## Weak hadronic decays of charmed mesons emitting pseudoscalar and axial-vector mesons

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In this paper, we investigate phenomenologically several weak decays of charmed mesons emitting a pseudoscalar meson and an axial-vector meson. Decay amplitudes are obtained using the factorization scheme in the spectator quark model. Branching ratios for the Cabibbo angle-favored, Cabibbo angle-suppressed, and Cabibbo angle-doubly-suppressed decays are obtained and compared with available experimental results.

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# I. INTRODUCTION

The spectator model using the factorization ansatz has achieved substantial success [1-4] in explaining most of the exclusive two-body *D*-meson decays emitting *s*-wave mesons. The model involves the expansion of transition amplitudes in terms of a few form factors, which provide essential information on the structure of the mesons. Therefore, it is natural to extend the phenomenological study to charmed meson decays emitting *p*-wave mesons [5-10]. Theoretically, these decays are expected to be suppressed, as there is less phase space available. However, the measured branching ratios for the observed modes are found to be rather large.

In the present paper, we study two-body weak hadronic decays emitting an axial-vector meson  $A(1^+)$  and a pseudoscalar meson  $P(0^-)$  in the Cabibbo-favored mode, the Cabibbo-suppressed mode, and the Cabibbo-doubly-suppressed mode. Using the factorization scheme to obtain the decay amplitudes, we calculate the branching ratios of these decay modes.

The paper is organized as follows. In Sec. II, we discuss the axial-vector meson spectroscopy. Methodology for calculating  $D \rightarrow PA$  decays is presented in Sec. II. Section IV gives numerical results, and discussions and conclusions are given in the last section.

## **II. AXIAL-VECTOR MESON SPECTROSCOPY**

Experimentally, two types of axial-vector mesons,  $A(J^{PC} = 1^{++})$  and  $A'(J^{PC} = 1^{+-})$ , behave well with respect to the quark model  $q\bar{q}$  expectations. The  $J^{PC} = 1^{++}$  nonet has two isoscalar states, besides the isovector nonstrange  $a_1(1.260)$  mesons and isospinor strange mesons. There exist three good  $1^{++}$  candidates,  $f_1(1.282)$ ,  $f_1(1.420)$ , and  $f'_1(1.512)$ , one more than the expected number. This indicates that one of the three mesons is a non- $q\bar{q}$  state. Particle Data Group [11] indicates that  $f_1(1.420)$  is a multiquark state in the form of a  $K\bar{K}\pi$  bound state [12] or a  $K\bar{K}^*$  deuteronlike state [13]. In the present analysis, we define mixing of the isoscalar states as

$$f_{1}(1.285) = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \cos\phi_{A} + (s\bar{s}) \sin\phi_{A},$$

$$f_{1}'(1.512) = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \sin\phi_{A} - (s\bar{s}) \cos\phi_{A}.$$
(1)

Similarly, mixing of two isoscalar mesons,  $h_1(1.170)$  and  $h'_1(1.380)$ , is defined as

$$h_1(1.170) = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \cos\phi_{A'} + (s\bar{s}) \sin\phi_{A'},$$

$$h_1'(1.380) = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \sin\phi_{A'} - (s\bar{s}) \cos\phi_{A'}.$$
(2)

Proximity of  $a_1(1.260)$  and  $f_1(1.285)$  and, to a lesser extent, that of  $b_1(1.235)$  and  $h_1(1.170)$  indicates the ideal mixing for both  $1^{++}$  and  $1^{+-}$  nonets, i.e.,

$$\phi_A = \phi_{A'} = 0^\circ. \tag{3}$$

This is also supported by their decay patterns.  $f_1(1.285)$  decays predominantly to  $4\pi$  and  $\eta\pi\pi$ , while  $f'_1(1.512)$  decays to  $K\bar{K}\pi$ . Similarly,  $h_1(1.170)$  decays predominantly to  $\rho\pi$ , and  $h'_1(1.380)$  decays to  $K\bar{K}^*$  and  $\bar{K}K^*$  states.

Experimentally, the isodoublet strange mesons  $K_1(1.270)$  and  $\underline{K}_1(1.400)$  are given by a mixture of  ${}^3P_1$  and  ${}^1P_1$  states [14],

$$K_{1} = K_{1A} \sin\theta + K_{1A'} \cos\theta,$$
  

$$\underline{K}_{1} = K_{1A} \cos\theta - K_{1A'} \sin\theta,$$
(4)

where  $K_{1A}$  and  $K_{1A'}$  are the strange partners of  $a_1(1.260)$ and  $b_1(1.235)$ , respectively. Particle Data Group [11] assumes maximal mixing ( $\theta = 45^\circ$ ). From the experimental information on masses and partial rates of  $K_1(1.270)$  and  $\underline{K}_1(1.400)$ , Suzuki [15] found two possible solutions with a twofold ambiguity,  $\theta \approx 33^\circ$  and 57°.

For  $\eta$  and  $\eta'$  pseudoscalar states, we use

$$\eta(0.547) = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \sin\phi_P - (s\bar{s}) \cos\phi_P,$$
  

$$\eta'(0.958) = \frac{1}{\sqrt{2}} (u\bar{u} + d\bar{d}) \cos\phi_P + (s\bar{s}) \sin\phi_P,$$
(5)

where  $\phi_P = \theta(\text{ideal}) - \theta_P(\text{physical}) \cdot \theta_P(\text{physical}) = -10^\circ$ 

and  $-23^{\circ}$  for quadratic and linear mass formulas [11], respectively.

### **III. METHODOLOGY**

#### A. Weak Hamiltonian

The general current  $\otimes$  current weak Hamiltonians  $H_W$  for charmed changing modes are classified as follows:

(i) Cabibbo-favored ( $\Delta C = \Delta S = -1$ ) decays,

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* [a(\bar{u}d)(\bar{s}c) + a_2(\bar{s}d)(\bar{u}c)]; \quad (6a)$$

(ii) Cabibbo-suppressed ( $\Delta C = -1$ ,  $\Delta S = 0$ ) decays,

$$H_W = \frac{G_F}{\sqrt{2}} V_{ud} V_{cd}^* [a\{(\bar{u}d)(\bar{d}c) - (\bar{u}s)(\bar{s}c)\} + a_2\{(\bar{d}d)(\bar{u}c) - (\bar{s}s)(\bar{u}c)\}],$$
(6b)

taking  $V_{us}V_{cs}^* = V_{ud}V_{cd}^*$ ;

(iii) Cabibbo-doubly-suppressed  $(\Delta C = -\Delta S = -1)$  decays,

$$H_W = \frac{G_F}{\sqrt{2}} V_{us} V_{cd}^* [a(\bar{u}s)(\bar{d}c) + a_2(\bar{d}s)(\bar{u}c)].$$
(6c)

Here,  $(q\bar{q})$  is shorthand for a color singlet combina-

tion  $\bar{q}\gamma_{\mu}(1-\gamma_5)q$ . The parameters  $a_1$  and  $a_2$  relate to the short-distance QCD Wilson coefficients. For hadronic charmed decays, we use  $a_1 = 1.26$  and  $a_2 = -0.51$ .

#### **B.** Decay amplitudes and rates

The decay rate formula for  $D \rightarrow PA$  decays is given by

$$\Gamma(D \to PA) = \frac{p_C^3}{8\pi m_A^2} |A(D \to PA)|^2, \tag{7}$$

where  $p_C$  is the magnitude of the three-momentum of a final-state particle in the rest frame of the *D* meson and  $m_A$  denotes the mass of the axial-vector meson.

The factorization scheme expresses the decay amplitudes as a product of the matrix elements of weak currents (up to the scale factor of  $\frac{G_F}{\sqrt{2}} \times \text{CKM}$  elements) as

$$A(D \to PA) = \langle P|J^{\mu}|0\rangle\langle A|J_{\mu}|D\rangle + \langle A|J^{\mu}|0\rangle\langle P|J_{\mu}|D\rangle,$$
  

$$A(D \to PA') = \langle P|J^{\mu}|0\rangle\langle A'|J_{\mu}|D\rangle + \langle A'|J^{\mu}|0\rangle\langle P|J_{\mu}|D\rangle.$$
(8)

Using Lorentz invariance, matrix elements of the current between meson states can be expressed [5,11] as

$$\langle P|J_{\mu}|0\rangle = -if_{P}k_{\mu}, \qquad \langle A|J_{\mu}|0\rangle = \in_{\mu}^{*} m_{A}f_{A}, \qquad \langle A'|J_{\mu}|0\rangle = \in_{\mu}^{*} m_{A'}f_{A'}, \langle A(P_{A})|J_{\mu}|D(P_{D})\rangle = l \in_{\mu}^{*} + c_{+}(\in^{*} \cdot P_{D})(P_{D} + P_{A})_{\mu} + c_{-}(\in^{*} \cdot P_{D})(P_{D} - P_{A})_{\mu},$$

$$\langle A'(P_{A'})|J_{\mu}|D(P_{D})\rangle = r \in_{\mu}^{*} + s_{+}(\in^{*} \cdot P_{D})(P_{D} + P_{A'})_{\mu} + s_{-}(\in^{*} \cdot P_{D})(P_{D} - P_{A'})_{\mu},$$

$$\langle A'(P_{A'})|J_{\mu}|D(P_{D})\rangle = r \in_{\mu}^{*} + s_{+}(\in^{*} \cdot P_{D})(P_{D} + P_{A'})_{\mu} + s_{-}(\in^{*} \cdot P_{D})(P_{D} - P_{A'})_{\mu},$$

$$\langle A'(P_{A'})|J_{\mu}|D(P_{D})\rangle = r \in_{\mu}^{*} + s_{+}(\in^{*} \cdot P_{D})(P_{D} + P_{A'})_{\mu} + s_{-}(\in^{*} \cdot P_{D})(P_{D} - P_{A'})_{\mu},$$

and

$$\begin{split} \langle P(P_P) | J_{\mu} | D(P_D) \rangle &= \left( P_{D\mu} + P_{P\mu} \right. \\ &- \frac{m_D^2 - m_P^2}{q^2} q_{\mu} \right) F_1^{DP}(q^2) \\ &+ \frac{m_D^2 - m_P^2}{q^2} q_{\mu} F_0^{DP}(q^2), \end{split}$$

which yield

<

$$A(D \to PA) = (2m_A f_A F_1^{D \to P}(m_A^2) + f_P F^{D \to A}(m_P^2)),$$
  

$$A(D \to PA') = (2m_{A'} f_{A'} F_1^{D \to P}(m_{A'}^2) + f_P F^{D \to A'}(m_P^2)),$$
(10)

where

$$F^{D \to A}(m_P^2) = l + (m_D^2 - m_A^2)c_+ + m_P^2c_-,$$
  

$$F^{D \to A'}(m_P^2) = r + (m_D^2 - m_{A'}^2)s_+ + m_P^2s_-.$$
(11)

#### C. Decay constants and form factors

Decay constants of pseudoscalar mesons are well known. However, for axial-vector mesons, decay constants for  $J^{PC} = 1^{+-}$  mesons may vanish due to the *C*-parity behavior. Under charge conjugation, the two types of axialvector mesons transform as

$$\begin{split} & M_b^a(1^{++}) \to + M_a^b(1^{++}) \\ & M_b^a(1^{+-}) \to - M_a^b(1^{+-}) \end{split} \qquad (a, b = 1, 2, 3) \end{split}$$

where  $M_b^a$  denotes meson  $3 \times 3$  matrix elements in SU(3)flavor symmetry. Since the weak axial-vector current transforms as  $(A_{\mu})_b^a \rightarrow + (A_{\mu})_a^b$  under charge conjugation, only the  $(1^{++})$  state can be produced through the axial-vector current in the SU(3) symmetry limit [15]. In this work, we use the following values of decay constants [11,16,17] of the axial-vector  $(1^{++})$  mesons and pseudoscalar  $(0^-)$  mesons:

$$f_{a_1} = 0.203 \text{ GeV}, \qquad f_{K_{1A}} = 0.175 \text{ GeV},$$
  
 $f_{f_1} = f_{f'_1} = 0.221 \text{ GeV}, \qquad f_{\pi} = 0.133 \text{ GeV},$   
 $f_K = 0.160 \text{ GeV},$   
 $f_{\eta} = 0.133 \text{ GeV}, \text{ and } f_{\eta'} = 0.126 \text{ GeV}.$ 

For  $F^{DP}$  form factors, experimental branching ratios [11] of semileptonic decays  $D \rightarrow K/\pi + l + v_l$  provide the following values:

$$F_1^{DK}(0) = F_0^{DK}(0) = 0.76,$$
  

$$F_1^{D\pi}(0) = F_0^{D\pi}(0) = 0.69, \qquad F_1^{D\eta}(0) = 0.68,$$
  

$$F_1^{D_s K}(0) = 0.64, \text{ and } F_1^{D_s \eta}(0) = 0.72$$

which match well with the form factors given by the Bauer-Stech-Wirbel (BSW) model [1]. Therefore, we take the BSW form factors for other  $F^{DP}$  transitions. Momentum dependence of the form factors is taken as

$$F(q^2) = \frac{F(0)}{(1 - q^2/m_*^2)^n}$$
(12)

with pole mass  $m_*$  given by vector meson masses. The original BSW model [1] assumes a monopole behavior

(n = 1) for both the form factors, which is not consistent with the heavy quark symmetry scaling relations for heavyto-light transitions. However, in the modified BSW model [18], consistency with the heavy quark symmetry is restored by taking dipole behavior for  $q^2$  dependence for the form factor  $F_1$ . Since the BSW model provides the form factors only for  $P(0^-) \rightarrow P(0^-)$  or  $P(0^-) \rightarrow V(1^-)$  transitions, we calculate form factors  $F^{DA}$  and  $F^{DA'}$  using the ISGW quark model [5].

### **IV. NUMERICAL RESULTS AND DISCUSSIONS**

Sandwiching the weak Hamiltonian (6) between the initial and the final states, the decay amplitudes for various  $D \rightarrow PA$  decay modes are obtained by using (8) and (9). The decays can be categorized as follows:

- (I) involving  $P(0^-) \rightarrow P(0^-)$  transitions only,
- (II) involving  $P(0^-) \rightarrow A(1^+)$  transitions only, and
- (III) involving both  $P(0^-) \rightarrow P(0^-)/A(1^+)$  transitions. Their respective decay amplitudes are given in Tables I, II, and III. Branching ratios obtained for these categories are given in Tables IV, V, and VI, respectively. For the Cabibbo-favored decays, though many decay channels are available for *D* mesons, the experimental measure-

TABLE I. Decay amplitudes for  $D \rightarrow PA$  decays involving  $P(0^-) \rightarrow P(0^-)$  transitions.

| (a) Cabibbo-favored decays $\times \frac{G_F}{g_L} \cos^2 \theta_c$ $D^0 \to \pi^0 \bar{K}_1^0$ $\sqrt{2} \sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{k_1}^2)$ $D^0 \to \pi^0 \bar{K}_1^0$ $\sqrt{2} \sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{k_1}^2)$ $D^0 \to \eta \bar{K}_1^0$ $\sqrt{2} \cos \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^* \to \eta \bar{K}_1^0$ $\sqrt{2} \sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^* \to \pi^+ \bar{K}_1^0$ $2\sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^* \to \pi^+ \bar{K}_1^0$ $2\cos \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^* \to \eta a^+$ $2\cos \phi_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^* \to \eta a^+$ $2\cos \phi_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^* \to \pi^- a^+$ $-2\cos \phi_a p_a m_{a_1} f_a F^{D \to \pi}(m_{a_1}^2)$ $D^0 \to \pi^- K_1^+$ $2\sin \theta_a 1m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{L_1}^2)$ $D^0 \to \pi^- K_1^+$ $2\sin \theta_a 1m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{L_1}^2)$ $D^+ \to \pi^+ f_1'$ $-2a_2m_{f_1} f_{f_1} F^{D \to \pi}(m_{L_1}^2)$ $D^* \to K^+ a_1^0$ $\sqrt{2} \sin \theta_a 1m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{L_1}^2)$ $D^* \to \pi^+ K_1^+$ $2\sin \theta_a 1m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{L_1}^2)$ $D^* \to \pi^+ f_1'$ $-2a_2m_{f_1} f_{f_1} F^{D \to \pi}(m_{L_1}^2)$ $D^* \to \pi^+ K_1^0$ $\sqrt{2} \sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^0 \to \pi^- K_1^+$ $2\sin \theta_a 1m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^0 \to \pi^- K_1^+$ $2\sin \theta_a 1m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^0 \to \pi^- K_1^+$ $2\sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^0 \to \pi^- K_1^+$ $2\sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ $D^0 \to \pi^- K_1^+$ $2\sin \theta_a 2m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^2)$ </th <th>Decay</th> <th>Decay amplitude</th>   | Decay  | Decay amplitude   |
|---|--|---|
| $ \begin{split} & D^{0} \to \eta \bar{K}_{1}^{0} & \sqrt{2} \sin\theta \sin\phi_{p} a_{2} m_{K_{1}} f_{K_{1A}} F^{D \to \eta}(m_{K_{1}}^{2}) \\ & D_{s}^{+} \to K^{+} \bar{K}_{1}^{0} & 2 \sin\theta a_{2} m_{K_{1}} f_{K_{1A}} F^{D \to -\eta}(m_{K_{1}}^{2}) \\ & D_{s}^{+} \to \eta a_{1}^{+} & 2 \cos\theta a_{2} m_{K_{1}} f_{K_{1A}} F^{D \to -\eta}(m_{a_{1}}^{2}) \\ & D_{s}^{+} \to \eta a_{1}^{+} & -2 \cos\phi_{p} a_{1} m_{a_{1}} f_{a_{1}} F^{D \to -\eta}(m_{a_{1}}^{2}) \\ & D_{s}^{+} \to \pi^{-} a_{1}^{+} & -2 \cos\phi_{p} a_{1} m_{a_{1}} f_{a_{1}} F^{D \to -\eta}(m_{a_{1}}^{2}) \\ & D_{s}^{0} \to \pi^{-} a_{1}^{+} & -2 \cos\phi_{p} a_{1} m_{a_{1}} f_{a_{1}} F^{D \to -\eta}(m_{a_{1}}^{2}) \\ & D_{s}^{0} \to \pi^{-} a_{1}^{+} & 2 \sin\theta a_{1} m_{K_{1}} f_{K_{1A}} F^{D \to -\kappa}(m_{K_{1}}^{2}) \\ & D_{s}^{0} \to \pi^{-} f_{1}^{+} & 2 \sin\theta a_{1} m_{K_{1}} f_{K_{1A}} F^{D \to -\kappa}(m_{K_{1}}^{2}) \\ & D_{s}^{+} \to K^{0} a_{1}^{+} & 2 \sin\theta a_{1} m_{K_{1}} f_{K_{1A}} F^{D \to -\kappa}(m_{K_{1}}^{2}) \\ & D_{s}^{+} \to K^{+} a_{1}^{0} & \sqrt{2} a_{2} m_{a_{1}} f_{a_{1}} F^{D \to -\kappa}(m_{a_{1}}^{2}) \\ & D_{s}^{+} \to K^{+} a_{1}^{0} & \sqrt{2} a_{2} m_{a_{1}} f_{a_{1}} F^{D \to -\kappa}(m_{a_{1}}^{2}) \\ & D_{s}^{+} \to K^{+} f_{1} & -2a_{2} m_{f_{1}} f_{f_{1}} F^{D \to -\kappa}(m_{a_{1}}^{2}) \\ & D_{s}^{+} \to K^{+} f_{1} & -\sqrt{2} a_{2} m_{a_{1}} f_{a_{1}} F^{D \to -\kappa}(m_{a_{1}}^{2}) \\ & D_{s}^{+} \to K^{+} f_{1} & -\sqrt{2} a_{2} m_{a_{1}} f_{a_{1}} F^{D \to -\kappa}(m_{a_{1}}^{2}) \\ & D_{s}^{0} \to \pi^{-} K_{1}^{+} & -\sqrt{2} a_{1} m_{a_{1}} f_{a_{1}} F^{D \to -\kappa}(m_{a_{1}}^{2}) \\ & D_{s}^{0} \to \pi^{-} K_{1}^{+} & -\sqrt{2} a_{1} m_{a_{1}} f_{a_{1}} F^{D \to -\pi}(m_{K_{1}}^{2}) \\ & D_{s}^{0} \to \pi^{-} K_{1}^{+} & 2 \sin\theta a_{1} m_{K_{1}} f_{K_{1A}} F^{D \to -\pi}(m_{K_{1}}^{2}) \\ & D_{s}^{0} \to \pi^{-} K_{1}^{+} & 2 \sin\theta a_{1} m_{K_{1}} f_{K_{1A}} F^{D \to -\pi}(m_{K_{1}}^{2}) \\ & D_{s}^{0} \to \eta K_{1}^{0} & \sqrt{2} \sin\theta \sin\phi_{p} a_{2} m_{K_{1}} f_{K_{1A}} F^{D \to -\pi}(m_{K_{1}}^{2}) \\ & D_{s}^{-} \to \pi^{+} K_{1}^{0} & 2 \sin\theta a_{2} m_{K_{1}} f_{K_{1A}} F^{D \to -\pi}(m_{K_{1}}^{2}) \\ & D_{s}^{+} \to \pi^{+} K_{1}^{0} & 2 \cos\theta a_{2} m_{K_{1}} f_{K_{1A}} F^{D \to -\pi}(m_{K_{1}}^{2}) \\ & D_{s}^{+} \to \pi^{+} K_{1}^{0} & 2 \cos\theta a_{2} m_{K_{1}} f_{K_{1A}} F^{D \to -\pi}(m_{K_{1}}^{2}) \\ & D_{s}^{+} \to \pi^{+$ | (a) Cabibbo-favored decays<br>$D^0 \rightarrow K^- a_1^+$<br>$D^0 \rightarrow \pi^0 \bar{K}_1^0$<br>$D^0 \rightarrow \pi^0 \underline{\bar{K}}_1^0$  | $ \begin{array}{l} \times \frac{G_{F}}{\sqrt{2}}\cos^{2}\theta_{c} \\ 2a_{1}m_{a_{1}}f_{a_{1}}F^{D\to K}(m_{a_{1}}^{2}) \\ \sqrt{2}\sin\theta a_{2}m_{K_{1}}f_{K_{1A}}F^{D\to \pi}(m_{K_{1}}^{2}) \\ \sqrt{2}\cos\theta a_{2}m_{K_{1}}f_{K_{1A}}F^{D\to \pi}(m_{K_{1}}^{2}) \end{array} $   |
| $ \begin{array}{llllllllllllllllllllllllllllllllllll$   | $D^{0} \rightarrow \eta \bar{K}_{1}^{0}$ $D_{s}^{+} \rightarrow K^{+} \bar{K}_{1}^{0}$ $D_{s}^{+} \rightarrow K^{+} \underline{K}_{1}^{0}$ $D_{s}^{+} \rightarrow \eta a_{1}^{+}$ (b) Cabibbo-suppressed decays  | $ \sqrt{2} \sin\theta \sin\phi_p a_2 m_{K_1} f_{K_{1A}} F^{D \rightarrow \eta}(m_{K_1}^2)  2 \sin\theta a_2 m_{K_1} f_{K_{1A}} F^{D_s \rightarrow K}(m_{K_1}^2)  2 \cos\theta a_2 m_{K_1} f_{K_{1A}} F^{D_s \rightarrow K}(m_{K_1}^2)  -2 \cos\phi_p a_1 m_{a_1} f_{a_1} F^{D_s \rightarrow \eta}(m_{a_1}^2)  \times \frac{G_F}{5} \sin\theta_c \cos\theta_c $  |
| $\begin{array}{llllllllllllllllllllllllllllllllllll$  | $D^{0} \rightarrow \pi^{-} a_{1}^{+}$ $D^{0} \rightarrow K^{-} K_{1}^{+}$ $D^{0} \rightarrow \pi^{0} f_{1}^{\prime}$ $D^{+} \rightarrow \bar{K}^{0} K_{1}^{+}$ $D_{s}^{+} \rightarrow K^{0} a_{1}^{+}$ $D_{s}^{+} \rightarrow K^{+} a_{1}^{0}$ $D_{s}^{+} \rightarrow K^{+} c$ | $-\frac{2a_{1}^{2}m_{a_{1}}f_{a_{1}}F^{D\to\pi}(m_{a_{1}}^{2})}{2\sin\theta a_{1}m_{K_{1}}f_{K_{1A}}F^{D\to\pi}(m_{K_{1}}^{2})}$ $-\sqrt{2}a_{2}m_{f_{1}'}f_{f_{1}'}F^{D\to\pi}(m_{f_{1}'}^{2})$ $2\sin\theta a_{1}m_{K_{1}}f_{K_{1A}}F^{D\to\pi}(m_{K_{1}}^{2})$ $-2a_{2}m_{f_{1}'}f_{f_{1}'}F^{D\to\pi}(m_{f_{1}'}^{2})$ $-2a_{1}m_{a_{1}}f_{a_{1}}F^{D_{3}\toK}(m_{a_{1}}^{2})$ $\sqrt{2}a_{2}m_{a_{1}}f_{a_{1}}F^{D_{3}\toK}(m_{a_{1}}^{2})$  |
| $ \begin{array}{llllllllllllllllllllllllllllllllllll$   | $D_s \to K^+ f_1$<br>(c) Cabibbo-doubly suppressed decays<br>$D^0 \to \pi^0 K_1^0$   | $-\sqrt{2}a_2m_{f_1}f_{f_1}F^{D_s}\pi(m_{f_1})$ $\times \frac{G_F}{\sqrt{2}}\sin^2\theta_c$ $\sqrt{2}\sin\theta a_2m_{K_1}f_{K_{1A}}F^{D\to\pi}(m_{K_1}^2)$   |
| $ \begin{array}{ll} D^0 \to \eta K_1^0 & \sqrt{2} \sin\theta \sin\phi_p a_2 m_{K_1} f_{K_{1A}} F^{D \to \eta} (m_{K_1}^2) \\ D^+ \to \pi^+ K_1^0 & 2 \sin\theta a_2 m_{K_1} f_{K_{1A}} F^{D \to \pi} (m_{K_1}^2) \\ D^+ \to \pi^+ \underline{K}_1^0 & 2 \cos\theta a_2 m_{\underline{K}_1} f_{\underline{K}_{1A}} F^{D \to \pi} (m_{\underline{K}_1}^2) \\ D^+ \to \pi^0 K_1^+ & -\sqrt{2} \sin\theta a_1 m_{K_1} f_{\underline{K}_{1A}} F^{D \to \pi} (m_{\underline{K}_1}^2) \end{array} $  | $D^{0} \rightarrow \pi^{0} \underline{K}_{1}^{0}$ $D^{0} \rightarrow \pi^{-} K_{1}^{+}$ $D^{0} \rightarrow \pi^{-} \underline{K}_{1}^{+}$  | $ \sqrt{2}\cos\theta a_2 m_{\underline{K}_1} f_{\underline{K}_{1A}} F^{D \to \pi}(m_{\underline{K}_1}^{2^{\prime}}) 2\sin\theta a_1 m_{K_1} f_{K_{1A}} F^{D \to \pi}(m_{K_1}^{2^{\prime}}) 2\cos\theta a_1 m_{\underline{K}_1} f_{\underline{K}_{1A}} F^{D \to \pi}(m_{\underline{K}_1}^{2^{\prime}}) $   |
| $D^+ \to \pi^0 \underline{K}_1^+ \qquad -\sqrt{2} \cos\theta_1 m_{\underline{K}_1} f \underline{K}_{\underline{L}_1} F^{D \to \pi}(m_{\underline{K}_1}^{X_1})$ $D^+ \to \pi^{K^+} \qquad (\overline{2} \sin\theta \sin t, \underline{K}_{\underline{L}_1} f \underline{K}_{\underline{L}_1} F^{D \to \pi}(m_{\underline{K}_1}^{X_1}))$  | $D^{\circ} \to \eta K_{1}^{\circ}$ $D^{+} \to \pi^{+} K_{1}^{0}$ $D^{+} \to \pi^{+} \underline{K}_{1}^{0}$ $D^{+} \to \pi^{0} K_{1}^{+}$ $D^{+} \to \pi^{0} \underline{K}_{1}^{+}$ $D^{+} \to \pi^{0} \underline{K}_{1}^{+}$   | $ \sqrt{2} \sin\theta \sin\phi_{p} a_{2} m_{K_{1}} f_{K_{1A}} F^{D\to\pi}(m_{K_{1}}^{2}) 2 \sin\theta a_{2} m_{K_{1}} f_{K_{1A}} F^{D\to\pi}(m_{K_{1}}^{2}) 2 \cos\theta a_{2} m_{K_{1}} f_{\underline{K}_{1A}} F^{D\to\pi}(m_{K_{1}}^{2}) - \sqrt{2} \sin\theta a_{1} m_{K_{1}} f_{K_{1A}} F^{D\to\pi}(m_{K_{1}}^{2}) - \sqrt{2} \cos\theta a_{1} m_{K_{1}} f_{\underline{K}_{1A}} F^{D\to\pi}(m_{K_{1}}^{2}) - \sqrt{2} \cos\theta a_{1} m_{K_{1}} f_{\underline{K}_{1A}} F^{D\to\pi}(m_{K_{1}}^{2}) - \sqrt{2} \sin\theta \sin\theta a_{1} m_{K_{1}} f_{\underline{K}_{1A}} F^{D\to\pi}(m_{K_{1}}^{2}) $ |

TABLE II. Decay amplitudes for  $D \rightarrow PA$  decays involving  $P(0^-) \rightarrow A(1^+)$  transitions.

| Decay  | Amplitude   |
|--|---|
| (a) Cabibbo-f  | avored decays   |
| $D^0 \rightarrow \bar{K}^0 a_1^0$                                    | $\frac{a_2 f_K}{\sqrt{2}} F^{D \to a_1}(m_K^2)$   |
| $D^0 \rightarrow \pi^+ \dot{K}_1^-$                                  | $\sin^{\sqrt{2}}\theta a_1 f_{\pi} F^{D \to K_{1A}}(m_{\pi}^2) + \cos^2\theta a_1 f_{\pi} F^{D \to K_{1A'}}(m_{\pi}^2)$   |
| $D^0 \rightarrow \pi^+ K_1^-$  | $\cos\theta a_1 f_{\pi} F^{D \to K_{1A}}(m_{\pi}^2) - \sin\theta a_1 f_{\pi} F^{D \to K_{1A'}}(m_{\pi}^2)$  |
| $D^0 \to \bar{K}^0 \bar{f}_1$  | $\frac{a_2 f_K}{\sqrt{2}} F^{D \to f_1}(m_K^2)$   |
| $D^0 \rightarrow \bar{K}^0 b_1^0$                                    | $\frac{a_{2f_K}}{\sqrt{2}}F^{D \to b_1}(m_K^2)$   |
| $D^0 \rightarrow \bar{K}^0 h_1$                                      | $\frac{a_2 f_K}{\sqrt{2}} F^{D \to h_1}(m_K^2)$   |
| $D^+ \rightarrow \bar{K}^0 b_1^+$                                    | $a_2 f_K F^{D \to b_1}(m_K^2)$  |
| $D_s^+ \rightarrow \bar{K}^0 K_1^+$                                  | $\sin\theta a_2 f_K F^{D_s \to K_{1A}}(m_\pi^2) + \cos\theta a_2 f_K F^{D_s \to K_{1A'}}(m_\pi^2)$  |
| $D_s^+ \to \bar{K}^0 \underline{K}_1^+$                              | $\cos\theta a_2 f_K F^{D_s \to K_{1A}}(m_\pi^2) - \sin\theta a_2 f_K F^{D_s \to K_{1A'}}(m_\pi^2)$  |
| $D_s^+ \rightarrow \pi^+ f_1'$                                       | $-a_1 f_{\pi} F^{D_s \to f'_1}(m_{\pi}^2)$  |
| $D_s^+ \rightarrow \pi^+ h_1'$                                       | $-a_1 f_{\pi} F^{D_s \to n_1}(m_{\pi}^2)$   |
| (b) Cabibbo-s  | suppressed decays   |
| $D^0 \rightarrow K^+ K_1^-$  | $\sin\theta a_1 f_K F^{D \to K_{1A}}(m_K^2) + \cos\theta a_1 f_K F^{D \to K_{1A'}}(m_K^2)$  |
| $D^0 \rightarrow \pi^+ a_1^-$  | $-a_1 f_{\pi} F^{D \to a_1}(m_{\pi}^2)$   |
| $D^0 \rightarrow \pi^+ b_1^-$  | $-a_1 f_{\pi} F^{D \to b_1}(m_{\pi}^2)$   |
| $D^0 \rightarrow \pi^0 b_1^0$  | $\frac{\frac{d_{2}f_{\pi}}{2}F^{D\to b_{1}}(m_{\pi}^{2})}{\cos\phi_{n}\sin\phi_{n}} \leftarrow F^{D\to b_{1}}(m_{\pi}^{2})$   |
| $D^0 \rightarrow \eta b_1^0$   | $(- \frac{-p}{2} - \frac{-p}{2}) a_2 f_{\eta} F^{D \to b_1}(m_{\eta}^2)$  |
| $D^0 \rightarrow \pi^0 h_1$  | $\frac{\frac{m_D}{2}\pi}{(2\cos\phi_p - \sin\phi_p)} = C  \nabla D \rightarrow h_1(-2)$   |
| $D^{\circ} \rightarrow \eta h_1$<br>$D^+ \rightarrow K^+ \bar{k}^0$  | $\left(-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)a_{2}f_{\eta}F^{D-m_{1}}(m_{\eta}^{2})$ $\sin\theta \sqrt{2}f_{\tau}F^{D\rightarrow K_{1}}(m^{2}) + \cos\theta \sqrt{2}f_{\tau}F^{D\rightarrow K_{1}}(m^{2})$ |
| $D^+ \rightarrow \kappa^+ \kappa_1^0$<br>$D^+ \rightarrow \pi^+ h^0$ | $\sin\theta a_1 f_K F^{D} = \sin(m_{\tilde{K}}) + \cos\theta a_1 f_K F^{D} = \sin(m_{\tilde{K}})$ $\frac{a_1 f_{\pi}}{2} F^{D \to b_1}(m^2)$  |
| $D^+$ $0I^+$   | $\sqrt{2}$ $I$ $(m_{\pi})$<br>$a_2 f_{\pi} r D \rightarrow b_1 (-2)$  |
| $D^+ \rightarrow \pi^0 b_1^+$  | $\frac{\omega \omega}{\sqrt{2}} F^{D} = m_{\pi}(m_{\pi}^{2})$   |
| $D' \rightarrow \eta b_1'$   | $(-\cos\phi_p - \frac{1}{\sqrt{2}})a_2f_{\eta}F^{D-b_1}(m_{\eta}^2)$  |
| $D^+ \rightarrow \pi^+ h_1$  | $-\frac{u_{1j\pi}}{\sqrt{2}}F^{D-n_{1}}(m_{\pi}^{2})$   |
| $D_s^+ \to \pi^+ K_1^0$  | $-\sin\theta a_1 f_{\pi} F^{D_s \to \kappa_{1A}}(m_{\pi}^2) - \cos\theta a_1 f_{\pi} F^{D_s \to \kappa_{1A'}}(m_{\pi}^2)$   |
| $D_s^+ \to \pi^0 K_1^+$  | $\frac{\cos \omega a_{2J\pi}}{\sqrt{2}} F^{D_s \to K_{1A}}(m_\pi^2) - \frac{\sin \omega a_{2J\pi}}{\sqrt{2}} F^{D_s \to K_{1A'}}(m_\pi^2)$  |
| $D_s^+ \to \pi^+ \underline{K}_1^0$                                  | $\sin\theta a_1 f_\pi F^{D_s \to K_{1A}}(m_\pi^2) - \cos\theta a_1 f_\pi F^{D_s - K_{1A'}}(m_\pi^2)$  |
| $D_s^+ \to \pi^0 \underline{K}_1^+$                                  | $\frac{\sin\theta a_2 f_\pi}{\sqrt{2}} F^{D_s \to K_{1A'}}(m_\pi^2) + \frac{\cos a_2 f_\pi}{\sqrt{2}} F^{D_s \to K_{1A}}(m_\pi^2)$  |
| $D_s^+ \to K^+ h_1'$   | $-a_1 f_K F^{D_s \to h_1'}(m_K^2)$  |
| (c) Cabibbo-c  | loubly-suppressed decays  |
| $D^0 \rightarrow K^+ a_1^-$  | $a_1 f_K F^{D \to a_1}(m_K^2)$  |
| $D^0 \rightarrow K^0 a_1^0$  | $\frac{a_2 f_K}{\sqrt{2}} F^{D \to a_1}(m_K^2)$   |
| $D^0 \rightarrow K^0 f_1$  | $\frac{a_2 f_K}{\sqrt{2}} F^{D \to f_1}(m_K^2)$   |
| $D^0 \to K^+ b_1^-$  | $a_1 f_K F^{D \to b_1}(m_K^2)$  |
| $D^0 \rightarrow K^0 b_1^0$  | $\frac{a_2 f_K}{\sqrt{2}} F^{D \to b_1}(m_K^2)$   |
| $D^0 \rightarrow K^0 h_1$  | $\frac{a_2 f_K}{2} F^{D \to h_1}(m_K^2)$  |
| $D^+ \rightarrow K^+ a_1^0$  | $-\frac{a_1 J_K}{\sqrt{2}} F^{D \to a_1}(m_K^2)$  |
| $D^+ \rightarrow K^0 a_1^+$  | $a_2 f_K F^{D \to a_1}(m_K^2)$  |
| $D^+ \rightarrow K^+ f_1$  | $\frac{a_{1JK}}{\sqrt{2}}F^{D\to f_1}(m_K^2)$   |
| $D^+ \rightarrow K^0 b_1^+$  | $a_2 f_K F^{D \to b_1}(m_K^2)$  |
| $D^+ \rightarrow K^+ b_1^0$  | $-\frac{a_{1JK}}{\sqrt{2}}F^{D\to b_1}(m_K^2)$  |
| $D^+ \rightarrow K^+ h_1$  | $\frac{a_1 f_K}{c} F^{D \to h_1}(m_K^2)$  |

ments for branching ratios are available only for  $D^0 \rightarrow K^- a_1^+, D^+ \rightarrow \bar{K}^0 a_1^+, D^0 \rightarrow \pi^+ K_1^-, D^+ \rightarrow \pi^+ \underline{K}_1^0$ , and experimental upper limits are available for  $D^0 \rightarrow \pi^0 \overline{K}_1^0$ ,  $D^0 \rightarrow \pi^0 \overline{K}_1^0$ ,  $D^0 \rightarrow \overline{K}^0 a_1^0$ ,  $D^0 \rightarrow \pi^+ \underline{K}_1^-$ , and  $D^+ \rightarrow \pi^+ \overline{K}_1^0$ .

In the earlier work [6],  $B(D^0 \rightarrow K^- a_1^+) = 1.46\%$  and  $B(D^+ \rightarrow \bar{K}^0 a_1^+) = 3.75\%$  were obtained using monopole  $q^2$  dependence of the  $F_1^{DP}(q^2)$  form factor, which are much less than the experimental values. Recently, Cheng [16] has pointed out that the  $q^2$  dependence of the form factor  $F_1^{DP}(q^2)$  should have dipole form rather than the monopole form in order to be consistent with the heavy quark symmetry, which yields a higher branching ratio for  $D \rightarrow \bar{K}a_1$ decays. In the present work, using the dipole  $q^2$  dependence, we calculate  $B(D^+ \rightarrow \bar{K}^0 a_1^+) = 9.45\%$  which agrees well with the experimental value  $(8.2 \pm 1.7)\%$ . The calculated branching ratio  $B(D^0 \rightarrow \bar{K}^0 a_1^0) = 0.004\%$ is consistent with the experimental upper limit <1.9%. However, branching ratio for  $D^0 \rightarrow K^- a_1^+$  increases to 3.28%, which is still lower than the experimental value  $(7.2 \pm 1.1)\%$ . For these decays, the large width of the emitted  $a_1$  meson increases the phase space for these decays. Taking a running mass of the  $a_1$  meson and using the Breit-Wigner measure, it has already been shown [7,8,16,17] that the branching ratio may get smearing enhancement by a factor 1.07–1.20 for  $\Gamma(a_1) =$ 0.3–0.6 GeV. However, the enhancement is not sufficient to have agreement with experiment.

It may be noted that *W*-annihilation and *W*-exchange diagrams may also contribute to the  $D^0$  decays under consideration [2,4,7]. Normally, such contributions are expected to be suppressed due to the helicity and color arguments. However, for the  $D \rightarrow \bar{K}a_1$  mode, dominance of the spectator quark diagram may not be justified. Including the factorizable contribution of such diagrams, the decay amplitudes of  $D \rightarrow \bar{K}a_1$  get modified to (putting aside the scale factor  $\frac{G_F}{\sqrt{2}}\cos^2\theta_C$ )

$$A(D^{0} \to K^{-}a_{1}^{+}) = a_{1}2m_{a_{1}}f_{a_{1}}F^{D\to K}(m_{a_{1}}^{2}) + a_{2}f_{D}F^{a_{1}\to K}(m_{D}^{2}),$$

$$A(D^{0} \to \bar{K}^{0}a_{1}^{0}) = \frac{1}{\sqrt{2}}(a_{2}f_{K}F^{D\to a_{1}}(m_{K}^{2}) - a_{2}f_{D}F^{a_{1}\to K}(m_{D}^{2})).$$
(13)

As it is not possible to evaluate the form factor  $F^{a_1 \rightarrow K}$  at  $m_D^2$  even in the phenomenological models, it is treated as a free parameter. Taking  $f_D \approx 0.3$  GeV, we find that the experimental branching ratio  $B(D^0 \rightarrow K^- a_1^+) = (7.2 \pm 1.1)\%$  requires  $F^{a_1 \rightarrow K}(m_D^2) = (-3.74 \pm 0.85)$  GeV. This in turn enhances the branching ratio for  $D^0 \rightarrow \bar{K}^0 a_1^0$  to  $(0.29 \pm 0.15)\%$ , which remains consistent with the experimental upper limit <1.9%. However, such a large value of  $F^{a_1 \rightarrow K}(m_D^2)$  is less likely.

It has been established that, since charmed meson masses lie in the resonance region, elastic final-state interactions (FSI) may also affect the decays significantly [6,7]. At the isospin level the elastic FSI introduce appropriate phase factors in the different isospin channels. Using the isospin-level analysis, FSI modified amplitudes for  $D \rightarrow \bar{K}a_1$  modes are given by

TABLE III. Decay amplitudes for  $D \to PA$  decays involving  $P(0^-) \to P(0^-)/A(1^+)$  transitions.

| Decay   | Amplitude   |   |
|---|---|---|
| (a) Cabibbo-favored                             | decays  |   |
| $D^+ \rightarrow \bar{K}^0 a_1^+$               | $2a_1m_{a_1}f_{a_1}F^{D\to K}(m_{a_1}^2)$   | $a_2 f_K F^{D \to a_1}(m_{K_1}^2)$  |
| $D^+ 	o \pi^+ ar{K}^0_1$                        | $2\sin\theta a_2 m_{K_1} f_{K_{1,k}} F^{D \to \pi}(m_{K_1}^2)$  | $\sin\theta a_1 f_{\pi} F^{D \to \tilde{K}_{1A}}(m_{\pi}^2) + \cos\theta a_1 f_{\pi} F^{D \to K_{1A'}}(m_{\pi}^2)$    |
| $D^+  ightarrow \pi^+ \overline{K}_1^{\dot{0}}$ | $2\cos\theta a_2 m_{K_1} f_{K_{14}}^{T} F^{D \to \pi}(m_{K_1}^{2^1})$   | $\cos\theta a_1 f_{\pi} F^{D \to K_{1A}}(m_{\pi}^2) - \sin\theta a_1 f_{\pi} F^{D \to K_{1A'}}(m_{\pi}^2)$            |
| (b) Cabibbo-suppress                            | sed decays  |   |
| $D^0 \rightarrow \pi^0 a_1^0$                   | $a_2 m_{a_1} f_{a_1} F^{D \to \pi}(m_{a_1}^2)$  | $\frac{a_2 f_{\pi}}{\sqrt{2}} F^{D \to a_1}(m_{\pi}^2)$   |
| $D^0  ightarrow \eta a_1^0$                     | $\sin\phi_p a_2 m_{a_1} f_{a_1} F^{D \to \eta}(m_{a_1}^2)$  | $\left(-\frac{\cos\phi_p}{\sqrt{2}} - \frac{\sin\phi_p}{2}\right)a_2f_{\eta}F^{D \to a_1}(m_{\eta}^2)$                |
| $D^0 \rightarrow \pi^0 f_1$                     | $-a_2 m_{f_1} f_{f_1} F^{D \to \pi}(m_{f_1}^2)$   | $\frac{a_2 f_{\pi}}{\sqrt{2}} F^{D \to f_1}(m_{\pi}^2)$   |
| $D^0 \rightarrow \eta f_1$                      | $-\sin\phi_n a_2 m_{f_1} f_{f_1} F^{D \to \eta}(m_{f_1}^2)$   | $\left(-\frac{\cos\phi_p}{2}-\frac{\sin\phi_p}{2}\right)a_2f_nF^{D\to f_1}(m_n^2)$                                    |
| $D^+ \rightarrow \pi^+ a_1^0$                   | $\sqrt{2}a_2f_{a_1}m_{a_1}F^{D\to\pi}(m_{a_1}^2)$   | $\frac{a_1 f_\pi}{\sqrt{2}} \vec{F}^{D \to a_1}(m_\pi^2)$   |
| $D^+ \rightarrow \pi^0 a_1^+$                   | $\sqrt{2}a_1f_{a_1}m_{a_1}F^{D\to\pi}(m_{a_1}^2)$   | $\frac{a_2 f_{\pi}}{\sqrt{2}} F^{D \to a_1}(m_{\pi}^2)$   |
| $D^+ \rightarrow \eta a_1^+$                    | $-\sqrt{2}\sin\phi_p a_1 m_{a_1} f_{a_1} F^{D \to \eta}(m_{a_1}^2)$   | $\left(-\cos\phi_p - \frac{\sin\phi_p}{\sqrt{2}}\right)a_2f_\eta F^{D\to a_1}(m_\eta^2)$                              |
| $D^+ \rightarrow \pi^+ f_1$                     | $-\sqrt{2}a_2m_{f_1}f_{f_1}F^{D\to\pi}(m_{f_1}^2)$  | $-\frac{a_1f_\pi}{2}F^{D\to f_1}(m_\pi^2)$  |
| $D_s^+ \rightarrow \eta K_1^+$                  | $-2\cos\phi_p\sin\theta a_1m_{K_1}f_{K_{1A}}F^{D_s\to\eta}(m_{K_1}^2)$  | $(-\cos^{\sqrt{2}}\phi_p\sin\theta - \frac{\sin\phi_p\sin\theta}{\sqrt{2}})a_2f_{\eta}F^{D_s \to K_{1A}}(m_{\eta}^2)$ |
|   |   | + $(-\cos\theta\cos\phi_p - \frac{\cos\theta\sin\phi_p}{\sqrt{2}})a_2f_{\eta}F^{D_s \to K_{1A'}}(m_{\eta}^2))$        |
| $D_s^+ \rightarrow \eta \underline{K}_1^+$      | $-2\cos\theta\cos\phi_p a_1 m_{\underline{K}_1} f_{\underline{K}_{1A}} F^{D_s \to \eta}(m_{\underline{K}_1}^2)$ | $(-\cos\theta\cos\phi_p - \frac{\cos\theta\sin\phi_p}{\sqrt{2}})a_2f_\eta F^{D_s \to K_{1A}}(m_\eta^2)$               |
|   | _   | $+(\cos\phi_n\sin\theta-\frac{\sin\theta\sin\phi_p}{5})a_2f_nF^{D_s\to K_{1A'}}(m_n^2))$                              |
| (c) Cabibbo-doubly-suppressed decays            |   | $\sqrt{2}$ $\sqrt{2}$ $\sqrt{2}$ $\sqrt{1}$   |
| $D_s^+ \rightarrow K^+ K_1^0$                   | $2\sin\theta a_2 m_{K_1} f_{K_{1A}} F^{D_s \to K}(m_{K_1}^2)$   | $\sin\theta a_1 f_K F^{D_s \to K_{1A}}(m_K^2) + \cos\theta a_1 f_K F^{D_s \to K_{1A'}}(m_K^2)$                        |
| $D_s^+ \longrightarrow K^+ \underline{K}_1^0$   | $2\cos\theta a_2 m_{\underline{K}_1} f_{\underline{K}_{1A}} F^{D_s \to K}(m_{\underline{K}_1}^2)$               | $\cos\theta a_1 f_K F^{D_s \to K_{1A}}(m_K^2) - \sin\theta a_1 f_K F^{D_s \to K_{1A'}}(m_K^2)$                        |
| $D_s^+ \rightarrow K^0 K_1^+$                   | $2\sin\theta a_1 m_{K_1} f_{K_{1A}} F^{D_s \to K}(m_{K_1}^2)$   | $\sin\theta a_2 f_K F^{D_s \to K_{1A}}(m_K^2) + \cos\theta a_2 f_K F^{D_s \to K_{1A'}}(m_K^2)$                        |
| $D_s^+ \rightarrow K^0 \underline{K}_1^+$       | $2\cos\theta a_1 m_{K,f_{K,r}} F^{D_s \to K}(m_{K,r}^2)$  | $\cos\theta a_2 f_K F^{D_s \rightarrow K_{1A}}(m_K^2) - \sin\theta a_1 f_K F^{D_s \rightarrow K_{1A'}}(m_K^2)$        |

$$A(D^{0} \to K^{-}a_{1}^{+}) = A^{\bar{K}a_{1}} \exp(i\delta_{1/2}^{\bar{K}a_{1}}) \left[ 1 + \frac{r^{Ka_{1}}}{2} \exp(-i\delta^{\bar{K}a_{1}}) \right],$$
  

$$A(D^{0} \to \bar{K}^{0}a_{1}^{0}) = -\frac{1}{\sqrt{2}}A^{\bar{K}a_{1}} \exp(i\delta_{1/2}^{\bar{K}a_{1}})$$
  

$$\times [1 - r^{\bar{K}a_{1}} \exp(-i\delta^{\bar{K}a_{1}})],$$
  

$$A(D^{+} \to \bar{K}^{0}a_{1}^{+}) = A_{3/2}^{\bar{K}a_{1}} \exp(i\delta_{3/2}^{\bar{K}a_{1}}),$$
  
(14)

where  $A^{\bar{K}a_1} = 2/3A_{1/2}^{\bar{K}a_1}$ ,  $r^{\bar{K}a_1} = A_{3/2}^{\bar{K}a_1}/A_{1/2}^{\bar{K}a_1}$ , and the phase difference

$$\delta^{ar{K}a_1} = \delta^{ar{K}a_1}_{1/2} - \delta^{ar{K}a_1}_{3/2}.$$

Thus, elastic FSI yield the following branching ratios (%):

$$B(D^0 \to K^- a_1^+) = 1.43(1.195 + \cos\delta^{\bar{K}a_1}),$$
  

$$B(D^0 \to \bar{K}^0 a_1^0) = 1.42(1.003 - \cos\delta^{\bar{K}a_1}).$$
(15)

It is obvious that any nonzero value of the phase difference  $\delta^{\bar{K}a_1}$  will enhance  $D^0 \rightarrow \bar{K}^0 a_1^0$  and deplete  $D^0 \rightarrow K^- a_1^+$ . Thus, we find that the elastic FSI works in the wrong direction here. Besides the elastic FSI, these decays may be affected by inelastic FSI involving quark exchange diagrams, because the produced quarks have enough time to rearrange before combining to form the final-state hadrons. Recently, Cheng [16] has shown that the FSI can induce large long-distance *W*-exchange terms. Assuming that the data analysis [4] for  $D \rightarrow \bar{K}\rho$  decay holds also for  $D \rightarrow \bar{K}a_1$ , he has obtained  $B(D^0 \rightarrow K^-a_1^+) = 6.2\%$  in good agreement with the experimental value  $(7.2 \pm 1.1)\%$ . Note that the branching ratio of  $B(D^+ \rightarrow \bar{K}^0a_1^+) = 9.45\%$  remains unaffected by the *W*-exchange process and the FSI effects.

For  $D \rightarrow \pi \bar{K}_1 / \pi \underline{\bar{K}}_1$  decays modes, we have calculated the branching ratios for different choices of the  $K_{1A}$ - $K_{1A'}$ mixing angle ( $\theta = 33^\circ, 45^\circ, 57^\circ$ ) using dipole  $q^2$  behavior of the  $F_1^{DP}(q^2)$  form factors. Theoretical values of branching ratios of  $D^0 \to \pi^+ \underline{K}_1^- / \pi^0 \underline{\bar{K}}_1^0, D^+ \to \pi^+ \underline{\bar{K}}_1^0$ , and  $D^0 \to$  $\pi^0 \bar{K}_1^0$  are consistent with experimental data for all the mixing angles. However, the experimental upper limit  $B(D^+ \rightarrow \pi^+ \bar{K}^0_1)$  favors the choice of  $\theta = 33^\circ$ , which in turn implies  $B(D^+ \to \pi^+ \underline{\bar{K}}_1^0) = 6.52\%$ ,  $B(D^0 \to \pi^+ \underline{\bar{K}}_1^0) = 0.17\%$ , and  $B(D^0 \to \pi^0 \underline{\bar{K}}_1^0) = 0.69\%$ . We wish to remark here that  $\theta = 33^{\circ}$  has been obtained earlier by Godfrey and Isgur [14] in a unified quark model analysis, and also favored by  $\underline{K}_1(1.400)$  production in  $\tau$  decays [15]. This choice of the mixing angle yields the largest branching ratio  $B(D^0 \rightarrow \pi^+ K_1^-) = 0.16\%$ , though it is still lower than the observed value  $(1.14 \pm 0.31)\%$ . Similar to the  $D^0 \rightarrow K^- a_1^+$  decays, this decay mode is also likely to have contribution from the W-annihilation and W-exchange processes. Including the factorizable con-

| Decays   | Br (%) $\theta = 33^{\circ}$ | Br (%) $\theta = 45^{\circ}$ | Br (%) $\theta = 57^{\circ}$ | Experiment (%) |
|--|------------------------------|------------------------------|------------------------------|----------------|
| (a) Cabibbo-favo                                 | red decays                   |                              |                              |                |
| $D^0 \rightarrow K^- a_1^+$                      | 3.28                         | 3.28                         | 3.28                         | $(72 \pm 1.1)$ |
| $D^0 \rightarrow \pi^0 \bar{K}_1^{\dot{0}}$      | 0.307                        | 0.518                        | 0.729                        | <2.0           |
| $D^0 \rightarrow \pi^0 \overline{K}_1^{\dot{0}}$ | 0.689                        | 0.490                        | 0.291                        | <3.7           |
| $D^0 \rightarrow \eta \bar{K}_1^0$               | 0.009 <sup>a</sup>           | 0.016 <sup>a</sup>           | $0.222^{a}$                  |                |
|  | 0.013 <sup>b</sup>           | 0.022 <sup>b</sup>           | 0.032 <sup>b</sup>           |                |
| $D_s^+ \rightarrow K^+ \bar{K}_1^0$              | 0.337                        | 0.567                        | 0.798                        |                |
| $D_s^+ \to K^+ \underline{\bar{K}}_1^{\bar{0}}$  | 0.301                        | 0.214                        | 0.127                        |                |
| $D_s^+ \rightarrow \eta a_1^+$                   | 3.30 <sup>a</sup>            | 3.30 <sup>a</sup>            | 3.30 <sup>a</sup>            |                |
| -  | 1.84 <sup>b</sup>            | 1.84 <sup>b</sup>            | 1.84 <sup>b</sup>            |                |
| (b) Cabibbo-supp                                 | pressed decays               |                              |                              |                |
| $D^0 \rightarrow \pi^- a_1^+$                    | 0.883                        | 0.883                        | 0.883                        |                |
| $D^0 \rightarrow K^- K_1^+$                      | 0.034                        | 0.057                        | 0.080                        |                |
| $D^0 \rightarrow \pi^0 f'_1$                     | 0.070                        | 0.070                        | 0.070                        |                |
| $D^+ \rightarrow \bar{K}^0 K_1^+$                | 0.088                        | 0.148                        | 0.208                        |                |
| $D^+ \rightarrow \pi^+ f_1'$                     | 0.367                        | 0.367                        | 0.367                        |                |
| $D_s^+ \rightarrow K^0 a_1^+$                    | 0.490                        | 0.490                        | 0.490                        |                |
| $D_s^+ \to K^+ f_1$                              | 0.045                        | 0.045                        | 0.045                        |                |
| (c) Cabibbo-doub                                 | oly-suppressed deca          | ays (×10 <sup>-2</sup> )%    |                              |                |
| $D^0 \rightarrow \pi^0 K_1^0$                    | 0.084                        | 0.141                        | 0.198                        |                |
| $D^0 \rightarrow \pi^0 \underline{K}^0_1$        | 0.187                        | 0.133                        | 0.079                        |                |
| $D^0 \rightarrow \pi^- K_1^+$                    | 1.01                         | 1.71                         | 2.40                         |                |
| $D^0 \rightarrow \pi^- \underline{K}_1^+$        | 2.26                         | 1.61                         | 0.956                        |                |
| $D^0 \rightarrow \eta K_1^0$                     | 0.003 <sup>a</sup>           | 0.004 <sup>a</sup>           | $0.006^{a}$                  |                |
|  | 0.004 <sup>b</sup>           | 0.006 <sup>b</sup>           | 0.009 <sup>b</sup>           |                |
| $D^+ \rightarrow \pi^+ K_1^0$                    | 0.432                        | 0.729                        | 1.02                         |                |
| $D^+ \rightarrow \pi^+ \underline{K}_1^0$        | 0.975                        | 0.693                        | 0.411                        |                |
| $D^+ \rightarrow \pi^0 K_1^+$                    | 1.33                         | 2.24                         | 3.15                         |                |
| $D^+ \rightarrow \pi^0 \underline{K}_1^+$        | 3.00                         | 2.13                         | 1.26                         |                |
| $D^+ \rightarrow \theta \eta K_1^+$              | 0.045 <sup>a</sup>           | 0.077 <sup>a</sup>           | 0.110 <sup>a</sup>           |                |
|  | 0.065 <sup>b</sup>           | 0.111 <sup>b</sup>           | 0.154 <sup>b</sup>           |                |

TABLE IV. Branching ratio for D,  $D_s$  meson decays involving  $P(0^-) \rightarrow P(0^-)$  transitions.

<sup>a</sup>for 
$$\theta_p = -10^{\circ}$$
  
<sup>b</sup>for  $\theta_p = -23^{\circ}$ 

tribution of such diagrams, the decay amplitudes of  $D \rightarrow \pi \bar{K}_1$  get modified to (leaving aside the scale factor  $\frac{G_F}{\sqrt{2}}\cos^2\theta_C$ )

$$A(D^{0} \to \pi^{+}K_{1}^{-}) = a_{1}f_{\pi}F^{D \to K_{1}}(m_{\pi}^{2}) + a_{2}f_{D}F^{K_{1} \to \pi}(m_{D}^{2}),$$

$$A(D^{0} \to \pi^{0}\bar{K}_{1}^{0}) = \frac{1}{\sqrt{2}}(a_{2}2m_{K_{1}}f_{K_{1A}}F^{D \to \pi}(m_{K_{1}}^{2}) - a_{2}f_{D}F^{K_{1} \to \pi}(m_{D}^{2})),$$
(16)

where

$$F^{D \to K_1} = \sin\theta F^{D \to K_{1A}} + \cos\theta F^{D \to K_{1A'}},$$
  
$$F^{K_1 \to \pi} = \sin\theta F^{K_{1A} \to \pi} + \cos\theta F^{K_{1A'} \to \pi},$$

and

$$f_{K_1} = f_{K_{1A}}\sin\theta + f_{K_{1A'}}\cos\theta.$$

For  $f_D \approx 0.3$  GeV and  $\theta = 33^\circ$ , we find that the experi-

mental value  $B(D^0 \to \pi^+ K_1^-) = (1.14 \pm 0.31)\%$  requires  $F^{K_1 \to \pi}(m_D^2) = (-1.33 \pm 0.29)$  GeV. This in turn enhances the branching ratio for  $D^0 \to \pi^0 \bar{K}_1^0$  to  $(0.55 \pm 0.06)\%$ , which remains well below the experimental upper limit <2.0%. It may be noted that  $B(D^+ \to \pi^+ \bar{K}_1^0)$  is larger than the  $B(D^+ \to \pi^+ \bar{K}_1^0)$  by 1 order of magnitude due to the constructive and destructive interference between colorallowed and color-suppressed amplitudes, respectively. In contrast to the  $D^0 \to \bar{K}^0 a_1^0$  mode, we observe that  $B(D^0 \to \pi^0 \bar{K}_1^0 / \pi^0 \bar{K}_1^0)$  are comparable or even greater than the branching of corresponding charge modes. Similar observations have also been made by Cheng [16].

For the Cabibbo-suppressed decays, dominant decay modes are  $D^0 \rightarrow \pi^- a_1^+$ ,  $D^+ \rightarrow \pi^+ f_1'$ ,  $D^+ \rightarrow \pi^+ a_1^0$ ,  $D^+ \rightarrow \pi^+ f_1$ ,  $D^+ \rightarrow \pi^0 a_1^+$ , and  $D_s^+ \rightarrow K^0 a_1^+$ . For the Cabibbo-doubly-suppressed decays, dominant decays are  $D^0 \rightarrow \pi^- \underline{K}_1^+$ ,  $D^0 \rightarrow \pi^- K_1^+$ ,  $D^0 \rightarrow K^+ b_1^-$ ,  $D^+ \rightarrow \pi^0 \underline{K}_1^+$ ,  $D^+ \rightarrow \pi^0 K_1^+$ ,  $D^+ \rightarrow K^+ h_1$ ,  $D^+ \rightarrow K^+ b_1^0$ ,  $D_s^+ \rightarrow K^0 K_1^+$ , and  $D_s^+ \rightarrow K^0 \underline{K}_1^+$ .

|   | -                                  |                                    |                              |                   |
|---|------------------------------------|------------------------------------|------------------------------|-------------------|
| Decays  | Br (%) $\theta = 33^{\circ}$       | Br (%) $\theta = 45^{\circ}$       | Br (%) $\theta = 57^{\circ}$ | Experiment (%)    |
| (a) Cabibbo-fa                                    | vored decays                       |                                    |                              |                   |
| $D^0 \rightarrow \bar{K}^0 a_1^0$                 | 0.004                              | 0.004                              | 0.004                        | <1.9              |
| $D^0 \rightarrow \pi^+ K_1^-$                     | 0.16                               | 0.06                               | 0.01                         | $(1.14 \pm 0.31)$ |
| $D^0 \rightarrow \pi^+ \underline{K}_1^-$         | 0.17                               | 0.20                               | 0.22                         | <1.2              |
| $D^0 \rightarrow \bar{K}^0 \overline{f_1}^1$      | 0.003                              | 0.003                              | 0.003                        |                   |
| $D^0 \rightarrow \bar{K}^0 b_1^0$                 | 0.03                               | 0.03                               | 0.03                         |                   |
| $D^0 \rightarrow \bar{K}^0 h_1^1$                 | 0.05                               | 0.05                               | 0.05                         |                   |
| $D^+ \rightarrow \bar{K}^0 \dot{b}_1^+$           | 0.14                               | 0.14                               | 0.14                         |                   |
| $D_s^+ \rightarrow \bar{K}^0 \bar{K}_1^+$         | 0.08                               | 0.03                               | 0.01                         |                   |
| $D_{s}^{3} \rightarrow \bar{K}^{0}K_{1}^{1}$      | 0.03                               | 0.03                               | 0.04                         |                   |
| $D_{s}^{3} \rightarrow \pi^{+} f_{1}^{\prime}$    | 0.05                               | 0.05                               | 0.05                         |                   |
| $D_{s}^{+} \rightarrow \pi^{+} h_{1}^{\prime}$    | 0.72                               | 0.72                               | 0.72                         |                   |
| (b) Cabibbo-si                                    | inpressed decays                   | 0.7.2                              | 0.7.2                        |                   |
| $D^0 \rightarrow K^+ K^-$                         | 0.003                              | 0.001                              | 0.0004                       |                   |
| $D^0 \rightarrow \pi^+ a^-$                       | 0.003                              | 0.001                              | 0.0001                       |                   |
| $D^0 \rightarrow \pi^+ h^-$                       | 0.039                              | 0.019                              | 0.019                        |                   |
| $D^0 \rightarrow \pi^0 h^0$                       | 0.000                              | 0.0016                             | 0.000                        |                   |
| $D^0 \rightarrow m h^0$                           | 0.0010                             | 0.0010                             | 0.0010                       |                   |
| $D \rightarrow \eta v_1$                          | 0.000 02<br>0.000 003 <sup>b</sup> | 0.000 02<br>0.000 003 <sup>b</sup> | 0.000 02                     |                   |
| $D^0 \rightarrow -0h$                             | 0.000 003                          | 0.000 003                          | 0.000 003                    |                   |
| $D^0 \rightarrow \pi^* n_1$                       | 0.002<br>0.000.05 <sup>a</sup>     | 0.002                              | 0.002                        |                   |
| $D^{\circ} \rightarrow \eta n_1$                  | 0.000 03                           | 0.00005                            | 0.000 05                     |                   |
| $\mathbf{D}^+$ $\mathbf{v}^+  \bar{\mathbf{v}}^0$ | 0.000 01                           | 0.000 01                           | 0.00001                      |                   |
| $D^+ \rightarrow K^+ K_1^0$                       | 0.010                              | 0.004                              | 0.0012                       |                   |
| $D^+ \rightarrow \pi^+ b_1^0$                     | 0.051                              | 0.051                              | 0.051                        |                   |
| $D^+ \rightarrow \pi^0 b_1^+$                     | 0.008                              | 0.008                              | 0.008                        |                   |
| $D^+ \rightarrow \eta b_1^+$                      | 0.0001°                            | 0.0001°                            | 0.0001"                      |                   |
|   | 0.000 02                           | 0.000 02                           | 0.000 02                     |                   |
| $D^+ \rightarrow \pi^+ h_1$                       | 0.071                              | 0.071                              | 0.071                        |                   |
| $D_s^+ \rightarrow \pi^+ K_1^0$                   | 0.026                              | 0.008                              | 0.0002                       |                   |
| $D_s^+ \rightarrow \pi^0 K_1^+$                   | 0.002                              | 0.0007                             | 0.000 02                     |                   |
| $D_s^+ \to \pi^+ \underline{K}_1^0$               | 0.051                              | 0.060                              | 0.064                        |                   |
| $D_s^+ \to \pi^0 \underline{K}_1^+$               | 0.004                              | 0.005                              | 0.005                        |                   |
| $D_s^+ \to K^+ h_1'$                              | 0.012                              | 0.012                              | 0.012                        |                   |
| (c) Cabibbo-do                                    | oubly-suppressed dec               | cays (×10 <sup>-2</sup> )%         |                              |                   |
| $D^0 \rightarrow K^+ a_1^-$                       | 0.015                              | 0.015                              | 0.015                        |                   |
| $D^0 \rightarrow K^0 f_1$                         | 0.001                              | 0.001                              | 0.001                        |                   |
| $D^0 \rightarrow K^0 f_1$                         | 0.001                              | 0.001                              | 0.001                        |                   |
| $D^0 \rightarrow K^+ b_1^-$                       | 0.090                              | 0.090                              | 0.090                        |                   |
| $D^0 \rightarrow K^0 b_1^0$                       | 0.007                              | 0.007                              | 0.007                        |                   |
| $D^0 \rightarrow K^0 h_1$                         | 0.014                              | 0.014                              | 0.014                        |                   |
| $D^+ \rightarrow K^+ a_1^0$                       | 0.021                              | 0.021                              | 0.021                        |                   |
| $D^+ \rightarrow K^0 a_1^+$                       | 0.0065                             | 0.0065                             | 0.0065                       |                   |
| $D^+ \rightarrow K^+ f_1$                         | 0.014                              | 0.014                              | 0.014                        |                   |
| $D^+ \rightarrow K^0 \dot{b}_1^+$                 | 0.038                              | 0.038                              | 0.038                        |                   |
| $D^+ \rightarrow K^+ h^0$                         | 0.121                              | 0.121                              | 0.121                        |                   |
| $D^+ \rightarrow K^+ h_1$                         | 0.234                              | 0.234                              | 0.234                        |                   |
| 1   |                                    | -                                  |                              |                   |

TABLE V. Branching ratio for D,  $D_s$  meson decays involving  $P(0^-) \rightarrow A(1^+)$  transitions.

<sup>a</sup>for  $\theta_p = -10^{\circ}$ <sup>b</sup>for  $\theta_p = -23^{\circ}$ 

## **V. CONCLUSIONS**

In this paper, we have studied hadronic weak decays of charmed mesons into pseudoscalar and axial-vector mesons in Cabibbo-favored, Cabibbo-suppressed, and Cabibbo-doubly-suppressed channels. At present, experimental information is available only for  $D \rightarrow$  $\bar{K}a_1/\pi\bar{K}_1/\pi\bar{K}_1$  decay modes. We make the following conclusions:

(i)  $D^+$  decays are well understood with the dipole be-

TABLE VI. Branching ratios for D,  $D_s$  meson decays involving  $P(0^-) \rightarrow P(0^-)/A(1^+)$  transitions.

| Decay   | Br (%) $\theta = 33^{\circ}$ | Br (%) $\theta = 45^{\circ}$ | Br (%) $\theta = 57^{\circ}$ | Experiment (%)  |
|---|------------------------------|------------------------------|------------------------------|-----------------|
| (a) Cabibbo-favored decays                      |                              |                              |                              |                 |
| $D^+ \rightarrow \bar{K}^0 a_1^+$               | 9.45                         | 9.45                         | 9.45                         | $(8.2 \pm 1.7)$ |
| $D^+  ightarrow \pi^+ ar{K}^0_1$                | 0.38                         | 1.52                         | 3.21                         | < 0.7           |
| $D^+ \rightarrow \pi^+ \underline{\bar{K}}_1^0$ | 6.52                         | 5.41                         | 3.99                         | $(5.0 \pm 1.3)$ |
| (b) Cabibbo-sup                                 | pressed decays               |                              |                              |                 |
| $D^0 \rightarrow \pi^0 a_1^0$                   | 0.028                        | 0.028                        | 0.028                        |                 |
| $D^0 \rightarrow \eta a_1^0$                    | 0.0013 <sup>a</sup>          | 0.0013 <sup>a</sup>          | 0.0013 <sup>a</sup>          |                 |
| . 1   | 0.0021 <sup>b</sup>          | 0.0021 <sup>b</sup>          | 0.0021 <sup>b</sup>          |                 |
| $D^0 \rightarrow \pi^0 f_1$                     | 0.052                        | 0.052                        | 0.052                        |                 |
| $D^0 \rightarrow \eta f_1$                      | $0.0010^{a}$                 | $0.0010^{a}$                 | $0.0010^{a}$                 |                 |
|   | 0.001 <sup>b</sup>           | 0.001 <sup>b</sup>           | 0.001 <sup>b</sup>           |                 |
| $D^+ \rightarrow \pi^+ a_1^0$                   | 0.319                        | 0.319                        | 0.319                        |                 |
| $D^+ \rightarrow \pi^0 a_1^+$                   | 1.27                         | 1.27                         | 1.27                         |                 |
| $D^+ \rightarrow \eta a_1^+$                    | $0.478^{\rm a}$              | $0.478^{a}$                  | $0.478^{a}$                  |                 |
| - 1   | 0.072 <sup>b</sup>           | 0.072 <sup>b</sup>           | 0.072 <sup>b</sup>           |                 |
| $D^+ \rightarrow \pi^+ f_1$                     | 0.353                        | 0.353                        | 0.353                        |                 |
| $D_s^+ \rightarrow \eta K_1^+$                  | $0.032^{a}$                  | $0.057^{a}$                  | 0.083 <sup>a</sup>           |                 |
|   | 0.021 <sup>b</sup>           | 0.034 <sup>b</sup>           | 0.048 <sup>b</sup>           |                 |
| $D_s^+ \rightarrow \eta \underline{K}_1^+$      | $0.007^{\rm a}$              | $0.005^{a}$                  | 0.003 <sup>a</sup>           |                 |
| - · ·   | 0.003 <sup>b</sup>           | 0.002 <sup>b</sup>           | 0.001 <sup>b</sup>           |                 |
| (c) Cabibbo-dou                                 | bly-suppressed deca          | ys (×10 <sup>-2</sup> )%     |                              |                 |
| $D_s^+ \rightarrow K^+ K_1^0$                   | 0.003                        | 0.025                        | 0.132                        |                 |
| $D_s^+ \to K^+ \underline{K}_1^0$               | 0.257                        | 0.239                        | 0.203                        |                 |
| $D_s^+ \rightarrow K^0 K_1^+$                   | 0.353                        | 0.746                        | 1.20                         |                 |
| $D_s^+ \to K^0 \underline{K}_1^+$               | 0.585                        | 0.447                        | 0.296                        |                 |

<sup>a</sup>for  $\theta_p = -10^{\circ}$ <sup>b</sup>for  $\theta_p = -23^{\circ}$ 

havior for  $q^2$  dependence of the form factor  $F_1^{DP}(q^2)$ , which is justified in light of the heavy quark symmetry arguments.

- (ii) Available data for  $D \to \pi \bar{K}_1 / \pi \underline{\bar{K}}_1$  decay modes favor  $\theta = 33^\circ$  for  $K_1 \underline{K}_1$  mixing.
- (iii) The  $D^0$  decays seem to require sizable W exchange to bridge the gap between theoretical and experimental values. Such W-exchange contributions may be factorizable or nonfactorizable, as these may arise

from final-state interactions via quark rescattering.

- (iv) Generally, the decays which involve only the form factor for charmed meson to axial-vector meson transitions are usually suppressed in comparison to the other decays.
- (v) Out of the  $D \to \pi \bar{K}_1 / \pi \underline{\bar{K}}_1$  decay modes,  $D \to \pi \underline{\bar{K}}_1$  decays are found to be dominant over the other decay mode, particularly for  $D^+$  decays.

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