

Semiinclusive hadronic B decays as null tests of the standard modelA. Soni¹ and J. Zupan^{2,3}¹*Physics Department, Brookhaven National Laboratory, Upton, New York 11973, USA*²*Department of Physics, Carnegie Mellon University, Pittsburgh, Pennsylvania 15213, USA*³*J. Stefan Institute, Jamova 39, P.O. Box 3000, 1001 Ljubljana, Slovenia*

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We propose a new set of observables that can be used as experimental null tests of the Standard Model in charged and neutral B decays. The CP asymmetries in hadronic decays of charged B mesons into inclusive final states containing at least one of the following mesons: $K_{S,L}$, η' , $c\bar{c}$ bound states or neutral K^* or D mesons, for all of which a U -spin rotation is equivalent to a CP conjugation, are CKM suppressed and furthermore vanish in the exact U -spin limit. We show how this reduces the theoretical error by using Soft Collinear Effective Theory to calculate the CP asymmetries for $K_{S,L}X_{s+d}$, K^*X_{s+d} and $\eta'X_{s+d}$ final states in the endpoint region. For these CP asymmetries only the flavor and not the charge of the decaying B meson needs to be tagged up to corrections of NLO in $1/m_b$, making the measurements more accessible experimentally.

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I. INTRODUCTION

Recently the experiments at the two asymmetric B -factories have helped us attain an important milestone in our understanding of CP violation phenomena. The standard model (SM) prediction of $\sin 2\beta = 0.742^{+0.072}_{-0.026}$ [1] is found to be in very good agreement with the directly measured value 0.674 ± 0.026 [2]. The effects of a CP -odd phase due to beyond the SM sources are thus expected to cause only a small perturbation. Consequently *null tests* of the SM gain special importance in our quest for new physics (NP). Since CP is not an exact symmetry of the SM, it is generally not possible to construct exact null tests of the Cabibbo-Kobayashi-Maskawa (CKM) paradigm of CP violation [3]; the best we can hope for are approximate null tests. One such null test that has attracted a lot of attention lately is the prediction that the difference of S parameters in time dependent $B(t) \rightarrow J/\Psi K_{S,L}$ and ‘‘penguin-dominated’’ decays such as $B(t) \rightarrow (\phi, \eta') K_{S,L}$ should be well below 5% [4–9], which is violated at present by about $1 - 2\sigma$ [10]. In this paper we propose a new set of observables that can be used as rather clean and stringent null tests of the SM.

The proposed tests involve *direct* CP violating partial width differences (PWD) of *untagged* semi-inclusive hadronic decays of charged B mesons

$$\Gamma(B^- \rightarrow M^0 X_{s+d}^-) - \Gamma(B^+ \rightarrow \bar{M}^0 X_{s+d}^+), \quad (1)$$

with the notation that $M^0 X$ is a final state containing *at least one* meson M^0 . For judicious choice of meson M^0 the PWD in Eq. (1) is doubly suppressed. In addition to the CKM suppression to be discussed in more detail below, it also vanishes in the limit of exact U -spin, if meson M^0 is either (i) an eigenstate of discrete transformation $s \leftrightarrow d$, such as $K_{S,L}$, η' or any $c\bar{c}$ bound state, or (ii) if M^0 and \bar{M}^0 are related through $s \leftrightarrow d$ transformation, e.g. M^0 can be K^0 , K^{0*} or D^0 . In this latter case a sum over the two related

states needs to be made, e.g. $\Gamma(B^- \rightarrow M^0 X_{s+d}^-) \rightarrow \Gamma(B^- \rightarrow (K^{0*} + \bar{K}^{0*}) X_{s+d}^-)$. Because of the double suppression SM predicts vanishingly small (i.e. $<1\%$) asymmetries in these modes, while theoretical uncertainties on the predictions are reduced due to approximate $SU(3)$ symmetry, so that they constitute useful approximate null tests.

Recall that, as a rule, it is difficult to reliably predict direct CP violating asymmetries for exclusive hadronic final states due to limited knowledge of strong phases. Indeed a novel feature of our proposed tests is that through a judicious use of U -spin and of inclusive final states a sizable class of direct CP asymmetries can be turned into precision tests of the SM. A stricter null test of the SM is also obtained, if in (1) M^0 is replaced with a photon, a possibility already discussed in the literature [11].

The proposed null tests also have additional experimental advantages. Firstly, since direct CP asymmetries are involved no time-dependent measurements are needed. Also, since *untagged* final states are used no separation of X_d from X_s is required, rendering PWDs in (1) rather powerful null test observables. Finally, if M^0 is a light meson (for instance K_S , η') the partial decay widths in the endpoint region where M^0 is very energetic, with $E_M \sim M_B/2 + O(\Lambda)$, do not depend on the spectator quark at LO in $1/m_b$ [12]. Thus the PWD for both charged and neutral semi-inclusive B meson decays

$$\Gamma(B^-/\bar{B}^0 \rightarrow M^0 X_{s+d}) - \Gamma(B^+/B^0 \rightarrow \bar{M}^0 X_{s+d}), \quad (2)$$

vanish in the endpoint region up to U -spin breaking and corrections of higher order in $1/m_b$. This has a big experimental advantage since for these decay configurations, of isolated energetic meson M^0 and a back-to-back inclusive hadronic jet, only the flavor but not the charge of B meson needs to be tagged.

The remainder of this paper is structured as follows: in Sec. II we show that (1) vanishes in the SU(3) limit and then give numerical estimates of the U -spin breaking effects for a few examples in Sec. III. Finally, conclusions are gathered in Sec. IV.

II. SU(3) LIMIT

Let us first show that PWD (1) vanishes in the exact U -spin limit [11, 13]. To simplify the notation we take M^0 to be a U -spin singlet η' or a $c\bar{c}$ bound state, while the end result applies also to the other choices of M^0 that were listed above. Using the decomposition of the $\Delta S = 1$ decay width

$$\Gamma(B^- \rightarrow M^0 X_s^-) = |\lambda_c^{(s)} A_c^s + \lambda_u^{(s)} A_u^s|^2, \quad (3)$$

where $A_{u,c}^s$ denote the terms in the amplitude proportional to the corresponding CKM matrix elements $\lambda_c^{(s)} = V_{cb} V_{cs}^* \sim \lambda^2$ and $\lambda_u^{(s)} = V_{ub} V_{us}^* \sim \lambda^4$ (with $\lambda = \sin\theta_c = 0.22$), the corresponding $\Delta S = 1$ PWD is

$$\begin{aligned} \Delta\Gamma^s &= \Gamma(B^- \rightarrow M^0 X_s^-) - \Gamma(B^+ \rightarrow M^0 X_s^+) \\ &= -4J \text{Im}[A_c^s A_u^{s*}], \end{aligned} \quad (4)$$

with $J = \text{Im}[\lambda_c^{(s)} \lambda_u^{(s)*}] = -\text{Im}[\lambda_c^{(d)} \lambda_u^{(d)*}]$, the Jarlskog invariant. Note that $A_{u,c}^s$ are complex since they carry strong phases. Similarly for the λ^2 suppressed $\Delta S = 0$ decay

$$\Gamma(B^- \rightarrow M^0 X_d^-) = |\lambda_c^{(d)} A_c^d + \lambda_u^{(d)} A_u^d|^2, \quad (5)$$

and

$$\begin{aligned} \Delta\Gamma^d &= \Gamma(B^- \rightarrow M^0 X_d^-) - \Gamma(B^+ \rightarrow M^0 X_d^+) \\ &= 4J \text{Im}[A_c^d A_u^{d*}]. \end{aligned} \quad (6)$$

The transformation $s \leftrightarrow d$ exchanges X_s and X_d final states, while it has no effect on B^\pm and M^0 states. In the limit of exact U -spin thus $A_{u,c}^s = A_{u,c}^d$, giving a vanishing PWD in flavor untagged inclusive decay

$$\Delta\Gamma^{s+d} = \Delta\Gamma^s + \Delta\Gamma^d = 4J \text{Im}[A_c^d A_u^{d*} - A_c^s A_u^{s*}] = 0. \quad (7)$$

To the extent that U -spin is a valid symmetry of strong interactions the observable $\Delta\Gamma^{s+d}$ constitutes a null test of SM. The breaking can be parameterized completely generally as

$$\Delta\Gamma^{s+d} \equiv \delta_{s \leftrightarrow d} \Delta\Gamma^s, \quad (8)$$

leading to an expectation for the CP asymmetry of the decay into untagged light flavor

$$\mathcal{A}_{CP}^{s+d} = \frac{\Delta\Gamma^s + \Delta\Gamma^d}{\bar{\Gamma}^{s+d} + \bar{\Gamma}^{s+d}} \sim \delta_{s \leftrightarrow d} \frac{\Delta\Gamma^s}{\bar{\Gamma}^s + \bar{\Gamma}^s}, \quad (9)$$

where in the last relation we have neglected the CKM suppressed $\Gamma^d \sim \lambda^2 \Gamma^s$ decay amplitudes. The size of U -spin breaking parameter $\delta_{s \leftrightarrow d}$ is channel dependent

with an order of magnitude expectation $\delta_{s \leftrightarrow d} \sim m_s/\Lambda \sim 0.3$. This is the gain in the theoretical accuracy that one obtains by summing the $\Delta S = 1$ and $\Delta S = 0$ PWDs. Summing the two PWDs on the other hand is not expected to reduce the effect of new physics operators, since unlike SM contributions there is in general no reason for them to give opposite contributions in $\Delta S = 1$ and $\Delta S = 0$ transitions.

III. CONCRETE EXAMPLES AND THE SU(3) BREAKING

We next give several examples of null tests in the semi-inclusive hadronic decays. For each of them we also discuss how reliably we can control the size of SU(3) breaking parameters $\delta_{s \leftrightarrow d}$.

A. PWDs in $B^- \rightarrow D^0(\bar{D}^0)X_{s+d}^-$

This is a special case, since each of the decays $B^- \rightarrow D^0 X_s^-$ and $B^- \rightarrow \bar{D}^0 X_s^-$ is a pure ‘‘tree’’ decay with only one CKM structure multiplying the amplitude. This gives vanishing CP asymmetries

$$\Delta\Gamma(B^- \rightarrow D^0 X_s^-) = \Delta\Gamma(B^- \rightarrow \bar{D}^0 X_s^-) = 0, \quad (10)$$

and similarly for the $\Delta S = 0$ decays $B^- \rightarrow D^0(\bar{D}^0)X_d^-$. One thus has trivially

$$\Delta\Gamma(B^- \rightarrow D^0 X_{s+d}^-) = \Delta\Gamma(B^- \rightarrow \bar{D}^0 X_{s+d}^-) = 0 \quad (11)$$

without any SU(3) breaking corrections. The nonzero CP asymmetries arise here only from higher order *electroweak* corrections, for instance from a box diagram, giving a CP asymmetry well below a permil level. This makes either the summed (11) or separate $\Delta S = 0, 1$ CP asymmetries (10) clean probes of NP contributions despite the fact that the branching ratios are dominated by the (CKM suppressed) tree level SM transitions.

Note that to measure $\Delta\Gamma(B^- \rightarrow D^0 X_{s,d}^-)$ one needs to tag the D^0 flavor, for instance using semileptonic D decays or flavor specific D decays. The experimental difficulties in tagging the \bar{D}^0 flavor in $\Delta\Gamma(B^- \rightarrow D^0 X_{s,d}^-)$ are the same as in $\Delta\Gamma(B^- \rightarrow D^0 K^-)$ and will not be repeated here [14]. If alternatively D^0 and \bar{D}^0 are decaying to a common final state f , one would still have $\Delta\Gamma(B^- \rightarrow [D]_f X_{s+d}^-) = 0$ in the SU(3) limit, but the SU(3) breaking corrections are hard to quantify in this case due to a lack of a reliable calculational tool.

We comment in passing that one also has $\Delta\Gamma(B^- \rightarrow D^0 K^-) = \Delta\Gamma(B^- \rightarrow D^0 \pi^-) = 0$ up to higher order *electroweak* corrections, so that these two body decay CP asymmetries can equally be used as null tests of SM. Another possibility that avoids D^0 flavor tagging is to sum over the D final states. Namely, the decay width for the $B^- \rightarrow DK^-$ decay where a sum over neutral D meson decays is taken, is an incoherent sum of the decays into D^0 and \bar{D}^0 , $\Gamma(B^- \rightarrow D^0 K^-) + \Gamma(B^- \rightarrow \bar{D}^0 K^-)$ (and simi-

larly for $B^- \rightarrow D\pi^-$ or $B^- \rightarrow DX_{s,d}^-$). Each of these decays is a tree decay with only one CKM structure multiplying the amplitude, giving a vanishing CP asymmetry.

The discussed CP asymmetries can become nonzero in the presence of NP, if the new contributions have a different weak phase from the SM one and lead to a different chiral structure of the effective four-quark operators, giving a nonzero strong phase difference to the SM contribution (the factorization in $B^- \rightarrow D^0 K^-$ indicates that these phase differences would be $1/m_{b,c}$ suppressed, while in $B^- \rightarrow \bar{D}^0 K^-$ they could be $O(1)$). For example, using the recent analysis of SUSY effects on γ extraction from $B \rightarrow DK$ decays [15], one can conclude that a generic contribution in R parity conserving MSSM that obeys other FCNC constraints from $B \rightarrow X_s \gamma$ and $D^0 - \bar{D}^0$ mixing could lead to a percent level CP asymmetry in $B^- \rightarrow D^0(\bar{D}^0)X_s^-$ (depending on the sizes of nonperturbative strong phase differences between SM and NP operators). Larger CP asymmetries are possible if gluinos are more massive than quarks or if $D^0 - \bar{D}^0$ constraints are avoided through partial cancellations between different terms. Similar percent level effects can be expected in many other extensions of the SM, for instance in the two-Higgs doublet models where the charged Higgs exchanges can lead to enhanced CP asymmetries at the level of several percents [16].

B. Decays into KX_{s+d}

As a next example let us turn to the $B \rightarrow K_{S,L}X_{s+d}$ decays. We restrict the discussion to the endpoint region of the decay phase space with energetic neutral kaon going in the $\bar{n} = (1, 0, 0, 1)$ direction, and the particles in $X_{s,d}^-$ forming an energetic jet with invariant mass $p_X^2 \sim \Lambda_{\text{QCD}} m_B$ going in the $n = (1, 0, 0, -1)$ direction. For this kinematic setup SCET [17,18] offers a theoretical framework [19] that will allow us to assess the size of U -spin breaking using already available nonperturbative input from lattice QCD (in the limit of perturbative charming penguins). For simplicity we concentrate on the $\Delta S = 1$ decay $B^- \rightarrow \bar{K}^0 X_d^-$, while the results will be easy to extend to the $\Delta S = 0$ decay $B^- \rightarrow K^0 X_s^-$, as well as to the decays involving K^* vector mesons. More importantly, the results will also apply to the $\bar{B}^0 \rightarrow \bar{K}^0 X^0$, $K^0 X_s^0$ decays at LO in $1/m_b$. In particular, the contributions where the spectator ends up in the energetic \bar{K}^0 meson are $1/m_b^2$ suppressed [12], while annihilation contributions arise already at $1/m_b$ [20,21].

The relevant part of the SCET_I effective weak Hamiltonian is

$$H_W = \frac{2G_F}{\sqrt{2}} \sum_{n,\bar{n}} \int [d\omega_j]_{j=1}^3 c_4^{(s)}(\omega_j) Q_{4s}^{(0)}(\omega_j) + \dots, \quad (12)$$

with the ellipses denoting operators that do not contribute to $B^- \rightarrow \bar{K}^0 X_d^-$. The only contributing operator of leading

order in $\sqrt{\Lambda/m_B} \sim 0.3$ expansion is

$$Q_{4s}^{(0)} = \sum_q [\bar{q}_{n,\omega_1} \not{P}_L b_v][\bar{s}_{\bar{n},\omega_2} \not{P}_L q_{\bar{n},\omega_3}]. \quad (13)$$

Here the same notation along with the numbering of the operators has been used as in [22]. Note that the Wilson coefficient $c_4^{(s)}$ in (15) already contains CKM elements [19,23,24]

$$c_4^{(s)}(x) = \lambda_u^{(s)} C_4^u(x) + \lambda_c^{(s)} C_4^c(x), \quad (14)$$

where to NLO in $\alpha_S(m_b)$ the hard kernels at $\mu = m_b$ are ($p = u, c$)

$$C_4^p(x) = C_4 + \frac{C_3}{N} + \frac{\alpha_S}{4\pi} \frac{C_F}{N} \left[C_1 \left(\frac{2}{3} - G(s_p) \right) - \frac{2C_g}{1-x} \right] + \dots, \quad (15)$$

with the ellipses denoting an order of magnitude smaller terms that can be found in Appendix A of [19]. In the matching of full QCD effective weak Hamiltonian to SCET_I weak Hamiltonian the strong phases are generated at the order $\alpha_S(m_b)$ from the configurations with on-shell intermediate quarks carrying a collective momentum $p^2 \sim m_b^2$. The largest contribution to the strong phase comes from the tree operator Q_1 with the \bar{u} , u or \bar{c} , c legs contracted, leading to a complex function ($s_c = m_c^2/m_b^2 - i\epsilon$, $s_u = 0 - i\epsilon$)

$$G(s_p) = -4 \int_0^1 dz z(1-z) \log[s_p - z(1-z)(1-x)], \quad (16)$$

with the parameter x denoting the momentum fraction carried by the s quark. This then leads to nonzero CP asymmetries even though only one SCET_I operator (16) contributes to the decays considered.

An open problem in the construction of SCET_I weak Hamiltonian is the size of the long distance contributions coming from intermediate charm quarks annihilating into two collinear quarks, where charm quarks are in the non-relativistic QCD regime with small relative velocity. The view of BBNS [23] is that the phase space suppression of the threshold region is strong enough so that nonperturbative contributions are subleading, while Bauer *et al.* [22] argue that the phase space suppression is not effective as numerically $2m_c/m_b \sim O(1)$. For semi-inclusive hadronic decays the factorization of charming penguin contributions into soft and collinear parts has been shown in [19]. We will first proceed as though charm quarks can be perturbatively integrated out leading to C_1 term in (15) for $p = c$. The effect of nonperturbative charming penguins will then be discussed at the end of present subsection.

An important observation in deriving the expression for the partial decay width is that the \bar{n} and n parts of the operator $Q_{4s}^{(0)}$ in (13) decouple from each other at leading order in $1/m_b$. Making redefinitions $q_{n,\bar{n}} \rightarrow Y_{n,\bar{n}} q_{n,\bar{n}}$ and

$A_{n,\bar{n}} \rightarrow Y_{n,\bar{n}} A_{n,\bar{n}} Y_{n,\bar{n}}^\dagger$ with $Y_{\{n,\bar{n}\}}$ a Wilson line of ultrasoft gluons $\{n, \bar{n}\} \cdot A_{\text{us}}$, the ultrasoft gluons decouple from collinear fields both in the leading order SCET Lagrangian as well as in $Q_{4s}^{(0)}$, where now the ultrasoft Wilson lines multiply only the b_ν fields, $Y_n^\dagger b_\nu$ [24,25]. At leading order in $1/m_b$ operator $Q_{4s}^{(0)}$ thus factorizes to

$$Q_n^q(\omega_1) = [\bar{q}_{n,\omega_1} \not{P}_L Y_n^\dagger b_\nu], \quad (17)$$

and the remaining $Q_{4s,q}^{\bar{n}}$ pieces that do not talk to each other. It is this factorization that makes predictions of semi-inclusive decays $B^- \rightarrow \bar{K}^0 X_d^-$ in SCET_I region possible. The decay amplitude then factorizes into matrix elements of operators in n and \bar{n} directions

$$\begin{aligned} \langle X_d^- \bar{K}^0 | H_W | B^- \rangle &= \frac{2G_F}{\sqrt{2}} \sum_q \int [d\omega_j]_{j=1}^3 c_4^{(s)}(\omega_j) \\ &\times [\langle X_d^- | Q_n^q(\omega_1) | B^- \rangle \langle \bar{K}^0 | Q_{4s,q}^{\bar{n}}(\omega_j) | 0 \rangle \\ &+ \langle \bar{K}^0 | Q_n^q(\omega_1) | B^- \rangle \langle X_d^- | Q_{4s,q}^{\bar{n}}(\omega_j) | 0 \rangle]. \end{aligned} \quad (18)$$

In the decay of B^- the spectator \bar{u} quark cannot end up in \bar{K}^0 , so that $\langle \bar{K}^0 | Q_n^q(\omega_1) | B^- \rangle = 0$ and the last term in (18) vanishes. In the sum thus only $q = d$ contribution in the first term is nonzero.

Since \bar{K}^0 decouples to leading order from the rest of the amplitude, one can calculate the inclusive decay width $\bar{\Gamma}_K^{(s)} \equiv \Gamma(B^- \rightarrow \bar{K}^0 X_d^-)^1$ following the same steps as in the SCET calculation of $\Gamma(B \rightarrow X_s \gamma)$ in the endpoint region [25] (with more details given in [19]). The inclusive decay width is simply a product of two terms, one coming from \bar{n} , the other from n operators

$$\begin{aligned} \frac{d\bar{\Gamma}_K^{(s)}}{dE_K} &= \frac{G_F^2}{\pi} E_K^3 \left| \int_0^1 dx f_K \phi_K(x) c_4^{(s)}(x) \right|^2 \\ &\times \int_{2E_K - m_b}^{\bar{\Lambda}} dk^+ S(k^+) J(k^+ + m_b - 2E_K). \end{aligned} \quad (19)$$

Multiplying (19) by an extra factor of $1/2$ gives $\Gamma(B^- \rightarrow K_{S,L} X_d^-)$. The $\Gamma(B^+ \rightarrow K^0 X_d^+) \equiv \Gamma_K^{(s)}$ decay width is obtained by changing in Eq. (19) $\lambda_u^{(s)} \rightarrow \lambda_u^{(s)*}$ in $c_4^{(s)}(x)$. The B meson shape function $S(k^+)$ and the perturbatively calculable jet function $J(k^+)$ are exactly the same as the ones found in the decay $B \rightarrow X_q \gamma$, with their definitions given in [25]. For decays with K^* vector meson only the decay into longitudinal polarization state is nonzero at leading order in $1/m_b$. The decay width is then obtained from (19) by replacing the kaon decay constant $f_K \rightarrow f_{\parallel K^*}$ and the light-cone distribution amplitude $\phi_K \rightarrow \phi_{\parallel K^*}$.

¹Note that the superscript $\bar{\Gamma}_K^{(s)}$ denotes that this is a $\Delta S = 1$ decay. It does not denote the net strangeness content of the inclusive jet X_d .

Normalizing the difference of the decay widths $\Delta\Gamma_K^{(s)} \equiv \Gamma(B^+ \rightarrow K^0 X_d^+) - \Gamma(B^- \rightarrow \bar{K}^0 X_d^-) = \Gamma_K^{(s)} - \bar{\Gamma}_K^{(s)}$ with their sum, the shape and jet functions drop out and so does the dependence on E_K . To first order in λ^2 suppressed terms thus, in SCET_I region

$$\mathcal{A}_{CP}^{(s)} = \frac{\bar{\Gamma}_K^{(s)} - \Gamma_K^{(s)}}{\Gamma_K^{(s)} + \bar{\Gamma}_K^{(s)}} = -2J \frac{\text{Im}[\mathcal{T}_{K,c}^{(s)} \mathcal{T}_{K,u}^{(s)*}]}{|\lambda_c^{(s)} \mathcal{T}_{K,c}^{(s)}|^2}, \quad (20)$$

where the hard kernels $\mathcal{T}_{K,p}^{(s)}$ are

$$\mathcal{T}_{K,p}^{(s)} = f_K \int dx \phi_K(x) \mathcal{C}_4^p(x), \quad (21)$$

with $\mathcal{C}_4^{u,c}(x)$ given in (14) and (15). If we can neglect nonperturbative charming penguin contributions, the CP asymmetry (20) depends only on one nonperturbative function, the kaon LCDA $\phi_K(x)$. In the numerical results we will use a recent lattice QCD determination of the first coefficient in the Gegenbauer expansion of $\phi_K(x)$ which at $\mu = 2.0$ GeV is $a_1^K = 0.055 \pm 0.005$ [26]. This value is in agreement with a recent QCD sum rule analysis [27], that gives at $\mu = 2$ GeV: $a_1^K = 0.05 \pm 0.03$, $a_2^K = 0.23 \pm 0.12$ (we use this value for a_2^K in the numerical analysis but conservatively double the errors). Using the values of CKM elements from [1] and running the SCET_I Wilson coefficients (14) and (15) to $\mu = 2.0$ GeV using NLL RG equations [19], we obtain

$$\begin{aligned} \mathcal{A}_{CP}^{(s)} &= (0.27 \pm 0.05) \times (2J/|\lambda_c^{(s)}|^2) \\ &= (1.0 \pm 0.2) \cdot 10^{-2}, \end{aligned} \quad (22)$$

where the errors reflect only the errors on Gegenbauer coefficients $a_{1,2}^K$, with the error on a_2^K dominating.

In $\phi_K(x)$ parameter x denotes the fraction of kaon momentum carried by the strange quark, while in the hard kernels $\mathcal{T}_{K,p}^{(s)}(x)$ it denotes the momentum carried by quark (antiquark) in the K meson starting with a \bar{B} (B) initial meson. Thus the difference of the $\Delta S = 0$ decay widths $\Delta\Gamma_K^{(d)} \equiv \Gamma(B^- \rightarrow \bar{K}^0 X_s^-) - \Gamma(B^+ \rightarrow K^0 X_s^+)$ is obtained from (20) by replacing $J \rightarrow -J$ and $\phi_K(x) \rightarrow \phi_K(1-x)$. It is therefore useful to decompose $\phi_K(x)$ into functions $\phi_K^\pm(x)$ that are even and odd under the $x \rightarrow 1-x$ exchange

$$\phi_K(x) = \phi_K^+(x) + \phi_K^-(x), \quad \phi_K^\pm(x) = \pm \phi_K^\pm(1-x). \quad (23)$$

In the Gegenbauer polynomial expansion ϕ_K^+ (ϕ_K^-) receives contributions only from even (odd) Gegenbauer polynomials. Defining similarly the corresponding hard kernels

$$\mathcal{T}_{u,c}^\pm = f_K \int dx \phi_K^\pm(x) \mathcal{C}_4^{u,c}(x), \quad (24)$$

we get for the sum of the CP asymmetries (to the first order

in the CKM suppressed terms)

$$\begin{aligned}\mathcal{A}_{CP}^{d+s} &= (\Delta\Gamma_K^{(s)} + \Delta\Gamma_K^{(d)})/(\Gamma_K^{d+s} + \bar{\Gamma}_K^{d+s}) \\ &= -4JIm[\mathcal{T}_c^+ \mathcal{T}_u^{*-} + \mathcal{T}_c^- \mathcal{T}_u^{*+}]/|\lambda_c^{(s)} \mathcal{T}_c|^2,\end{aligned}\quad (25)$$

which using the same input values as in (22) gives

$$\begin{aligned}\mathcal{A}_{CP}^{d+s} &= (7.6 \pm 6.4) \cdot 10^{-2} \times 4J/|\lambda_c^{(s)}|^2 \\ &= (0.28 \pm 0.23) \cdot 10^{-2},\end{aligned}\quad (26)$$

where again the errors only show the dependence on kaon LCDA. Note that in the limit of exact U -spin $\mathcal{T}_c^- = \mathcal{T}_u^- = 0$ and therefore $\mathcal{A}_{CP}^{d+s} = 0$. Equation (25) encompasses the U -spin breaking effect due to asymmetric

$\phi_K(x)$, and gives a reasonable estimate for the size of $|\mathcal{A}_{CP}^{d+s}|$ also in the case of nonperturbative charming penguins as we discuss next. The remaining $SU(3)$ breaking due to m_s suppressed SCET operators lead to additional jet functions of order $m_s^2/\Lambda m_b$ and can be safely neglected [28].

To LO in Λ/m_c the nonperturbative charming penguin contributions in semi-inclusive hadronic decays factorize into an \bar{n} collinear factor that depends on meson M and a universal convolution \mathcal{F}_{cc} of soft ‘‘charm shape function’’ and an n -collinear jet function as shown in [19]. Normalizing the decay width to the $B \rightarrow X_s \gamma$ decay, so that the jet function and the shape function in (19) cancel for the perturbative hard kernels, we find that in the end-point region

$$\begin{aligned}\frac{d\Gamma(B^- \rightarrow \bar{K}^0 X_d^-)}{d\Gamma(B \rightarrow X_s \gamma)} &= \frac{2\pi^3}{\alpha m_b^2} \frac{1}{|\lambda_c^{(s)} C_\gamma (c_9^{\text{eff}} + 1/2c_{12}^{\text{eff}})|^2} \\ &\times \left\{ \left| \sum_{p=u,c} \lambda_p^{(s)} \mathcal{T}_{K,p}^{(s)} \right|^2 + f_K^2 |\lambda_c^{(s)}|^2 \bar{F}_{cc} + 2\mathcal{R}e \left[f_K \lambda_c^{(s)} \bar{f}_{cc} \left(\sum_{p=u,c} \lambda_p^{(s)} \mathcal{T}_{K,p}^{(s)} \right)^* \right] \right\},\end{aligned}\quad (27)$$

where one sets $E_\gamma = E_K$. The SCET Wilson coefficients are $c_9^{\text{eff}} = 1$, $c_{12}^{\text{eff}} = 0$ at LO with NLO calculated in [18], while C_γ is given e.g. in Eq. (13) of [21]. The complex parameter \bar{f}_{cc} that describes the interference of nonperturbative charming penguin with the perturbative hard kernels is related to the soft charm shape function \mathcal{F}_{cc} defined in [19]

$$\bar{f}_{cc} = \frac{\alpha_S(2m_c)}{m_b} \frac{\mathcal{F}_{cc} \phi_M (1 - \frac{2m_c^2}{E_M m_b})}{S(k^+) \otimes_{k^+} J(k^+ + m_b - 2E_K)}, \quad (28)$$

where \otimes_{k^+} denotes the integration over $k^+ \in [2E_K - m_b, \bar{\Lambda}]$. The parameter \mathcal{F}_{cc} is universal for any $\Gamma(B \rightarrow MX)$ up to $O(\Lambda/m_c)$ corrections, while \bar{f}_{cc} depends on meson M 's LCDA ϕ_M . The positive real parameter \bar{F}_{cc} in (27) on the other hand describes the square of nonperturbative charming penguin contributions. As a rule of thumb we can thus take $\bar{f}_{cc}^2 \sim \bar{F}_{cc}$. Similarly to \bar{f}_{cc} the parameter \bar{F}_{cc} depends on meson M through $\phi_M^2 (1 - 2m_c^2/E_M m_b)$. If hard kernels dominate the amplitudes, the term with \bar{f}_{cc} in (27) is subleading, while \bar{F}_{cc} term is even more suppressed and can be neglected as was done in [19]. It should, however, be kept in penguin-dominated modes. A prediction for $\Delta S = 0$ decay width $\Gamma(B^- \rightarrow K^0 X_s^-)$ in the presence of nonperturbative charming penguins is obtained from (27) by making a replacement $s \rightarrow d$, where $\mathcal{T}_{K,p}^{(d)}$ is obtained from (21) through a replacement $\phi_K(x) \rightarrow \phi_K(1-x)$. The B^+ decay widths are obtained by making a replacement $\lambda_p^{(q)} \rightarrow \lambda_p^{(q)*}$.

The results derived in this subsection are valid also for $B^0 \rightarrow K_S X_{s,d}^0$ semi-inclusive hadronic decays in the end-point region up to power suppressed corrections that in-

volve spectator quark and start at $1/m_b$ order. Up to these corrections all results, including numerical ones, are the same for charged and neutral $B \rightarrow K_S X_{s,d}$ decays.

We can use this fact to determine the charming penguin parameters from the presently available experimental data. Recently the first measurement of $B \rightarrow K^0 X$ branching ratio was reported by *BABAR* [29]

$$\text{Br}(B \rightarrow K^0 X) = (154_{-48}^{+55+55}) \cdot 10^{-6}, \quad (29)$$

where the lower cut on the K momentum of 2.34 GeV in the B rest frame was used. Normalizing to the $B \rightarrow X_s \gamma$ branching ratio with the same photon momentum cut one has [10,30]

$$\frac{\text{Br}(B \rightarrow K^0 X)}{\text{Br}(B \rightarrow X_s \gamma)} = 0.89 \pm 0.43, \quad (30)$$

The prediction for this ratio is given in (27) once it is CP averaged (alternatively, to accuracy we are working one can neglect λ_u suppressed terms). It depends on three nonperturbative parameters, \bar{F}_{cc} and magnitude and phase of \bar{f}_{cc} . At present there is not enough experimental information to determine all three of them. Quite generally one expects $|\bar{f}_{cc}|^2 \sim \bar{F}_{cc}$. As a starting point, we take this relation to be exact, which leads to $\sqrt{\bar{F}_{cc}} = (8.9 \pm 6.6) \cdot 10^{-2}$, where the error is a sum of experimental error and the variation of $\arg(\bar{f}_{cc}) \in [0, 2\pi)$. This value of $\sqrt{\bar{F}_{cc}}$ is about a factor of 4 ± 3 larger than the perturbative prediction for the charming penguin (15) (with $\sqrt{\bar{F}_{cc}} = 0$ corresponding to purely perturbative charming penguin). Experimentally, there is therefore a possible indication

for sizable nonperturbative charming penguin, but the data are at present also consistent with $\sqrt{\bar{F}_{cc}} = 0$ at a little above 1σ . For instance, neglecting nonperturbative charming penguins gives 0.19 ± 0.04 for the ratio in (30).

For nonzero nonperturbative charming penguin contributions the CP asymmetry \mathcal{A}_{CP}^{d+s} is governed by the size of $SU(3)$ breaking in the charming penguin. Taking a 30% $SU(3)$ breaking with $\bar{f}_{cc}^2 \sim \bar{F}_{cc}$, gives

$$\mathcal{A}_{CP}^{d+s} \in [-0.6\%, 0.9\%], \quad (31)$$

to be compared with

$$\mathcal{A}_{CP}^s \in [-2.3\%, 2.3\%], \quad (32)$$

that is obtained for the same set of input parameters. This illustrates the benefit of using combined CP asymmetry \mathcal{A}_{CP}^{d+s} , where the theoretical uncertainties are reduced in two ways: (i) the central value is reduced, since \mathcal{A}_{CP}^{d+s} vanishes in $SU(3)$ limit, while \mathcal{A}_{CP}^s does not, and (ii) the error on the prediction is reduced. To understand how this happens, let us look at the contribution of nonperturbative charming penguins to the rate asymmetries

$$\Delta\Gamma_K^{(s)} \propto \text{Im}[(\mathcal{T}_u^+ + \mathcal{T}_u^-)(\bar{f}_{cc}^K)^*], \quad (33)$$

$$\Delta\Gamma_K^{(s+d)} \propto \text{Im}[\mathcal{T}_u^+(\bar{f}_{cc}^K - \bar{f}_{cc}^{\bar{K}})^* + \mathcal{T}_u^-(\bar{f}_{cc}^K + \bar{f}_{cc}^{\bar{K}})^*], \quad (34)$$

where we have explicitly denoted the dependence of \bar{f}_{cc} on $M = K, \bar{K}$ (cf. Equation (28)). In the $SU(3)$ limit $\bar{f}_{cc}^K = \bar{f}_{cc}^{\bar{K}}$ and $\mathcal{T}_u^- = 0$, so that the contribution of nonperturbative charming penguins to $\Delta\Gamma_K^{(s+d)}$ vanishes as expected. Furthermore, if in the future \bar{f}_{cc}^K and $\bar{f}_{cc}^{\bar{K}}$ are determined from some other decay modes such as semi-inclusive decays involving charged kaons, the resulting error on the prediction of $\Delta\Gamma_K^{(s+d)}$ will be smaller than for $\Delta\Gamma_K^{(s)}$ since the error in the difference $\bar{f}_{cc}^K - \bar{f}_{cc}^{\bar{K}}$ partially cancels, while the error on $(\bar{f}_{cc}^K + \bar{f}_{cc}^{\bar{K}})^*$ comes multiplied by the $SU(3)$ breaking factor \mathcal{T}_u^- .

Finally, we also give the results for $B \rightarrow (K^{*0} + \bar{K}^{*0})X_{s,d}$ decays that can be trivially obtained from the above results with the replacement $\phi_K(x) \rightarrow \phi_{K^*}(x)$. Using $a_1^{K^*} = 0.08 \pm 0.13$, $a_2^{K^*} = 0.07 \pm 0.08$ at $\mu = 2.0$ GeV obtained by conservatively doubling the errors of [31], we get

$$\begin{aligned} \mathcal{A}_{CP,K^*}^{(s)} &= (0.24 \pm 0.02) \times (2J/|\lambda_c^{(s)}|^2) \\ &= (0.86 \pm 0.07) \cdot 10^{-2} \end{aligned} \quad (35)$$

and

$$\begin{aligned} \mathcal{A}_{CP,K^*}^{d+s} &= (3.2 \pm 2.6) \cdot 10^{-2} \times 4J/|\lambda_c^{(s)}|^2 \\ &= (0.12 \pm 0.10) \cdot 10^{-2}, \end{aligned} \quad (36)$$

and for the ratio of decay widths

$$\frac{d\Gamma(B^- \rightarrow \bar{K}^{*0} X_d^-)}{d\Gamma(B^- \rightarrow X_s \gamma)} = 0.33 \pm 0.11, \quad (37)$$

where as before the errors are only due to error on K^* LCDA. These predictions do not include effects of nonperturbative charming penguins. Using the determination of \bar{F}_{cc} from Eq. (30) and taking a 30% $SU(3)$ breaking with $\bar{f}_{cc}^2 \sim \bar{F}_{cc}$, gives

$$\mathcal{A}_{CP,K^*}^{d+s} \in [-0.6\%, 0.8\%], \quad (38)$$

$$\mathcal{A}_{CP,K^*}^s \in [-2.1\%, 2.1\%], \quad (39)$$

and

$$\frac{d\Gamma(B^- \rightarrow \bar{K}^{*0} X_d^-)}{d\Gamma(B^- \rightarrow X_s \gamma)} = 1.66 \pm 0.86, \quad (40)$$

where the Λ/m_c suppressed contributions from decays into transversely polarized K^* have been neglected.

C. $B^- \rightarrow \eta' X_{s+d}^-$

We next turn to the case of $B^- \rightarrow \eta' X_{s+d}^-$ decay, by first showing that the U -spin breaking $\delta_{s \leftrightarrow d}$ is still linear in m_s/Λ . In particular the $\eta - \eta'$ mixing does not introduce anomalously large breakings. We use the FKS mixing scheme [32] in which the mass eigenstates η, η' are related to the flavor basis through $\eta = \eta_q \cos\varphi - \eta_s \sin\varphi$, and $\eta' = \eta_q \sin\varphi + \eta_s \cos\varphi$, with $\varphi = (39.3 \pm 1.0)^\circ$ and $\eta_q = (\eta_u + \eta_d)/\sqrt{2}$. We start by rewriting (7)

$$\Delta\Gamma(\eta' X_{s+d}^\pm) = -4J \text{Im}[\Delta A_c A_u^{s*} + A_c^d \Delta A_u^*], \quad (41)$$

where the flavor breaking difference $\Delta A_c = A_c^s - A_c^d$ is

$$\begin{aligned} \Delta A_c &= \frac{\sin\varphi}{\sqrt{2}} \{ [A_c(\eta_u X_s^-) + A_c(\eta_d X_s^-) + A_c(\eta_s X_s^-)] \\ &\quad - [s \leftrightarrow d] \} + \left(\cos\varphi - \frac{\sin\varphi}{\sqrt{2}} \right) \\ &\quad \times (A_c(\eta_s X_s^-) - A_c(\eta_s X_d^-)), \end{aligned} \quad (42)$$

and similarly for $\Delta A_u = A_u^s - A_u^d$. Here $A_c(\eta_q X_s^-)$ denotes a term in the amplitude due to a $q\bar{q}$ part of η' wave function. The terms in the curly brackets in (42) cancel in the limit of exact $s \leftrightarrow d$ symmetry. The difference $A_c(\eta_s X_s^-) - A_c(\eta_s X_d^-)$ in the last term on the contrary, does not vanish in the exact U -spin limit (even though there is a partial cancellation). However, the term multiplying it, $\cos\varphi - \frac{\sin\varphi}{\sqrt{2}} = 0.33$, makes its size a typical $SU(3)$ breaking effect, and would vanish for $SU(3)$ singlet η' since then $\tan\varphi = \sqrt{2}$. Thus the corresponding $\delta_{s \leftrightarrow d}$ is of typical size, $O(m_s/\Lambda)$.

A more quantitative analysis can be made in the end-point region of the inclusive decay using $SCET_I$ in the same way as in the previous subsection. However, in order to make progress, we do make an additional approximation

of neglecting contributions from O_{8g} since they lead to shape functions with additional soft gluon that are completely unknown. Working in this approximation and neglecting the $1/m_b$ suppressed spectator interactions as in previous section one finds for the hard kernels ($p = u, c$, while \otimes denotes a convolution over x)

$$\mathcal{T}_{\eta',p}^{(s)} = f_{\eta_s} \cos\varphi \phi_{\eta_s} \otimes (C_4^p + C_5 - C_6) + f_{\eta_q} \frac{\sin\varphi}{\sqrt{2}} \phi_{\eta_q} \otimes (C_2^p - C_3 + 2C_5 - 2C_6), \quad (43)$$

$$\mathcal{T}_{\eta',p}^{(d)} = f_{\eta_s} \cos\varphi \phi_{\eta_s} \otimes (C_5 - C_6) + f_{\eta_q} \frac{\sin\varphi}{\sqrt{2}} \phi_{\eta_q} \otimes (C_2^p - C_3 + C_4^p + 2C_5 - 2C_6), \quad (44)$$

in terms of which the CP asymmetries are (neglecting the CKM suppressed terms)

$$\mathcal{A}_{CP,\eta'}^{(s)} = \frac{-2J}{|\lambda_c^{(s)}|^2} \text{Im}[(\mathcal{T}_{\eta',c}^{(s)} + \cos\varphi f_{\eta_s} \bar{f}_{cc}) \mathcal{T}_{\eta',u}^{(s)*}] D^{-1}, \quad (45)$$

and

$$\mathcal{A}_{CP,\eta'}^{(s+d)} = \frac{-2J}{|\lambda_c^{(s)}|^2} \left\{ \text{Im}[(\mathcal{T}_{\eta',c}^{(s)} + \cos\varphi f_{\eta_s} \bar{f}_{cc}) \mathcal{T}_{\eta',u}^{(s)*}] - \text{Im}\left[\left(\mathcal{T}_{\eta',c}^{(d)} + \frac{\sin\varphi}{\sqrt{2}} f_{\eta_q} \bar{f}_{cc}\right) \mathcal{T}_{\eta',u}^{(d)*}\right] \right\} D^{-1}, \quad (46)$$

with

$$D = [|\mathcal{T}_{\eta',c}^{(s)}|^2 + 2 \cos\varphi f_{\eta_s} \text{Re}(\mathcal{T}_{\eta',c}^{(s)} \bar{f}_{cc}) + \bar{F}_{cc} (\cos\varphi f_{\eta_s})^2]. \quad (47)$$

Additional contributions arise because of isosinglet nature of η' .

Here the complex parameter \bar{f}_{cc} and the real positive parameter \bar{F}_{cc} parameterize the charming penguin contributions in the same way as described in the previous subsection. They can be constrained using the measurements of *BABAR* [33] and *CLEO* [34] of $B \rightarrow \eta' X_s$ branching ratio. Combining the two measurements gives

$$\text{Br}(B \rightarrow \eta' X_s) = (420 \pm 94) \cdot 10^{-6}, \quad (48)$$

for a lower cut on η' energy of $E_{\eta'} > 2.218$ GeV. Normalizing to the $B \rightarrow X_s \gamma$ branching ratio with $E_\gamma > 2.218$ GeV [10]

$$\frac{\text{Br}(B \rightarrow \eta' X_s)}{B \rightarrow X_s \gamma} = 1.74 \pm 0.42, \quad (49)$$

we can use the expression

$$\frac{d\Gamma(B^- \rightarrow \bar{\eta}' X_s)}{d\Gamma(B \rightarrow X_s \gamma)} = \frac{2\pi^3}{\alpha m_b^2} \frac{D}{|\lambda_t^{(s)} C_\gamma (c_9^{\text{eff}} + 1/2c_{12}^{\text{eff}})|^2}, \quad (50)$$

to constrain \bar{F}_{cc} , while bounds on \bar{f}_{cc} obtained in this way are very loose. Naively one expects $\bar{F}_{cc} \sim |f_{cc}|^2$. If this is satisfied, then \bar{F}_{cc} dominates in $\text{Br}(B \rightarrow \eta' X_s)$ leading to a determination $\sqrt{\bar{F}_{cc}} = 0.15 \pm 0.03$, where the error is a combination of experimental one and due to a variation of $\arg(\bar{f}_{cc})$ in the determination. This corresponds to a non-perturbative charming penguin, which is about 5–8 times larger than the perturbative contribution. Whether this is the correct interpretation of the enhancement of $\text{Br}(B \rightarrow \eta' X_s)$ over the perturbative prediction could be clarified once other semi-inclusive hadronic decays are measured and our approximation of neglecting contributions from O_{8g} is relaxed. Then the parameters $\sqrt{\bar{F}_{cc}}$ determined in $\text{Br}(B \rightarrow \eta' X_s)$ and $\text{Br}(B \rightarrow K^0 X)$ should be the same up to corrections of order Λ/m_c .

Using $\bar{F}_{cc} \sim |f_{cc}|^2$ together with the hard kernels calculated using NLO matching at $\mu \sim m_b$ with NLL running to $\mu = 2.0$ GeV, setting $f_{\eta_q} = 140 \pm 3$ MeV, $f_{\eta_s} = 176 \pm 8$ MeV [32] and taking $\phi_{\eta_q}(x) = \phi_{\eta_s}(x) = \phi_\pi(x)$ in the lack of better information, while varying the phase $\arg(\bar{f}_{cc}) \in [0, 2\pi)$, we obtain

$$\mathcal{A}_{CP,\eta'}^{(s)} \in [-1.7\%, 1.7\%], \quad (51)$$

and

$$\mathcal{A}_{CP,\eta'}^{(s+d)} \in [-1.2\%, 0.9\%]. \quad (52)$$

This is in agreement with a result for CP asymmetry of this mode $\mathcal{A}_{CP,\eta'}^{(s)} \sim 1\%$ from [35].

In addition to the null tests discussed above there are also other null test that one could consider. For instance neglecting annihilation diagrams also neutral decay $\bar{B}^0 \rightarrow \pi^+ X_{s+d}^-$ has vanishing PWD (1) in U -spin limit, see e.g. [36]. Another interesting case is ϕX_{s+d}^- . Since ϕ is not a U -spin singlet the PWD (1) does not vanish in the exact U -spin limit. Nevertheless, in the SM this decay is penguin dominated with very small direct CP asymmetry $\approx 1\%$ [35] providing a valuable probe of NP.

IV. CONCLUSIONS

In light of B -factories' results it is becoming increasingly clear that deviations from the CKM-paradigm due to NP are likely to be small. Therefore null tests of the SM can be very valuable in search of NP. Bearing that in mind, we are proposing a new class of null tests involving CP asymmetries of untagged, semi-inclusive decays, $B \rightarrow M^0 X_{s+d}$ where M^0 is either a U -spin singlet (for instance D^0 or $c\bar{c}$ bound state) or a meson that is related to its antiparticle through a U -spin rotation. In general the CP asymmetries vanish in U -spin limit only for charged B

decays. However, if M^0 is an energetic light meson such as $K_{S,L}$, η' or K^* (taken together with the decay to its CP conjugate \bar{K}^*), the decaying B can be taken to be either charged or neutral up to $1/m_b$ corrections. In the examples discussed we showed that these CP asymmetries are very small, $<1\%$. Experimentally, to perform a completely inclusive measurement for these decays the flavor but not the charge of decaying B meson needs to be tagged. Recently the first measurement of $B \rightarrow K^0 X$ branching ratio was performed by BaBar using fully reconstructed B decays [29], suggesting that the proposed observables are experimentally measurable in practice.

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