_i^c\}(\mathbf{10})$ on the $x^6 = 0$ fixed line, where $i = 1, 2, 3$ is the generation index. Imposing the boundary conditions as in Eqs. (26)–(30), the only low-energy states below R_5^{-1} are the three generations of the 4D $N = 1$ quark and lepton supermultiplets, $Q_i, U_i, D_i,$ $L_i,$ and E_i. Locating two Higgs hypermultiplets $\{\mathcal{H}, \mathcal{H}^c\}(\mathbf{1}, \mathbf{2})_{1/2}$ and $\{\bar{\mathcal{H}}, \bar{\mathcal{H}}^c\}(\mathbf{1}, \mathbf{2})_{-1/2}$ on the $x^5 = 0$ fixed line, with the boundary conditions given by $\mathcal{H}(+, +)$, $\mathcal{H}^c(-, -)$, $\bar{\mathcal{H}}(+, +)$, and $\bar{\mathcal{H}}^c(-, -)$ along the sixth direction, we obtain two 4D $N = 1$ Higgs doublet supermultiplets H_u and H_d from these multiplets at low energies.[21] The Yukawa couplings $W \sim QUH_u + QDH_d$ and $W \sim LEH_d$ must be located at $(x^5, x^6) = (0, \pi R_6)$ and $(0, 0)$, respectively.[22] This setup realizes a geometrical separation between the quarks and leptons in an extra dimension, and thus may be used to suppresses possible tree-level proton decay operators. There is, however, a possible tension coming from the fact that we cannot make R_6 very large, since it would increase incalculable nonuniversal contributions to the low-energy 321 gauge couplings that arise from radiatively generated gauge kinetic operators on the $x^5 = 0$ line. Thus we will still need to make some assumptions on the spectrum for the heavy states around the cutoff scale.

The story for supersymmetry breaking can be similar to that in the minimal model. For R_5 larger than R_6, the T field corresponds mainly to the modulus controlling the size for R_5, although it has a small mixture with that for R_6. Assuming that the source of supersymmetry breaking is in nonzero $F_T, D_R,$ and anomaly mediation, the supersymmetry breaking masses at the scale M_c are given by Eqs. (45), (47), and (49) (although $a_{\mathcal{H}}, a_{\bar{\mathcal{H}}}, c_{\mathcal{H}},$ and $c_{\bar{\mathcal{H}}}$ will now be suppressed somewhat because the Higgs fields are brane fields in the 5D effective theory).

Higher-dimensional setups may also be used analogously to understand the quark and lepton masses and mixings in terms of the wave function profiles of matter fields, e.g., in the sixth dimension, although the issue of flavor changing neutral currents must be carefully addressed in such cases. Alternatively, the 6D setup described here can be used to realize the scenario discussed at the end of Sec. III A. For example, we can locate all the quark and lepton supermultiplets on the $x^6 = 0$ fixed line and the supersymmetry breaking field Z on the $(x^5, x^6) = (0, \pi R_6)$ fixed point. In this case the 321 gaugino masses

can be all independent, and the squark and slepton masses are generated through gauge loops and thus flavor universal. Another use of the setup includes geometrically separating the ϕ field from the quark and lepton supermultiplets, ensuring the absence of potential flavor violating operators of the form $\int d^4\theta(\phi^\dagger e^{2r_\phi V_R}\phi) \times (Q_i^\dagger e^{4V_R/3}Q_j)$.

C. Models with warping

So far, we have considered that the extra dimension(s) in which the 321 gauge fields propagate is (approximately) flat. It is, however, possible that there are nonnegligible warping effects. This is, in fact, a natural possibility because the gauging of $U(1)_R$ is associated with breaking of higher-dimensional Lorentz invariance. Here we study the effect of warping, taking as an example the minimal 5D $SU(5)$ model of Sec. IVA.

We consider that the 5D spacetime of the model of Sec. IVA has a nontrivial warping. The metric is given by

$$ds^2 = e^{-2k|y|}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2, \tag{54}$$

where k is the inverse curvature radius of the warped (AdS) space, which is taken to be somewhat smaller than the fundamental scale M_*. This corresponds to choosing the $y = 0$ (321) and $y = \pi R$ [$SU(5)$] branes to be the ultraviolet (UV) and infrared (IR) branes, respectively. Unlike the case of Ref. [50], however, here we take the scale of the UV brane, k, to be of $O(10 \sim 100 \text{ TeV})$. We consider that the warp factor $e^{-\pi kR}$ is of $O(0.01 \sim 0.1)$, so that the scale of the IR brane is given by $k' \equiv ke^{-\pi kR} = O(1 \sim 10 \text{ TeV})$.

The gauge group, matter content, and boundary conditions are taken to be identical to those in Sec. IVA. The model then works analogously to the flat space case. An important difference is that the wave function of the lightest (zero) mode of T is now localized to the IR brane, so that its F-term VEV is suppressed in 4D by the warp factor $e^{-\pi kR}$. This can thus be used to explain the small hierarchy between M_* and F_T ($\approx$ the weak scale) without using any small parameter. The soft supersymmetry breaking masses are still given by Eqs. (45), (47), and (49), although the explicit expressions for a_Φ and c_Φ are changed.[23] Another difference is that, since the KK towers in warped space are localized to the IR brane, where the active gauge group is $SU(5)$, their spectrum is approximately $SU(5)$ symmetric. In particular, the masses of the KK gauge states associated with $SU(3)_C, SU(2)_L, U(1)_Y,$ and $SU(5)/321$ are now roughly universal $M_3' \simeq M_2' \simeq M_1' \simeq M_X'$. Splittings among these masses, however, arise from the UV brane gauge

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[21]Quantization of $U(1)_Y$ hypercharges for these multiplets must come from physics above the cutoff scale, M_*.

[22]The S field can be introduced either on the $x_5 = 0$, $x_6 = 0$, or $x_6 = \pi R_6$ fixed line, or on the $(x^5, x^6) = (0, 0)$ or $(0, \pi R_6)$ fixed point. The Yukawa coupling $W \sim SH_uH_d$ arises from the $(x^5, x^6) = (0, 0)$ and/or $(0, \pi R_6)$ fixed point, while $W \sim S^3$ from the $(0, 0)$, $(0, \pi R_6)$, $(\pi R_5, 0)$ and/or $(\pi R_5, \pi R_6)$ fixed point.

[23]In a warped space model, the contribution from anomaly mediation may be naturally suppressed $\hat{m}_C/8\pi^2 \ll \hat{m}_T, \hat{m}_D$. This is because the T field is expected not to feel (effectively) the large gravitational dimensions, as suggested by the "dual" description of the theory (given below). The scale of fundamental supersymmetry breaking is then small, of $O(\text{TeV})$, giving a very small gravitino mass, $m_{3/2} = O(\text{TeV}^2/M_{\text{Pl}})$.

kinetic operators, which are determined by the observed 321 gauge couplings (for a fixed value of ζ'; see Sec. IVA). Precise relations are given by

$$M'_I \simeq M'_X \left(1 + \frac{2}{3} \frac{g_I^2}{g_U'^2 \ln(k/k')} \right), \qquad (55)$$

where M'_I ($I = 1, 2, 3$) are the masses of the first KK excitations for the 321 gauge bosons, M'_X the mass of the lightest $SU(5)/321$ gauge boson, $g_U'^2 = g_*^2/\pi R$ with g_* the 5D $SU(5)$ gauge coupling, and g_I the 321 gauge couplings at the scale $M'_X \simeq (3\pi/4)k'$. We thus find that $M'_3 > M'_2 > M'_1 > M'_X$ with each mass splitting typically of about 10%. [The second term in the bracket is $\simeq 0.25 g_I^2 (\ln(10)/\ln(k/k'))$ for $\zeta' = 1$.] The model described here is similar to that in Ref. [51], but now the scale of the UV brane is much smaller $k = O(10 \sim 100 \text{ TeV})$ and the gauge coupling evolution above this scale is mimicked by the M VEV through the operators of Eq. (38). This has the advantage that the 5D theory is (more) weakly coupled, since we can have a larger value of $M_*/\pi k$ with fixed values of the 4D gauge couplings. Relatively large mass splittings of $O(10\%)$ among the KK gauge boson states can be a consequence of this lowered fundamental scale. [The corresponding mass splittings are of order a few percent in the model of Ref. [51], since the second term in the bracket of Eq. (55) is then $\approx 0.05 g_I^2$.]

The present model can be interpreted as a purely 4D theory (except for possible gravitational dimensions) through the AdS/CFT correspondence [52]. In the 4D picture, the theory contains a strongly interacting (quasi-)conformal gauge sector G, whose conformality is spontaneously broken at the scale k'. The G sector possesses a $SU(5)$ global flavor symmetry, of which the $SU(3) \times SU(2) \times U(1)$ subgroup is explicitly gauged and identified as the standard model gauge group 321. There are quark, lepton, and Higgs supermultiplets, Q_i, U_i, D_i, L_i, E_i, H_u, H_d, and S, which are charged under 321 (except for S) and neutral under G. At the fundamental scale M_*, the gauge kinetic terms (gauge couplings) of 321 come purely from the terms of the form $\mathcal{L} = -\sum_{I=1,2,3} \int d^2\theta (\zeta_I \langle M \rangle/c) \times \mathcal{W}_I^{a\alpha} \mathcal{W}_{I\alpha}^a + \text{H.c.}$, where $\zeta_I = A_I - \zeta'$ with $\zeta' \gtrsim 1$. The remaining universal piece then comes from an asymptotically nonfree contribution from the G sector: $\delta(1/g_I^2)|_G = (b_G/8\pi^2) \ln(k/k')$, where b_G (> 0) is the beta-function coefficient for the contribution from G, and the universality of the contribution is guaranteed by the global $SU(5)$ symmetry. The required numerology can be read off from Eqs. (42) and (43) in the case of $\zeta' = 1$: $\langle M \rangle/c \simeq 0.25$ and $(b_G/8\pi^2) \ln(k/k') \simeq 1$. Note that the understanding of the coefficient ζ_I is quite simple in this context. The quantities A_I and $-\zeta'$ represent the mixed $U(1)_R$-321 gauge anomalies carried by the elementary (quark, lepton, Higgs, and 321 gauge) states and the G states, respectively, and the sum of them, $A_I - \zeta' = \zeta_I$, is canceled by the shift of M. Supersymmetry breaking is caused by the IR dynamics of

the G sector at the scale k', and is transmitted to the MSSM (and S) states through mixings between the elementary and G states and by 321 gauge loops.

The 4D description of the theory given above allows us to make a simple estimate for the scales appearing in the theory. First, from the observed values of the 321 gauge couplings, we find that the contribution from the G sector to the 321 gauge couplings should be

$$\delta\frac{1}{g_I^2} \bigg|_G = \frac{b_G}{8\pi^2} \ln\frac{k}{k'} \simeq 2 - \zeta'. \qquad (56)$$

Let us now focus on the case with $\zeta' = 1$ for simplicity, although similar results are also obtained for other values of ζ' unless ζ' is tuned such that $\delta(1/g_I^2)|_G \ll 1$. (Note that $\zeta' \lesssim 2$ to reproduce the observed 321 gauge couplings with $b_G > 0$.) Since $k/k' = e^{\pi k R} = O(10 \sim 100)$ in the present setup, Eq. (56) gives $b_G \approx (20 \sim 30)$, implying that the G sector is quite "large" when viewed from the 321 sector. (This is important to guarantee that the 5D picture is weakly coupled.) Now, let us assume that supersymmetry breaking is induced by the strong G dynamics that do not involve any particularly small or large numbers. In this case, the mass of the superparticles can be estimated using large-N scaling [53]; in particular, we find that the 321 gaugino masses M_I are given by

$$M_I \simeq \frac{g_I^2}{16\pi^2} b_G M_\rho, \qquad (57)$$

where $M_\rho \approx \pi k'$ represents the mass scale for the resonances in the G sector, i.e. the mass scale for the KK towers in the 5D picture. (For similar analyses, see [54].) This equation implies that for a large G sector of $b_G \approx (20 \sim 30)$, the mass hierarchy between the superparticles and the KK resonances is only of a factor of ≈ 10. We thus expect that the masses of the KK gauge bosons, which are approximately $SU(5)$ symmetric, are (only) of order a few TeV. Note that in the conventional desert case, e.g. in the model of [51], the large desert of $\ln(k/k') \simeq 30$ leads to $b_G \lesssim (4 \sim 5)$, making the KK masses much higher. This opens the exciting possibility that some of these states may be observed at the Large Hadron Collider (LHC). Moreover, the value of b_G may be measured from mass splittings among these states. Using the AdS/CFT correspondence, we find that Eq. (55) can be written as $M'_I \simeq M'_X(1 + g_I^2 b_G/12\pi^2)$. If we find a value of b_G that is significantly larger than $\simeq 5$ from these mass relations [or through Eq. (57)], it would provide a strong experimental suggestion, possibly together with large values of λ and/or κ, that the fundamental scale of nature is not that far above the weak scale.

V. DISCUSSION AND CONCLUSIONS

Unraveling the physical origin of electroweak symmetry breaking will be one of the central themes in particle physics in the next few years. The LHC will start exploring

an energy region well above the masses of the electroweak gauge bosons within two years, which may reveal some new physics beyond the standard model whose existence is intimately related to electroweak symmetry breaking. It is then crucial that we extract as much information as possible from this data both experimentally and theoretically. In particular, it will be important to explore possible interpretations of the data, which may eventually allow us to uncover some basic aspects of the fundamental theory of nature.

Supersymmetry is one of the leading candidates for new physics at the electroweak scale. Its successes are often stated in the context of the supersymmetric desert—weak scale supersymmetry stabilizes the large hierarchy between the Planck and the weak scales, and it leads to a successful unification of the three gauge couplings at a high energy close to the Planck scale. In our view, the most *robust* success of supersymmetry, however, lies in the fact that it stabilizes the hierarchy between the weak scale and the scale that suppresses most of higher-dimension operators of the standard model. From the LEP data, we know that this (small) hierarchy almost certainly exists. Supersymmetry provides one of the best ways to stabilize it without leading to an immediate conflict with the precision electroweak data, since the electroweak symmetry is still broken by the VEVs of Higgs fields that are perturbative at the weak scale. While aspects associated with the existence of the desert may well be an illusion, being the result of a vast extrapolation in many orders of magnitude in energy, we feel that the feature of supersymmetry just described should play a role if weak scale supersymmetry is actually realized in nature.

In this paper we have studied a framework of weak scale supersymmetry in which (most of) the virtues of the supersymmetric desert are naturally reproduced without actually having a large energy interval above the weak scale. Such a picture may, in fact, be suggested from naturalness of electroweak symmetry breaking, since fine-tuning in conventional supersymmetric theories often arises from the large logarithmic running of supersymmetry breaking masses and/or the conflict of a large coupling(s) with the Landau pole constraint, neither of which applies in the absence of the large desert. We have shown that a (gauged) $U(1)_R$ symmetry that assigns the same charge for all the matter and Higgs supermultiplets may play an important role—it may reproduce the successful supersymmetric prediction for the low-energy gauge couplings because of its relation to conformal symmetry. We have demonstrated that this can indeed be realized in effective field theory and have constructed classes of explicit models based on higher-dimensional unified field theories. The $U(1)_R$ symmetry can have important consequences for the form of the observable sector superpotential; in particular, the Higgs sector superpotential is expected not to contain any dimensionful parameters. This allows, together with a low fundamental scale, for making the mass of the lightest Higgs boson rather large, $\approx (200 \sim 300)$ GeV, helping to eliminate fine-tuning in electroweak symmetry breaking. The consistency with the precision electroweak data can be recovered through the contributions from the Higgs sector and/or new states at a multi-TeV region, such as the KK states associated with the standard model gauge fields.

There are many natural sources of supersymmetry breaking in this framework—the auxiliary component VEVs of various singlet (moduli) fields, $U(1)_R$ D-term VEV, and anomaly mediation. An interesting aspect here is that what is usually referred to (broadly) as gravity mediation now occurs at a scale not much far above the weak scale. We have calculated the resulting pattern of supersymmetry breaking masses and find that it can be quite distinct. For example, in the case that these masses are dominated by the $U(1)_R$ D-term VEV and the auxiliary component VEV of the field giving the universal contribution to the standard model gauge couplings, we obtain gaugino masses proportional to the square of the gauge couplings and squark and slepton masses proportional to their anomalous dimensions at the scale where these masses are generated (between of order a few and a hundred TeV). Since such a pattern of superparticle masses does not arise very naturally in the conventional desert framework, its observation at the LHC may provide a nontrivial hint for the absence of the desert.

Finally we comment on possible variations of the basic framework presented in this paper. We have mainly considered the fundamental scale M_* to be in the range of $O(10 \sim 100 \text{ TeV})$. It should, however, be obvious that there is no real upper bound on this scale, except that it should probably be smaller than the 4D (not reduced) Planck scale $\approx 10^{19}$ GeV. While lower values of M_* may be preferred from naturalness of electroweak symmetry breaking, there is nothing really wrong with other values; for example, the case with intermediate scale M_* may be an interesting possibility.[24] Such a variation of M_* will have interesting consequences on the superparticle spectrum and the resulting phenomenology. The charge assignment for $U(1)_R$ may also be modified. For example, one may consider a family-dependent charge assignment, which does not modify the prediction for the gauge couplings as long as it commutes with $SU(5)$. The Yukawa couplings are then generated through the VEV of the ϕ field, which may partly explain the origin of the observed Yukawa hierarchy if the ϕ VEV is somewhat smaller than

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[24]An alternative, amusing application of our models is to consider M_* to be at the gravitational (or the conventional string) scale with R^{-1} close to M_*. The effective gauge coupling unification scale can then be *lower* than M_* if $\langle M \rangle/c$ is negative, reproducing the conventional unification scale of $M_U \simeq 2 \times 10^{16}$ GeV for $\langle M \rangle/c \approx -(0.03 \sim 0.05)$. Such a value of $\langle M \rangle/c$ can, in fact, be generated through stabilization discussed in this paper or through standard gaugino condensation.

the cutoff scale, due to, e.g., a large $U(1)_R$ charge for ϕ. Theories which use a non-R $U(1)$ gauge symmetry, instead of an $U(1)_R$ gauge symmetry, can also be considered, and we present this possibility in Appendix B. Finally, the large gravitational dimension may be strongly warped, in which case our basic picture may have to be modified. For example, we can attach a strongly warped gravitational dimension to the basic module of Sec. II. The resulting theory can then be a 5D supersymmetric theory with the metric given by Eq. (54) in which all the MSSM and S states are localized to the IR brane at $y = \pi R$. The scales at the UV and IR branes can be chosen to be of order the 4D Planck scale and $(10 \sim 100)$ TeV, respectively. This picture is then close to that of Ref. [50], but with the small hierarchy between the IR brane cutoff scale and the weak scale stabilized by supersymmetry and with the successful gauge coupling prediction reproduced through interactions of the form of Eq. (3). The way this theory avoids the Landau pole constraints is similar to that in [7], although the absence of the standard model gauge fields in the bulk allows for the 5D theory to be more weakly coupled. Note that this theory in fact possesses a large energy desert, as can be clearly seen in the 4D dual picture. The desert, however, is simply not relevant for the MSSM and S states in this picture because they are composite states generated at the scale of $(10 \sim 100)$ TeV.

In summary, we have presented a simple and realistic framework for supersymmetry which does not possess a large energy desert above the weak scale. The framework has rich phenomenological implications and allows for detailed analyses of, e.g., electroweak symmetry breaking. If the LHC finds superparticles whose spectrum shows features discussed in this paper and/or if it discovers a Higgs sector which indicates some of the couplings are so large that they hit the Landau pole before the conventional unification scale, then it would provide a strong suggestion that the fundamental scale of nature may not be far above the weak scale. It is exciting that we may indeed be able to explore this possibility in the next few years.

ACKNOWLEDGMENTS

APPENDIX A: SOFT SUPERSYMMETRY BREAKING PARAMETERS IN THE MINIMAL MODEL

In this appendix we present explicit expressions for the soft supersymmetry breaking terms in the model of Sec. IVA. The gaugino masses are given by Eq. (45).

The scalar trilinear interactions are given in general by Eq. (47). Defining $\mathcal{L}_{\text{soft}} = -(a_u)_{ij}\tilde{q}_i\tilde{u}_j h_u - (a_d)_{ij}\tilde{q}_i\tilde{d}_j h_d - (a_e)_{ij}\tilde{l}_i\tilde{e}_j h_d - (a_\nu)_{ij}\tilde{l}_i\tilde{n}_j h_u - a_\lambda s h_u h_d - (a_\kappa/3)s^3 + \text{H.c.}$, this gives

$$(a_u)_{ij} = -(y_u)_{ij}(a_{\mathcal{H}}\hat{m}_T + (\gamma_{Q_i} + \gamma_{U_j} + \gamma_{H_u})\hat{m}_C), \quad \text{(A1)}$$

$$(a_d)_{ij} = -(y_d)_{ij}(a_{\bar{\mathcal{H}}}\hat{m}_T + (\gamma_{Q_i} + \gamma_{D_j} + \gamma_{H_d})\hat{m}_C), \quad \text{(A2)}$$

$$(a_e)_{ij} = -(y_e)_{ij}(a_{\bar{\mathcal{H}}}\hat{m}_T + (\gamma_{L_i} + \gamma_{E_j} + \gamma_{H_d})\hat{m}_C), \quad \text{(A3)}$$

$$[(a_\nu)_{ij} = -(y_\nu)_{ij}(a_{\mathcal{H}}\hat{m}_T + (\gamma_{L_i} + \gamma_{N_j} + \gamma_{H_u})\hat{m}_C)], \quad \text{(A4)}$$

$$a_\lambda = -\lambda((a_{\mathcal{S}} + a_{\mathcal{H}} + a_{\bar{\mathcal{H}}})\hat{m}_T + (\gamma_{\mathcal{S}} + \gamma_{H_u} + \gamma_{H_d})\hat{m}_C), \quad \text{(A5)}$$

$$a_\kappa = -3\kappa(a_{\mathcal{S}}\hat{m}_T + \gamma_{\mathcal{S}}\hat{m}_C), \quad \text{(A6)}$$

where a_φ ($\varphi = \mathcal{H}, \bar{\mathcal{H}}, \mathcal{S}$) are given by $a_\varphi = 2\pi R M_\varphi/(e^{2\pi R M_\varphi} - 1)$ [see Eq. (48)]. Here, we have used Eq. (44), and the neutrino Yukawa couplings $(y_\nu)_{ij}$ in Eq. (A4) are defined analogously to the other Yukawa couplings in Eq. (1).

The soft supersymmetry breaking scalar squared masses are given by Eq. (49). For the squarks and sleptons, this gives

$$m_{\tilde{F}_i}^2 = -\gamma_{F_i}\hat{m}_D^2 + \frac{1}{2}\frac{d\gamma_{F_i}}{d\ln\mu_R}|\hat{m}_C|^2 - \sum_I \frac{C_I^F}{8\pi^2}\frac{g_I^4}{g_U'^2}(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \quad \text{(A7)}$$

$$m_{\tilde{Q}_3}^2 = -\gamma_{Q_3}\hat{m}_D^2 + \frac{1}{2}\frac{d\gamma_{Q_3}}{d\ln\mu_R}|\hat{m}_C|^2 + \frac{1}{8\pi^2}\left(\frac{1}{2}(y_t^2 a_{\mathcal{H}} + y_b^2 a_{\bar{\mathcal{H}}}) - \sum_I C_I^Q \frac{g_I^4}{g_U'^2}\right)(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \quad \text{(A8)}$$

$$m_{\tilde{U}_3}^2 = -\gamma_{U_3}\hat{m}_D^2 + \frac{1}{2}\frac{d\gamma_{U_3}}{d\ln\mu_R}|\hat{m}_C|^2 + \frac{1}{8\pi^2}\left(y_t^2 a_{\mathcal{H}} - \sum_I C_I^U \frac{g_I^4}{g_U'^2}\right)(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \quad \text{(A9)}$$

$$m_{\tilde{D}_3}^2 = -\gamma_{D_3}\hat{m}_D^2 + \frac{1}{2}\frac{d\gamma_{D_3}}{d\ln\mu_R}|\hat{m}_C|^2 + \frac{1}{8\pi^2}\left(y_b^2 a_{\bar{\mathcal{H}}} - \sum_I C_I^D \frac{g_I^4}{g_U'^2}\right)(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \quad \text{(A10)}$$

$$m_{\tilde{L}_3}^2 = -\gamma_{L_3}\hat{m}_D^2 + \frac{1}{2}\frac{d\gamma_{L_3}}{d\ln\mu_R}|\hat{m}_C|^2 + \frac{1}{8\pi^2}\left(\frac{1}{2}(y_\tau^2 a_{\bar{\mathcal{H}}} + y_{\nu_3}^2 a_{\mathcal{H}}) - \sum_I C_I^L \frac{g_I^4}{g_U'^2}\right)(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \quad \text{(A11)}$$

$$m_{\tilde{E}_3}^2 = -\gamma_{E_3}\hat{m}_D^2 + \frac{1}{2}\frac{d\gamma_{E_3}}{d\ln\mu_R}|\hat{m}_C|^2 + \frac{1}{8\pi^2}\left(y_\tau^2 a_{\tilde{\mathcal{H}}}\right.$$

$$\left. - \sum_I C_I^E \frac{g_I^4}{g_U'^2}\right)(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \qquad (A12)$$

$$\left[m_{\tilde{N}_3}^2 = -\gamma_{N_3}\hat{m}_D^2 + \frac{1}{2}\frac{d\gamma_{N_3}}{d\ln\mu_R}|\hat{m}_C|^2 \right.$$

$$\left. + \frac{1}{8\pi^2}y_{\nu_3}^2 a_{\mathcal{H}}(\hat{m}_T\hat{m}_C^* + \text{H.c.})\right], \quad (A13)$$

where $F = Q, U, D, L, E$ (and N), the index $i = 1, 2$ runs over the first two generations in Eq. (A7), and C_I^F are the group theoretical factors given by $(C_1^F, C_2^F, C_3^F) = (1/60, 3/4, 4/3)$, $(4/15, 0, 4/3)$, $(1/15, 0, 4/3)$, $(3/20, 3/4, 0)$, $(3/5, 0, 0)$, and $(0, 0, 0)$ for $F = Q, U, D, L, E$, and N, respectively. Here, we have retained only the third generation Yukawa couplings $y_t \equiv (y_u)_{33}$, $y_b \equiv (y_d)_{33}$, $y_\tau \equiv (y_e)_{33}$, and $y_{\nu_3} \equiv (y_\nu)_{33}$ (which may not be valid for the neutrino Yukawa couplings). We have also used one-loop expressions for γ_Φ in the last terms in the right-hand sides of Eqs. (A7)–(A13) for illustrative purposes. The first and second terms are the contributions from a nonvanishing $U(1)_R$ D-term VEV and pure anomaly mediation, respectively.

For the Higgs fields h_u, h_d, and s, we find

$$m_{h_u}^2 = -\gamma_{H_u}\hat{m}_D^2 + c_{\mathcal{H}}|\hat{m}_T|^2 + \frac{1}{2}\frac{d\gamma_{H_u}}{d\ln\mu_R}|\hat{m}_C|^2$$

$$+ \frac{1}{8\pi^2}\left(\frac{1}{2}\{(3y_t^2 + y_{\nu_3}^2 + \lambda^2)a_{\mathcal{H}} + \lambda^2 a_{\tilde{\mathcal{H}}} + \lambda^2 a_{\mathcal{S}}\}\right.$$

$$\left. - \sum_I C_I^H \frac{g_I^4}{g_U'^2}\right)(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \qquad (A14)$$

$$m_{h_d}^2 = -\gamma_{H_d}\hat{m}_D^2 + c_{\tilde{\mathcal{H}}}|\hat{m}_T|^2 + \frac{1}{2}\frac{d\gamma_{H_d}}{d\ln\mu_R}|\hat{m}_C|^2$$

$$+ \frac{1}{8\pi^2}\left(\frac{1}{2}\{(3y_b^2 + y_\tau^2 + \lambda^2)a_{\tilde{\mathcal{H}}} + \lambda^2 a_{\mathcal{H}} + \lambda^2 a_{\mathcal{S}}\}\right.$$

$$\left. - \sum_I C_I^H \frac{g_I^4}{g_U'^2}\right)(\hat{m}_T\hat{m}_C^* + \text{H.c.}), \qquad (A15)$$

$$m_s^2 = -\gamma_S\hat{m}_D^2 + c_{\mathcal{S}}|\hat{m}_T|^2 + \frac{1}{2}\frac{d\gamma_S}{d\ln\mu_R}|\hat{m}_C|^2$$

$$+ \frac{1}{8\pi^2}\{(\lambda^2 + 3\kappa^2)a_{\mathcal{S}} + \lambda^2 a_{\mathcal{H}} + \lambda^2 a_{\tilde{\mathcal{H}}}\}$$

$$\times (\hat{m}_T\hat{m}_C^* + \text{H.c.}), \qquad (A16)$$

where $(C_1^H, C_2^H, C_3^H) = (3/20, 3/4, 0)$. The coefficients c_φ ($\varphi = \mathcal{H}, \tilde{\mathcal{H}}, \mathcal{S}$) are given (in flat space) by $c_\varphi = (\pi R M_\varphi/\sinh(\pi R M_\varphi))^2$ [see Eq. (51)].

APPENDIX B: THEORIES WITH A NON-R $U(1)$ GAUGE SYMMETRY

In this appendix we present an alternative class of theories in which a non-R pseudoanomalous $U(1)$ gauge sym-

metry is used instead of the gauged R symmetry. This class of theories has a potential advantage in that the cutoff scale does not have to be charged under the $U(1)$ gauge symmetry, i.e. one can regulate the theory in such a way that there is no anomalous transformation of the superspace density under a supersymmetric $U(1)$ transformation. This may allow us to rely less on the structure of the ultraviolet theory above M_* to explain the form of the low-energy effective Lagrangian.[25] As we will see below, this is achieved at the expense of somewhat arbitrary choices of the matter content and the $U(1)$ charge assignment.

Let us first consider the effective theory below M_c. We introduce, as usual, the standard model quark, lepton, and Higgs chiral superfields, Q_i, U_i, D_i, L_i, E_i, H_u, and H_d ($i = 1, 2, 3$), which have the Yukawa couplings of Eq. (1). We then introduce a (pseudoanomalous) $U(1)_A$ gauge symmetry, under which the matter and Higgs fields carry charges of $+1/2$ and -1, respectively. While the Yukawa couplings are invariant under this charge assignment, the fundamental mass term for the Higgsinos, $W = \mu H_u H_d$, is not. We thus introduce a singlet field $S(+2)$ together with the interaction $W = \lambda S H_u H_d$, which gives a necessary mass term for the Higgsinos after S obtains a VEV and also provides an extra contribution to the physical Higgs boson mass.

At this point, the mixed $U(1)_A$-321 gauge anomalies are given by $A_1 = 12/5$, $A_2 = 2$, and $A_3 = 3$, which do not take the necessary form of $A_I \propto b_I + b$ ($I = 1, 2, 3$), where b_I are the MSSM beta-function coefficients, $(b_1, b_2, b_3) = (33/5, 1, -3)$, and b is a constant which does not depend on the gauge group I. To fix this "problem," we introduce fields X_1, X_2, and X_3 which transform as adjoints under the 321 gauge group, i.e. $X_1(\mathbf{1}, \mathbf{1})_0$, $X_2(\mathbf{1}, \mathbf{3})_0$, and $X_3(\mathbf{8}, \mathbf{1})_0$, and carry an $U(1)_A$ charge of $+3$. The mixed anomalies are then given by $A_1 = 12/5$, $A_2 = 8$, and $A_3 = 12$, which satisfies

$$A_I = -b_I + 9. \qquad (B1)$$

These mixed anomalies are canceled by terms of the form of Eq. (3) upon introduction of a moduli field M that transforms as $M \to M + (c/16\pi^2)\Lambda$ under $U(1)_A$, where Λ is the gauge transformation parameter superfield for $U(1)_A$ in the normalization that a chiral superfield with an $U(1)_A$ charge q transforms as $\Phi \to e^{q\Lambda}\Phi$. [The $U(1)_Y$-$U(1)_A^2$ anomaly is automatically vanishing, and we do not consider the $U(1)_A^3$ or $U(1)_A$-(gravity)^2 anomalies since they depend on unknown fields that are singlet under the 321 gauge group.] Introducing the universal contribu-

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[25]For example, it may be easier in these theories to conceive that $U(1)$ invariance of the (observable sector) superspace density is ensured purely by the $U(1)$ gauge multiplet V, while that of the gauge kinetic functions (at the quantum level) by a moduli field M. This will be the case, e.g., if the matter and Higgs fields are geometrically separated from the M field in extra spatial dimensions in which the standard model gauge fields propagate.

tion $\langle T \rangle$ to the 321 gauge kinetic functions, as in Eq. (4), we find that the 321 gauge couplings at M_c, g_I, take exactly the same form as that arising in the conventional supersymmetric desert picture, Eq. (7). The correspondence between the two theories is now given by

$$\langle T \rangle - \frac{36}{c} \langle M \rangle \leftrightarrow \frac{1}{g_U^2}, \tag{B2}$$

$$\langle M \rangle \leftrightarrow \frac{c}{32\pi^2} \ln \frac{M_U}{M_c}, \tag{B3}$$

instead of Eqs. (8) and (9), and the relation among the low-energy gauge couplings is given by the standard supersymmetric one in Eq. (10). The required VEVs of $\langle T \rangle = O(1) > 0$ and $\langle M \rangle / c = O(0.1) > 0$ can be generated in the way discussed in Sec. II B. With $c = O(1)$, this can be done within the regime of effective field theory.

In order for the model to be realistic, the adjoint fields X_1, X_2, and X_3 must obtain masses of order the weak scale or larger (at least for X_2 and X_3). We thus introduce a singlet field ϕ with an $U(1)_A$ charge of -6, together with the superpotential interactions $W = (\lambda_1/2)\phi X_1^2 + (\lambda_2/2)\phi X_2^2 + (\lambda_3/2)\phi X_3^2$, where the couplings λ_I's may or may not respect the $SU(5)$ relation, $\lambda_1 = \lambda_2 = \lambda_3$, at a scale $\approx M_c$. The ϕ field obtains a VEV of order $M_*/4\pi$ to cancel the Fayet-Iliopoulos term for $U(1)_A$. The Fayet-Iliopoulos term is generated after the M field obtains a

VEV, since its Kähler potential is given by $K = M_*^2 \mathcal{F}(M + M^\dagger + (c/8\pi^2)V_A)$, where V_A is the $U(1)_A$ gauge multiplet and $\mathcal{F}(x)$ is an arbitrary polynomial in x with coefficients of $O(1)$ up to symmetry factors. We assume that the generated Fayet-Iliopoulos term is positive, in which case ϕ can absorb (most of) this term. The fact that ϕ absorbs, and not H_u and H_d, should be determined energetically after supersymmetry breaking effects are included. We can alternatively generate a larger VEV for ϕ, of order M_*, if we introduce the superpotential $W = Y(\phi\bar{\phi} - M_*^2)$, where Y and $\bar{\phi}$ are chiral superfields with $U(1)_A$ charges of 0 and $+6$, respectively. A nonvanishing $\langle \phi \rangle$ gives masses to the adjoint fields X_1, X_2, and X_3. It also provides the S^3 term in the superpotential through $W = \eta \phi S^3 / M_*$.

Summarizing, the superpotential of the model is given by

$$W = W_{\text{Yukawa}} + \lambda S H_u H_d + \frac{\eta}{3M_*} \phi S^3 + \frac{\lambda_1}{2} \phi X_1^2$$
$$+ \frac{\lambda_2}{2} \phi X_2^2 + \frac{\lambda_3}{2} \phi X_3^2, \tag{B4}$$

where ϕ obtains a nonvanishing VEV of $O(M_*/4\pi \sim M_*)$ and W_{Yukawa} is given by Eq. (1). The $U(1)_A$ charge assignment for the fields is given by

$$\begin{array}{ccccccccccccc} & Q_i & U_i & D_i & L_i & E_i & (N_i) & H_u & H_d & S & \phi & X_1 & X_2 & X_3 \\ U(1)_A: & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & 1/2 & -1 & -1 & 2 & -6 & 3 & 3 & 3 \end{array}. \tag{B5}$$

An interesting point is that, despite the absence of $U(1)_R$, we are again led to the form of the superpotential of the next-to-minimal supersymmetric standard model. (The absence of M in the superpotential is assumed, as always.) Some of the dangerous higher-dimension operators in the superpotential, such as the one leading to large Majorana neutrino masses, are suppressed by the (broken) $U(1)_A$ symmetry. The standard matter parity (R parity) also arises automatically as a discrete subgroup of $U(1)_A$, if the Kähler potential is made $U(1)_A$ invariant purely by the $U(1)_A$ gauge multiplet, V_A, and not by the combination $-(8\pi^2/c)(M + M^\dagger)$. (The stability of protons can be ensured in a way discussed in Sec. III B.) Note that a constant term in the superpotential, necessary to cancel the cosmological constant, can be introduced without breaking $U(1)_A$.

The model can be embedded into higher dimensions analogously to Sec. IV. Both flat space and warped space models can be constructed. The $U(1)_A$ charge assignment for 5D supermultiplets $\{\Phi, \Phi^c\}$ ($\Phi = Q_i, U_i, \cdots$) is determined by Eq. (B5), with a conjugated field carrying the opposite $U(1)_A$ charge from a nonconjugated field. With the help of an extra moduli field M' or vectorlike states

located on the $SU(5)$-preserving brane, we can have a consistent effective field theory describing physics above the compactification scale M_c.

The story for supersymmetry breaking is similar to the $U(1)_R$ case. We have in general contributions from F_T, F_M, anomaly mediation, as well as a nonvanishing D-term for $U(1)_A$, D_A, which is generated, e.g., if the ϕ field has a supersymmetry breaking mass of order the weak scale. In the case that the contributions arise from F_T, D_A, and anomaly mediation, the supersymmetry breaking masses at the scale M_c are given by Eqs. (45), (47), and (49) but with the first term in the right-hand side of Eq. (49) replaced as

$$-\gamma_\Phi \hat{m}_D^2 \to q_\Phi \hat{m}_D^2, \tag{B6}$$

where q_Φ is the $U(1)_A$ charge of a superfield Φ, and $\hat{m}_D^2$ is now given by $\hat{m}_D^2 = -D_A$ instead of Eq. (50). In the presence of gauge kinetic mixing between $U(1)_A$ and $U(1)_Y$ at tree level, this term is given by $(q_\Phi - \epsilon g_1^2 Y_\Phi) \hat{m}_D^2$, where ϵ is the coefficient of the kinetic mixing term and Y_Φ the $U(1)_Y$ hypercharge of the chiral superfield Φ in the $SU(5)$ normalization (see footnote 19).

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